Today: Syllabus & section 11.1

$\boxed{L1}$

$\boxed{L1}$

Today: Syllabus & section 11.1

Thursday: Sections 11.2

L1

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HW#1: Due January 19th {Next Tuesday}

11.2, 11.3, 11.4a, 11.16a, 11.23a, 11.28a	§11.1
11.34, 11.36, 11.44, 11.47, 11.51, 11.57	§11.2

L1

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Thursday: Sections 11.2

HW#1: Due January 19th {Next Tuesday}

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11.34, 11.36, 11.44, 11.47, 11.51, 11.57	§11.2

* Syllabus

Who are we?

1 Computer gaming

Who are we?$_x$

1 Computer gaming

5 Engineering - Auto systems

Who are we?

1 Computer gaming

5 Engineering – Auto systems

11 Engineering – Mechanical systems

Who are we?

1 Computer gaming

5 Engineering – Auto systems

11 Engineering – Mechanical systems

6 Engineering – Robotics

Who are we?

1 Computer gaming

5 Engineering – Auto systems

11 Engineering – Mechanical systems

6 Engineering – Robotics

1 Applied physics

Who are we?

1 Computer gaming

5 Engineering – Auto systems

11 Engineering – Mechanical systems

6 Engineering – Robotics

1 Applied physics

Academic level

1 Sophomore

Who are we?

1 Computer gaming

5 Engineering – Auto systems

11 Engineering – Mechanical systems

6 Engineering – Robotics

1 Applied physics

Academic level

1 Sophomore

9 Junior

Who are we?

1 Computer gaming

5 Engineering – Auto systems
11 Engineering – Mechanical systems

6 Engineering – Robotics
1 Applied physics

Academic level

1 Sophomore
9 Junior
14 Senior

Kinematics

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* Want to describe 3d motion

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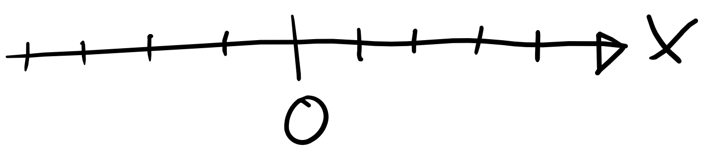

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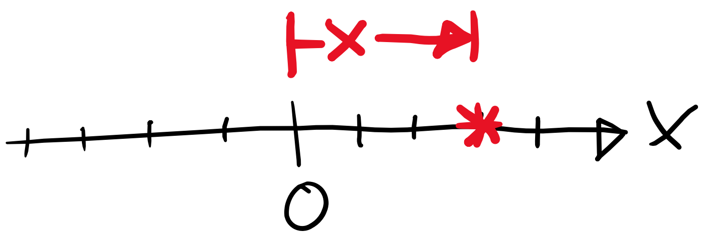

$x = +3$ units

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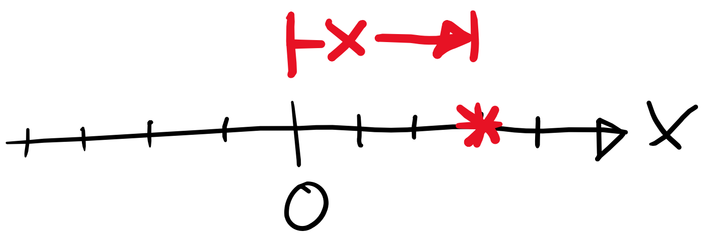

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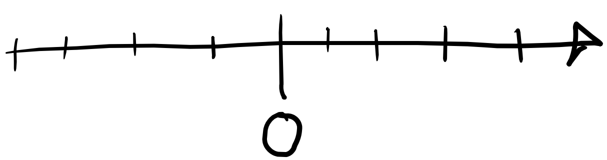

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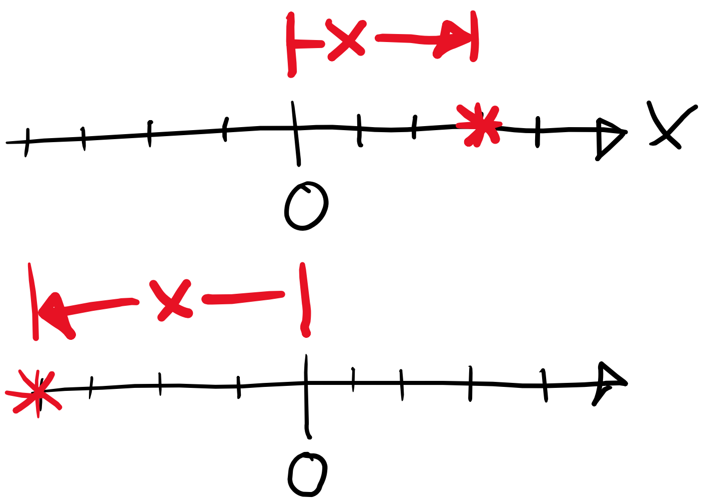

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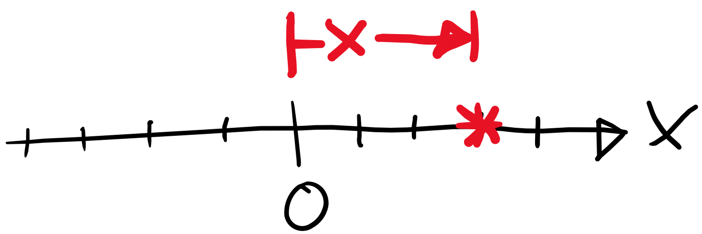

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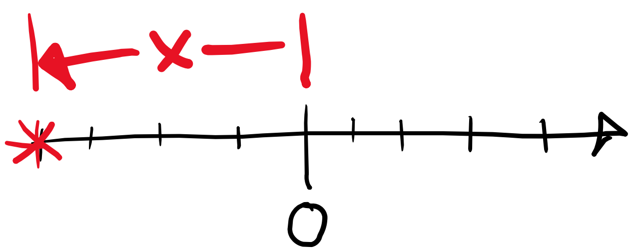

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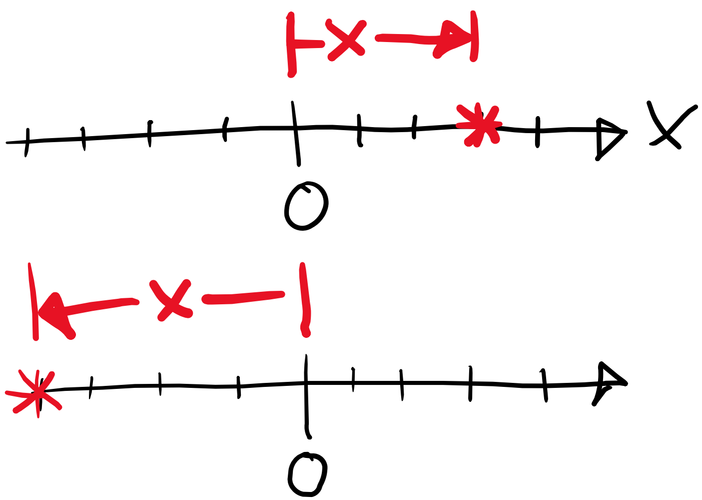

$x = +3$ units

$x = -4$ units

Units could be ft, m, km, Miles, cm, ...

Imagine the motion of a flashing object constrained to move in the vertical direction

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Average velocity $= \frac{\Delta y}{\Delta t}$

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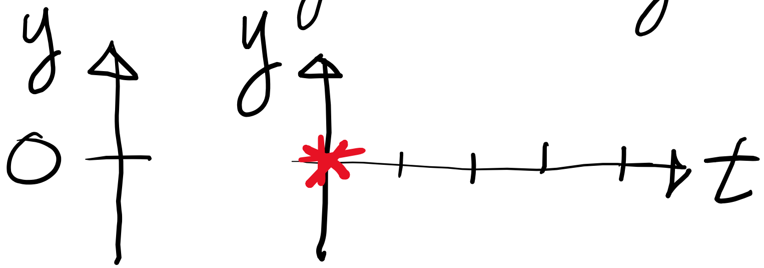

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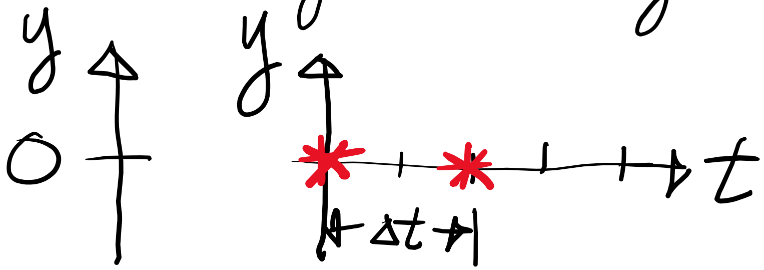

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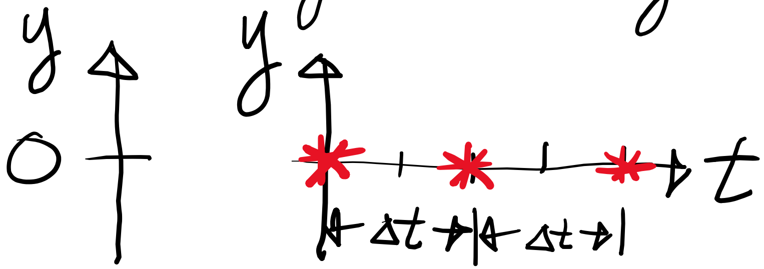

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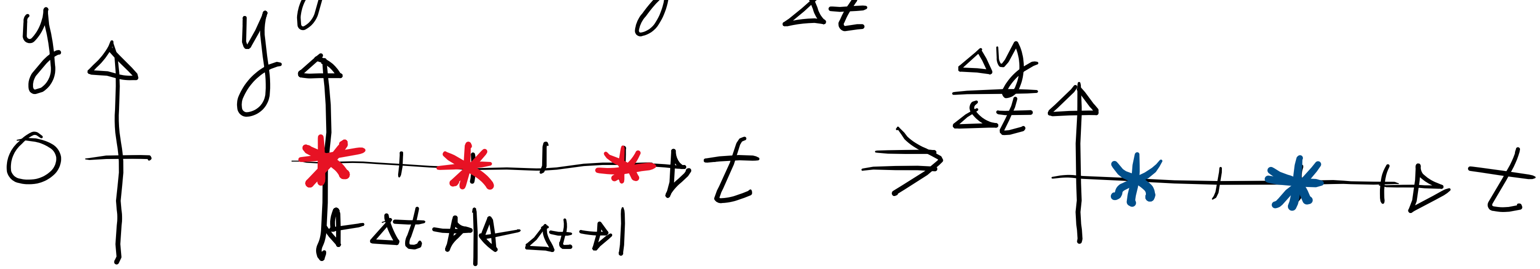

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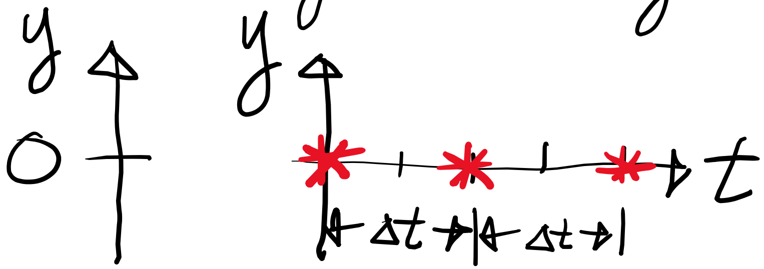

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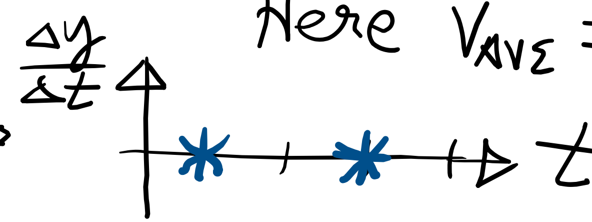

Here $V_{AVE} = 0$

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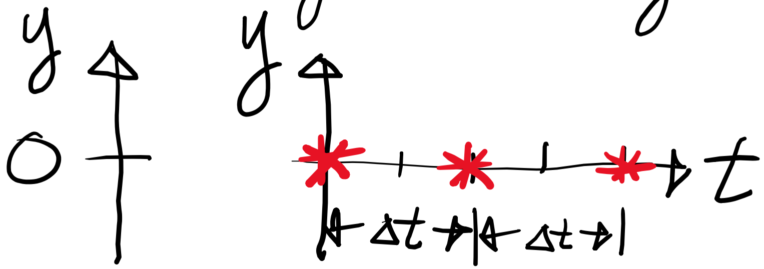

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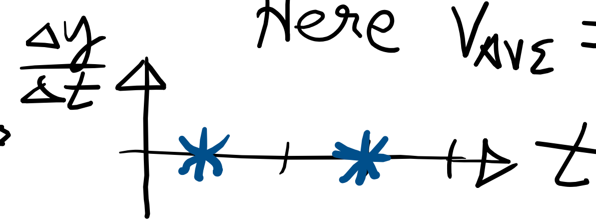

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Now double the rate

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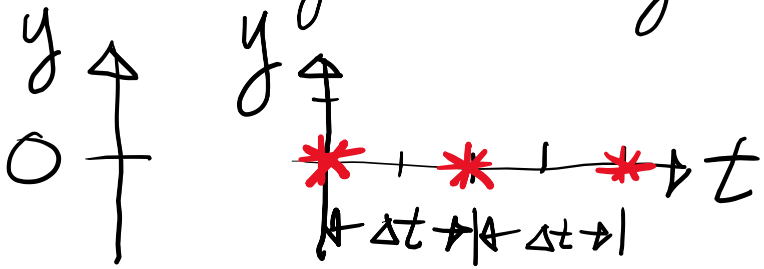

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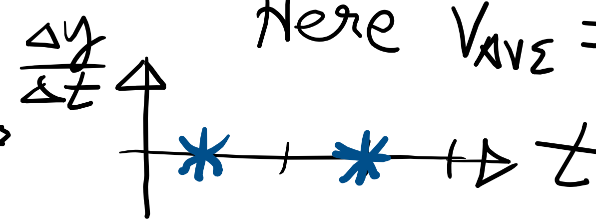

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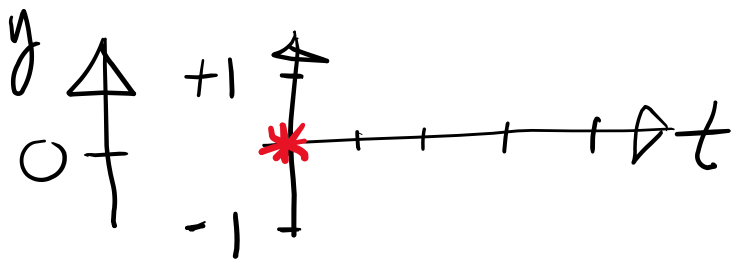

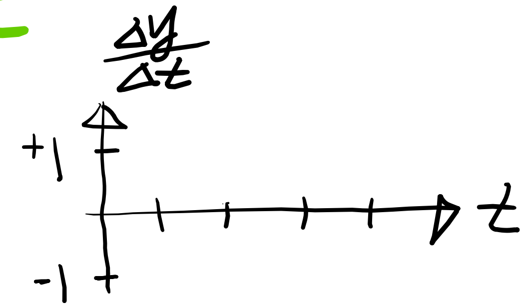

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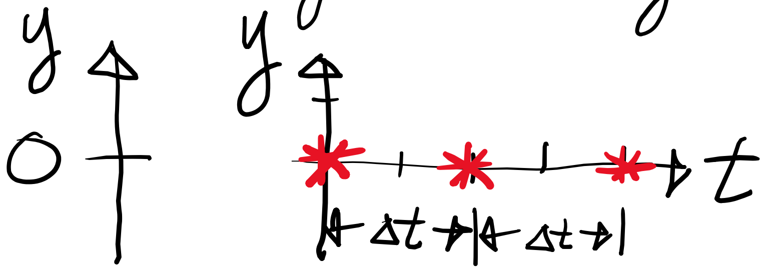

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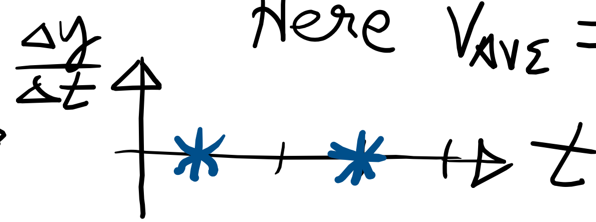

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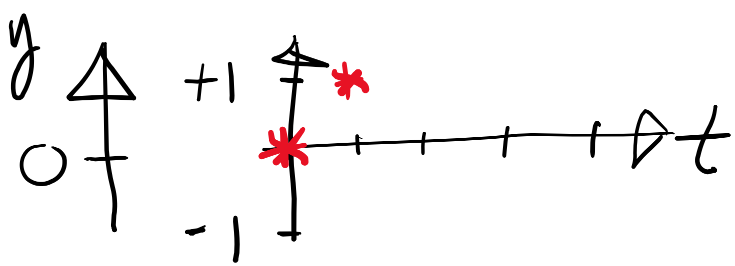

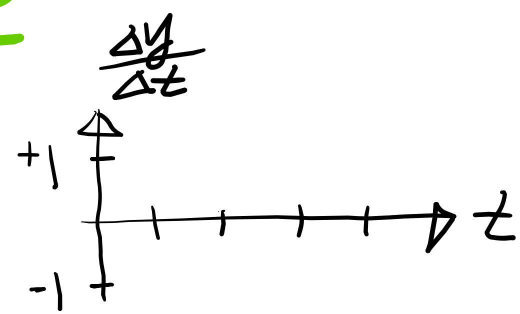

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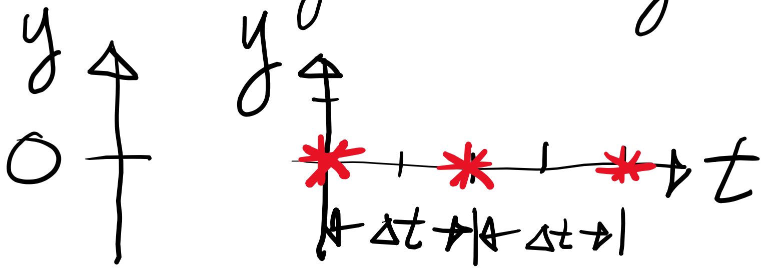

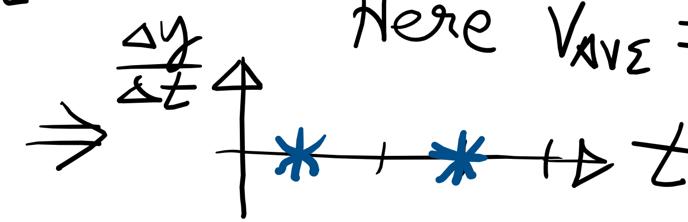

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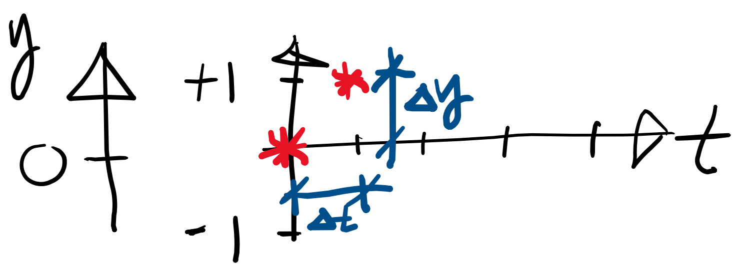

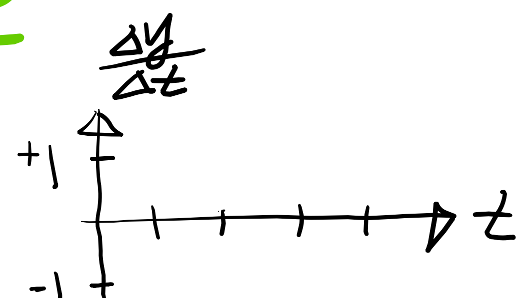

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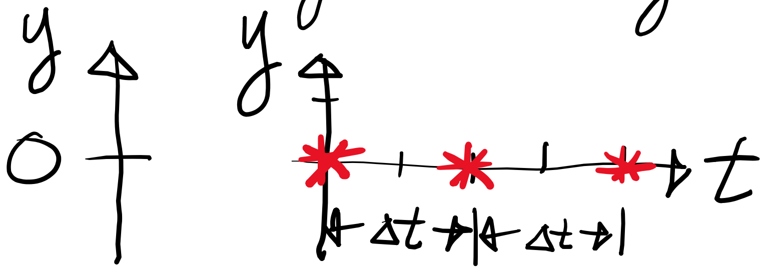

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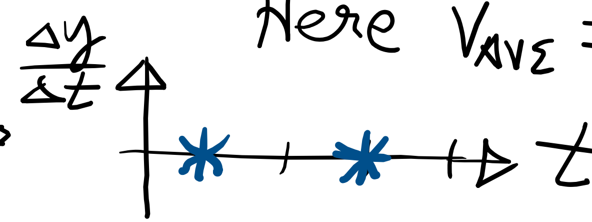

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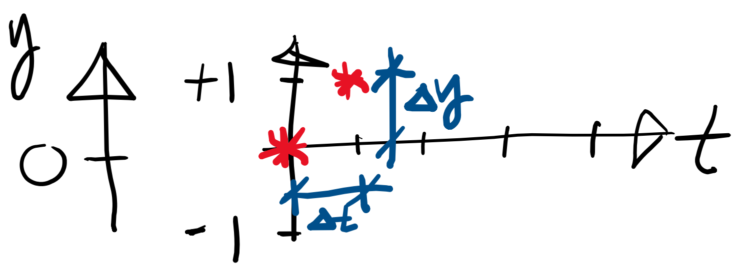

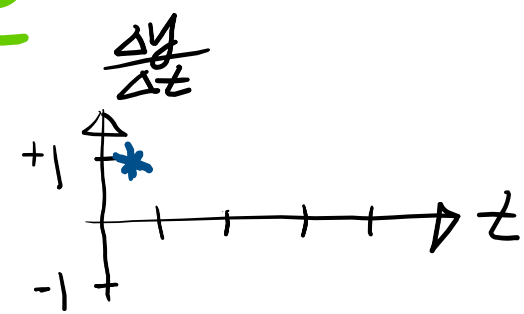

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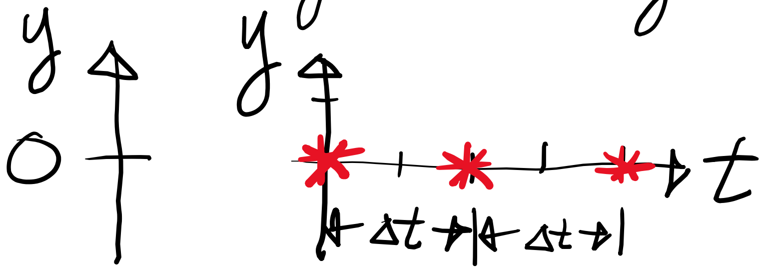

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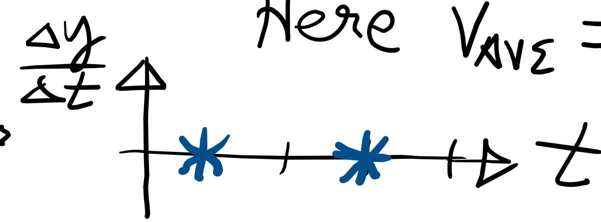

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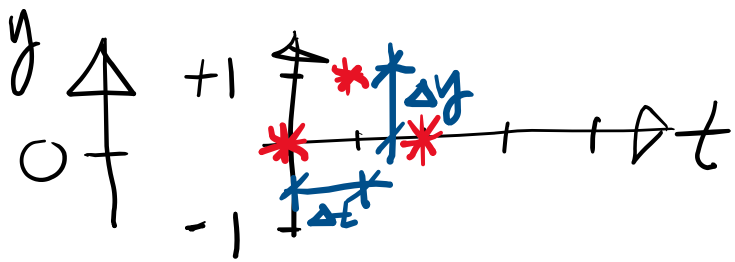

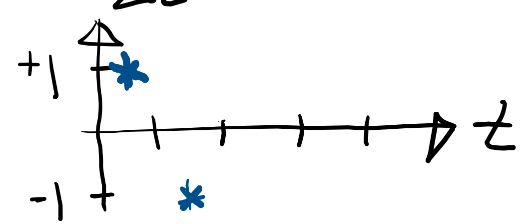

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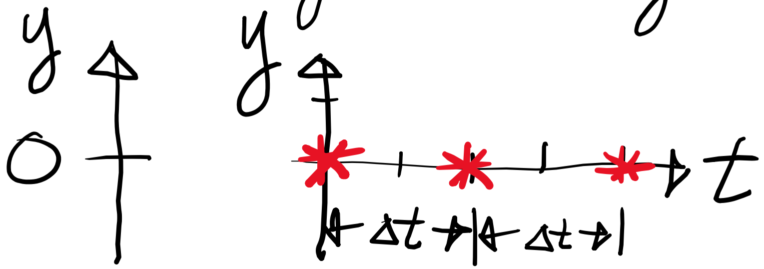

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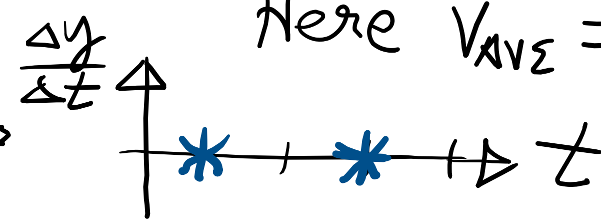

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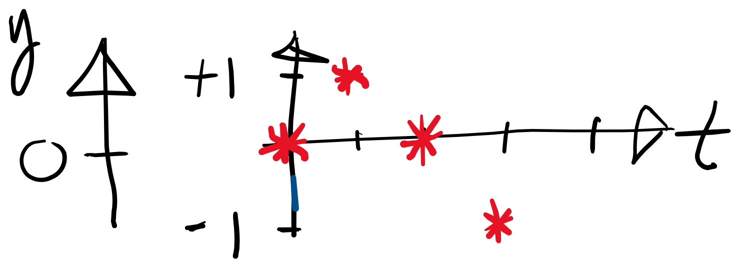

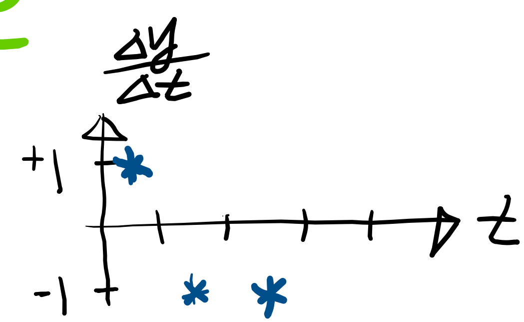

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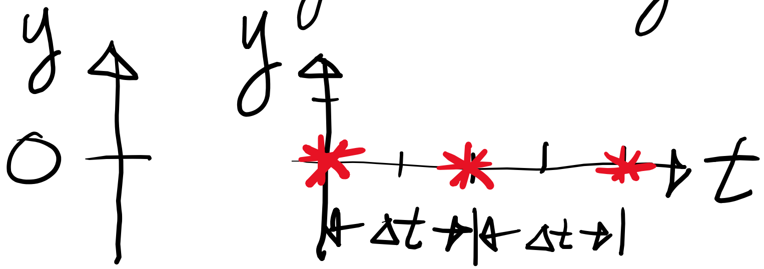

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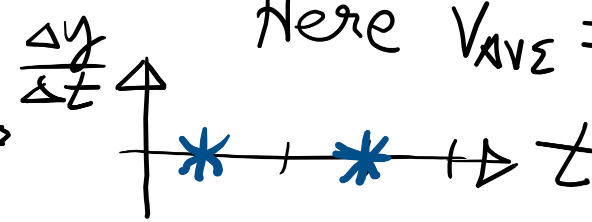

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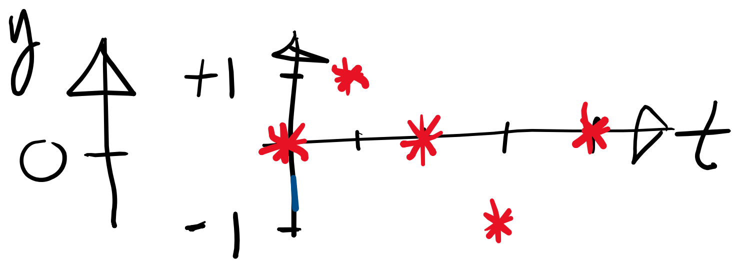

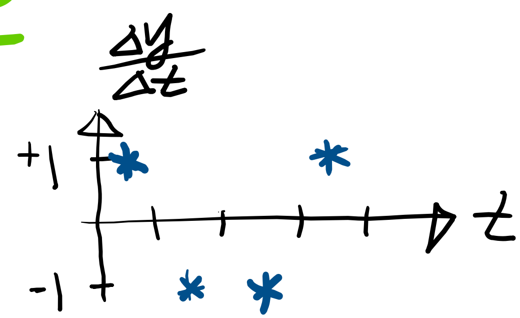

ASU

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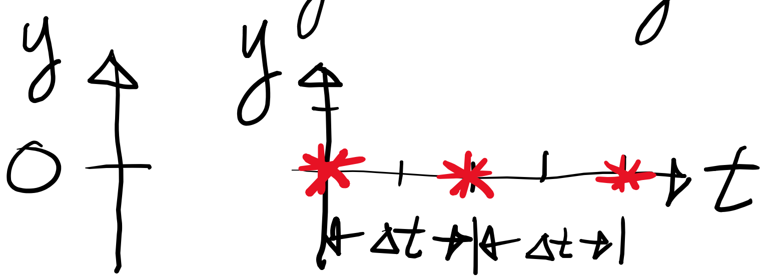

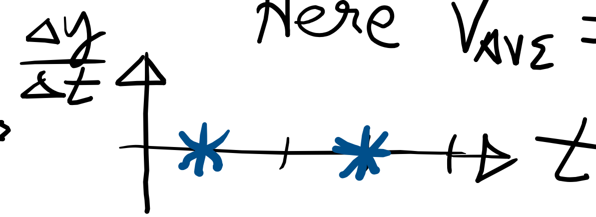

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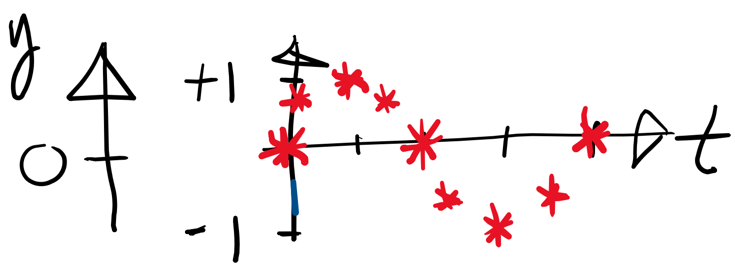
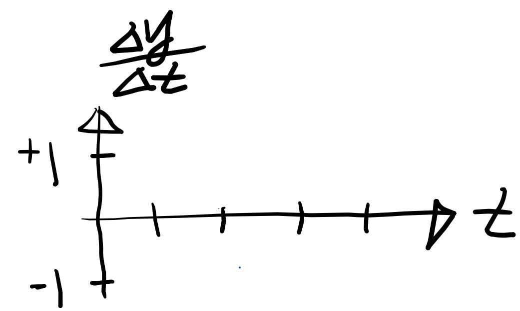
QUAD RATE





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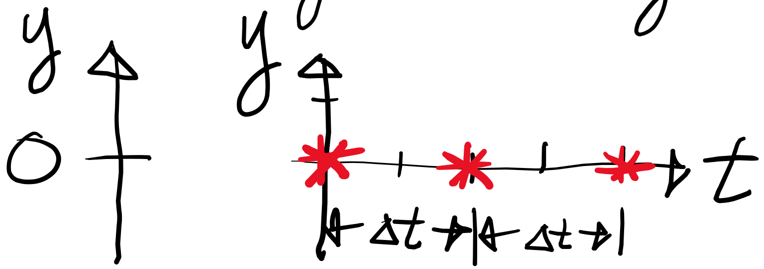

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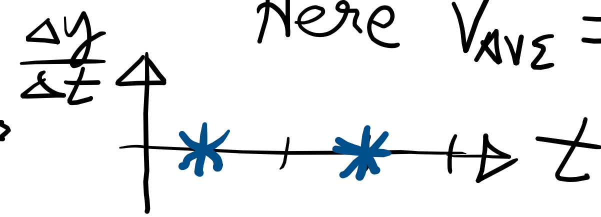

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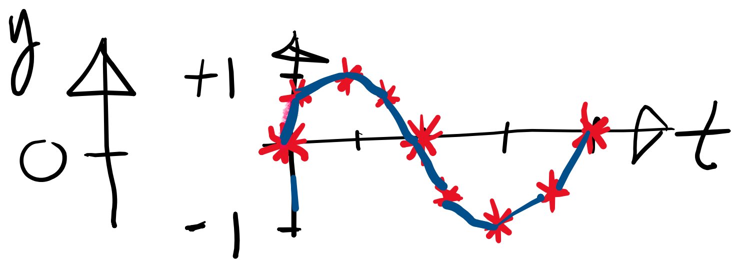
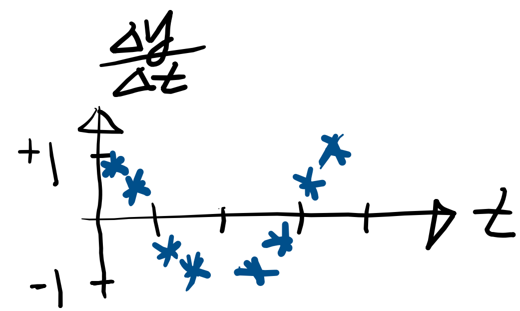
QUAD RATE





Continuous light

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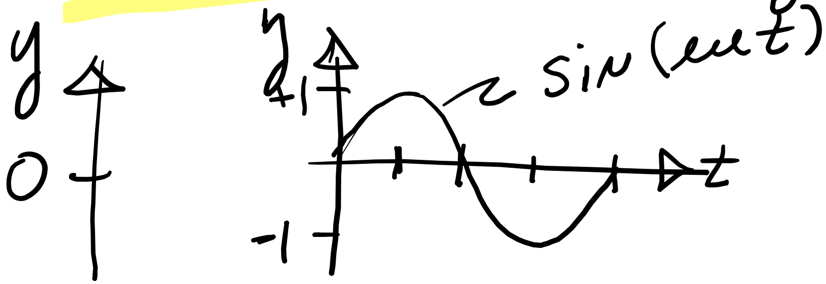

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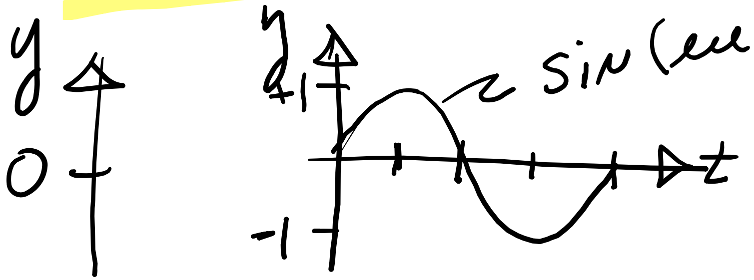
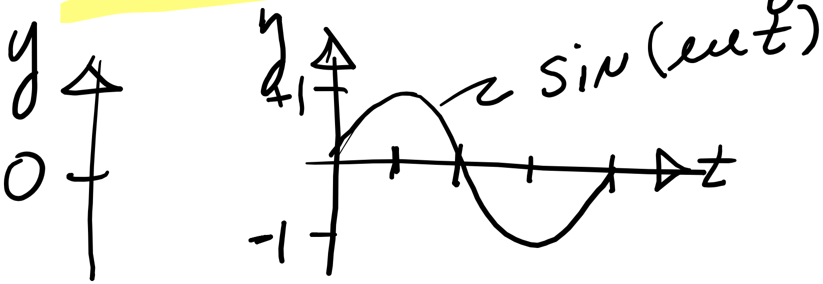


Now there is no nice Δt between flashes of light

Continuous light

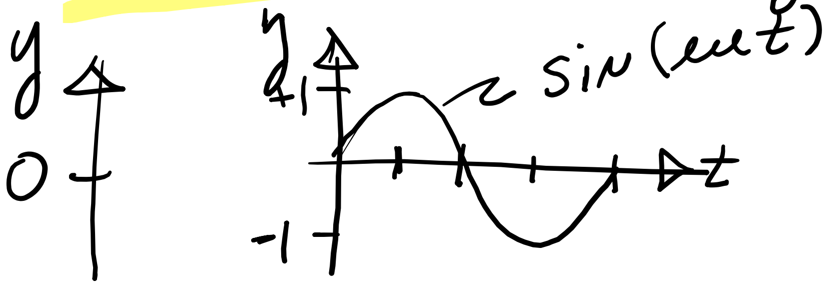


Now there is no nice Δt between flashes of light

Solution: Take limit as $\Delta t \to 0$:

$$v = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} \equiv \frac{dy}{dt}$$

Continuous light



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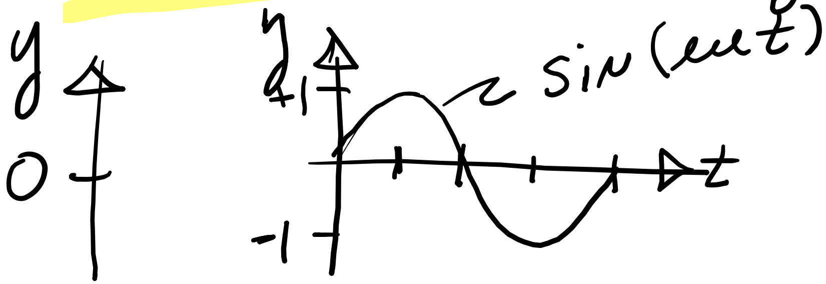
$$v = \frac{d}{dt}\sin(\omega t) = \omega \cos(\omega t)$$

Continuous light



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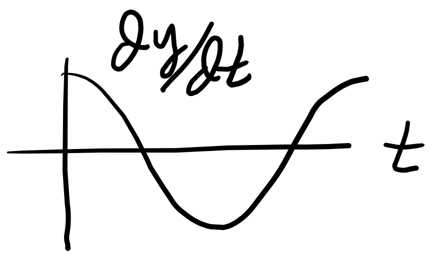

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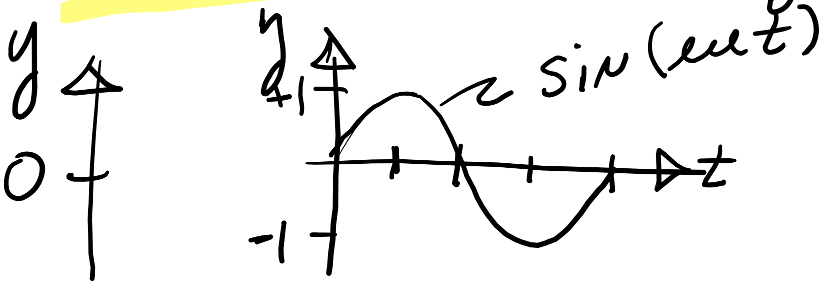

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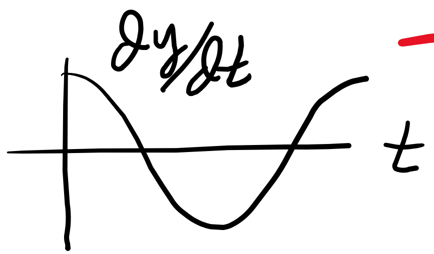
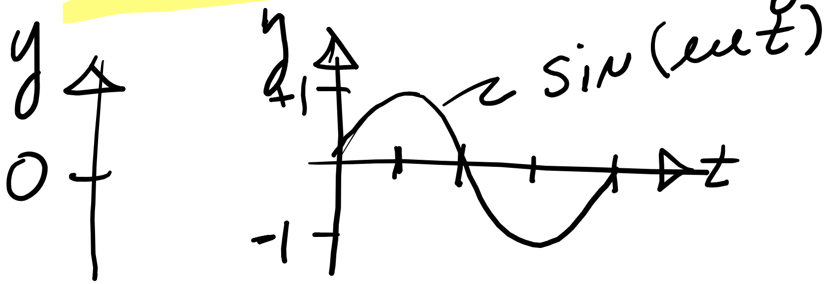


We can see that the velocity is never constant (in this example)

Continuous light



Now there is no nice Δt between flashes of light

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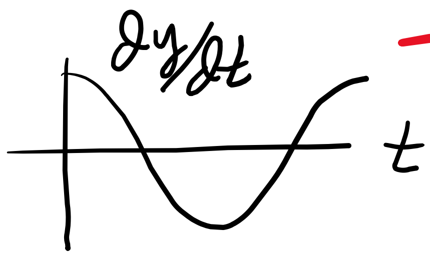

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ASU

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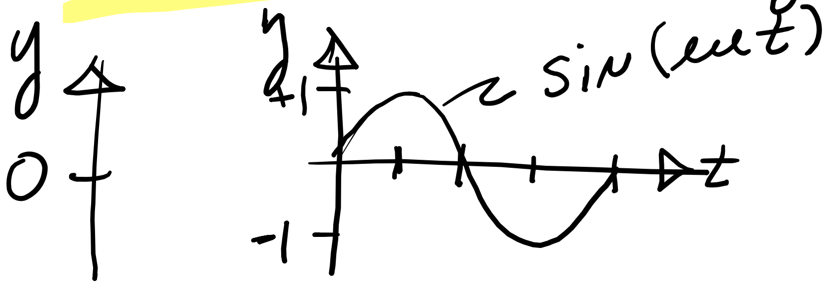

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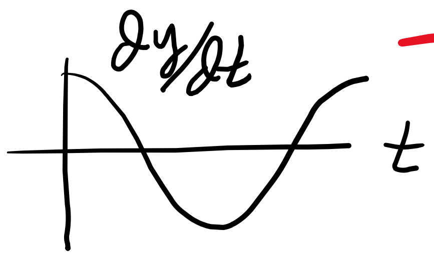

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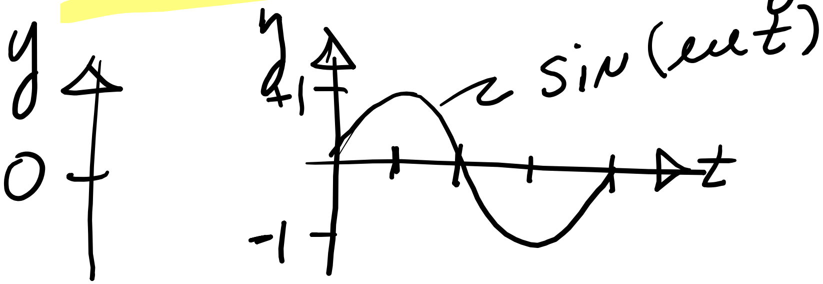

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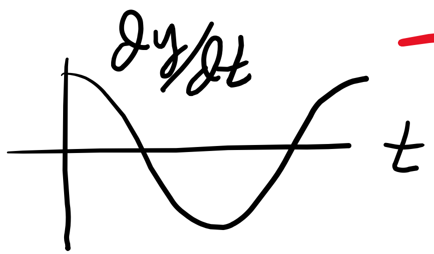

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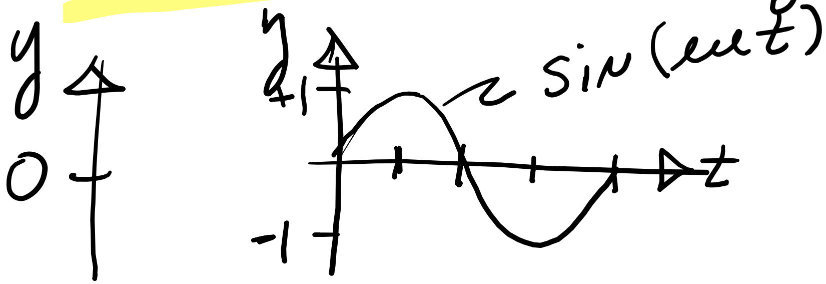

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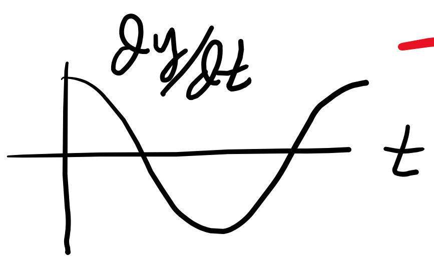

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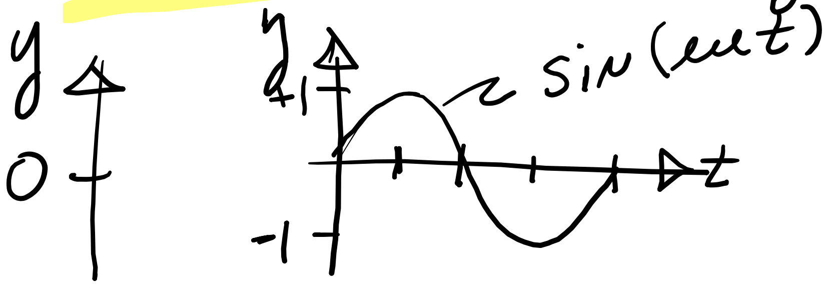

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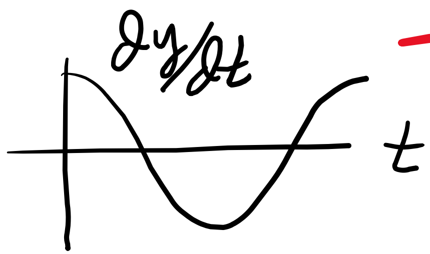

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Starting with $x(t)$

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x as a function of t

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Differential elements

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Differential elements

If y is a function of x, the differential element of y is $dy = \frac{dy}{dx} dx$

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Then $$v = v_0 + \int_0^t a\, dt$$

Example: $a = -9.81 \frac{m}{s^2}$

Find v & x

Example: $a = -9.81 \frac{m}{s^2} \Rightarrow$

$$V = V_0 + \int_0^t \left(-9.81 \frac{m}{s^2}\right) dt = V_0 - \left(9.81 \frac{m}{s^2}\right) t$$

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ASU

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$$\Rightarrow x = x_0 + v_0 t - 9.81\frac{m}{s^2}\left(\frac{t^2}{2}\right)$$

ASU

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$\Rightarrow x = x_0 + V_0 t - 9.81\frac{m}{s^2}\left(\frac{t^2}{2}\right)$ or rearrange to get $x = -\frac{1}{2}\left(9.81\frac{m}{s^2}\right)t^2 + V_0 t + x_0$

What if x is a function of t but v is a function of x? x

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We could write $v(x(t))$

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ASU

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Now $a = \frac{dv}{dt} = \boxed{\left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right)}$ Chain rule

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$\frac{1}{2}(v^2 - v_0^2) = \int_{x_0}^{x} a\,dx$

ASU

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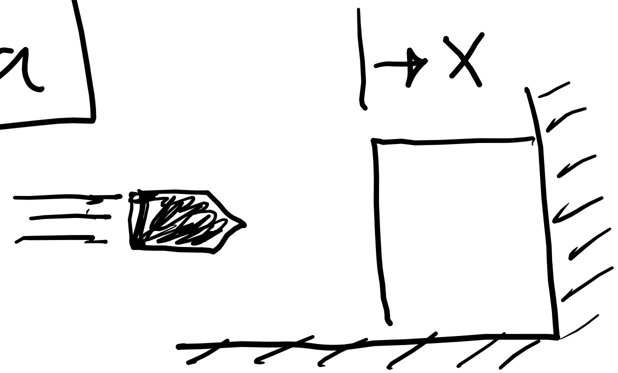
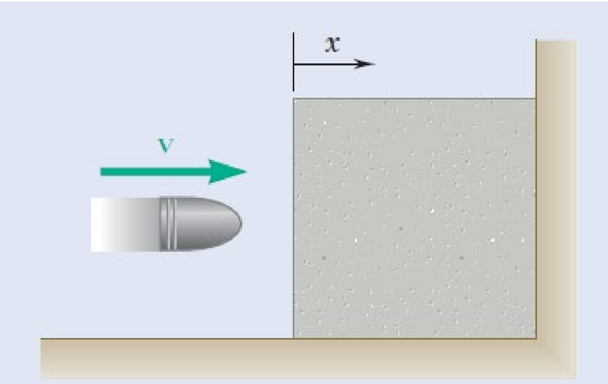
Notes on problems:

$\boxed{11.16a}$



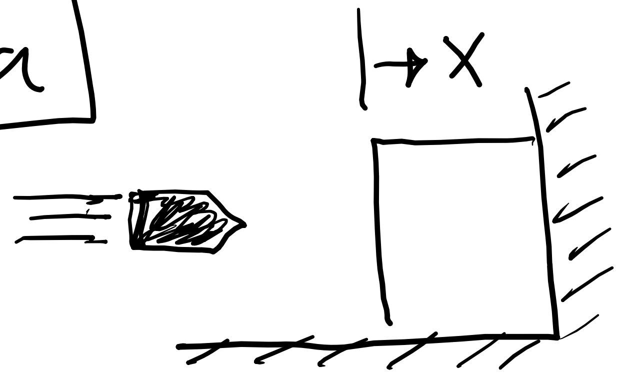
Given: $\left\{ \begin{array}{l} V_0 = 900\,\frac{ft}{s} \\ \quad \text{at } x_0 = 0 \\ V_1 = 0 \text{ at} \\ \quad x_1 = 4\,in \\ \text{\& } V = V_0 - kx \end{array} \right.$

11.16 A projectile enters a resisting medium at $x = 0$ with an initial velocity $v_0 = 900$ ft/s and travels 4 in. before coming to rest. Assuming that the velocity of the projectile is defined by the relation $v = v_0 - kx$, where v is expressed in ft/s and x is in feet, determine (*a*) the initial acceleration of the projectile, (*b*) the time required for the projectile to penetrate 3.9 in. into the resisting medium.



Notes on problems:

$\boxed{11.16a}$



Given:
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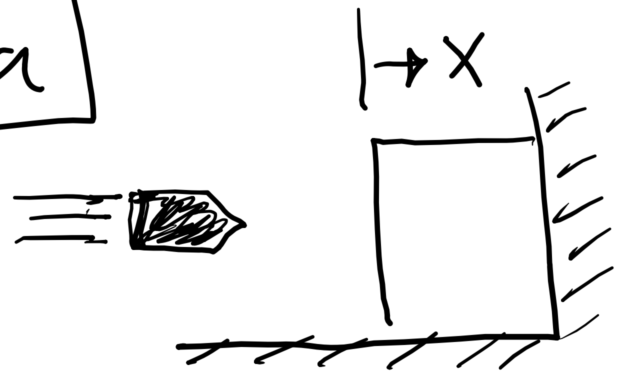
note unit change

Notes on problems:

$\boxed{11.16a}$



Given:

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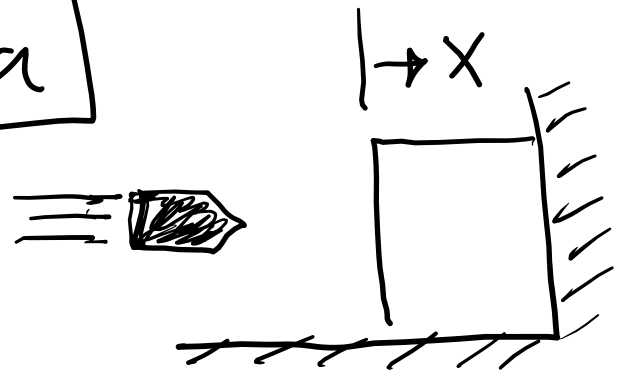
Find $a_0 = a(t=0)$

Notes on problems:

11.16a

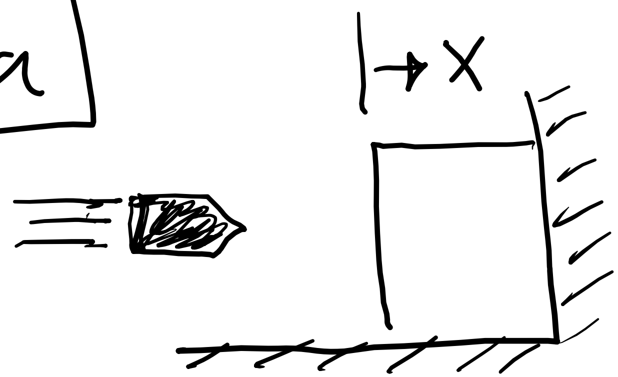


Given: $\begin{cases} v_0 = 900\,\frac{ft}{s} \text{ at } x_0 = 0 \\ v_1 = 0 \text{ at } x_1 = 4\,in \\ \& \; v = v_0 - kx \end{cases}$

Find $a_0 = a(t=0)$ We have v as a function of x.

Notes on problems:

$\boxed{11.16a}$



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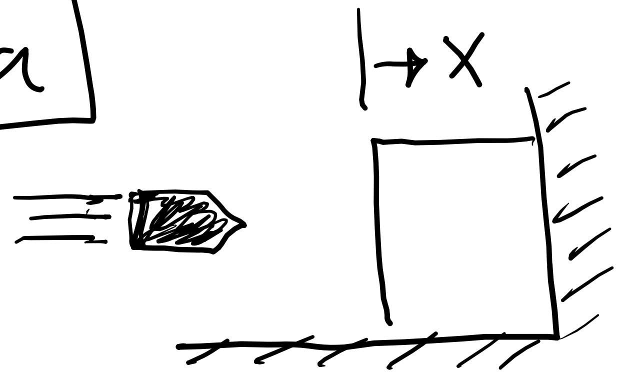
Find $a_0 = a(t=0)$ We have v as a function of x. This suggests we use $a = v\frac{dv}{dx}$.

Notes on problems:

$\boxed{11.16a}$



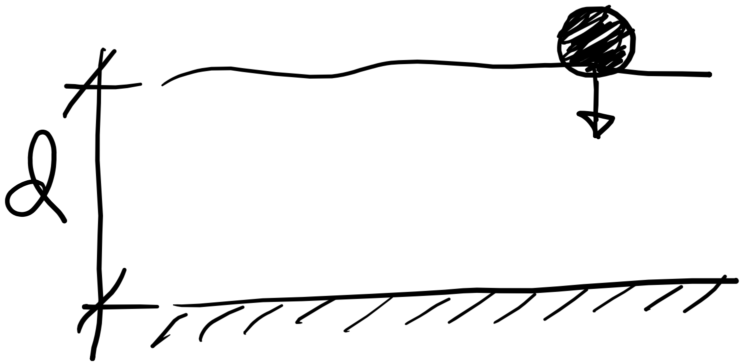
Given: $\left\{ \begin{array}{l} v_0 = 900 \frac{ft}{s} \\ \quad \text{at } x_0 = 0 \\ v_1 = 0 \text{ at} \\ \quad x_1 = 4 in \\ \quad \& \ v = v_0 - kx \end{array} \right.$

Find $a_0 = a(t=0)$ We have v as a function of x. This suggests we use $a = v\frac{dv}{dx}$.

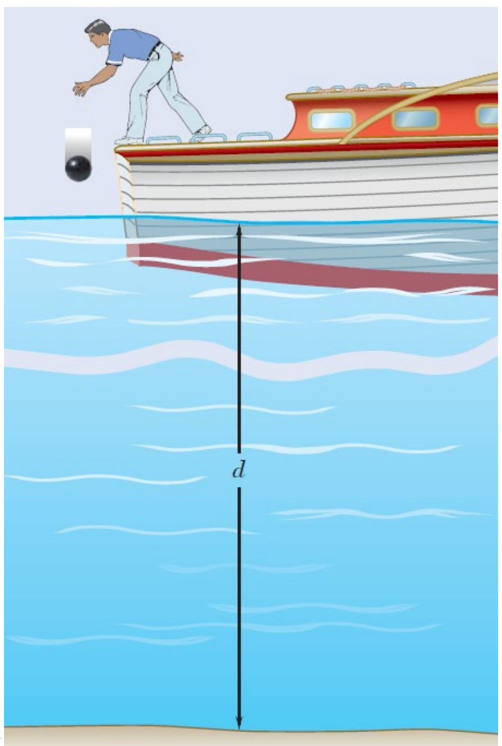
Will need to find k first

$\boxed{11.23}$

Given: $\begin{cases} V_0 = 16.5 \frac{ft}{s} \\ a = 10 - 0.8v \\ \& \Delta t = 3s \text{ to hit bottom} \end{cases}$



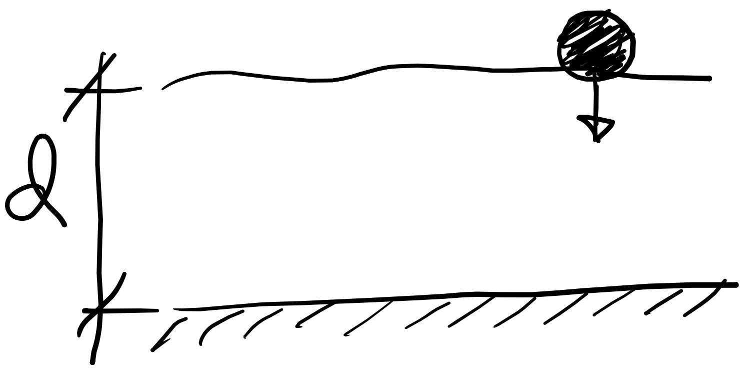
Find d



11.23 A ball is dropped from a boat so that it strikes the surface of a lake with a speed of 16.5 ft/s. While in the water the ball experiences an acceleration of $a = 10 - 0.8v$, where a and v are expressed in ft/s^2 and ft/s, respectively. Knowing the ball takes 3 s to reach the bottom of the lake, determine (*a*) the depth of the lake, (*b*) the speed of the ball when it hits the bottom of the lake.

$\boxed{11.23}$

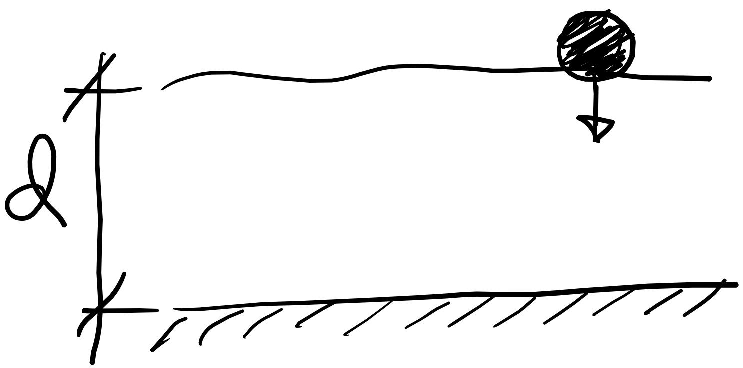
Given: $\begin{cases} V_0 = 16.5\,\frac{ft}{s} \\ a = 10 - 0.8v \\ \text{\& } \Delta t = 3s \text{ to hit bottom} \end{cases}$



Find d

Here we have a as a function of v

11.23

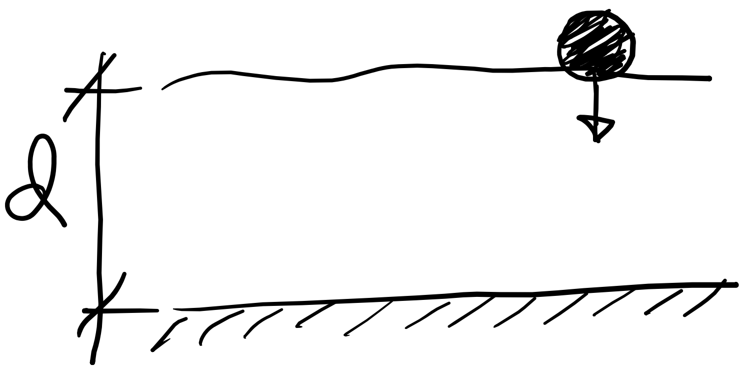


Given: $\begin{cases} V_0 = 16.5 \frac{ft}{s} \\ a = 10 - 0.8v \\ \Delta t = 3s \text{ to hit bottom} \end{cases}$

Find d

Here we have a as a function of v. One way to solve is to note that $a = \frac{dv}{dt} \Rightarrow \int_0^t dt = \int_{v_0}^{v} \frac{dv}{a}$, where $a = 10 - 0.8v$.

$\boxed{11.23}$



Given: $\begin{cases} v_0 = 16.5\,\frac{ft}{s} \\ a = 10 - 0.8v \\ \& \ \Delta t = 3s \text{ to hit bottom} \end{cases}$

Find d

Here we have a as a function of v. One way to solve is to note that $a = \frac{dv}{dt} \Rightarrow \int_0^t dt = \int_{v_0}^{v} \frac{dv}{a}$, where $a = 10 - 0.8v$. Solve for v then use $\frac{dx}{dt} = v \Rightarrow \int dx = \int v\,dt$

$\boxed{11.28a}$

Given: $\begin{cases} v = 7.5(1-0.04x)^{0.3} \\ \& \ x=0 \text{ at } t=0 \end{cases}$

Find $x(t=1h)$

11.28 Based on observations, the speed of a jogger can be approximated by the relation $v = 7.5(1 - 0.04x)^{0.3}$, where v and x are expressed in mi/h and miles, respectively. Knowing that $x = 0$ at $t = 0$, determine (a) the distance the jogger has run when $t = 1$ h, ~~(b) the jogger's acceleration in ft/s^2 at $t = 0$, (c) the time required for the jogger to run 6 mi.~~

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$\boxed{11.28a}$

Given: $\begin{cases} v = 7.5(1-0.04x)^{0.3} \\ \text{\& } x = 0 \text{ at } t = 0 \end{cases}$

Find $x(t=1h)$

We have v as a function of x

11.28a

Given: $\begin{cases} v = 7.5(1 - 0.04x)^{0.3} \\ \& \; x = 0 \text{ at } t = 0 \end{cases}$

Find $x(t = 1h)$

We have v as a function of x & know

$$v = \frac{dx}{dt} \Rightarrow \int dt = \int \frac{dx}{v}$$

Section 11.2

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Special cases & Relative motion

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$v = \text{constant}$

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$v = \text{constant} \Rightarrow a = 0$ & $x = vt + x_0$

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$v = \text{constant} \Rightarrow a = 0$ & $x = vt + x_0$

$a = \text{constant} \Rightarrow v = at + v_0$ & $x = \frac{1}{2}at^2 + v_0 t + x_0$

Section 11.2

Special cases & Relative motion

$v = \text{constant} \Rightarrow a = 0$ & $x = vt + x_0$

$a = \text{constant} \Rightarrow v = at + v_0$ & $x = \frac{1}{2}at^2 + v_0 t + x_0$

Also $a = v\frac{dv}{dx} \Rightarrow a\int_{x_0}^{x} dx = \int_{v_0}^{v} v\,dv$

Section 11.2

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$v = \text{constant} \Rightarrow a = \cancel{0}$ & $x = vt + x_0$

$a = \text{constant} \Rightarrow v = at + v_0$ & $x = \frac{1}{2}at^2 + v_0 t + x_0$

Also $a = v\frac{dv}{dx} \Rightarrow a\int_{x_0}^{x} dx = \int_{v_0}^{v} v\,dv$

$\Rightarrow a(x - x_0) = \frac{1}{2}(v^2 - v_0^2)$

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Section 11.2

Special cases & Relative motion

$v = \text{constant} \Rightarrow a = \cancel{0}$ & $x = vt + x_0$

$a = \text{constant} \Rightarrow v = at + v_0$ & $x = \frac{1}{2}at^2 + v_0 t + x_0$

Also $a = v\dfrac{dv}{dx} \Rightarrow a\displaystyle\int_{x_0}^{x} dx = \int_{v_0}^{v} v\,dv$

$\Rightarrow a(x - x_0) = \frac{1}{2}(v^2 - v_0^2)$ or

$v^2 = 2a(x - x_0) + v_0^2$