Today 13.4, 14.1

$\boxed{L10}$

Today 13.4, 14.1

$\boxed{\text{L10}}$

Impacts

Today 13.4, 14.1

$\boxed{L10}$

System of particles: Newton's 2nd law

Today 13.4, 14.1 $\boxed{L10}$

Thursday 14.2

Today 13.4, 14.1

Thursday 14.2

$\boxed{\text{L10}}$

Energy & work for system of particles

Section 13.4

Elastic impact

Section 13.4

Elastic impact $\Rightarrow$ momentum is conserved

Section 13.4

Elastic impact $\Rightarrow$ momentum is conserved

AND kinetic energy is conserved

Section 13.4

Elastic impact $\Rightarrow$ momentum is conserved
AND kinetic energy is conserved

Non-elastic impact

Section 13.4

Elastic impact ⇒ momentum is conserved AND kinetic energy is conserved

Non-elastic impact ⇒ momentum is conserved

Section 13.4

Elastic impact ⇒ momentum is conserved <u>AND</u> kinetic energy is conserved

Non-elastic impact ⇒ momentum is conserved <u>But</u> kinetic energy is <u>NOT</u> conserved

Section 13.4

Elastic impact ⇒ Momentum is conserved **AND** Kinetic energy is conserved

Non-elastic impact ⇒ Momentum is conserved **But** Kinetic energy is **NOT** conserved

Note: Non-elastic is also called "plastic".

Section 13.4

Elastic impact ⇒ Momentum is conserved **AND** Kinetic energy is conserved

Non-elastic impact ⇒ Momentum is conserved **But** Kinetic energy is **NOT** conserved

Note: Non-elastic is also called "plastic".

A completely non-elastic {completely plastic} impact is one where the two items stick together

Section 14.1 : System of particles

Section 14.1 : System of particles

For this material there is going to be more than one particle. In fact, there could be any number of particles and sometimes they will collide.

Section 14.1 : System of particles

For this material there is going to be more than one particle. In fact, there could be any number of particles and sometimes they will collide.

Define: $\vec{f}_{ij} \equiv$ force on particle i exerted by particle j

Section 14.1: System of particles

For this material there is going to be more than one particle. In fact, there could be any number of particles and sometimes they will collide.

Define: $\vec{f}_{ij} \equiv$ force on particle i exerted by particle j

This means that $f_{ji} = -f_{ij}$.

Section 14.1: System of particles

For this material there is going to be more than one particle. In fact, there could be any number of particles and sometimes they will collide.

Define: $\vec{f}_{ij} \equiv$ force on particle i exerted by particle j

This means that $f_{ji} = -f_{ij}$.

Example: particle i hits particle j

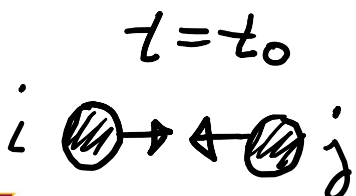
Section 14.1 : System of particles

For this material there is going to be more than one particle. In fact, there could be any number of particles and sometimes they will collide.

Define: $\vec{f}_{ij} \equiv$ force on particle i exerted by particle j

This means that $f_{ji} = -f_{ij}$.

Example: particle i hits particle j



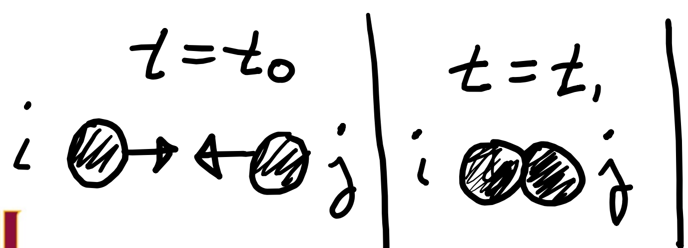
Section 14.1 : System of particles

For this material there is going to be more than one particle. In fact, there could be any number of particles and sometimes they will collide.

Define: $\vec{f}_{ij} \equiv$ force on particle i exerted by particle j

This means that $f_{ji} = -f_{ij}$.

Example: particle i hits particle j



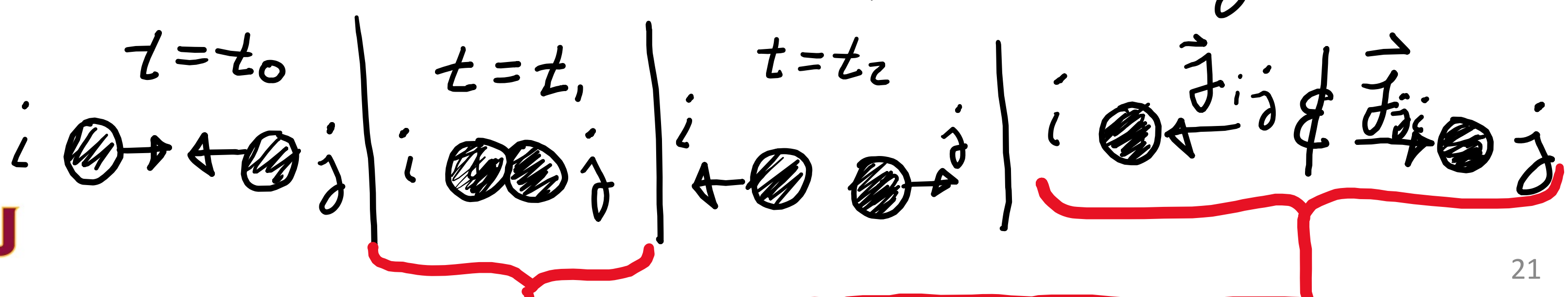
Section 14.1: System of particles

For this material there is going to be more than one particle. In fact, there could be any number of particles and sometimes they will collide.

Define: $\vec{f}_{ij} \equiv$ force on particle i exerted by particle j

This means that $f_{ji} = -f_{ij}$.

Example: particle i hits particle j



For some particle i, assume some external force $\vec{F}_i$ acting on it.

For some particle i, assume some external force $\vec{F}_i$ acting on it. We can write $\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{ij} = m_i \vec{a}_i$

For some particle i, assume some external force $\vec{F}_i$ acting on it. We can write $\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{i,j} = m_i \vec{a}_i$. This says that the acceleration of particle i and mass m_i is due to the external force $\vec{F}_i$ and any interaction forces $\vec{f}_{i,j}$ from the other particles.

ASU

For some particle i, assume some external force $\vec{F}_i$ acting on it. We can write $\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{i,j} = m_i \vec{a}_i$. This says that the acceleration of particle i and mass m_i is due to the external force $\vec{F}_i$ and any interaction forces $\vec{f}_{i,j}$ from the other particles.

Let us now take the cross product: $\vec{r}_i \times [\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{i,j}] = \vec{r}_i \times [m_i \vec{a}_i]$, where $\vec{r}_i$ is the position vector of particle i.

For some particle i, assume some external force $\vec{F}_i$ acting on it. We can write $\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{ij} = m_i \vec{a}_i$. This says that the acceleration of particle i and mass m_i is due to the external force $\vec{F}_i$ and any interaction forces $\vec{f}_{ij}$ from the other particles.

Let us now take the cross product:
$\vec{r}_i \times [\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{ij}] = \vec{r}_i \times [m_i \vec{a}_i]$, where $\vec{r}_i$ is the position vector of particle i. $\Rightarrow$

$$\vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{f}_{ij} = m_i \vec{r}_i \times \vec{a}_i$$

From the previous slide, we have

$$\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{ij} = m_i \vec{a}_i \quad \& \quad \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{f}_{ij} = m_i \vec{r}_i \times \vec{a}_i$$

From the previous slide, we have

$$\vec{F}_i + \sum_{j=1}^{N} \vec{J}_{ij} = m_i \vec{a}_i \quad \& \quad \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{J}_{ij} = m_i \vec{r}_i \times \vec{a}_i$$

Now sum over index i:

$$\sum_i \vec{F}_i + \sum_i \sum_j \vec{J}_{ij} = \sum_i m_i \vec{a}_i \quad (1)$$

$$\& \sum_i \vec{r}_i \times \vec{F}_i + \sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = \sum_i m_i \vec{r}_i \times \vec{a}_i \quad (2)$$

From the previous slide, we have

$$\vec{F}_i + \sum_{j=1}^{N} \vec{J}_{ij} = m_i \vec{a}_i \quad \& \quad \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{J}_{ij} = m_i \vec{r}_i \times \vec{a}_i$$

Now sum over index i:

$$\sum_i \vec{F}_i + \sum_i \sum_j \vec{J}_{ij} = \sum_i m_i \vec{a}_i \quad (1)$$

$$\& \sum_i \vec{r}_i \times \vec{F}_i + \sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = \sum_i m_i \vec{r}_i \times \vec{a}_i \quad (2)$$

Since $\vec{J}_{ij} = -\vec{J}_{ji}$

From the previous slide, we have

$$\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{ij} = m_i \vec{a}_i \quad \& \quad \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{f}_{ij} = m_i \vec{r}_i \times \vec{a}_i$$

Now sum over index i:

$$\sum_i \vec{F}_i + \sum_i \sum_j \vec{f}_{ij} = \sum_i m_i \vec{a}_i \quad (1)$$

$$\& \sum_i \vec{r}_i \times \vec{F}_i + \sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = \sum_i m_i \vec{r}_i \times \vec{a}_i \quad (2)$$

Since $\vec{f}_{ij} = -\vec{f}_{ji}$ & $\vec{f}_{ii} = 0$ particle does <u>not</u> act on itself

From the previous slide, we have

$$\vec{F}_i + \sum_{j=1}^{N} \vec{f}_{ij} = m_i \vec{a}_i \quad \& \quad \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{f}_{ij} = m_i \vec{r}_i \times \vec{a}_i$$

Now sum over index i:

$$\sum_i \vec{F}_i + \sum_i \sum_j \vec{f}_{ij} = \sum_i m_i \vec{a}_i \quad (1)$$

$$\& \sum_i \vec{r}_i \times \vec{F}_i + \sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = \sum_i m_i \vec{r}_i \times \vec{a}_i \quad (2)$$

Since $\vec{f}_{ij} = -\vec{f}_{ji}$ & $\vec{f}_{ii} = \theta$ particle does <u>not</u> act on itself

then $\sum_i \sum_j \vec{f}_{ij} = \theta$

From the previous slide, we have

$$\vec{F}_i + \sum_{j=1}^{N} \vec{J}_{ij} = m_i \vec{a}_i \quad \& \quad \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{J}_{ij} = m_i \vec{r}_i \times \vec{a}_i$$

Now sum over index i:

$$\sum_i \vec{F}_i + \sum_i \sum_j \vec{J}_{ij} = \sum_i m_i \vec{a}_i \quad (1)$$

$$\& \sum_i \vec{r}_i \times \vec{F}_i + \sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = \sum_i m_i \vec{r}_i \times \vec{a}_i \quad (2)$$

Since $\vec{J}_{ij} = -\vec{J}_{ji}$ & $\vec{J}_{ii} = \vec{0}$ particle does <u>not</u> act on itself

then $\sum_i \sum_j \vec{J}_{ij} = \vec{0}$ and equation 1

becomes $\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i$

From the previous slide, we have

$$\vec{F}_i + \sum_{j=1}^{N} \vec{J}_{ij} = m_i \vec{a}_i \quad \& \quad \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{N} \vec{r}_i \times \vec{J}_{ij} = m_i \vec{r}_i \times \vec{a}_i$$

Now sum over index i:

$$\sum_i \vec{F}_i + \sum_i \sum_j \vec{J}_{ij} = \sum_i m_i \vec{a}_i \quad (1)$$

$$\& \sum_i \vec{r}_i \times \vec{F}_i + \sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = \sum_i m_i \vec{r}_i \times \vec{a}_i \quad (2)$$

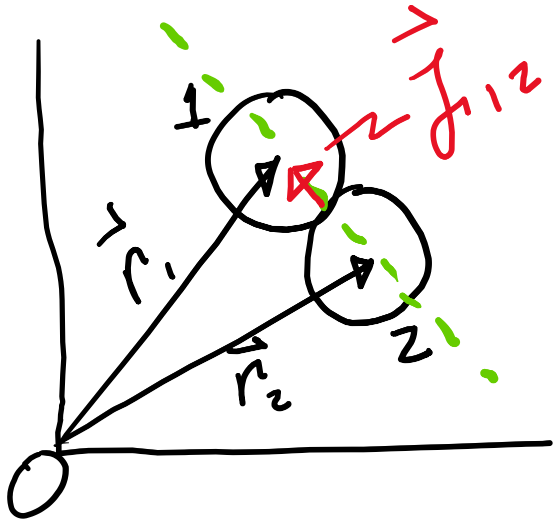
Since $\vec{J}_{ij} = -\vec{J}_{ji}$ & $\vec{J}_{ii} = 0$ particle does not act on itself

then $\sum_i \sum_j \vec{J}_{ij} = 0$ and equation 1 becomes $\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i$

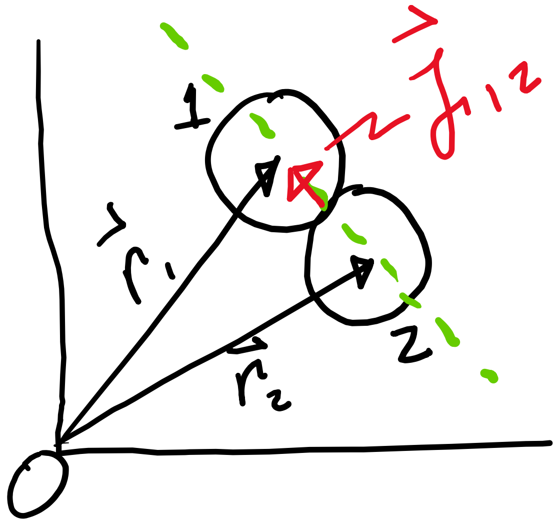
Next, we want to get rid of the term $\sum_i \sum_j \vec{r}_i \times \vec{J}_{ij}$ from equation 2 $\rightarrow$

First we look at the case where there are only two particles:

First we look at the case where there are only two particles: $\sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = \vec{r}_1 \times \vec{f}_{12} + \vec{r}_2 \times \vec{f}_{21}$

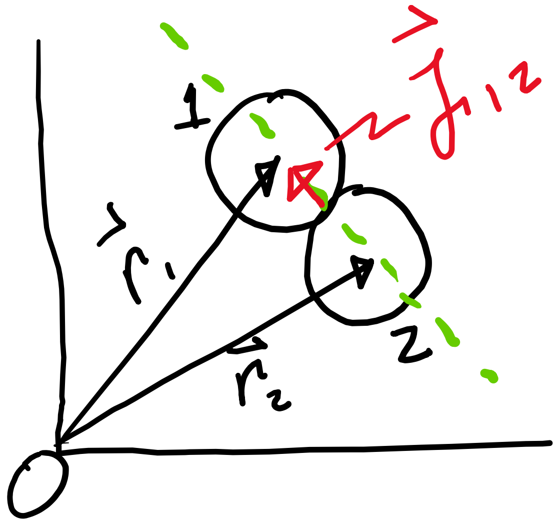


First we look at the case where there are only two particles: $\sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = \vec{r}_1 \times \vec{f}_{12} + \vec{r}_2 \times \vec{f}_{21}$



But $\vec{f}_{12} = -\vec{f}_{21}$

First we look at the case where there are only two particles: $\sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = \vec{r}_1 \times \vec{f}_{12} + \vec{r}_2 \times \vec{f}_{21}$

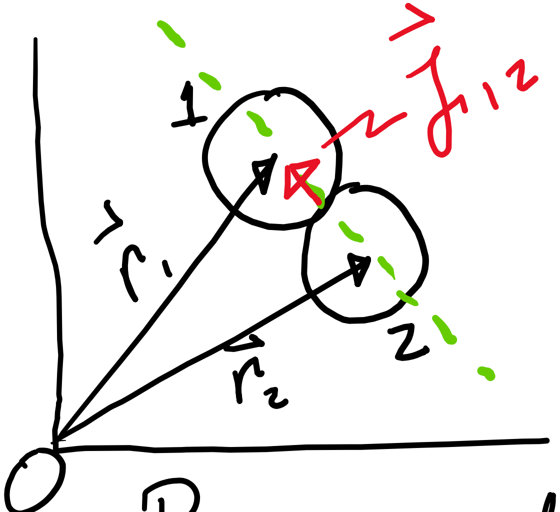


But $\vec{f}_{12} = -\vec{f}_{21}$ so

$$\sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = \vec{r}_1 \times \vec{f}_{12} - \vec{r}_2 \times \vec{f}_{12}$$

$$= (\vec{r}_1 - \vec{r}_2) \times \vec{f}_{12}$$

First we look at the case where there are only two particles: $\sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = \vec{r}_1 \times \vec{f}_{12} + \vec{r}_2 \times \vec{f}_{21}$

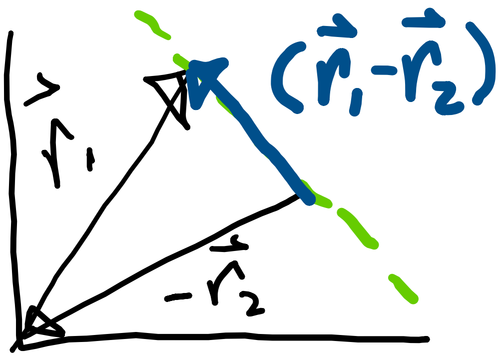


But $\vec{f}_{12} = -\vec{f}_{21}$ so

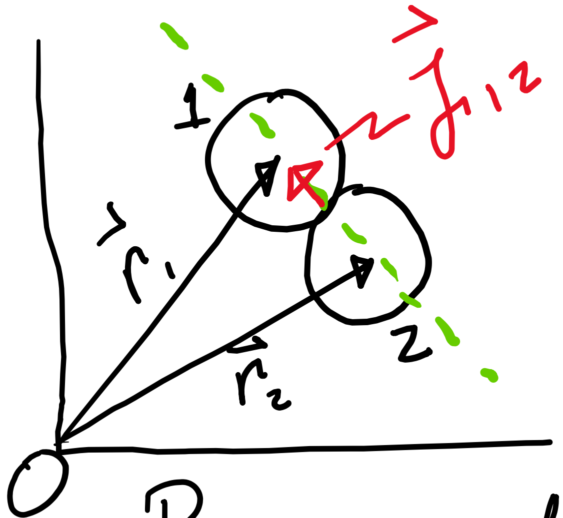
$$\sum_i \sum_j \vec{r}_i \times \vec{f}_{ij} = \vec{r}_1 \times \vec{f}_{12} - \vec{r}_2 \times \vec{f}_{12}$$

$$= (\vec{r}_1 - \vec{r}_2) \times \vec{f}_{12}$$

Remove objects & reset $\vec{r}_2$ to $-\vec{r}_2$



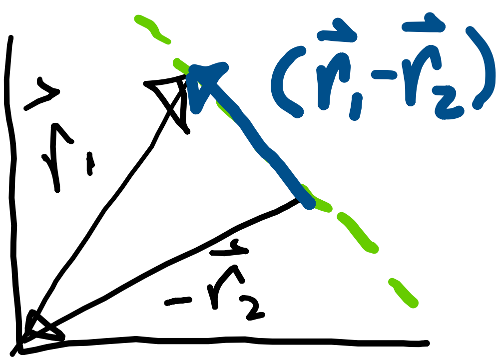
First we look at the case where there are only two particles: $\sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = \vec{r}_1 \times \vec{J}_{12} + \vec{r}_2 \times \vec{J}_{21}$



But $\vec{J}_{12} = -\vec{J}_{21}$ so

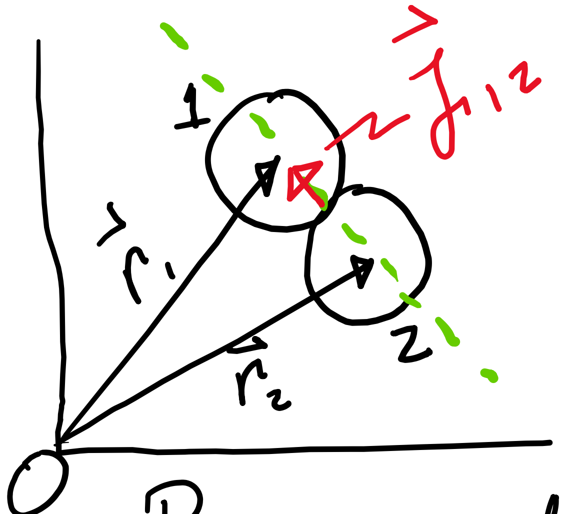
$$\sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = \vec{r}_1 \times \vec{J}_{12} - \vec{r}_2 \times \vec{J}_{12}$$
$$= (\vec{r}_1 - \vec{r}_2) \times \vec{J}_{12}$$

Remove objects & reset $\vec{r}_2$ to $-\vec{r}_2$



$\vec{r}_1 - \vec{r}_2$ is in same direction as $\vec{J}_{12}$

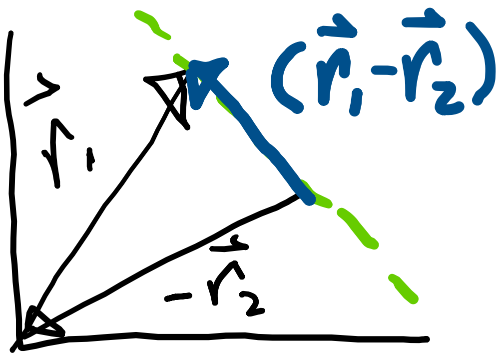
First we look at the case where there are only two particles: $\sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = \vec{r}_1 \times \vec{J}_{12} + \vec{r}_2 \times \vec{J}_{21}$



But $\vec{J}_{12} = -\vec{J}_{21}$ so

$$\sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = \vec{r}_1 \times \vec{J}_{12} - \vec{r}_2 \times \vec{J}_{12}$$
$$= (\vec{r}_1 - \vec{r}_2) \times \vec{J}_{12}$$

Remove objects & reset $\vec{r}_2$ to $-\vec{r}_2$



$\vec{r}_1 - \vec{r}_2$ is in same direction as $\vec{J}_{12}$

$\Rightarrow$

$$\sum_i \sum_j \vec{r}_i \times \vec{J}_{ij} = (\vec{r}_1 - \vec{r}_2) \times \vec{J}_{12} = 0$$

Now we can write

$$\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i \quad \& \quad \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times m \vec{a}_i$$

Now we can write

$$\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i \quad \& \quad \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times m \vec{a}_i$$

But $\vec{L}_i = m_i \vec{v}_i \Rightarrow \dot{\vec{L}}_i = m_i \vec{a}_i$

Now we can write

$$\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i \;\;\&\;\; \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times m \vec{a}_i$$

But $\vec{L}_i = m_i \vec{v}_i \Rightarrow \dot{\vec{L}}_i = m_i \vec{a}_i$

& $\vec{H}_{oi} = \vec{r}_i \times m \vec{v}_i \Rightarrow \dot{\vec{H}}_{oi} = (\dot{\vec{r}}_i \times m \vec{v}_i) + (\vec{r}_i \times m \vec{a}_i)$

Now we can write

$$\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i \quad \& \quad \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times m \vec{a}_i$$

But $\vec{L}_i = m_i \vec{v}_i \Rightarrow \dot{\vec{L}}_i = m_i \vec{a}_i$

& $\vec{H}_{o_i} = \vec{r}_i \times m \vec{v}_i \Rightarrow \vec{H}_{o_i} = \underbrace{\left(\dot{\vec{r}}_i \times m \vec{v}_i\right)}_{\text{Equals zero since } \dot{\vec{r}}_i = \vec{v}_i \;\&\; \vec{v}_i \times \vec{v}_i = 0} + \left(\vec{r}_i \times m \vec{a}_i\right)$

Now we can write

$$\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i \;\; \& \;\; \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times m \vec{a}_i$$

But $\vec{L}_i = m_i \vec{v}_i \Rightarrow \dot{\vec{L}}_i = m_i \vec{a}_i$

& $\vec{H}_{o_i} = \vec{r}_i \times m \vec{v}_i \Rightarrow \dot{\vec{H}}_{o_i} = \underbrace{(\dot{\vec{r}}_i \times m \dot{\vec{v}}_i)} + (\vec{r}_i \times m \vec{a}_i)$

↳ Equals zero since $\dot{\vec{r}}_i = \vec{v}_i$ & $\vec{v}_i \times \vec{v}_i = 0$

Now let $\sum \vec{L}_i$ & $\sum \vec{H}_{o_i}$ for some system be simply written as $\vec{L}$ & $\vec{H}$ respectively.

Now we can write

$$\sum_i \vec{F}_i = \sum_i m_i \vec{a}_i \; \& \; \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times m \vec{a}_i$$

But $\vec{L}_i = m_i \vec{v}_i \Rightarrow \dot{\vec{L}}_i = m_i \vec{a}_i$

$\& \quad \vec{H}_{o_i} = \vec{r}_i \times m \vec{v}_i \Rightarrow \dot{\vec{H}}_{o_i} = \underbrace{(\dot{\vec{r}}_i \times m \vec{v}_i)}_{} + (\vec{r}_i \times m \vec{a}_i)$

↳ Equals zero since $\dot{\vec{r}}_i = \vec{v}_i$ & $\vec{v}_i \times \vec{v}_i = \theta$

Now let $\sum \vec{L}_i$ & $\sum \vec{H}_{o_i}$ for some system be simply written as $\vec{L}$ & $\vec{H}$ respectively. Now we get

$$\boxed{\sum \vec{F} = \dot{\vec{L}} \quad \& \quad \sum \vec{M}_o = \dot{\vec{H}}_o}$$

Mass center

Mass center

$$m\bar{\vec{r}} = \sum_i m_i \vec{r}_i \, ,$$

Mass center

$m\vec{\bar{r}} = \sum_i m_i \vec{r}_i$, where $\vec{\bar{r}} \equiv$ mass center

Mass center

$m\vec{\bar{r}} = \sum_i m_i \vec{r}_i$, where $\vec{\bar{r}} \equiv$ mass center & $m \equiv \sum_i m_i$

Mass center

$m\vec{\bar{r}} = \sum_i m_i \vec{r}_i$, where $\vec{\bar{r}} \equiv$ mass center & $m \equiv \sum_i m_i$

In rectangular coordinates:

Mass center

$m\bar{\vec{r}} = \sum_i m_i \vec{r}_i$, where $\bar{\vec{r}} \equiv$ mass center & $m \equiv \sum_i m_i$

In rectangular coordinates:

$$m\bar{x} = \sum_i m_i x_i \ , \ m\bar{y} = \sum_i m_i y_i \ , \ m\bar{z} = \sum_i m_i z_i$$

Mass center

$m\bar{\vec{r}} = \sum_i m_i \vec{r}_i$, where $\bar{\vec{r}} \equiv$ mass center & $m \equiv \sum_i m_i$

In rectangular coordinates:

$$m\bar{x} = \sum_i m_i x_i \; , \; m\bar{y} = \sum_i m_i y_i \; , \; m\bar{z} = \sum_i m_i z_i$$

We also have $m\dot{\bar{\vec{r}}} = \sum m_i \dot{\vec{r}}_i \Rightarrow m\bar{\vec{V}} = \sum m_i \vec{V}_i$

Mass center

$m\vec{\bar{r}} = \sum_i m_i \vec{r}_i$, where $\vec{\bar{r}} \equiv$ mass center & $m \equiv \sum_i m_i$

In rectangular coordinates:

$m\bar{x} = \sum_i m_i x_i$, $m\bar{y} = \sum_i m_i y_i$, $m\bar{z} = \sum_i m_i z_i$

We also have $m\dot{\vec{\bar{r}}} = \sum m_i \dot{\vec{r}}_i \Rightarrow m\vec{\bar{v}} = \sum m_i \vec{v}_i$

This means that $\vec{L} = m\vec{\bar{v}}$

Mass center

$m\vec{\bar{r}} = \sum_i m_i \vec{r}_i$, where $\vec{\bar{r}} \equiv$ mass center & $m \equiv \sum_i m_i$

In rectangular coordinates:

$m\bar{x} = \sum_i m_i x_i$, $m\bar{y} = \sum_i m_i y_i$, $m\bar{z} = \sum_i m_i z_i$

We also have $m\dot{\vec{\bar{r}}} = \sum m_i \dot{\vec{r}}_i \Rightarrow m\vec{\bar{v}} = \sum m_i \vec{v}_i$

This means that $\vec{L} = m\vec{\bar{v}}$ & $\dot{\vec{L}} = m\vec{\bar{a}}$

Mass center

$m\bar{\vec{r}} = \sum_i m_i \vec{r}_i$, where $\bar{\vec{r}} \equiv$ mass center & $m \equiv \sum_i m_i$

In rectangular coordinates:

$m\bar{x} = \sum_i m_i x_i$, $m\bar{y} = \sum_i m_i y_i$, $m\bar{z} = \sum_i m_i z_i$

We also have $m\dot{\bar{\vec{r}}} = \sum m_i \dot{\vec{r}}_i \Rightarrow m\bar{\vec{v}} = \sum m_i \vec{v}_i$

This means that $\vec{L} = m\bar{\vec{v}}$ & $\dot{\vec{L}} = m\bar{\vec{a}}$

& $\sum \vec{F} = m\bar{\vec{a}}$

Mass center

$m\vec{\bar{r}} = \sum_i m_i \vec{r}_i$, where $\vec{\bar{r}} \equiv$ mass center & $m \equiv \sum_i m_i$

In rectangular coordinates:

$m\bar{x} = \sum_i m_i x_i$, $m\bar{y} = \sum_i m_i y_i$, $m\bar{z} = \sum_i m_i z_i$

We also have $m\dot{\vec{\bar{r}}} = \sum m_i \dot{\vec{r}}_i \Rightarrow m\vec{\bar{v}} = \sum m_i \vec{v}_i$

This means that $\vec{L} = m\vec{\bar{v}}$ & $\dot{\vec{L}} = m\vec{\bar{a}}$

& $\sum \vec{F} = m\vec{\bar{a}}$ $\Rightarrow$ We can now treat the entire system of particles as a single particle located at $\vec{\bar{r}}$ with mass $m \equiv \sum_i m_i$ ☺

<u>Things to note</u>: You could run into a problem with a system of particles where $\sum \vec{F}_i = \vec{0}$ [no external forces] and $\sum \vec{M}_0 = \vec{0}$ [no external torques].

Things to note: You could run into a problem with a system of particles where $\sum \vec{F}_i = \vec{0}$ [No external forces] and $\sum \vec{M}_0 = \vec{0}$ [No external torques]. In such a case $\vec{L} = \sum_i \vec{L}_i$ is conserved

Things to note: You could run into a problem with a system of particles where $\sum \vec{F}_i = \vec{0}$ [no external forces] and $\sum \vec{M}_o = \vec{0}$ [no external torques]. In such a case $\vec{L} = \sum_i \vec{L}_i$ is conserved & $\vec{H}_o = \sum_i \vec{H}_{oi}$ is conserved

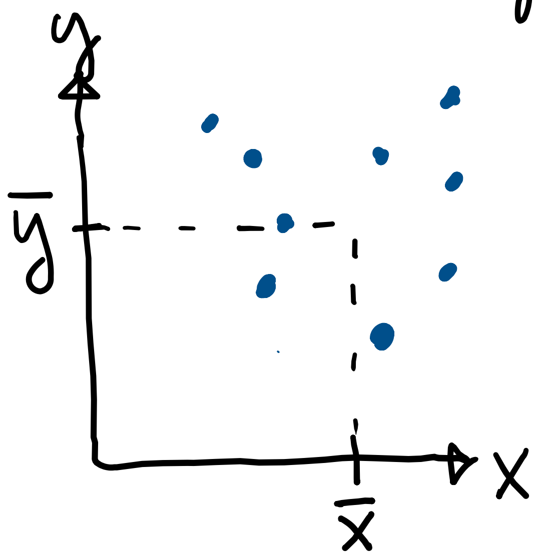
Things to note: You could run into a problem with a system of particles where $\sum \vec{F}_i = \emptyset$ [no external forces] and $\sum \vec{M}_o = \emptyset$ [no external torques]. In such a case $\vec{L} = \sum_i \vec{L}_i$ is conserved & $\vec{H}_o = \sum_i \vec{H}_{oi}$ is conserved $\Rightarrow \vec{L}_I = \vec{L}_F$

<u>Things to note</u>: You could run into a problem with a system of particles where $\sum \vec{F}_i = \emptyset$ [No external forces] and $\sum \vec{M}_0 = \emptyset$ [No external torques]. In such a case $\vec{L} = \sum_i \vec{L}_i$ is conserved & $\vec{H}_0 = \sum_i \vec{H}_{0i}$ is conserved $\Rightarrow$ $\vec{L}_I = \vec{L}_F$ & $\vec{H}_I = \vec{H}_F$

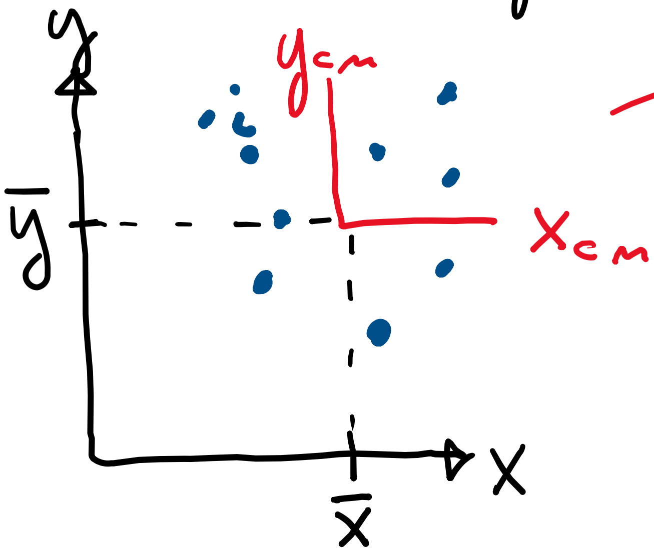
<u>Things to note</u>: You could run into a problem with a system of particles where $\sum \vec{F}_i = \vec{0}$ [no external forces] and $\sum \vec{M}_O = \vec{0}$ [no external torques]. In such a case $\vec{L} = \sum_i \vec{L}_i$ is conserved & $\vec{H}_O = \sum_i \vec{H}_{O_i}$ is conserved $\Rightarrow \vec{L}_I = \vec{L}_F$ & $\vec{H}_I = \vec{H}_F$

{A good example is the universe}

The center-of-mass system has some nice properties:

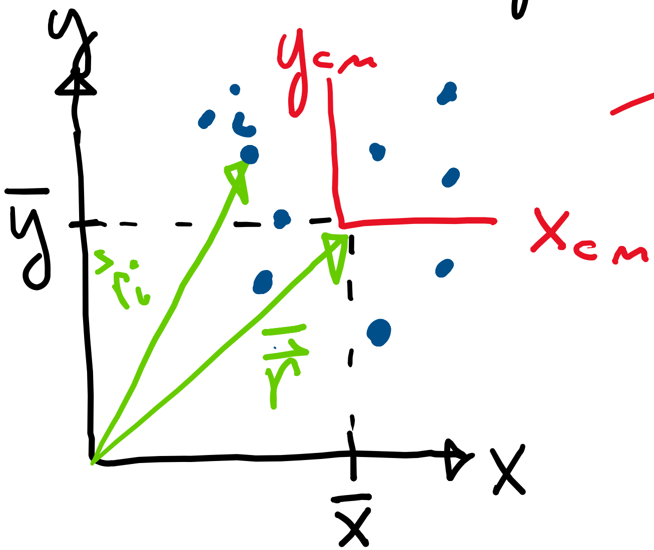


The center-of-mass system has some nice properties:



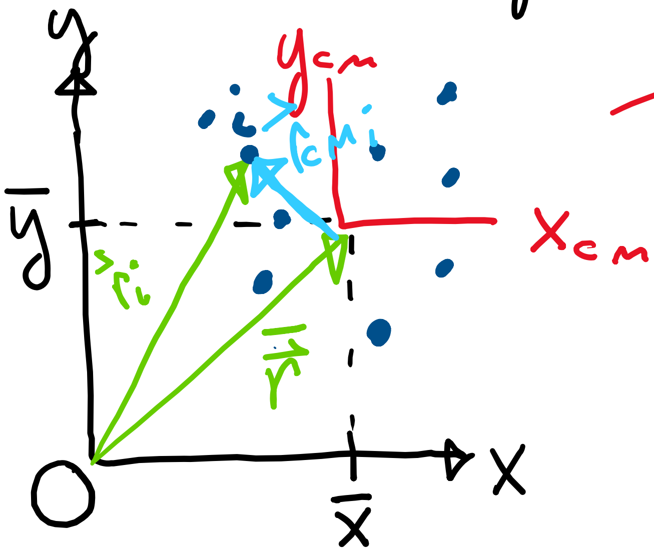
Center-of-mass system
[origin set at $\bar{x}, \bar{y}$]

The center-of-mass system has some nice properties:



Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

The center-of-mass system has some nice properties:

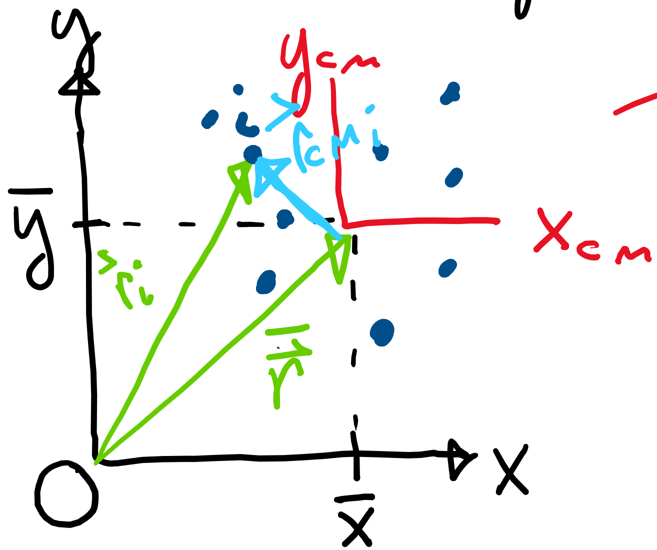


Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

* non-stop along $\vec{r}_i$
* Stop at CM origin $= \vec{r} + \vec{r}_{cm\,i}$

The center-of-mass system has some nice properties:



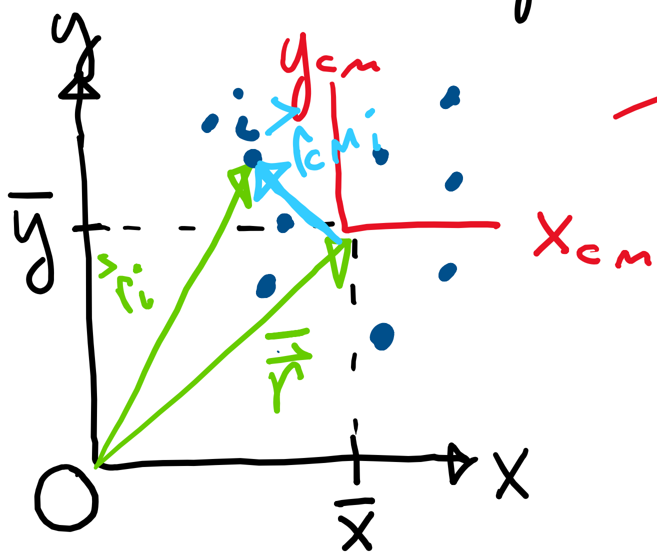
→ Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

* Non-stop along $\vec{r}_i$
* Stop at CM origin $= \vec{\bar{r}} + \vec{r}_{cm_i}$

$$\Rightarrow \quad \vec{r}_i = \vec{\bar{r}} + \vec{r}_{cm_i}$$

The center-of-mass system has some nice properties:



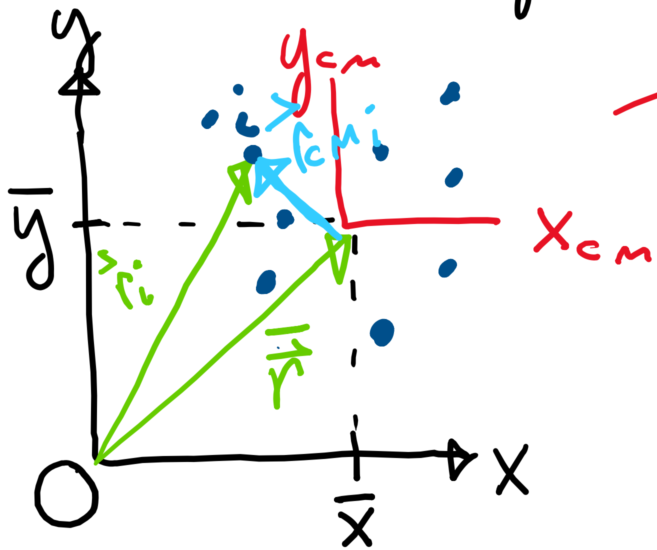
→ Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

* Non-stop along $\vec{r}_i$
* Stop at CM origin $= \vec{\bar{r}} + \vec{r}_{cm_i}$

$$\Rightarrow \quad \vec{r}_i = \vec{\bar{r}} + \vec{r}_{cm_i}$$

The center-of-mass system has some nice properties:



→ Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

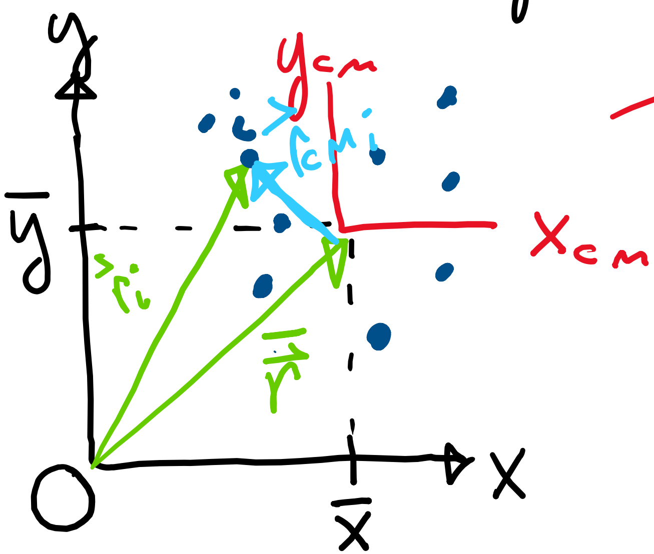
* NON-stop along $\vec{r}_i$
* Stop at CM origin $= \vec{\bar{r}} + \vec{r}_{cm_i}$

$$\Rightarrow \quad \vec{r}_i = \vec{\bar{r}} + \vec{r}_{cm_i}$$

By construction

$$\sum m_i \vec{r}_{cm_i} = \sum_i m_i (\vec{r}_i - \vec{\bar{r}})$$

The center-of-mass system has some nice properties:



Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

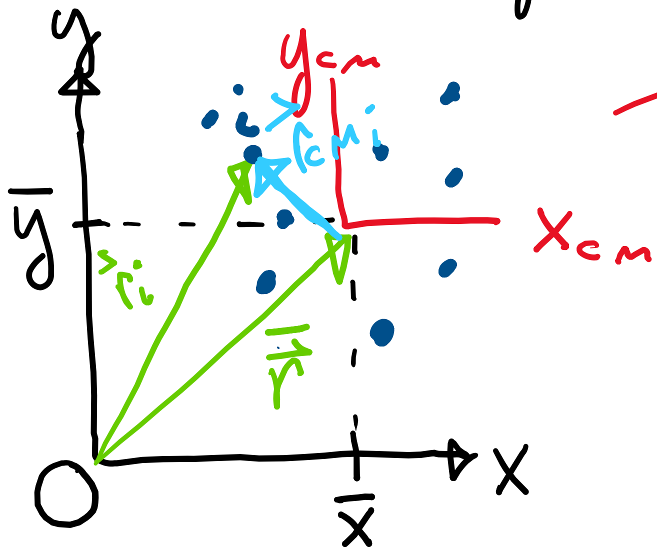
* non-stop along $\vec{r}_i$
* Stop at CM origin $= \vec{\bar{r}} + \vec{r}_{cm_i}$

$$\Rightarrow \quad \vec{r}_i = \vec{\bar{r}} + \vec{r}_{cm_i}$$

By construction

$$\sum m_i \vec{r}_{cm_i} = \sum_i m_i (\vec{r}_i - \vec{\bar{r}}) = \left(\sum_i m_i \vec{r}_i\right) - \left(\sum m_i \vec{\bar{r}}\right)$$

The center-of-mass system has some nice properties:



→ Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

* non-stop along $\vec{r}_i$
* Stop at cm origin $= \vec{\bar{r}} + \vec{r}_{cm_i}$

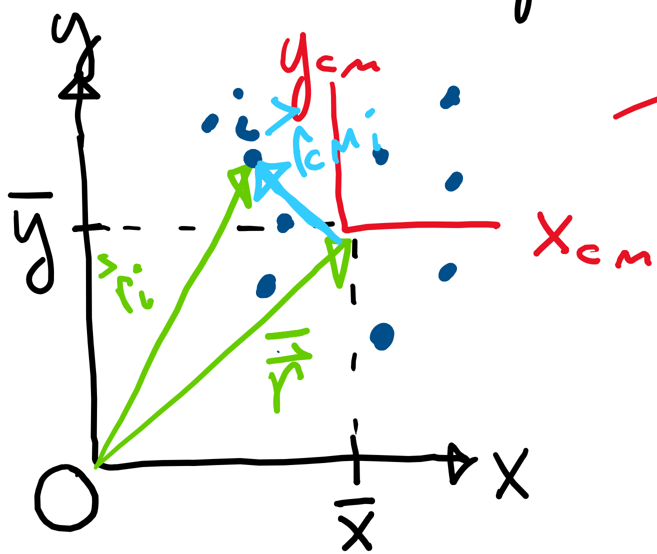
$$\Rightarrow \quad \vec{r}_i = \vec{\bar{r}} + \vec{r}_{cm_i}$$

By construction

$$\sum m_i \vec{r}_{cm_i} = \sum_i m_i (\vec{r}_i - \vec{\bar{r}}) = \left(\sum_i m_i \vec{r}_i\right) - \left(\sum m_i \vec{\bar{r}}\right) = M\vec{\bar{r}} - M\vec{\bar{r}}$$

$$= 0$$

The center-of-mass system has some nice properties:



Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

* non-stop along $\vec{r}_i$
* Stop at cm origin $= \vec{\bar{r}} + \vec{r}_{cm\,i}$

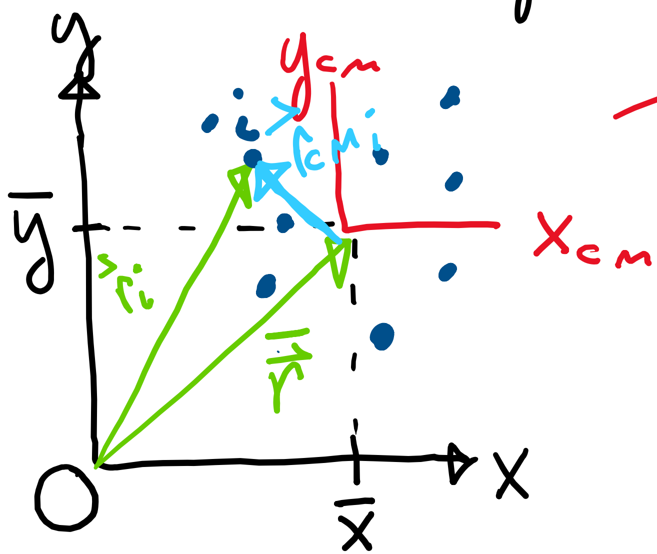
$\Rightarrow \quad \vec{r}_i = \vec{\bar{r}} + \vec{r}_{cm\,i}$

By construction

$$\sum m_i \vec{r}_{cm\,i} = \sum_i m_i (\vec{r}_i - \vec{\bar{r}}) = \Big(\sum_i m_i \vec{r}_i\Big) - \Big(\sum m_i \vec{\bar{r}}\Big) = M\vec{\bar{r}} - M\vec{\bar{r}}$$

$$= 0 \quad \text{so} \quad M\vec{\bar{r}}_{cm} = \sum m_i \vec{r}_{cm\,i} = 0$$

The center-of-mass system has some nice properties:



→ Center-of-mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

* non-stop along $\vec{r}_i$
* Stop at cm origin $= \vec{\bar{r}} + \vec{r}_{cm_i}$

$$\Rightarrow \quad \vec{r}_i = \vec{\bar{r}} + \vec{r}_{cm_i}$$

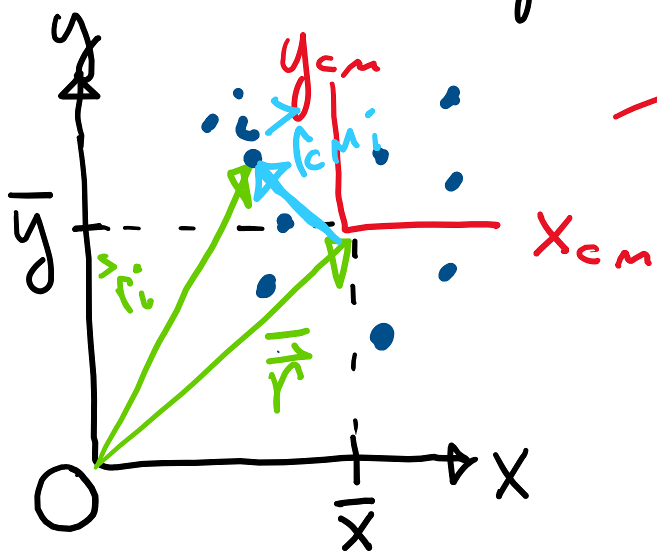
By construction

$$\sum m_i \vec{r}_{cm_i} = \sum_i m_i (\vec{r}_i - \vec{\bar{r}}) = \left(\sum_i m_i \vec{r}_i\right) - \left(\sum m_i \vec{\bar{r}}\right) = M\vec{\bar{r}} - M\vec{\bar{r}}$$

$$= \emptyset \quad \text{so} \quad M\vec{\bar{r}}_{cm} = \sum M_i \vec{r}_{cm_i} = \emptyset$$

This means that $M\vec{\bar{V}}_{cm} = \emptyset$

The center-of-mass system has some nice properties:



Center-of-Mass system [origin set at $\bar{x}, \bar{y}$]

Two ways to get from origin O to point i:

* non-stop along $\vec{r}_i$
* Stop at CM origin $= \vec{\bar{r}} + \vec{r}_{cm\,i}$

$$\Rightarrow \quad \vec{r}_i = \vec{\bar{r}} + \vec{r}_{cm\,i}$$

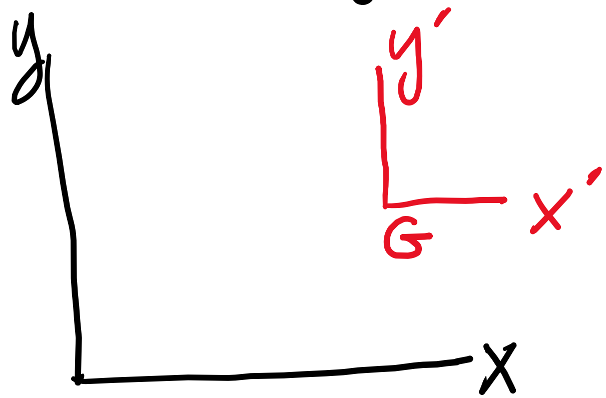
By construction

$$\sum m_i \vec{r}_{cm\,i} = \sum_i m_i (\vec{r}_i - \vec{\bar{r}}) = \left(\sum_i m_i \vec{r}_i\right) - \left(\sum m_i \vec{\bar{r}}\right) = M\vec{\bar{r}} - M\vec{\bar{r}}$$

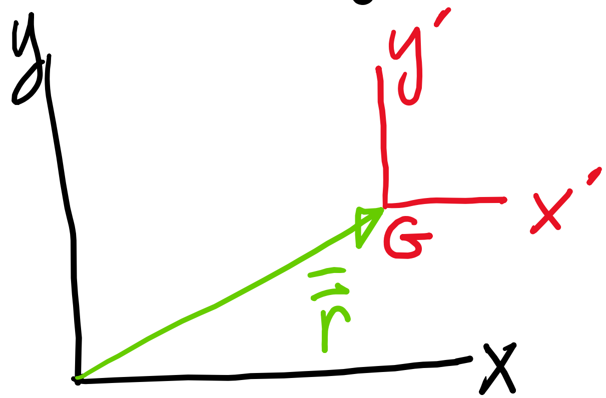
$$= \emptyset \quad \text{so} \quad M\vec{\bar{r}}_{cm} = \sum m_i \vec{r}_{cm\,i} = \emptyset$$

This means that $M\vec{\bar{v}}_{cm} = \emptyset \Rightarrow$ Center of mass system is also center of momentum system

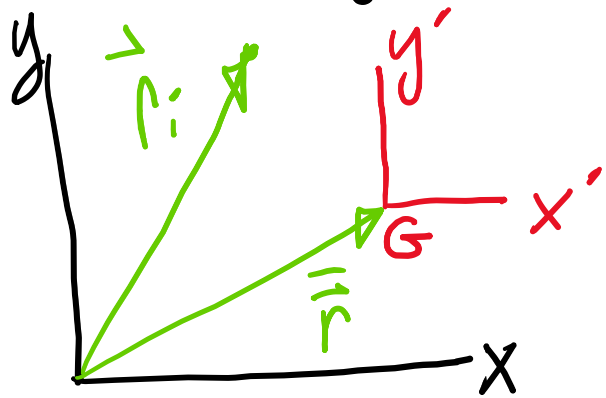
We will define reference frame G as the center-of-mass frame



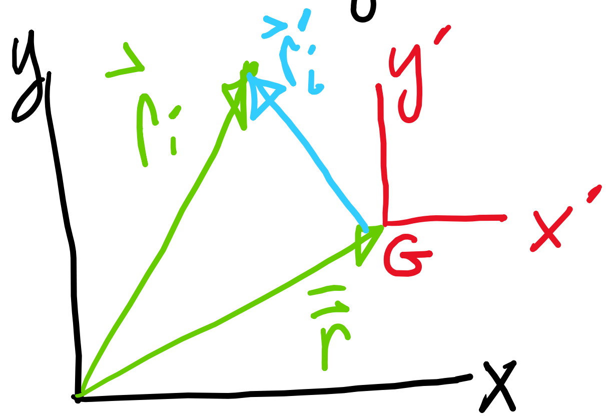
We will define reference frame G as the center-of-mass frame



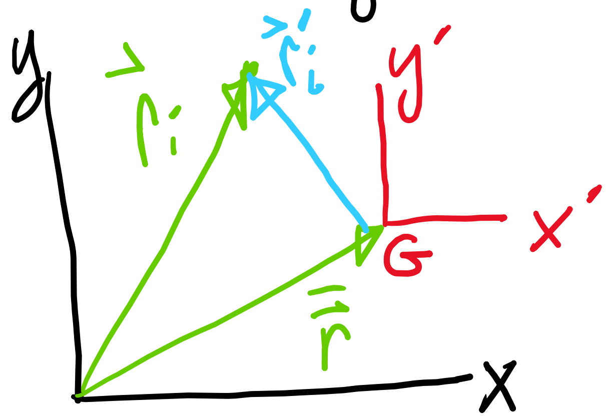
We will define reference frame G as the center-of-mass frame



We will define reference frame G as the center-of-mass frame

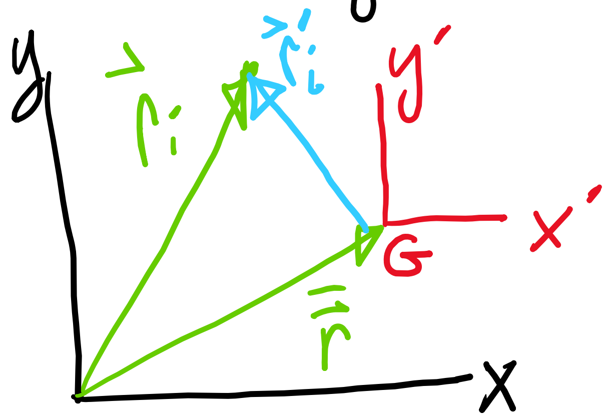


We will define reference frame G as the center-of-mass frame. For this frame



$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i'$$

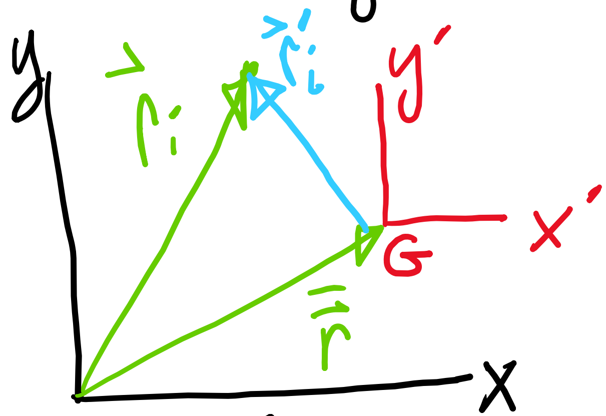
We will define reference frame G as the center-of-mass frame. For this frame



$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i' \times m_i \vec{v}_i' + \sum_i \vec{r}_i' \times m_i \vec{a}_i'$$

We will define reference frame G as the center-of-mass frame. For this frame

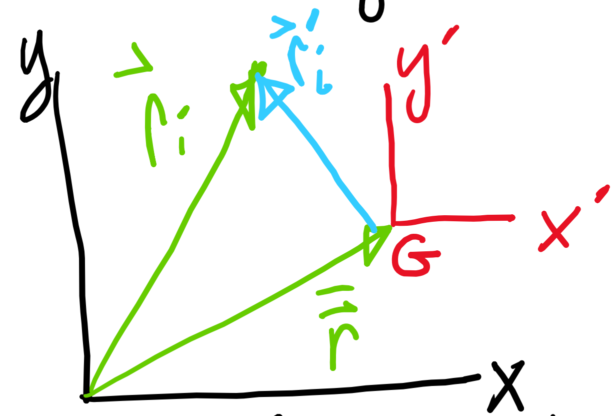


$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i' \times m_i \vec{v}_i' + \sum_i \vec{r}_i' \times m_i \vec{a}_i'$$

But $\dot{\vec{r}}_i' = \vec{v}_i'$

We will define reference frame G as the center-of-mass frame. For this frame

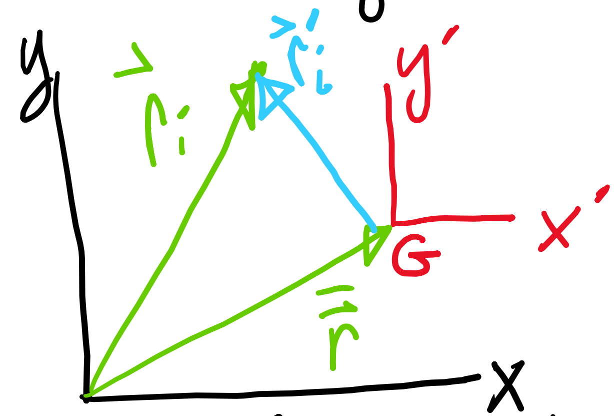


$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i' \times m_i \vec{v}_i' + \sum_i \vec{r}_i' \times m_i \vec{a}_i'$$

But $\dot{\vec{r}}_i' = \vec{v}_i'$ & $\vec{v}_i' \times \vec{v}_i' = 0$

We will define reference frame G as the center-of-mass frame. For this frame

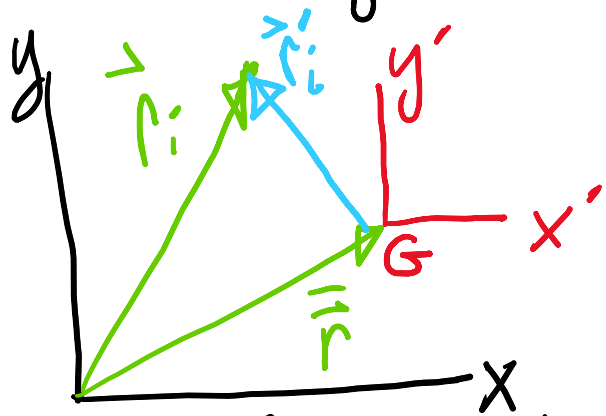


$$\vec{H}_G = \sum_i \vec{r}_i{}' \times m_i \vec{v}_i{}' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i{}' \times m_i \vec{v}_i{}' + \sum_i \vec{r}_i{}' \times m_i \vec{a}_i{}'$$

But $\dot{\vec{r}}_i{}' = \vec{v}_i{}'$ & $\vec{v}_i{}' \times \vec{v}_i{}' = 0$ so $\dot{\vec{H}}_G = \sum_i \vec{r}_i{}' \times m_i \vec{a}_i{}'$

We will define reference frame G as the center-of-mass frame. For this frame



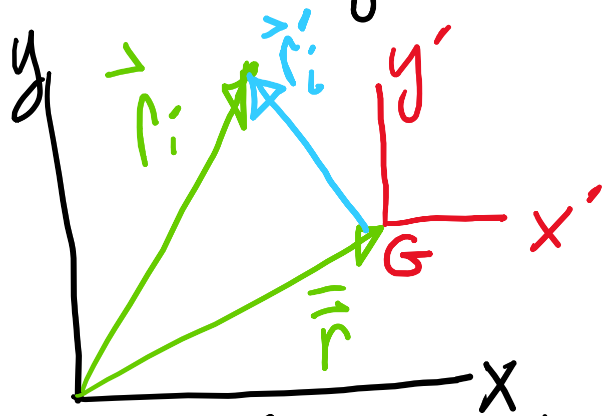
$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i' \times m_i \vec{v}_i' + \sum_i \vec{r}_i' \times m_i \vec{a}_i'$$

But $\dot{\vec{r}}_i' = \vec{v}_i'$ & $\vec{v}_i' \times \vec{v}_i' = 0$ so $\dot{\vec{H}}_G = \sum_i \vec{r}_i' \times m_i \vec{a}_i'$

But $\vec{r}_i = \bar{\vec{r}} + \vec{r}_i' \Rightarrow \vec{r}_i' = \vec{r}_i - \bar{\vec{r}} \Rightarrow \vec{a}_i' = \vec{a}_i - \bar{\vec{a}}$

We will define reference frame G as the center-of-mass frame. For this frame



$$\vec{H}_G = \sum_i \vec{r}_i{}' \times m_i \vec{v}_i{}' \Rightarrow$$

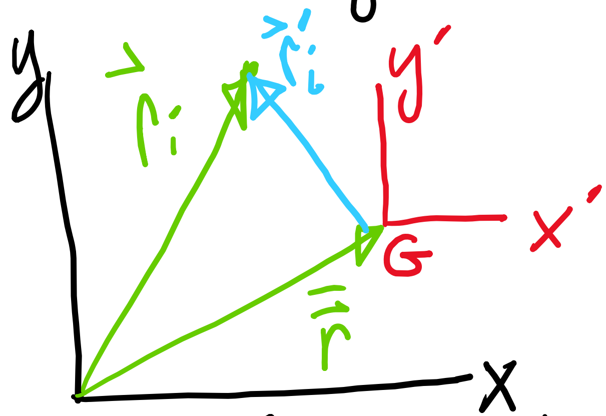
$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i{}' \times m_i \vec{v}_i{}' + \sum_i \vec{r}_i{}' \times m_i \vec{a}_i{}'$$

But $\dot{\vec{r}}_i{}' = \vec{v}_i{}'$ & $\vec{v}_i{}' \times \vec{v}_i{}' = 0$ so $\dot{\vec{H}}_G = \sum_i \vec{r}_i{}' \times m_i \vec{a}_i{}'$

But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i{}' \Rightarrow \vec{r}_i{}' = \vec{r}_i - \vec{\bar{r}} \Rightarrow \vec{a}_i{}' = \vec{a}_i - \vec{\bar{a}}$

So $\dot{\vec{H}}_G = \sum_i \vec{r}_i{}' \times m_i (\vec{a}_i - \vec{\bar{a}})$

We will define reference frame G as the center-of-mass frame. For this frame



$$\vec H_G = \sum_i \vec r_i' \times m_i \vec v_i' \Rightarrow$$

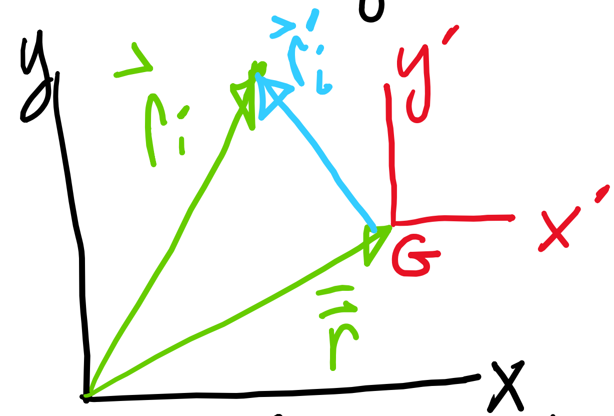
$$\frac{d}{dt}\vec H_G = \sum_i \dot{\vec r}_i' \times m_i \vec v_i' + \sum_i \vec r_i' \times m_i \vec a_i'$$

But $\dot{\vec r}_i' = \vec v_i'$ & $\vec v_i' \times \vec v_i' = \cancel{0}$ so $\dot{\vec H}_G = \sum_i \vec r_i' \times m_i \vec a_i'$

But $\vec r_i = \bar{\vec r} + \vec r_i' \Rightarrow \vec r_i' = \vec r_i - \bar{\vec r} \Rightarrow \vec a_i' = \vec a_i - \bar{\vec a}$

So $\dot{\vec H}_G = \sum_i \vec r_i' \times m_i (\vec a_i - \bar{\vec a}) = \sum_i m_i \vec r_i' \times \vec a_i - \left(\sum m_i \vec r_i'\right) \bar{\vec a}$

We will define reference frame G as the center-of-mass frame. For this frame



$$\vec{H}_G = \sum_i \vec{r}_i{}' \times m_i \vec{v}_i{}' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i{}' \times m_i \vec{v}_i{}' + \sum_i \vec{r}_i{}' \times m_i \vec{a}_i{}'$$

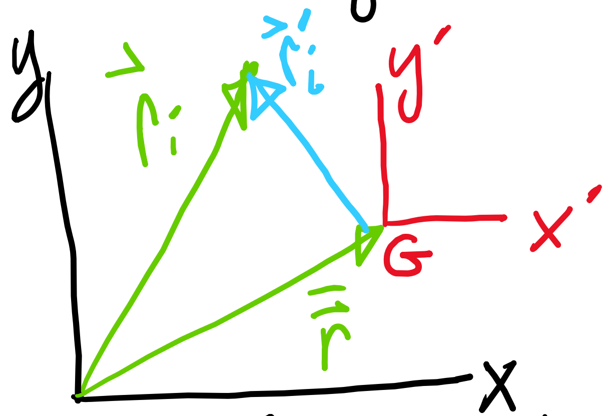
But $\dot{\vec{r}}_i{}' = \vec{v}_i{}'$ & $\vec{v}_i{}' \times \vec{v}_i{}' = 0$ so $\dot{\vec{H}}_G = \sum_i \vec{r}_i{}' \times m_i \vec{a}_i{}'$

But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i{}' \Rightarrow \vec{r}_i{}' = \vec{r}_i - \vec{\bar{r}} \Rightarrow \vec{a}_i{}' = \vec{a}_i - \vec{\bar{a}}$

So $\dot{\vec{H}}_G = \sum_i \vec{r}_i{}' \times m_i (\vec{a}_i - \vec{\bar{a}}) = \sum_i m_i \vec{r}_i{}' \times \vec{a}_i - \overbrace{\left(\sum m_i \vec{r}_i{}'\right)}^{0} \vec{\bar{a}}$

$$\Rightarrow \dot{\vec{H}}_G = \sum \vec{r}_i{}' \times m_i \vec{a}_i$$

We will define reference frame G as the center-of-mass frame. For this frame



$$\vec H_G = \sum_i \vec r_i' \times m_i \vec v_i' \Rightarrow$$

$$\frac{d}{dt}\vec H_G = \sum_i \dot{\vec r}_i' \times m_i \vec v_i' + \sum_i \vec r_i' \times m_i \vec a_i'$$

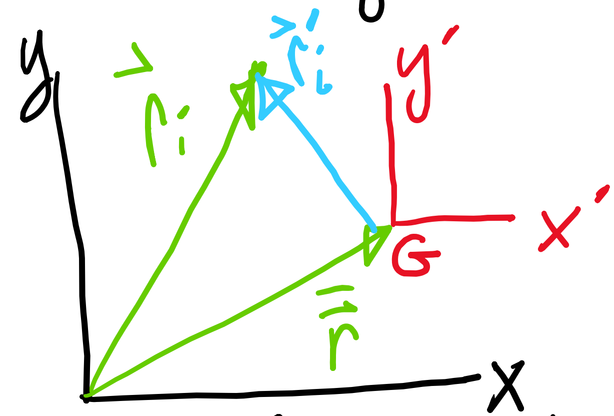
But $\dot{\vec r}_i' = \vec v_i'$ & $\vec v_i' \times \vec v_i' = 0$ so $\dot{\vec H}_G = \sum_i \vec r_i' \times m_i \vec a_i'$

But $\vec r_i = \bar{\vec r} + \vec r_i' \Rightarrow \vec r_i' = \vec r_i - \bar{\vec r} \Rightarrow \vec a_i' = \vec a_i + \bar{\vec a}$

So $\dot{\vec H}_G = \sum_i \vec r_i' \times m_i (\vec a_i + \bar{\vec a}) = \sum_i m_i \vec r_i' \times \vec a_i + \overbrace{\left(\sum m_i \vec r_i'\right)}^{0} \bar{\vec a}$

$$\Rightarrow \dot{\vec H}_G = \sum \vec r_i' \times m_i \vec a_i = \sum \vec r_i' \times \vec F_i$$

We will define reference frame G as the center-of-mass frame. For this frame



$$\vec{H}_G = \sum_i \vec{r}_i' \times m_i \vec{v}_i' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i' \times m_i \vec{v}_i' + \sum_i \vec{r}_i' \times m_i \vec{a}_i'$$

But $\dot{\vec{r}}_i' = \vec{v}_i'$ & $\vec{v}_i' \times \vec{v}_i' = 0$ so $\dot{\vec{H}}_G = \sum_i \vec{r}_i' \times m_i \vec{a}_i'$

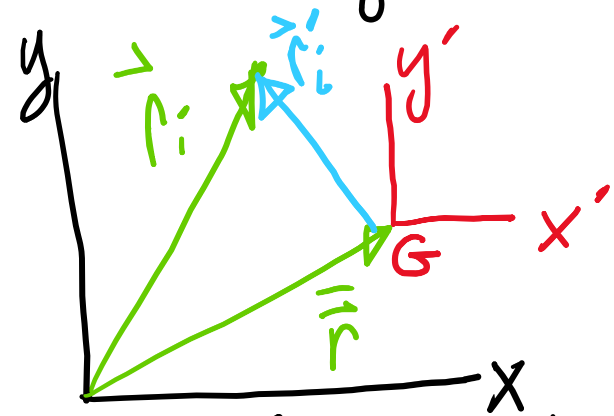
But $\vec{r}_i = \bar{\vec{r}} + \vec{r}_i' \Rightarrow \vec{r}_i' = \vec{r}_i - \bar{\vec{r}} \Rightarrow \vec{a}_i' = \vec{a}_i + \bar{\vec{a}}$

So $\dot{\vec{H}}_G = \sum_i \vec{r}_i' \times m_i(\vec{a}_i + \bar{\vec{a}}) = \sum_i m_i \vec{r}_i' \times \vec{a}_i + \overbrace{\left(\sum m_i \vec{r}_i'\right)}^{0} \bar{\vec{a}}$

$$\Rightarrow \dot{\vec{H}}_G = \sum \vec{r}_i' \times m_i \vec{a}_i = \sum \vec{r}_i' \times \vec{F}_i \Rightarrow$$

$$\sum \vec{M}_G = \dot{\vec{H}}_G$$

We will define reference frame G as the center-of-mass frame. For this frame



$$\vec{H}_G = \sum_i \vec{r}_i{}' \times m_i \vec{v}_i{}' \Rightarrow$$

$$\frac{d}{dt}\vec{H}_G = \sum_i \dot{\vec{r}}_i{}' \times m_i \vec{v}_i{}' + \sum_i \vec{r}_i{}' \times m_i \vec{a}_i{}'$$

But $\dot{\vec{r}}_i{}' = \vec{v}_i{}'$ & $\vec{v}_i{}' \times \vec{v}_i{}' = 0$ so $\dot{\vec{H}}_G = \sum_i \vec{r}_i{}' \times m_i \vec{a}_i{}'$

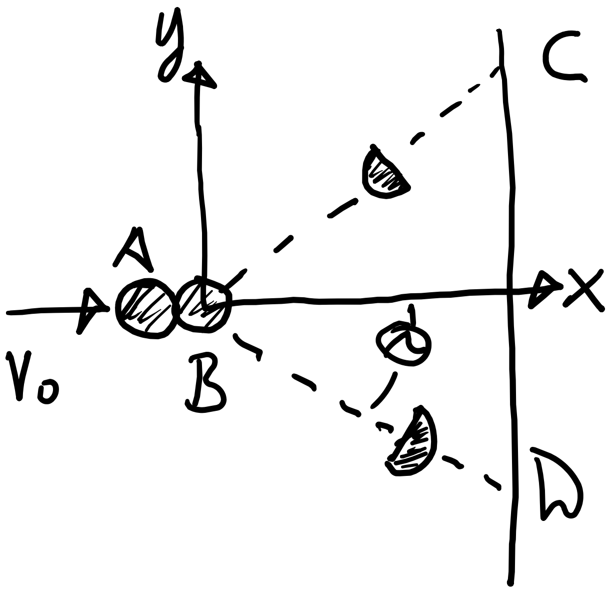
But $\vec{r}_i = \vec{\bar{r}} + \vec{r}_i{}' \Rightarrow \vec{r}_i{}' = \vec{r}_i - \vec{\bar{r}} \Rightarrow \vec{a}_i{}' = \vec{a}_i + \vec{\bar{a}}$

So $\dot{\vec{H}}_G = \sum_i \vec{r}_i{}' \times m_i (\vec{a}_i + \vec{\bar{a}}) = \sum_i m_i \vec{r}_i{}' \times \vec{a}_i + \overbrace{\left(\sum m_i \vec{r}_i{}'\right)}^{0} \vec{\bar{a}}$

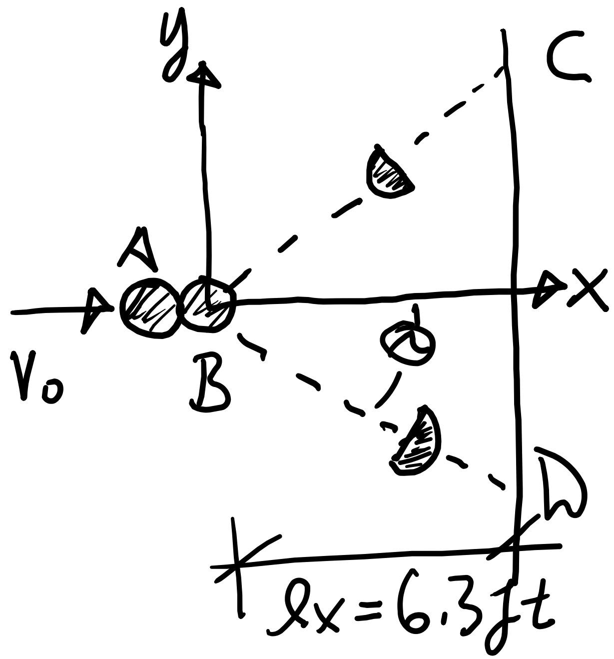
$$\Rightarrow \dot{\vec{H}}_G = \sum \vec{r}_i{}' \times m_i \vec{a}_i = \sum \vec{r}_i{}' \times \vec{F}_i \Rightarrow$$

$$\sum \vec{M}_G = \dot{\vec{H}}_G$$

Example problem: No friction, Horizontal plane [don't worry about gravity]. Mass A hits mass B ($M_A = M_B = M$). Mass B breaks into 2-pieces, each of mass $= \frac{1}{2}M$.

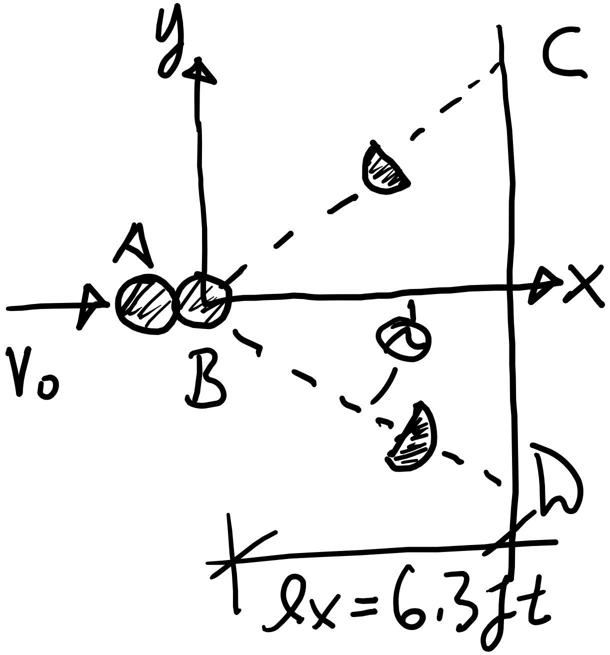


Example problem: No friction, Horizontal plane [don't worry about gravity]. Mass A hits mass B ($M_A = M_B = M$). Mass B breaks into 2-pieces, each of mass $= \frac{1}{2}M$. Given



$V_0 = 16\ \mathrm{ft/s}$, hits C at $\Delta t_C = 0.7\,\mathrm{s}$, $\vec{V}_{AF} = V_{AF}\,\hat{i}$, hits D at $\Delta t_D = 0.9\,\mathrm{s}$.

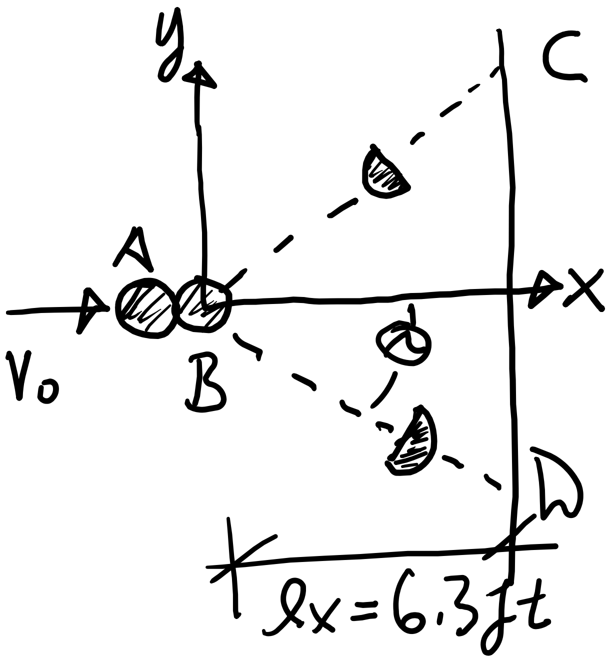
Example problem: No friction, Horizontal plane [don't worry about gravity]. Mass A hits mass B ($M_A = M_B = M$). Mass B breaks into 2-pieces, each of mass $= \frac{1}{2}M$. Given



$V_0 = 16 \, ft/s$, hits C at $\Delta t_C = 0.7 s$, $\vec{V}_{AF} = V_{AF}\,\hat{i}$, hits D at $\Delta t_D = 0.9 s$.

<u>Find V_{AF}</u>:

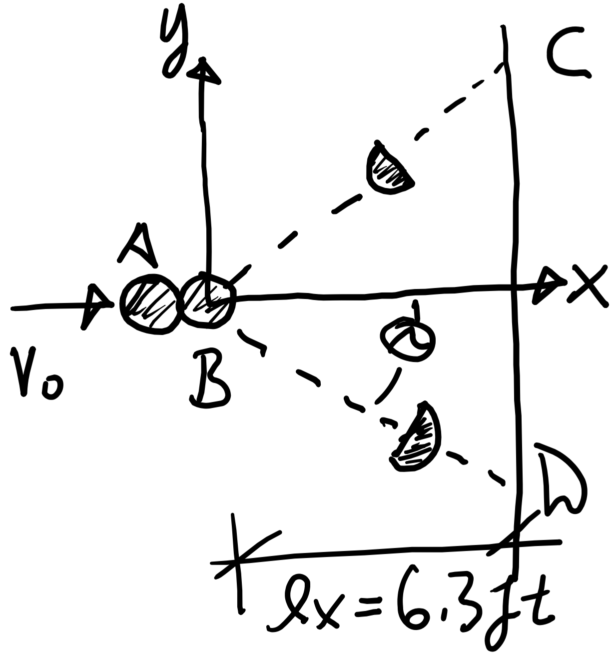
Example problem: No friction, Horizontal plane [don't worry about gravity]. Mass A hits mass B ($M_A = M_B = M$). Mass B breaks into 2-pieces, each of mass $= \frac{1}{2}M$. Given



$V_0 = 16\,ft/s$, hits C at $\Delta t_C = 0.7\,s$, $\vec{V}_{AF} = V_{AF}\,\hat{i}$, hits D at $\Delta t_D = 0.9\,s$.

<u>Find V_{AF}</u>: No external forces so $\vec{L}_I = \vec{L}_F$

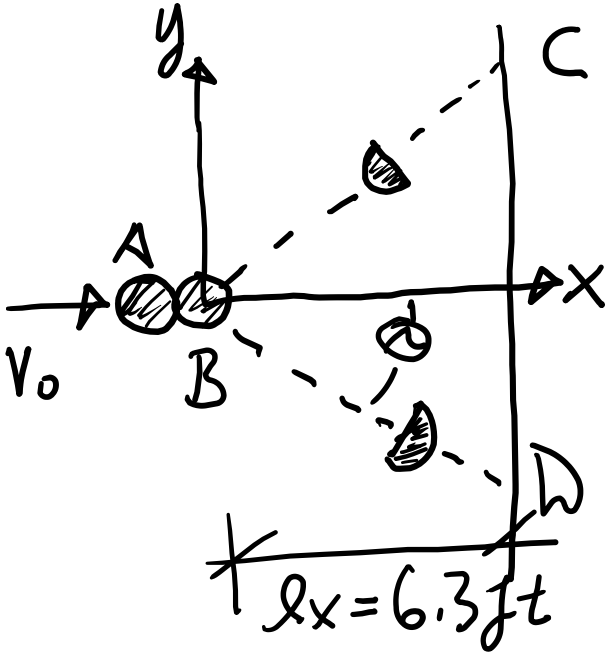
Example problem: No friction, Horizontal plane [don't worry about gravity]. Mass A hits mass B ($M_A = M_B = M$). Mass B breaks into 2-pieces, each of mass $= \frac{1}{2}M$. Given



$V_0 = 16\,ft/s$, hits C at $\Delta t_C = 0.7\,s$, $\vec{V}_{AF} = V_{AF}\,\hat{i}$, hits D at $\Delta t_D = 0.9\,s$.

<u>Find V_{AF}</u>: No external forces so $\vec{L}_I = \vec{L}_F$ & $\vec{L}_I = M\vec{V}_{AI} = MV_0\,\hat{i}$

Example problem: No friction, Horizontal plane [don't worry about gravity]. Mass A hits mass B ($M_A = M_B = M$). Mass B breaks into 2-pieces, each of mass $= \frac{1}{2}M$. Given



$V_0 = 16\,ft/s$, hits C at $\Delta t_C = 0.7\,s$, $\vec{V}_{AF} = V_{AF}\,\hat{i}$, hits D at $\Delta t_D = 0.9\,s$.

<u>Find V_{AF}</u>: No external forces

so $\vec{L}_I = \vec{L}_F$ & $\vec{L}_I = M\vec{V}_{AI} = M V_0\,\hat{i}$

& $\vec{L}_F = \frac{1}{2}M\vec{V}_C + \frac{1}{2}M\vec{V}_D + M\vec{V}_A$

So $\hat{i} M V_0 = \frac{1}{2} M \left(V_{cx} \hat{i} + V_{cy} \hat{j} + V_{Dx} \hat{i} + V_{Dy} \hat{j} \right) + M V_{AF} \hat{i}$

$\int O \quad MV_0 = \frac{1}{2}M\left(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}\right) + MV_{AF}\hat{i}$

$\Rightarrow \quad MV_o = \frac{1}{2}M(V_{cx} + V_{cy}) + MV_{AF}$

$\circlearrowleft O \quad MV_0 = \frac{1}{2}M\left(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}\right) + MV_{AF}\hat{i}$

$\Rightarrow MV_o = \frac{1}{2}M(V_{cx} + V_{Dx}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}(V_{cx} + V_{Dx}) + V_A$

ASU

$\circlearrowleft O \quad MV_0 = \frac{1}{2}M\left(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}\right) + MV_{AF}\hat{i}$

$\Rightarrow MV_o = \frac{1}{2}M(V_{cx} + V_{DX}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}\left(V_{cx} + V_{Dx}\right) + V_A$

$\Rightarrow V_A = V_0 - \frac{1}{2}\left(V_{cx} + V_{Dx}\right)$

So $MV_0 = \frac{1}{2}M(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}) + MV_{AF}\hat{i}$

$\Rightarrow MV_0 = \frac{1}{2}M(V_{cx} + V_{DX}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}(V_{cx} + V_{Dx}) + V_A$

$\Rightarrow V_A = V_0 - \frac{1}{2}(V_{cx} + V_{Dx})$, where $V_0 = 16\ ft/s$. We just need V_{cx} & V_{Dx}: Use kinematics

So $MV_0 = \frac{1}{2}M(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}) + MV_{AF}\hat{i}$

$\Rightarrow MV_0 = \frac{1}{2}M(V_{cx} + V_{Dx}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}(V_{cx} + V_{Dx}) + V_A$

$\Rightarrow V_A = V_0 - \frac{1}{2}(V_{cx} + V_{Dx})$, where $V_0 = 16\ ft/s$. We just need V_{cx} & V_{Dx}: Use kinematics $\Rightarrow$

$\vec{V}_c \Delta t = \vec{r}_c$

So $MV_0 = \frac{1}{2}M(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}) + MV_{AF}\hat{i}$

$\Rightarrow MV_0 = \frac{1}{2}M(V_{cx} + V_{Dx}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}(V_{cx} + V_{Dx}) + V_A$

$\Rightarrow V_A = V_0 - \frac{1}{2}(V_{cx} + V_{Dx})$, where $V_0 = 16\ ft/s$. We just need V_{cx} & V_{Dx}: Use kinematics $\Rightarrow$

$\vec{V}_c \Delta t = \vec{r}_c \Rightarrow (V_{cx}\hat{i} + V_{cy}\hat{j})\Delta t_c = r_{cx}\hat{i} + r_{cy}\hat{j}$

So $MV_0 = \frac{1}{2}M(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}) + MV_{AF}\hat{i}$

$\Rightarrow MV_0 = \frac{1}{2}M(V_{cx} + V_{Dx}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}(V_{cx} + V_{Dx}) + V_A$

$\Rightarrow V_A = V_0 - \frac{1}{2}(V_{cx} + V_{Dx})$, where $V_0 = 16\ ft/s$. We just need V_{cx} & V_{Dx}: Use kinematics $\Rightarrow$

$\vec{V}_c \Delta t = \vec{r}_c \Rightarrow (V_{cx}\hat{i} + V_{cy}\hat{j})\Delta t_c = r_{cx}\hat{i} + r_{cy}\hat{j} \Rightarrow$

$V_{cx}\Delta t_c = r_{cx}$

So $MV_0 = \frac{1}{2}M\left(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}\right) + MV_{AF}\hat{i}$

$\Rightarrow MV_0 = \frac{1}{2}M(V_{cx} + V_{Dx}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}(V_{cx} + V_{Dx}) + V_A$

$\Rightarrow V_A = V_0 - \frac{1}{2}(V_{cx} + V_{Dx})$, where $V_0 = 16\,ft/s$. We just need V_{cx} & V_{Dx}: Use Kinematics $\Rightarrow$

$\vec{V}_c \Delta t = \vec{r}_c \Rightarrow (V_{cx}\hat{i} + V_{cy}\hat{j})\Delta t_c = r_{cx}\hat{i} + r_{cy}\hat{j} \Rightarrow$

$V_{cx}\Delta t_c = r_{cx}$ <u>But</u> $r_{cx} = \ell_x = 6.3\,ft \Rightarrow V_{cx} = \frac{\ell_x}{\Delta t_c}$

So $MV_0 = \frac{1}{2}M(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}) + MV_{AF}\hat{i}$

$\Rightarrow MV_0 = \frac{1}{2}M(V_{cx} + V_{Dx}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}(V_{cx} + V_{Dx}) + V_A$

$\Rightarrow V_A = V_0 - \frac{1}{2}(V_{cx} + V_{Dx})$, where $V_0 = 16\ ft/s$. We just need V_{cx} & V_{Dx}: Use Kinematics $\Rightarrow$

$\vec{V}_c \Delta t = \vec{r}_c \Rightarrow (V_{cx}\hat{i} + V_{cy}\hat{j})\Delta t_c = r_{cx}\hat{i} + r_{cy}\hat{j} \Rightarrow$

$V_{cx}\Delta t_c = r_{cx}$ <u>But</u> $r_{cx} = l_x = 6.3\ ft \Rightarrow V_{cx} = \frac{l_x}{\Delta t_c}$

Similarly $V_{Dx} = l_x / \Delta t_D$

So $MV_0 = \frac{1}{2}M(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}) + MV_{AF}\hat{i}$

$\Rightarrow MV_0 = \frac{1}{2}M(V_{cx} + V_{Dx}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}(V_{cx} + V_{Dx}) + V_A$

$\Rightarrow V_A = V_0 - \frac{1}{2}(V_{cx} + V_{Dx})$, where $V_0 = 16\,ft/s$. We just need V_{cx} & V_{Dx} : Use kinematics $\Rightarrow$

$\vec{V}_c \Delta t = \vec{r}_c \Rightarrow (V_{cx}\hat{i} + V_{cy}\hat{j})\Delta t_c = r_{cx}\hat{i} + r_{cy}\hat{j} \Rightarrow$

$V_{cx}\Delta t_c = r_{cx}$ But $r_{cx} = l_x = 6.3\,ft \Rightarrow V_{cx} = \frac{l_x}{\Delta t_c}$

Similarly $V_{Dx} = l_x/\Delta t_D$ Now

$V_A = V_0 - \frac{1}{2}l_x\left(\frac{1}{\Delta t_c} + \frac{1}{\Delta t_D}\right)$

So $MV_0 = \frac{1}{2}M(V_{cx}\hat{i} + V_{cy}\hat{j} + V_{Dx}\hat{i} + V_{Dy}\hat{j}) + MV_{AF}\hat{i}$

$\Rightarrow MV_0 = \frac{1}{2}M(V_{cx}+V_{Dx}) + MV_{AF} \Rightarrow V_0 = \frac{1}{2}(V_{cx}+V_{Dx}) + V_A$

$\Rightarrow V_A = V_0 - \frac{1}{2}(V_{cx}+V_{Dx})$, where $V_0 = 16\,ft/s$. We just need V_{cx} & V_{Dx}: Use kinematics $\Rightarrow$

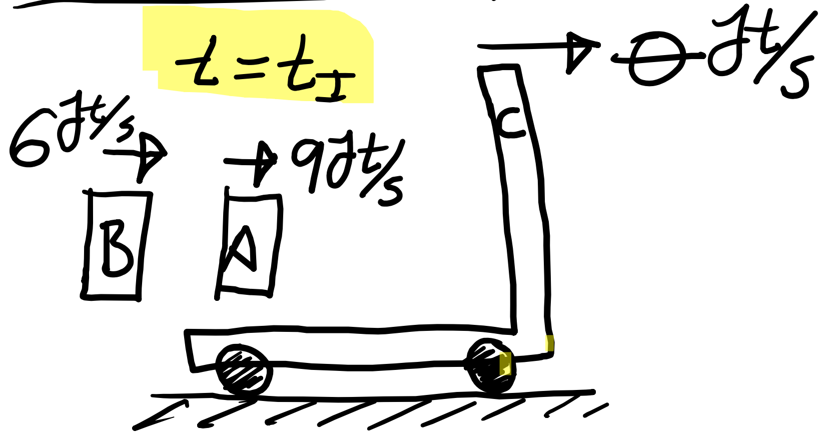
$\vec{V}_c \Delta t = \vec{r}_c \Rightarrow (V_{cx}\hat{i} + V_{cy}\hat{j})\Delta t_c = r_{cx}\hat{i} + r_{cy}\hat{j} \Rightarrow$

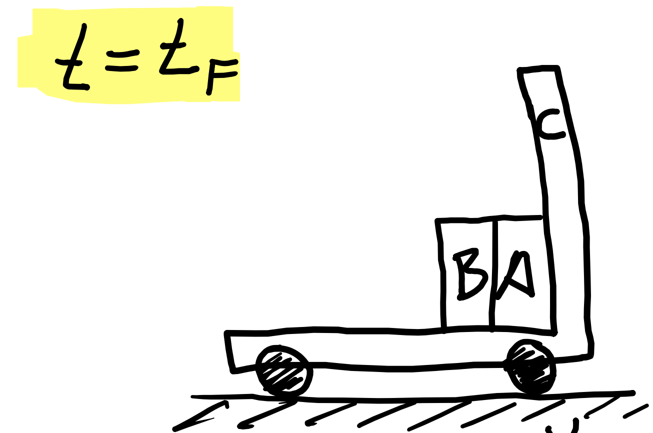
$V_{cx}\Delta t_c = r_{cx}$ <u>But</u> $r_{cx} = l_x = 6.3\,ft \Rightarrow V_{cx} = \frac{l_x}{\Delta t_c}$

Similarly $V_{Dx} = l_x/\Delta t_D$ Now

$$\boxed{\begin{aligned} V_A &= V_0 - \tfrac{1}{2}l_x\left(\frac{1}{\Delta t_c} + \frac{1}{\Delta t_D}\right) = \left[16 - \left(\frac{6.3}{2}\right)\left(\frac{1}{0.7} + \frac{1}{0.9}\right)\right] ft/s \\ &= \left[16 - \left(\frac{6.3}{2}\right)\left(\frac{0.9+0.7}{0.63}\right)\right] ft/s = \left[16 - (5)(1.6)\right] ft/s = 8\,ft/s \end{aligned}}$$

Another example: Two masses thrown onto carrier



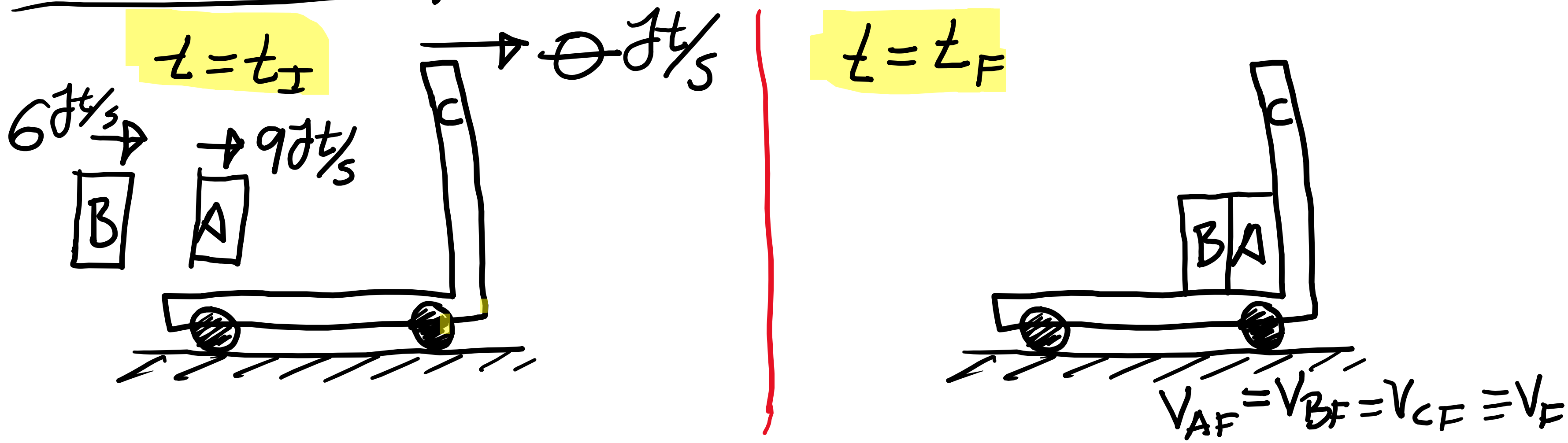


$V_{AF} = V_{BF} = V_{CF} \equiv V_F$

$W_A = 30\,lb, \quad W_B = 40\,lb, \quad W_C = 50\,lb$

Find V_F:

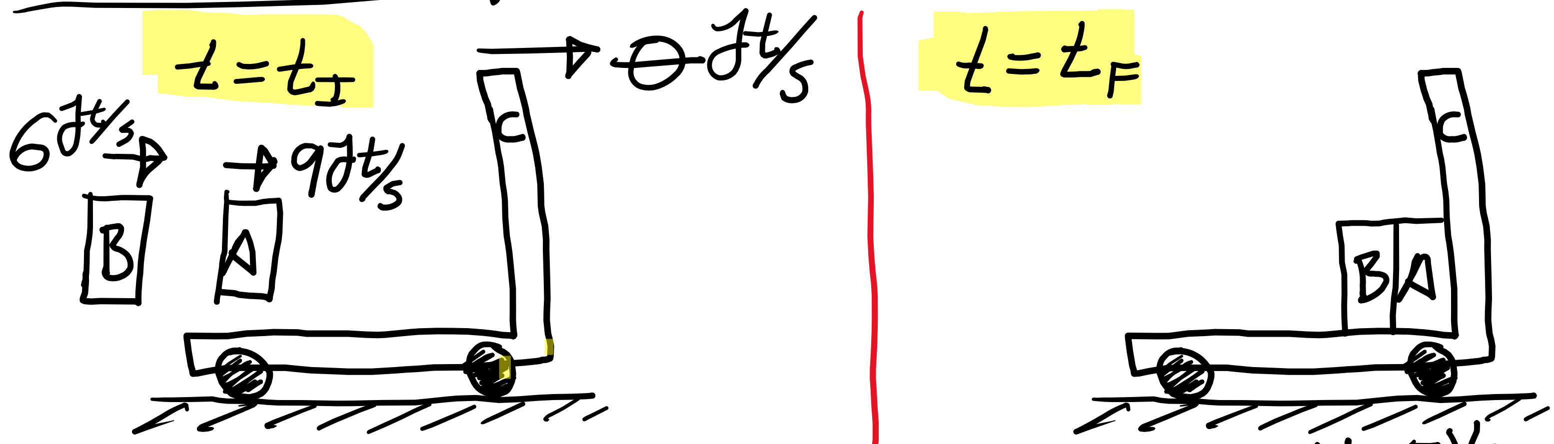
Another example: Two masses thrown onto carrier



$$W_A = 30\,lb, \quad W_B = 40\,lb, \quad W_c = 50\,lb$$

Find v_F: Conservation of momentum $\vec{L}_I = \vec{L}_F$

Another example: Two masses thrown onto carrier



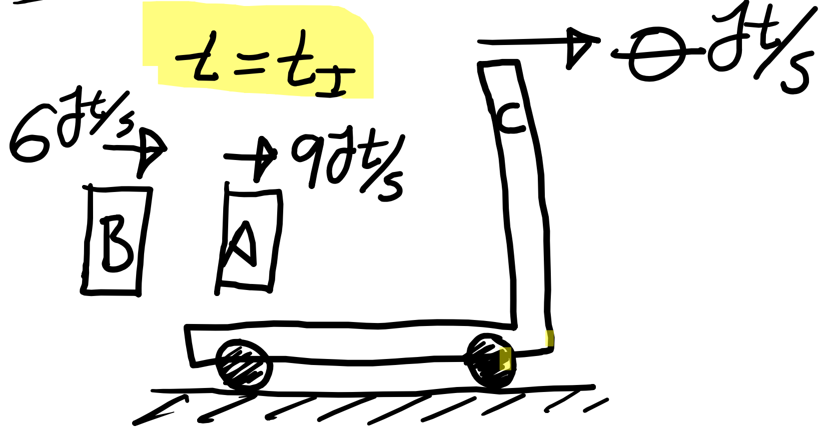
$V_{AF} = V_{BF} = V_{CF} = V_F$

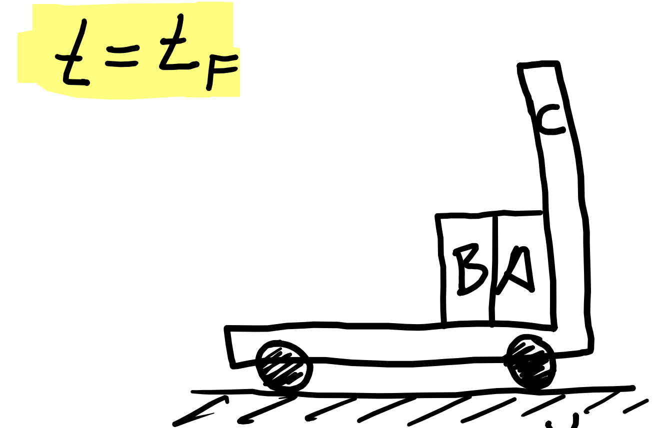
$W_A = 30\,lb, \; W_B = 40\,lb, \; W_c = 50\,lb$

Find V_F: Conservation of momentum $\vec{L}_I = \vec{L}_F$

$\Rightarrow \quad M_A V_{AI} + M_B V_{BI} + M_c V_{CI} = M_A V_{AF} + M_B V_{BF} + M_c V_{CF}$

Another example: Two masses thrown onto carrier





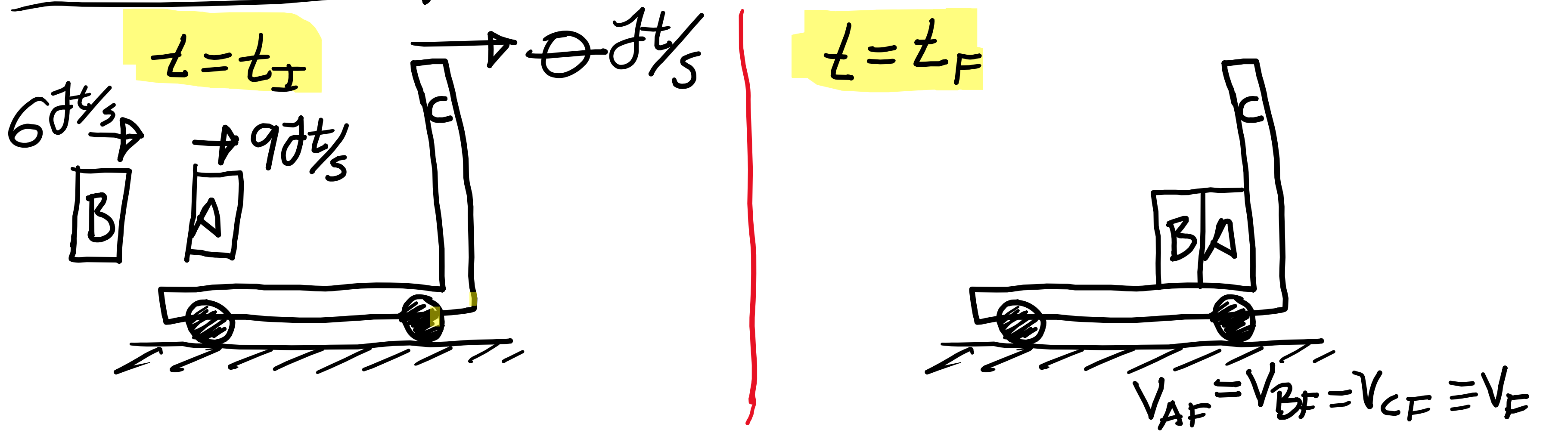
$V_{AF} = V_{BF} = V_{CF} = V_F$

$$W_A = 30\,lb,\quad W_B = 40\,lb,\quad W_C = 50\,lb$$

Find V_F: Conservation of momentum $\vec{L}_I = \vec{L}_F$

$$\Rightarrow M_A V_{AI} + M_B V_{BI} + M_C \cancelto{0}{V_{CI}} = M_A V_{AF} + M_B V_{BF} + M_C V_{CF}$$

Another example: Two masses thrown onto carrier



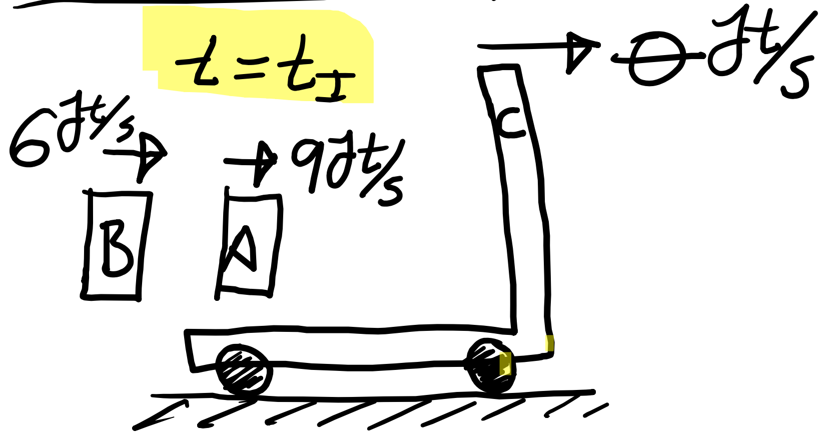
$$W_A = 30\,lb, \quad W_B = 40\,lb, \quad W_C = 50\,lb$$

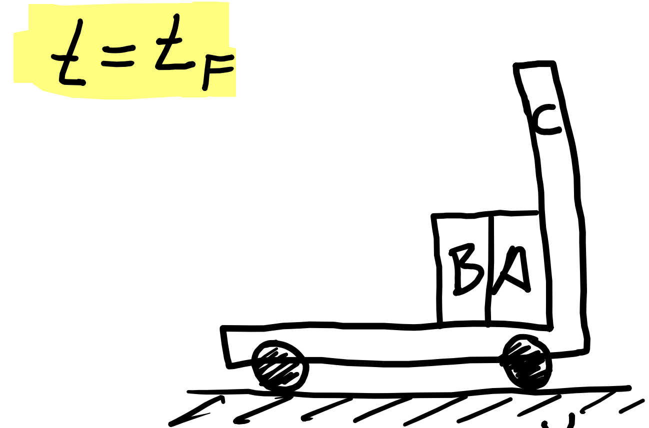
Find V_F: Conservation of momentum $\vec{L}_I = \vec{L}_F$

$$\Rightarrow \quad M_A V_{AI} + M_B V_{BI} + \cancelto{0}{M_C V_{CI}} = M_A V_{AF} + M_B V_{BF} + M_C V_{CF}$$

$$\Rightarrow \quad M_A V_{AI} + M_B V_{BI} = (M_A + M_B + M_C) V_F$$

Another example: Two masses thrown onto carrier





$V_{AF} = V_{BF} = V_{CF} = V_F$

$$W_A = 30\,lb,\quad W_B = 40\,lb,\quad W_c = 50\,lb$$

Find V_F: Conservation of momentum $\vec{L}_I = \vec{L}_F$

$$\Rightarrow \quad M_A V_{AI} + M_B V_{BI} + M_c V_{CI} = M_A V_{AF} + M_B V_{BF} + M_c V_{CF}$$

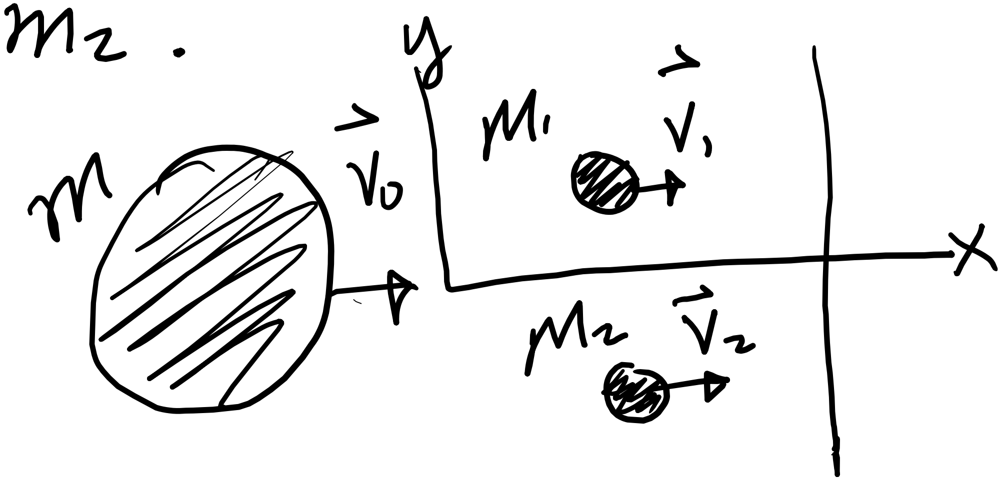
$V_{CI} = 0$

$$\Rightarrow \quad M_A V_{AI} + M_B V_{BI} = (M_A + M_B + M_c) V_F \quad \Rightarrow$$

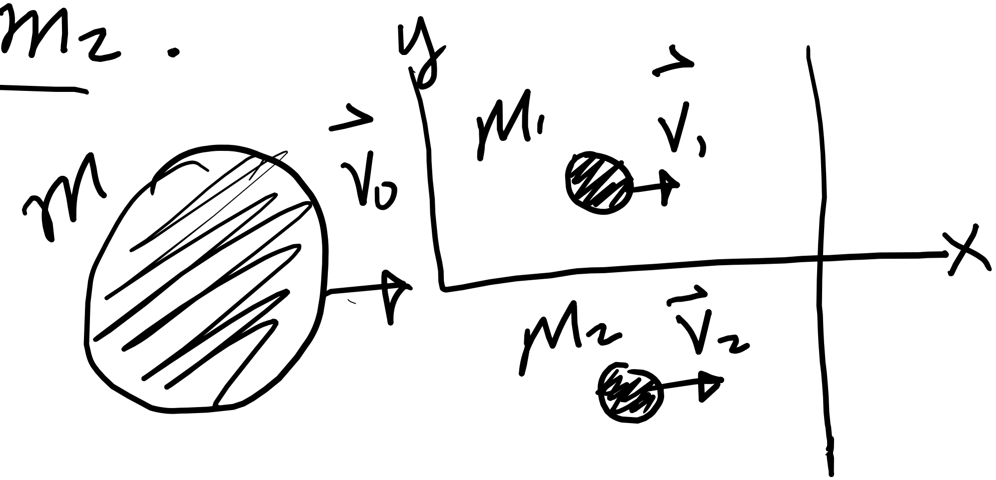
$$V_F = \left(\frac{M_A V_{AI} + M_B V_{BI}}{M_A + M_B + M_c}\right)\left(\frac{g}{g}\right) = \frac{W_A V_{AI} + W_B V_{BF}}{W_A + W_B + W_c} = \left(\frac{30*9 + 40*6}{30+40+50}\right) ft/s$$

$$\Rightarrow \quad V_F = 4.25\ ft/s$$

Mass m with velocity $\vec{v}_0$ breaks into two objects, one with mass m_1 & the other with mass m_2.



Mass m with velocity $\vec{v}_0$ breaks into two objects, one with mass m_1 & the other with mass m_2.



Given: $M = M_1 + M_2$ &

$M_1 = 6\text{Kg}$, $M_2 = 4\text{Kg}$ &

$\vec{V}_0 = 10\frac{m}{s}\hat{i}$, $\vec{V}_1 = 6\frac{m}{s}\hat{i}$,

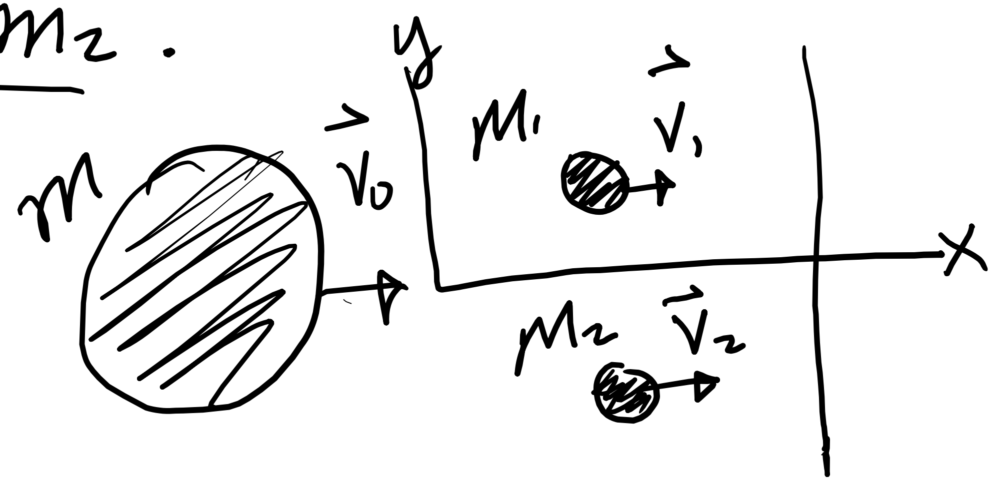
m_1 hits wall at $y_1 = \frac{1}{4}m$ & m_2 hits wall at $y_2 = -\frac{1}{4}m$.

Mass m with velocity $\vec{v}_0$ breaks into two objects, one with mass m_1 & the other with mass m_2.

Given: $M = M_1 + M_2$ &

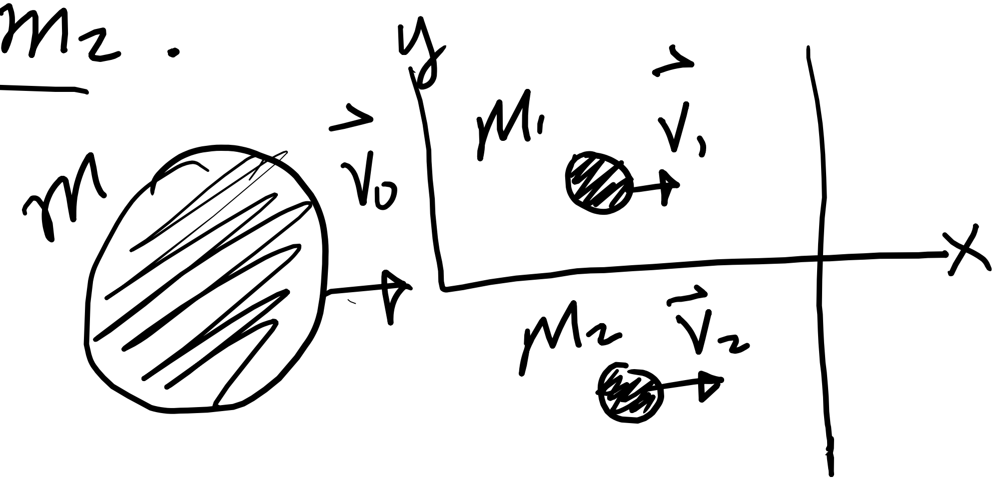
$M_1 = 6\,\text{kg}$, $M_2 = 4\,\text{kg}$ &

$\vec{V}_0 = 10\frac{m}{s}\hat{i}$, $\vec{V}_1 = 6\frac{m}{s}\hat{i}$,



m_1 hits wall at $y_1 = \frac{1}{4}m$ & m_2 hits wall at $y_2 = -\frac{1}{4}m$. Find the angular momentum about the center-of-mass of the original object.

Mass m with velocity $\vec{v}_0$ breaks into two objects, one with mass m_1 & the other with mass m_2.



Given: $M = M_1 + M_2$ &

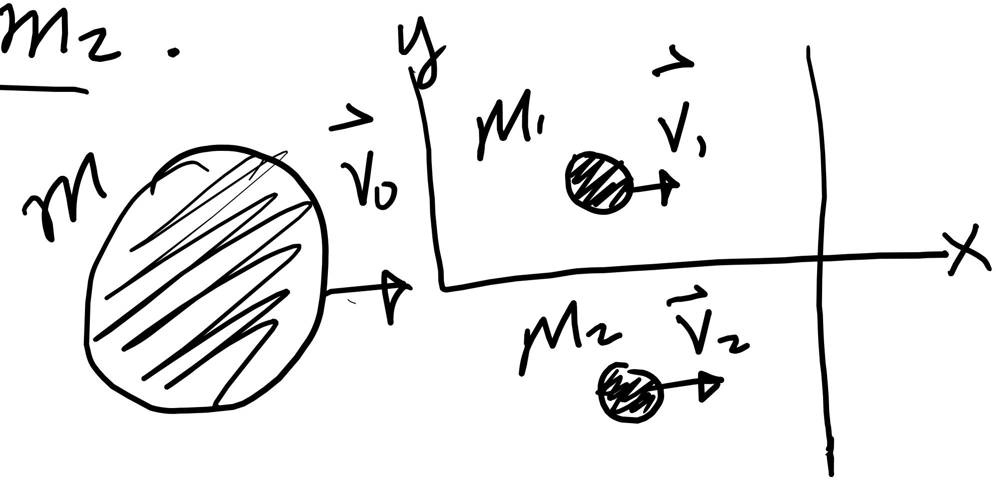
$M_1 = 6\,kg$, $M_2 = 4\,kg$ &

$\vec{V}_0 = 10\frac{m}{s}\hat{i}$, $\vec{V}_1 = 6\frac{m}{s}\hat{i}$,

m_1 hits wall at $y_1 = \frac{1}{4}m$ & m_2 hits wall at $y_2 = -\frac{1}{4}m$. Find the angular momentum about the center-of-mass of the original object.

We will use conservation of angular momentum & conservation of linear momentum.

Mass m with velocity $\vec{v}_0$ breaks into two objects, one with mass m_1 & the other with mass m_2.



Given: $M = M_1 + M_2$ &

$M_1 = 6\,kg$, $M_2 = 4\,kg$ &

$\vec{v}_0 = 10\frac{m}{s}\hat{i}$, $\vec{v}_1 = 6\frac{m}{s}\hat{i}$,

m_1 hits wall at $y_1 = \frac{1}{4}m$ & m_2 hits wall at $y_2 = -\frac{1}{4}m$. Find the angular momentum about the center-of-mass of the original object.

We will use conservation of angular momentum & conservation of linear momentum.

$$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$$

$$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2 \,, \quad \text{where} \quad \vec{L}_1 = m_1\vec{v}_1$$

$$\& \quad \vec{L}_2 = m_2\vec{v}_2 \,.$$

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1 + m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1+m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$$\Rightarrow \quad \vec{L}_2 = (m_1+m_2)\vec{v}_0 - m_1\vec{v}_1$$

ASU

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1+m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$$\Rightarrow \quad \vec{L}_2 = (m_1+m_2)\vec{v}_0 - m_1\vec{v}_1 = (10\,kg)10\tfrac{m}{s}\hat{i} - (6\,kg)6\tfrac{m}{s}\hat{i}$$

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1+m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$$\Rightarrow \quad \vec{L}_2 = (m_1+m_2)\vec{v}_0 - m_1\vec{v}_1 = (10\,\text{kg})10\tfrac{\text{m}}{\text{s}}\hat{i} - (6\,\text{kg})6\tfrac{\text{m}}{\text{s}}\hat{i}$$

$$= [100-36]\frac{\text{kg}\cdot\text{m}}{\text{s}}\hat{i} = 64\frac{\text{kg}\cdot\text{m}}{\text{s}}\hat{i}$$

ASU

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1+m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$$\Rightarrow \vec{L}_2 = (m_1+m_2)\vec{v}_0 - m_1\vec{v}_1 = (10\,\text{kg})10\tfrac{\text{m}}{\text{s}}\hat{i} - (6\,\text{kg})6\tfrac{\text{m}}{\text{s}}\hat{i}$$

$$= [100-36]\tfrac{\text{kg}\cdot\text{m}}{\text{s}}\hat{i} = 64\tfrac{\text{kg}\cdot\text{m}}{\text{s}}\hat{i}$$

$$\& \ \vec{L}_1 = (6\,\text{kg})(6\tfrac{\text{m}}{\text{s}})\hat{i} = 36\tfrac{\text{kg}\cdot\text{m}}{\text{s}}\hat{i}$$

ASU

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1+m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$$\Rightarrow \vec{L}_2 = (m_1+m_2)\vec{v}_0 - m_1\vec{v}_1 = (10\,kg)10\tfrac{m}{s}\hat{i} - (6\,kg)6\tfrac{m}{s}\hat{i}$$

$$= [100-36]\frac{kg\cdot m}{s}\hat{i} = 64\frac{kg\cdot m}{s}\hat{i}$$

$$\&\ \vec{L}_1 = (6\,kg)(6\tfrac{m}{s})\hat{i} = 36\frac{kg\cdot m}{s}\hat{i}$$

Now $\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2 = (r_{1x}\hat{i} + r_{1y}\hat{j}) \times \left(36\frac{kg\cdot m}{s}\right)\hat{i}$

$$+ (r_{2x}\hat{i} + r_{2y}\hat{j}) \times \left(64\frac{kg\cdot m}{s}\right)\hat{i}$$

$\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2$, where $\vec{L}_1 = m_1\vec{v}_1$ & $\vec{L}_2 = m_2\vec{v}_2$. We can calculate $\vec{L}_1$ from our knowledge of m_1 & $\vec{v}_1$, but we need to find $\vec{L}_2$.

$$L_I = L_F \Rightarrow (m_1+m_2)\vec{v}_0 = m_1\vec{v}_1 + \vec{L}_2$$

$$\Rightarrow \vec{L}_2 = (m_1+m_2)\vec{v}_0 - m_1\vec{v}_1 = (10\,kg)10\tfrac{m}{s}\hat{i} - (6\,kg)6\tfrac{m}{s}\hat{i}$$

$$= [100-36]\tfrac{kg \cdot m}{s}\hat{i} = 64\tfrac{kg \cdot m}{s}\hat{i}$$

$$\& \quad \vec{L}_1 = (6\,kg)(6\tfrac{m}{s})\hat{i} = 36\tfrac{kg \cdot m}{s}\hat{i}$$

Now $\vec{H}_G = \vec{r}_1 \times \vec{L}_1 + \vec{r}_2 \times \vec{L}_2 = (r_{1x}\hat{i} + r_{1y}\hat{j}) \times (36\tfrac{kg \cdot m}{s})\hat{i}$

$+ (r_{2x}\hat{i} + r_{2y}\hat{j}) \times (64\tfrac{kg \cdot m}{s})\hat{i}$ But $\hat{i} \times \hat{i} = 0$

& $\hat{j} \times \hat{i} = -\hat{k}$ So $\longrightarrow$

$\vec{H}_G = r_{1y}\left(36\,\frac{kg \cdot m}{s}\right)(-\hat{k}) + r_{2y}\left(64\,\frac{kg \cdot m}{s}\right)(-\hat{k})$, where

$r_{1y} = y_1 = \frac{1}{4}\,m$ & $r_{2y} = y_2 = -\frac{1}{4}\,m$

$$\vec{H}_G = r_{1y}\left(36\,\frac{kg\cdot m}{s}\right)(-\hat{k}) + r_{2y}\left(64\,\frac{kg\cdot m}{s}\right)(-\hat{k}),\ \text{where}$$

$$r_{1y} = y_1 = \frac{1}{4}\,m \quad \& \quad r_{2y} = y_2 = -\frac{1}{4}\,m$$

$$\Rightarrow \vec{H}_G = \left(\frac{36}{4} - \frac{64}{4}\right)(-\hat{k})\,\frac{kg\cdot m^2}{s}$$

$$\vec{H}_G = r_{1y}\left(36\,\frac{kg\cdot m}{s}\right)(-\hat{k}) + r_{2y}\left(64\,\frac{kg\cdot m}{s}\right)(-\hat{k}), \text{ where}$$

$$r_{1y} = y_1 = \frac{1}{4}\,m \quad \& \quad r_{2y} = y_2 = -\frac{1}{4}\,m$$

$$\Rightarrow \vec{H}_G = \left(\frac{36}{4} - \frac{64}{4}\right)(-\hat{k})\,\frac{kg\cdot m^2}{s} = (9-16)(-\hat{k})\,\frac{kg\cdot m^2}{s}$$

$$\vec{H}_G = r_{1y}\left(36\,\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)(-\hat{k}) + r_{2y}\left(64\,\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)(-\hat{k}),$$ where

$$r_{1y} = y_1 = \frac{1}{4}\,\text{m} \quad \& \quad r_{2y} = y_2 = -\frac{1}{4}\,\text{m}$$

$$\Rightarrow \vec{H}_G = \left(\frac{36}{4} - \frac{64}{4}\right)(-\hat{k})\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}} = (9-16)(-\hat{k})\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\Rightarrow \vec{H}_G = 7\hat{k}\left(\frac{\text{kg}\cdot\text{m}^2}{\text{s}}\right)$$

ASU