

Today 15.1

L13



Today 15.1

L13

Translation &
fixed axis
rotation of
rigid bodies

Today 15.1
Thursday 15.2

L13



Today 15.1

Thursday 15.2

L13

General plane
motion for rigid bodies

Grade calculation at end of semester

Grade calculation at end of semester

$$\text{Final score} = (\text{Exam ave}) *$$

Grade calculation at end of semester

$$\text{Final score} = (\text{Exam ave}) * 4$$

↳ of 4 highest exams out of 5

Grade calculation at end of semester

$$\text{Final score} = (\text{Exam ave}) * 4$$

Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

IF

Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

IF $\text{HW}_{\text{AVE}} = 100$

Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

If $\text{HW}_{\text{ave}} = 100$, then

$$\text{Exam ave} = \left[\frac{5 * (\text{Final score}) - 100}{4} \right] \%$$

Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

IF $\text{HW}_{\text{ave}} = 100$, then

$$\text{Exam ave} = \left[\frac{5 * (\text{Final score}) - 100}{4} \right] \%$$

so IF $\text{HW}_{\text{ave}} = 100\%$

Grade calculation at end of semester

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So IF $\text{HW}_{\text{ave}} = 100\%$ Final score Exam ave

Grade calculation at end of semester

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$$\text{Exam ave} = \left[\frac{5 * (\text{Final score}) - 100}{4} \right] \%$$

So IF $\text{HW}_{\text{ave}} = 100\%$

Final score	Exam ave
A: 90%	87.5%

Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

IF $\text{HW}_{\text{ave}} = 100$, then

$$\text{Exam ave} = \left[\frac{5 * (\text{Final score}) - 100}{4} \right] \%$$

So IF $\text{HW}_{\text{ave}} = 100\%$

Final score	Exam ave
A: 90%	87.5%
B: 80%	75%

Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

IF $\text{HW}_{\text{ave}} = 100$, then

$$\text{Exam ave} = \left[\frac{5 * (\text{Final score}) - 100}{4} \right] \%$$

So IF $\text{HW}_{\text{ave}} = 100\%$

Final score	Exam ave
A: 90%	87.5%
B: 80%	75%
C: 70%	62.5%

Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

IF $\text{HW}_{\text{ave}} = 100$, then

$$\text{Exam ave} = \left[\frac{5 * (\text{Final score}) - 100}{4} \right] \%$$

So IF $\text{HW}_{\text{ave}} = 100\%$

Final score	Exam ave
A: 90%	87.5%
B: 80%	75%
C: 70%	62.5%
D: 60%	50%

Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

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So IF $\text{HW}_{\text{ave}} = 100\%$

Final score	Exam ave
A: 90%	87.5%
B: 80%	75%
C: 70%	62.5%
D: 60%	50%

+ Keep your homework scores high +



Grade calculation at end of semester

$$\text{Final score} = \left[\frac{(\text{Exam ave}) * 4 + (\text{HW ave})}{5} \right] \%$$

IF $\text{HW}_{\text{ave}} = 100$, then

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So IF $\text{HW}_{\text{ave}} = 100\%$

Final score	Exam ave
A: 90%	87.5%
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C: 70%	62.5%
D: 60%	50%

+ Keep your homework scores high +



Rigid bodies

Rigid bodies

Definitions:

Translation if no rotation of body

Rigid bodies

Definitions:

Translation if no rotation of body

Rigid bodies

Definitions:

Translation if no rotation of body

Two types of translation

Rigid bodies

Definitions:

Translation if no rotation of body

Two types of translation

* Rectilinear translation

Rigid bodies

Definitions:

Translation if no rotation of body

Two types of translation

* Rectilinear translation
AND

Rigid bodies

Definitions:

Translation if no rotation of body

Two types of translation

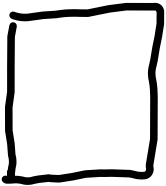
* Rectilinear translation
AND

* Curvilinear translation

Rectilinear translation

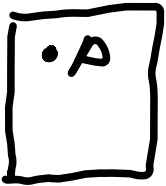
Rectilinear translation

y
└
x



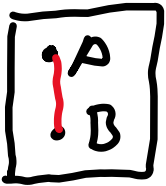
Rectilinear translation

y
└
x

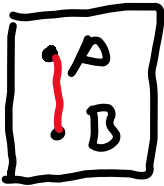


Rectilinear translation

y
└
x

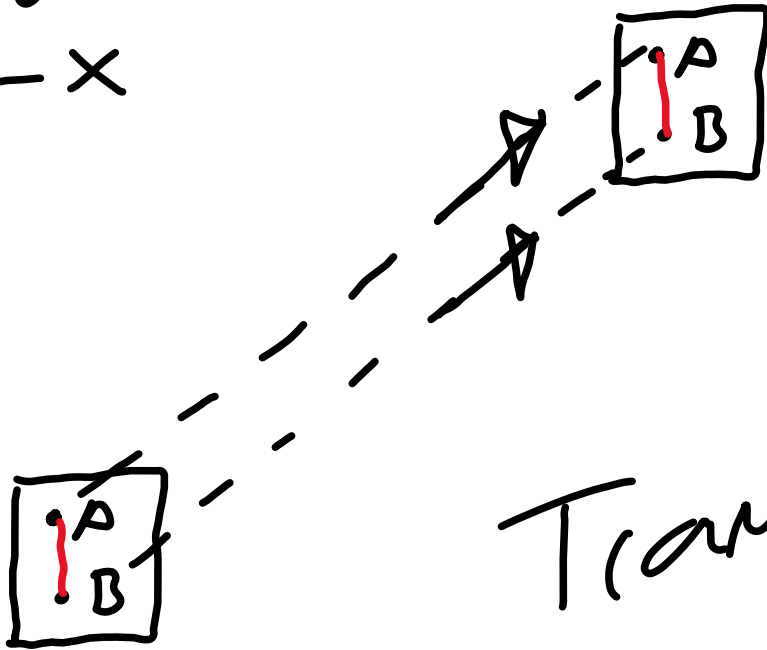


Rectilinear translation



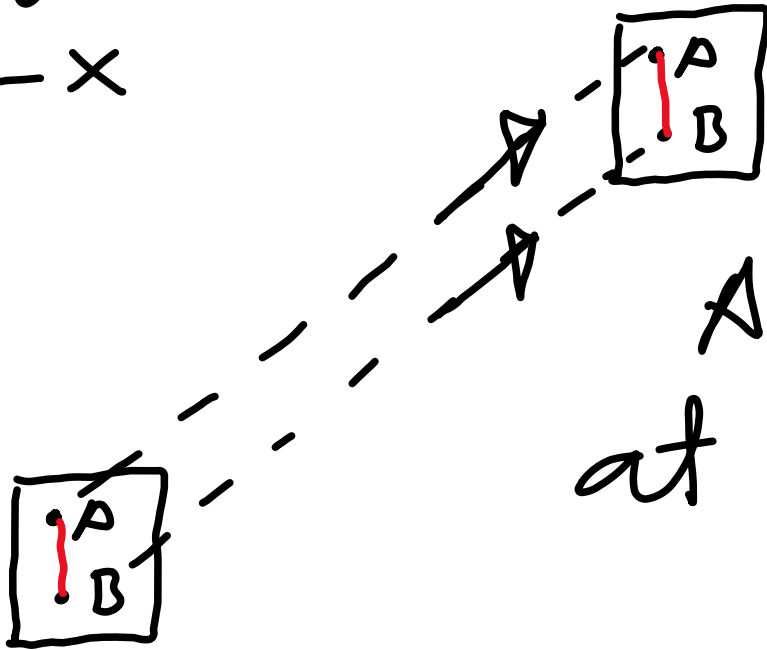
Note: red line connecting points A & B

Rectilinear translation



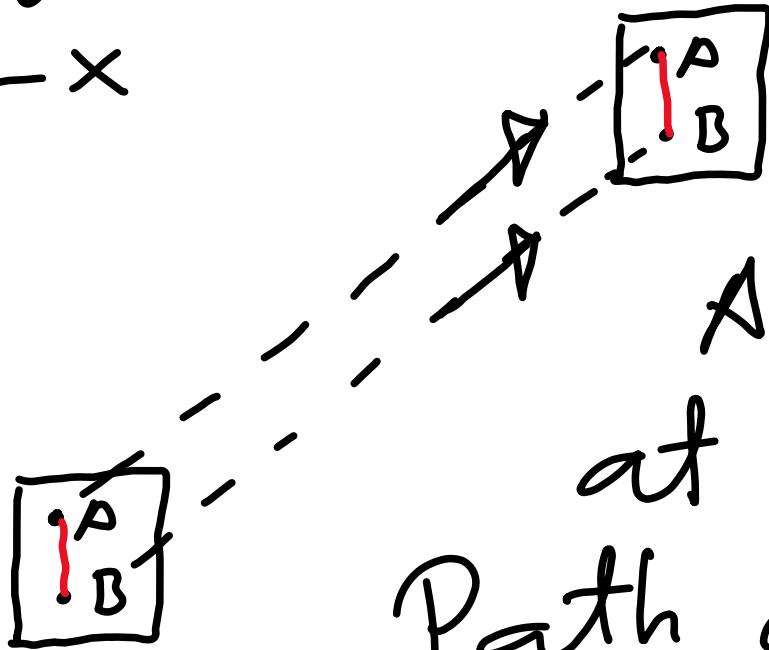
Translate
as shown

Rectilinear translation



line connecting
A & B always
at same angle.

Rectilinear translation



line connecting

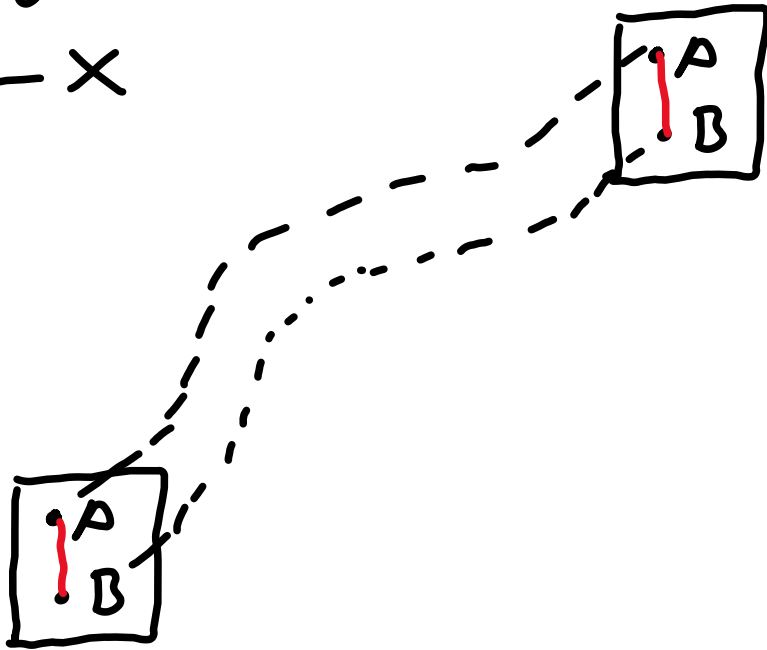
A & B always

at same angle.

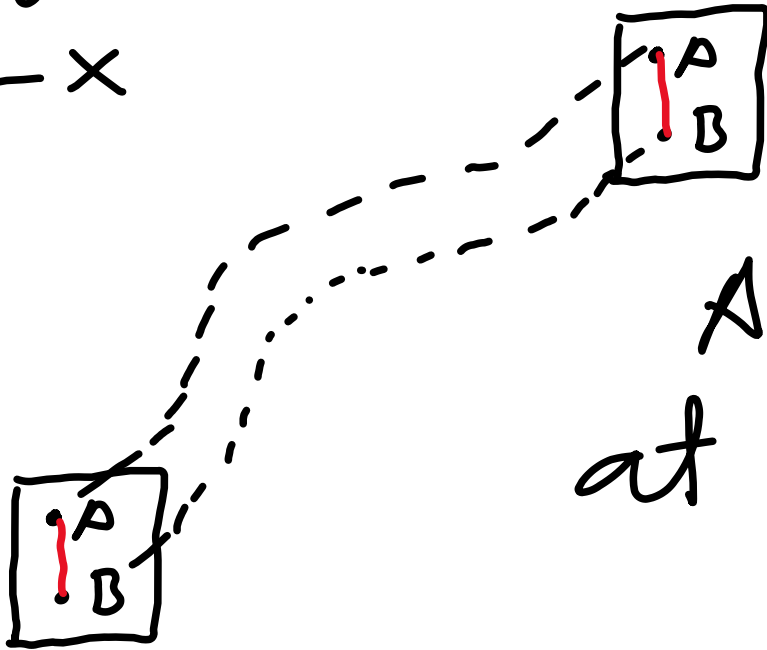
Path of object is
a straight line

Curvilinear translation

Curvilinear translation

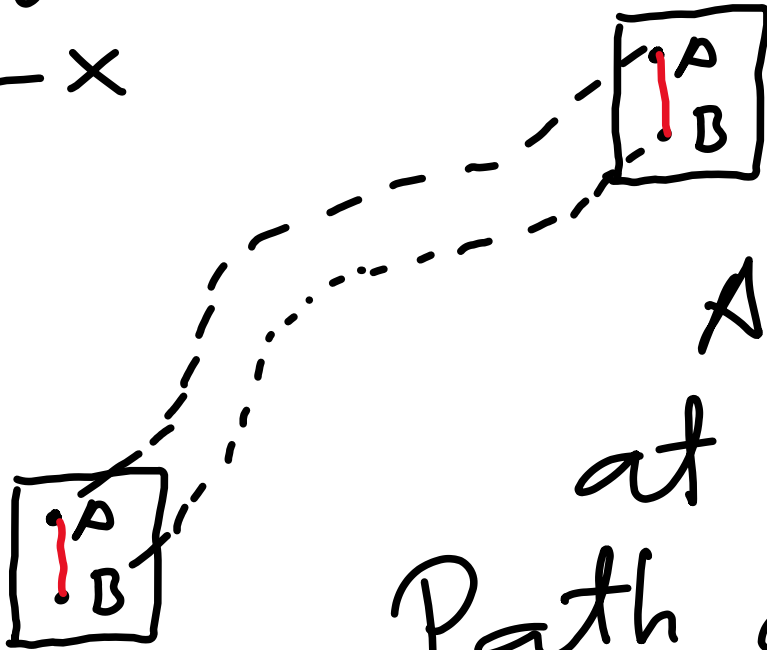


Curvilinear translation



line connecting
A & B always
at same angle.

Curvilinear translation



line connecting

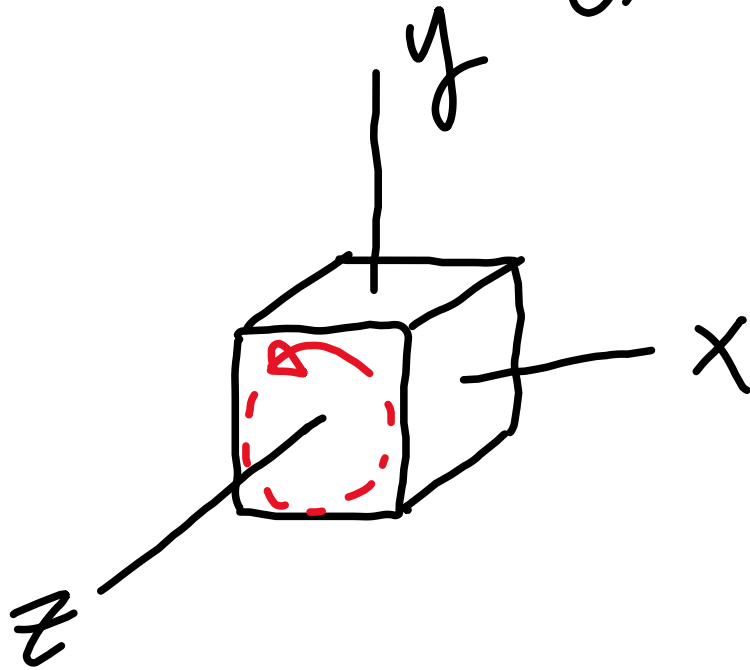
A & B always

at same angle.

Path of object is
a curved line

Rotation about fixed axis

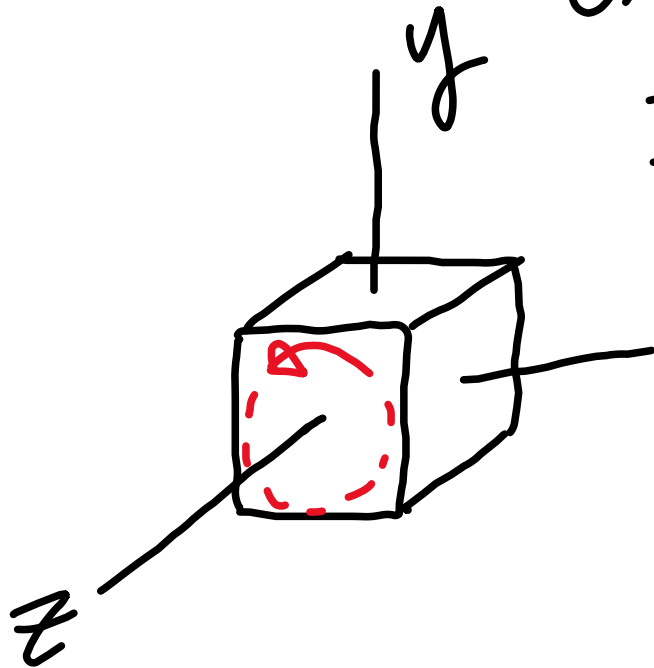
Motion along circles centered on axis of rotation.



Rotation about fixed axis

Motion along circles centered on axis of rotation.

In this, a cube is rotated about the z-axis.

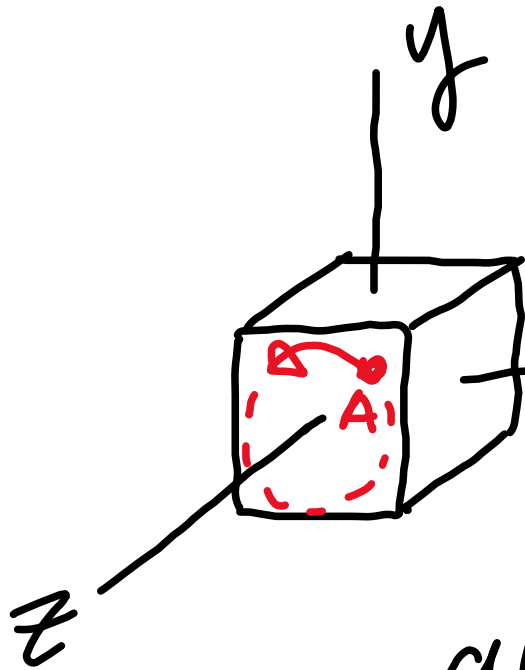


Rotation about fixed axis

Motion along circles centered on axis of rotation.

In this, a cube is rotated

about the z-axis.



point **A** rotates about this axis as shown

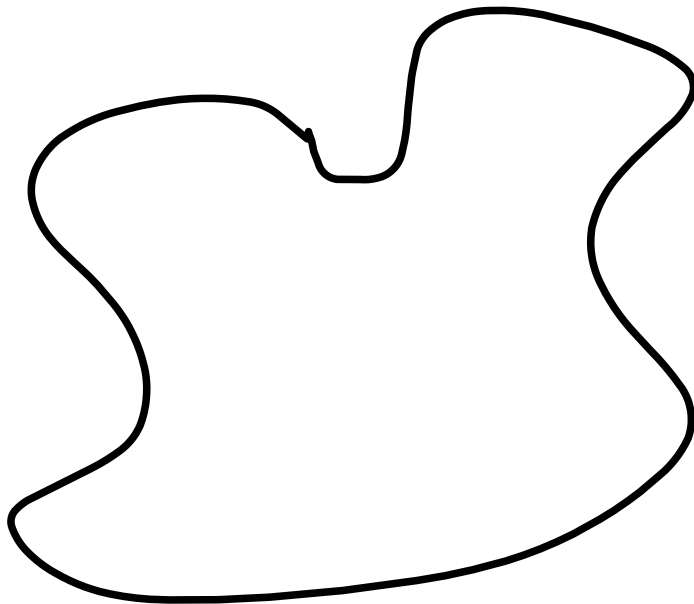
For translation of rigid body

For translation of rigid body

On a rigid body,

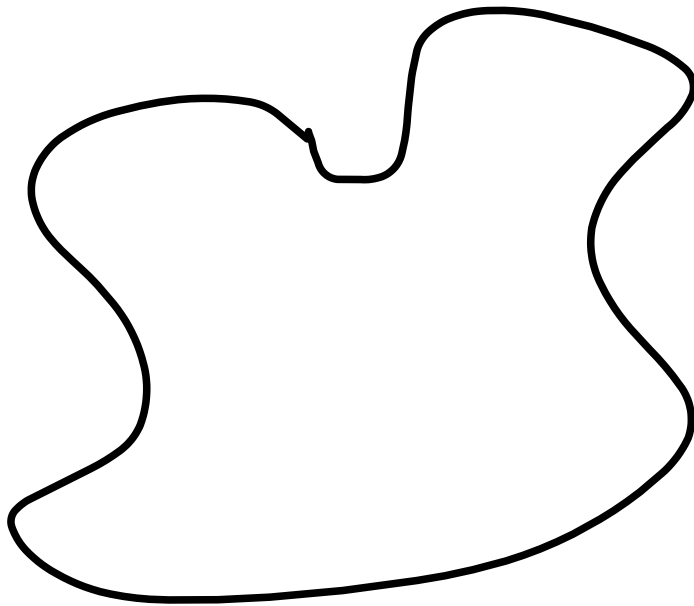
For translation of rigid body

On a rigid body,



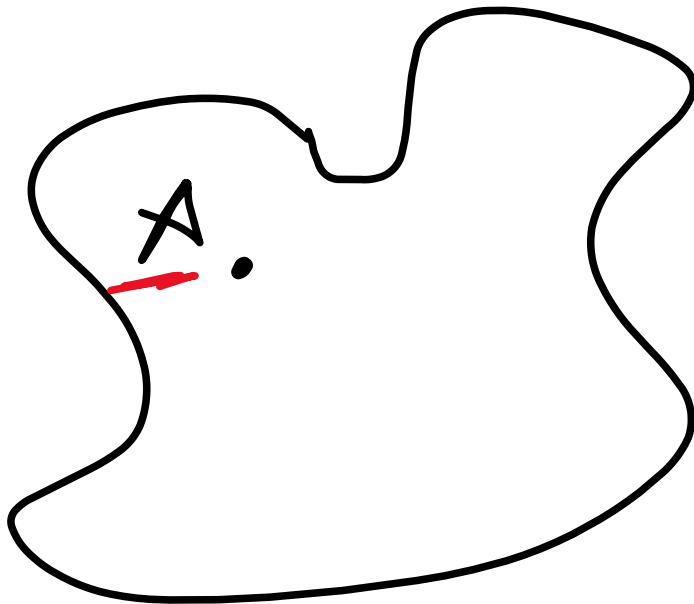
For translation of rigid body

On a rigid body, if we choose any two points A & B



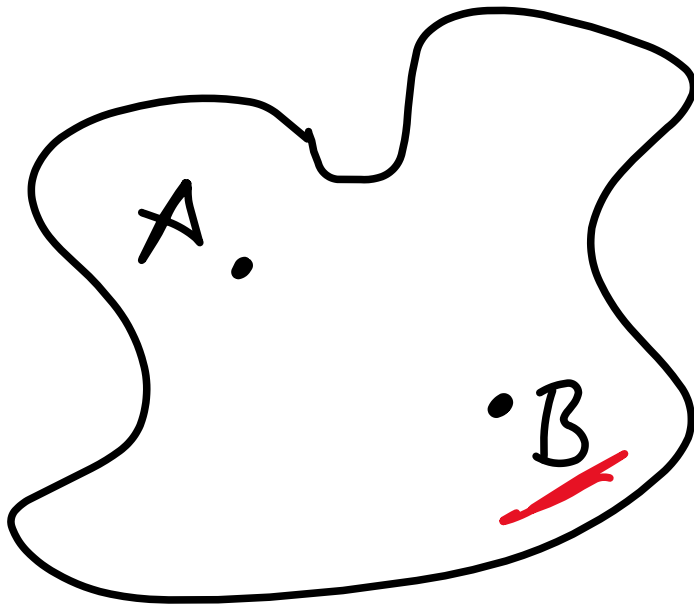
For translation of rigid body

On a rigid body, if we choose any two points A & B



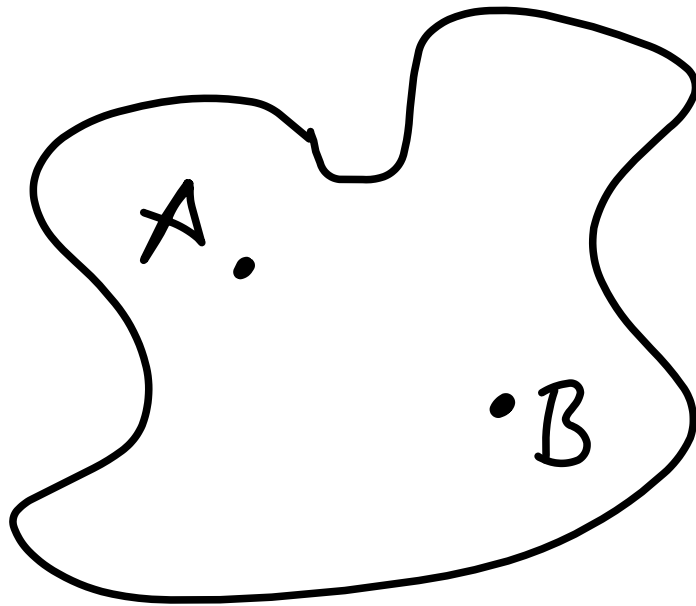
For translation of rigid body

On a rigid body, if we choose any two points A & B



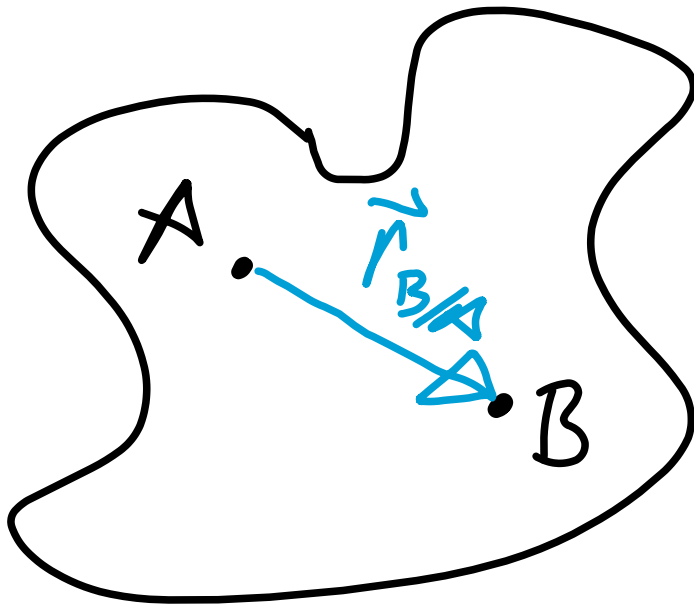
For translation of rigid body

On a rigid body, if we choose any two points A & B , then we can relate the position of B to that of point A as $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$



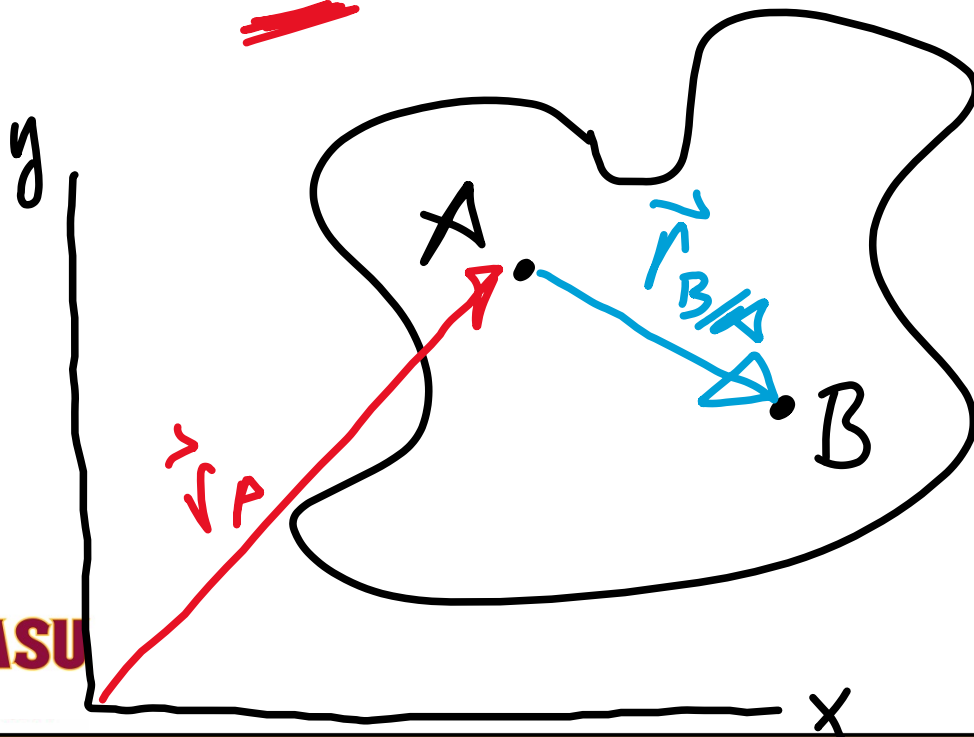
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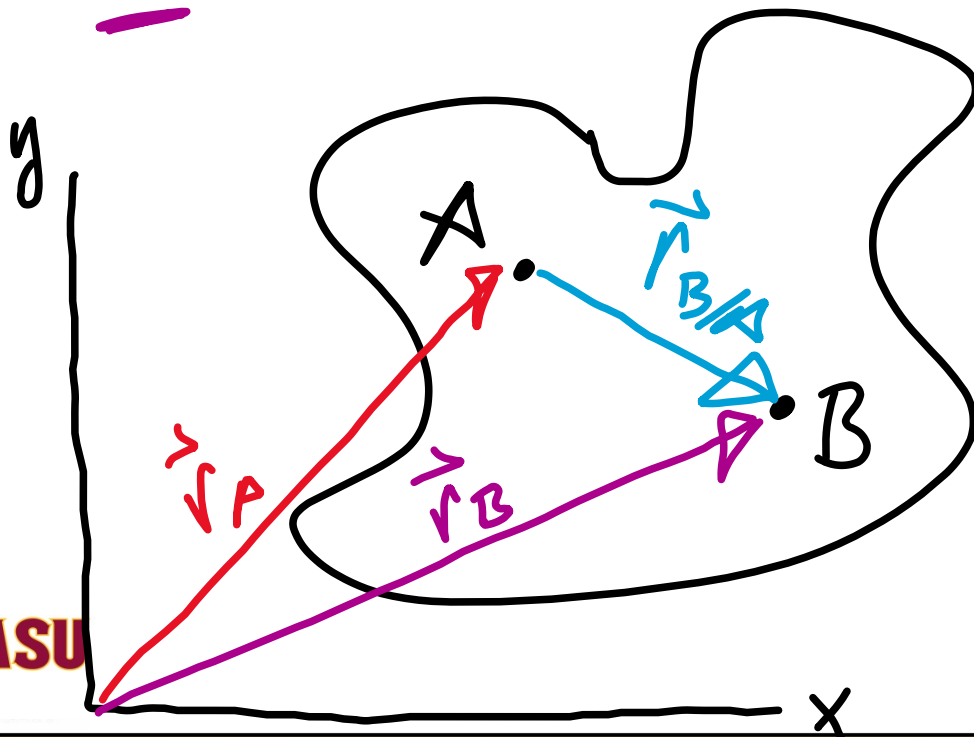
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For translation of rigid body

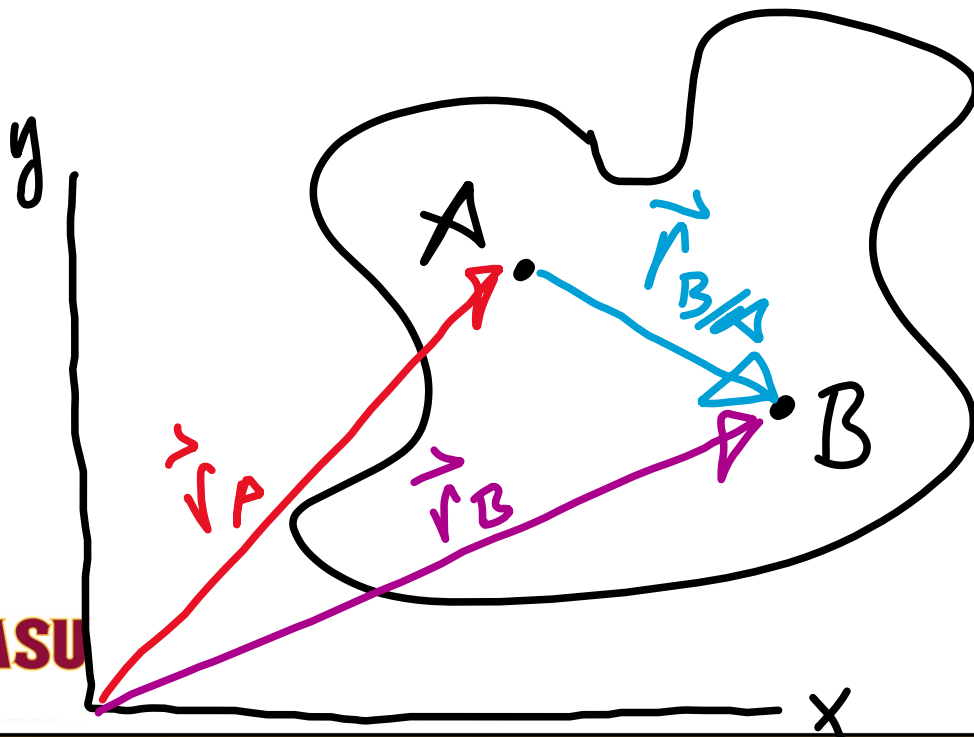
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For translation of rigid body

On a rigid body, if we choose any two points A & B , then we can relate the position of B to that of point A as $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$. Since A & B are fixed on rigid body

$$|\vec{r}_{B/A}| = \text{const.}$$

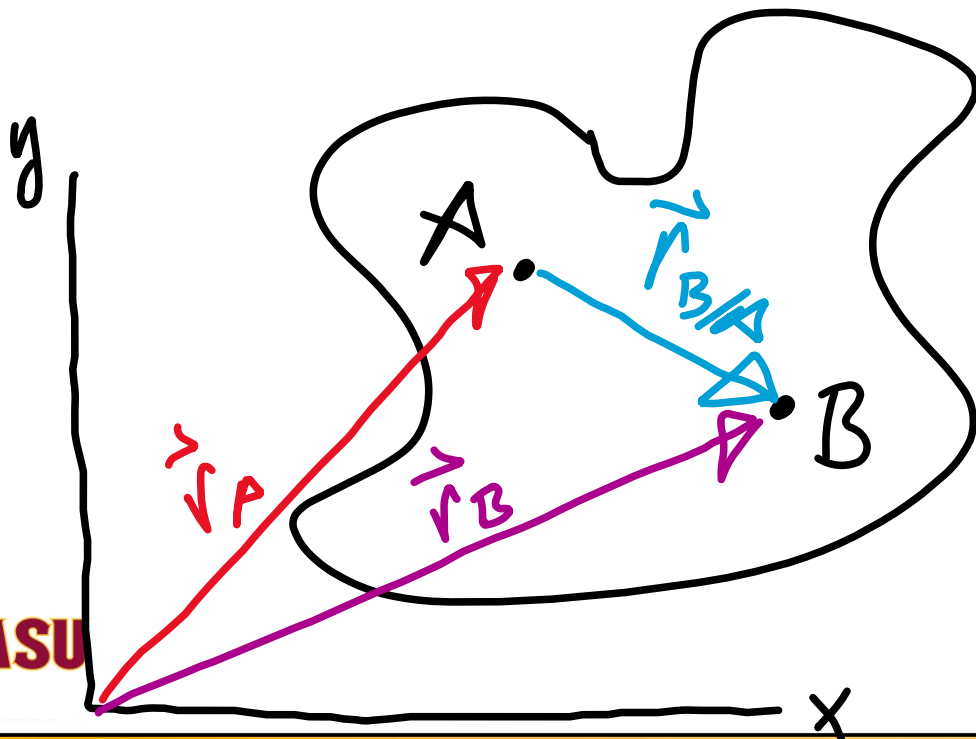


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$|\vec{r}_{B/A}| = \text{const.}$ & since no rotation

$$\frac{d\vec{r}_{B/A}}{dt} = \theta$$



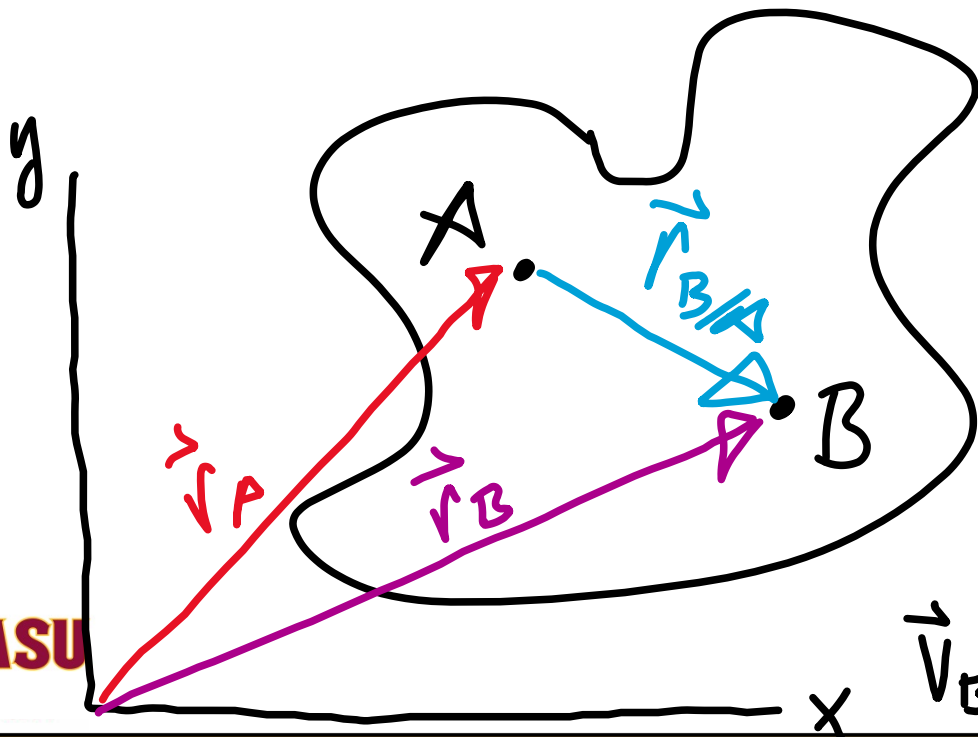
For translation of rigid body

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$|\vec{r}_{B/A}| = \text{const.}$ & since no rotation

$$\frac{d\vec{r}_{B/A}}{dt} = \vec{0} \quad \text{so}$$

$$\vec{v}_B = \vec{v}_A$$



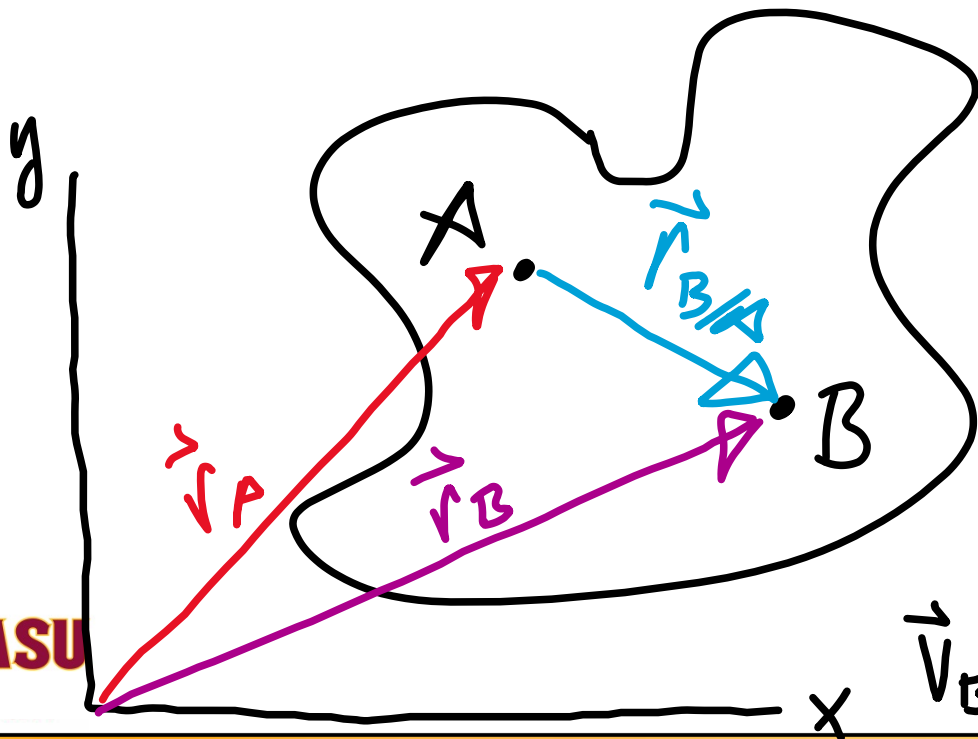
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$|\vec{r}_{B/A}| = \text{const.}$ & since no rotation

$$\frac{d\vec{r}_{B/A}}{dt} = 0 \text{ so}$$

$$\vec{v}_B = \vec{v}_A \text{ \& } \vec{a}_B = \vec{a}_A$$



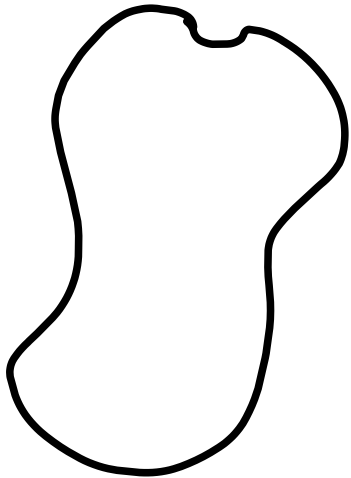
For rotation about a fixed axis

For rotation about a fixed axis

* On a rigid body

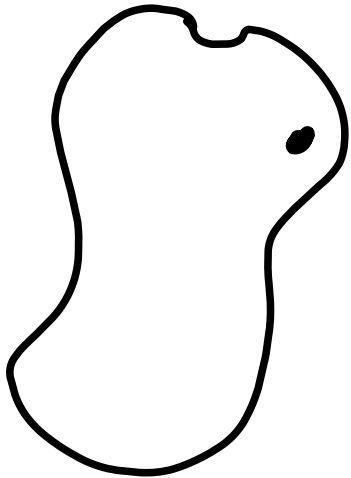
For rotation about a fixed axis

* On a rigid body



For rotation about a fixed axis

* On a rigid body take a point



For rotation about a fixed axis

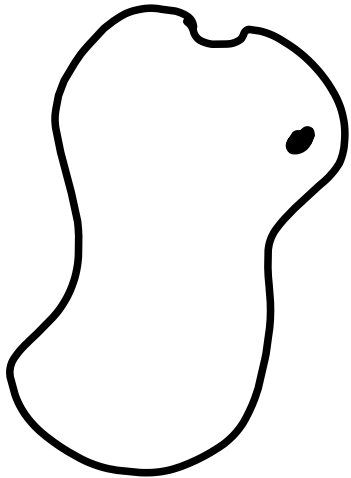
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For rotation about a fixed axis

* On a rigid body take a point

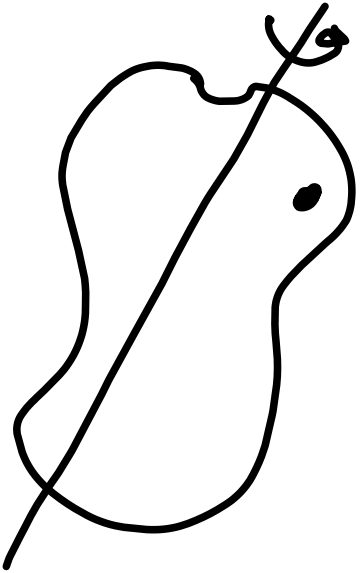
* Along the axis of rotation



For rotation about a fixed axis

* On a rigid body take a point

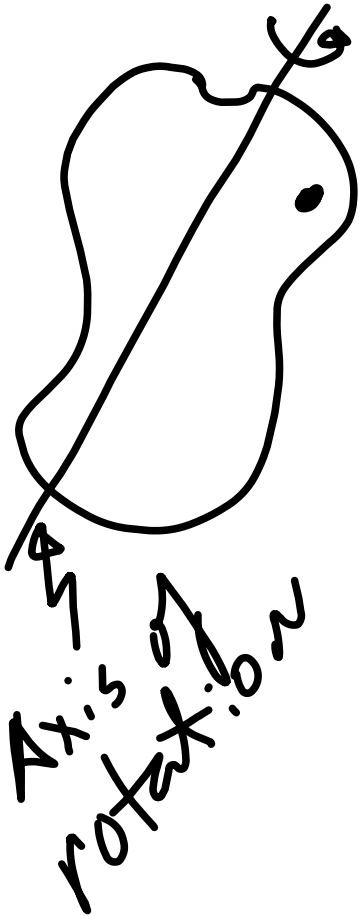
* Along the axis of rotation



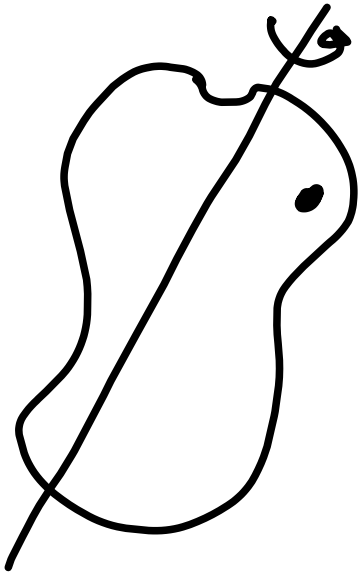
For rotation about a fixed axis

* On a rigid body take a point

* Along the axis of rotation



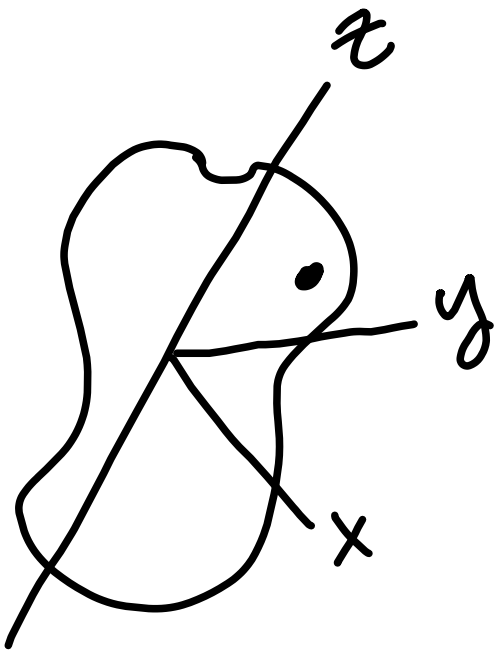
For rotation about a fixed axis



* On a rigid body take a point

* Along the axis of rotation
Define the z -axis

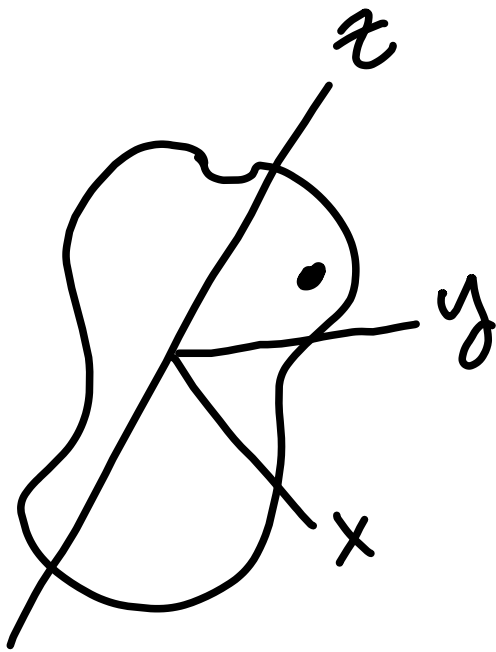
For rotation about a fixed axis



* On a rigid body take a point

* Along the axis of rotation
Define the z-axis

For rotation about a fixed axis

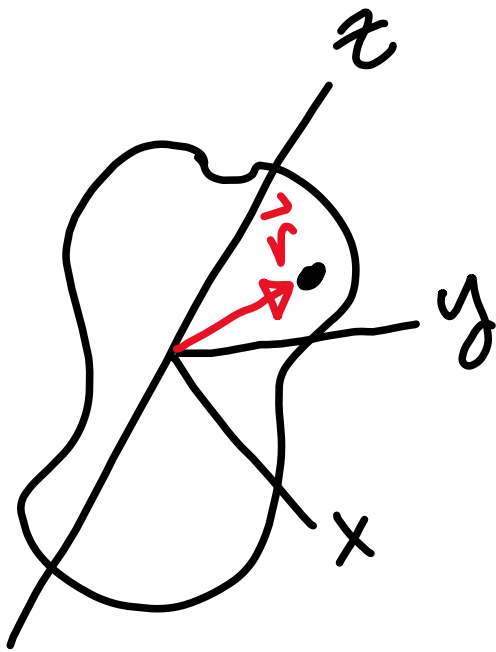


* On a rigid body take a point

* Along the axis of rotation
Define the z -axis

* angle \vec{r} makes with
 z -axis is φ

For rotation about a fixed axis

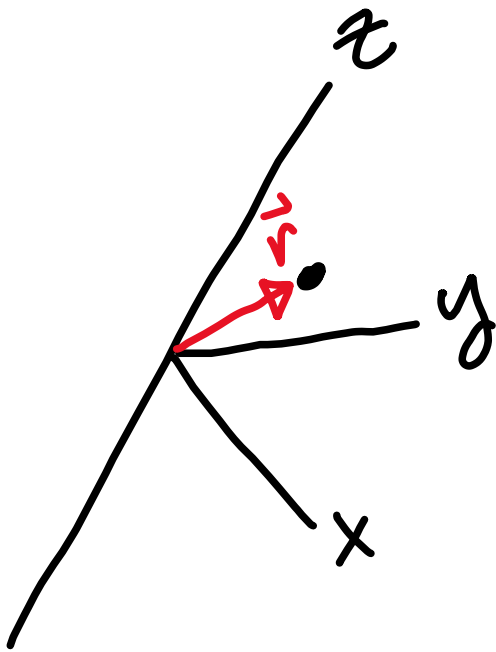


* On a rigid body take a point

* Along the axis of rotation Define the z-axis

* angle \vec{r} makes with z-axis is φ

For rotation about a fixed axis

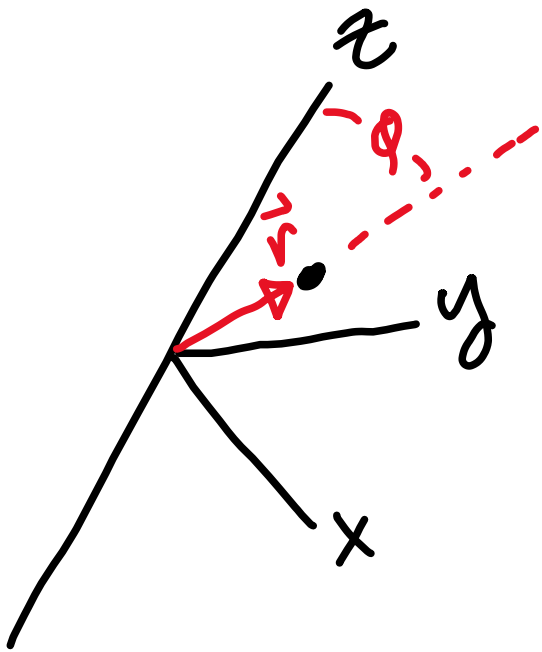


* On a rigid body take a point

* Along the axis of rotation Define the z-axis

* angle \vec{r} makes with z-axis is ϕ

For rotation about a fixed axis

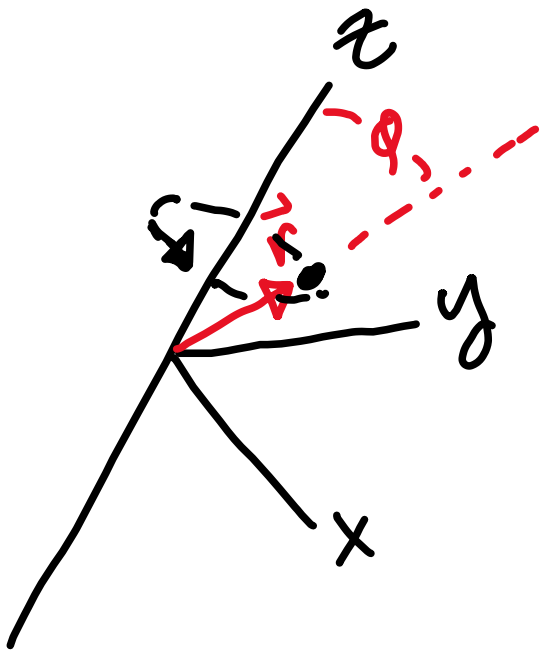


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For rotation about a fixed axis

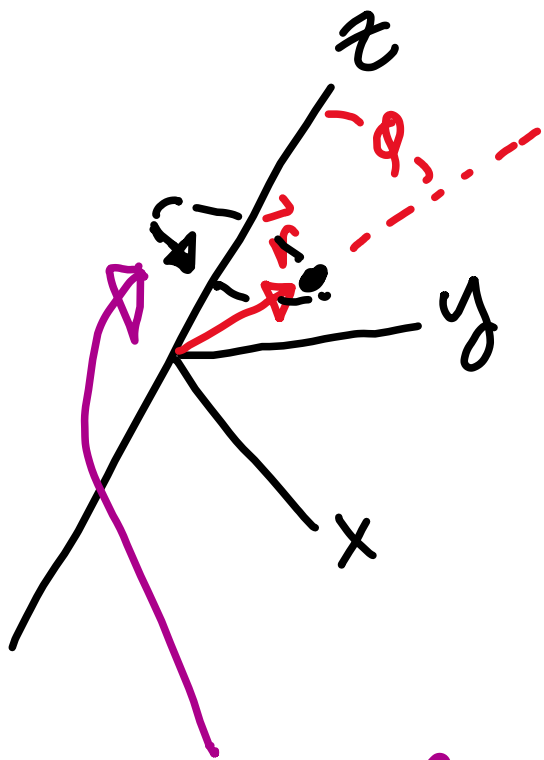


* On a rigid body take a point

* Along the axis of rotation Define the z-axis

* angle \vec{r} makes with z-axis is φ

For rotation about a fixed axis



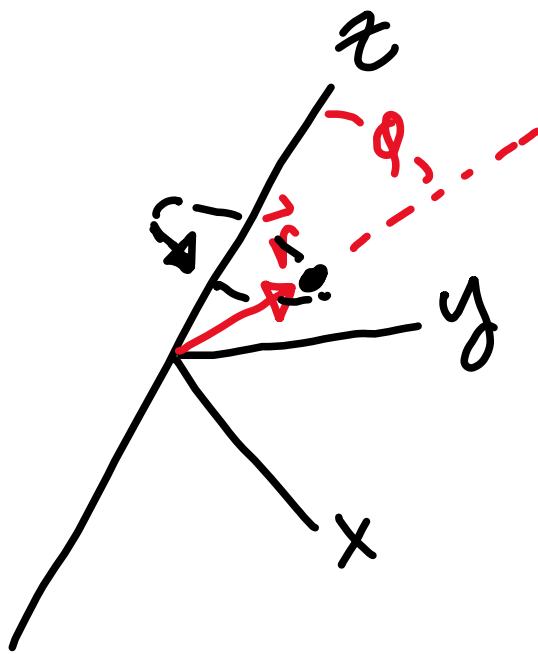
* On a rigid body take a point

* Along the axis of rotation Define the z-axis

* angle \vec{r} makes with z-axis is φ

Circular path swept by point of interest about axis of rotation

For rotation about a fixed axis

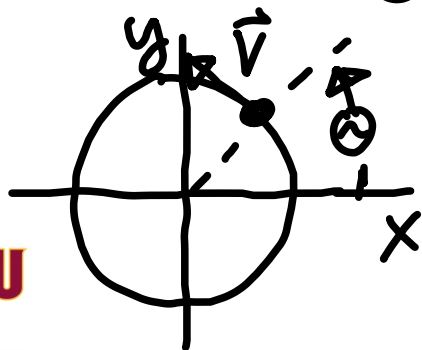


* On a rigid body take a point

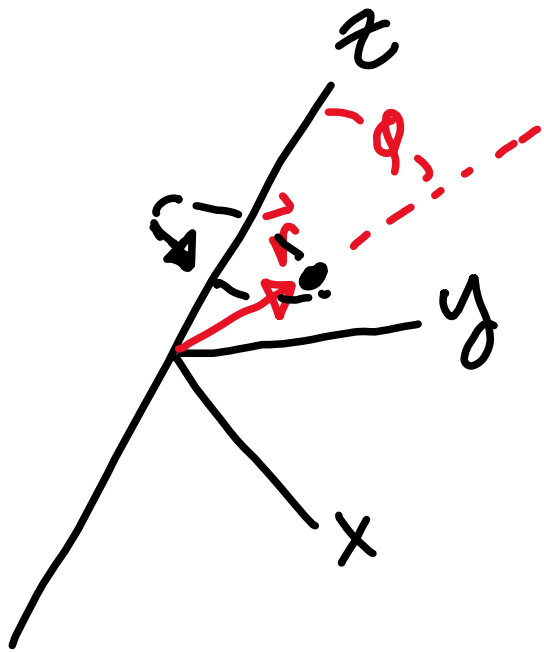
* Along the axis of rotation Define the z-axis

* angle \vec{r} makes with z-axis is ϕ

* Angle on xy-plane between \vec{r} &



For rotation about a fixed axis



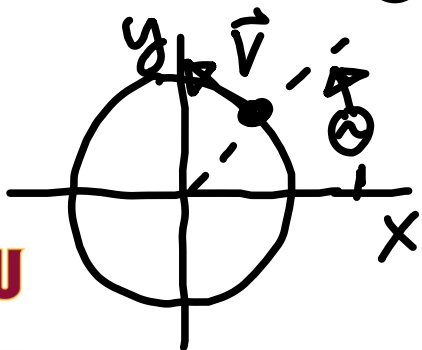
* On a rigid body take a point

* Along the axis of rotation Define the z-axis

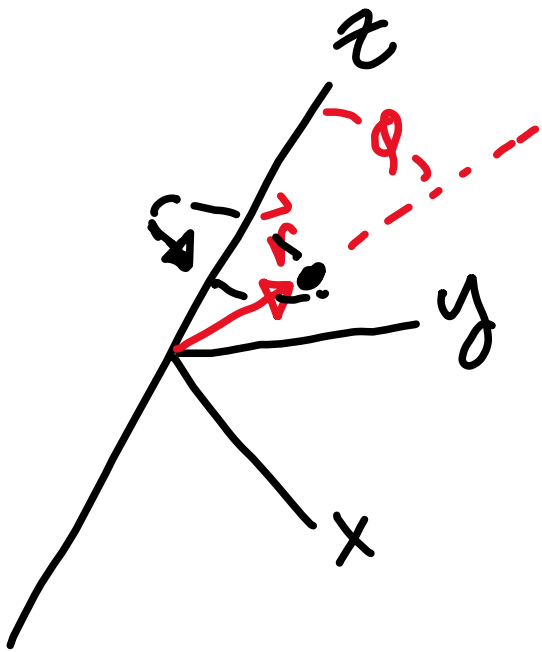
* angle \vec{r} makes with z-axis is ϕ

* Angle on xy-plane between \vec{r} & x-axis is θ

* radius of curvature is ρ



For rotation about a fixed axis



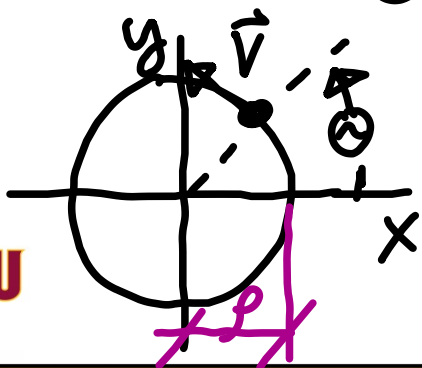
* On a rigid body take a point

* Along the axis of rotation Define the z-axis

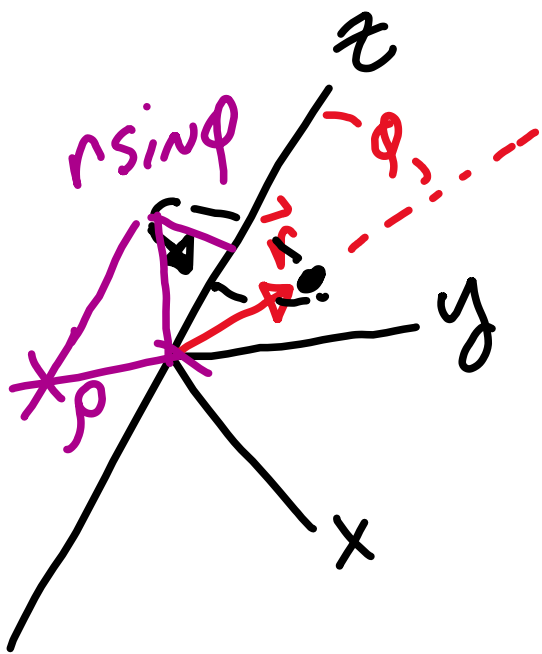
* angle \vec{r} makes with z-axis is ϕ

* Angle on xy-plane between \vec{r} & x-axis is θ

* radius of curvature is ρ



For rotation about a fixed axis



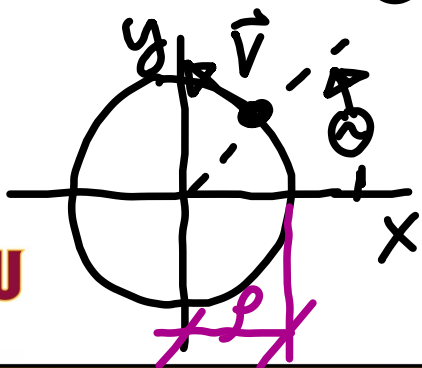
* On a rigid body take a point

* Along the axis of rotation Define the z-axis

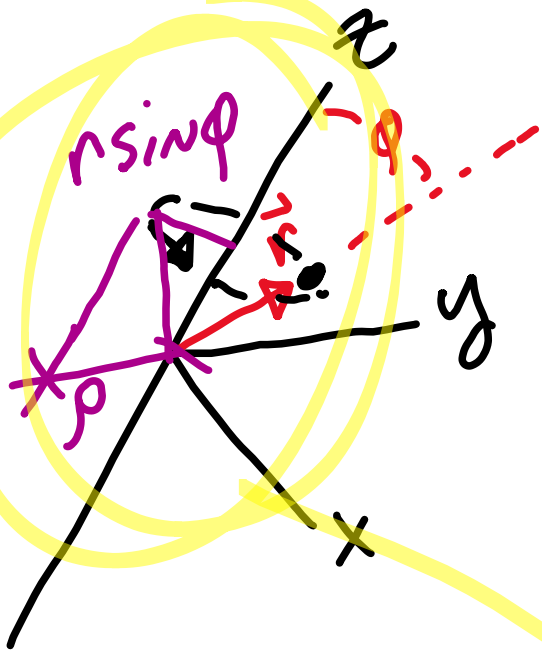
* angle \vec{r} makes with z-axis is ϕ

* Angle on xy-plane between \vec{r} & x-axis is θ

* radius of curvature is ρ & $\rho = r \sin \phi$



For rotation about a fixed axis



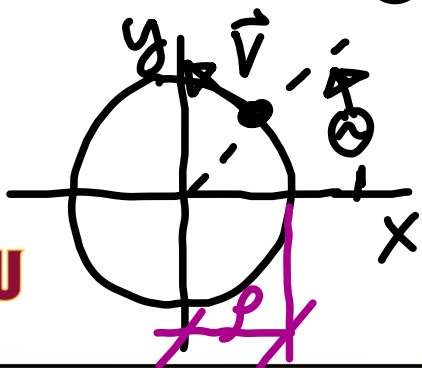
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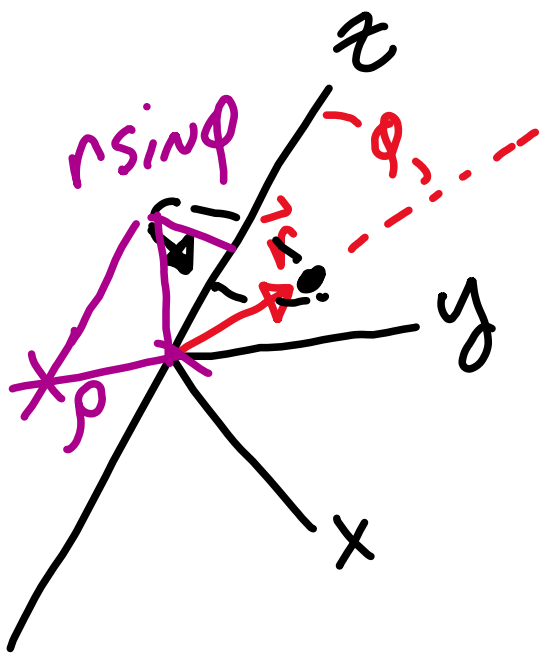
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For rotation about a fixed axis



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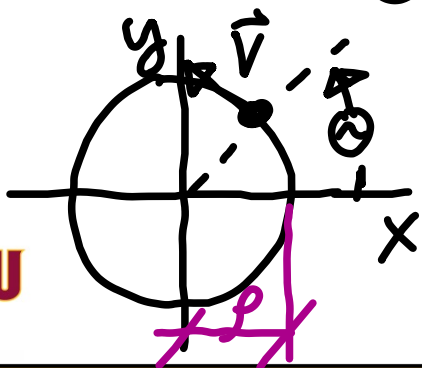
* Along the axis of rotation Define the z-axis

* angle \vec{r} makes with z-axis is ϕ

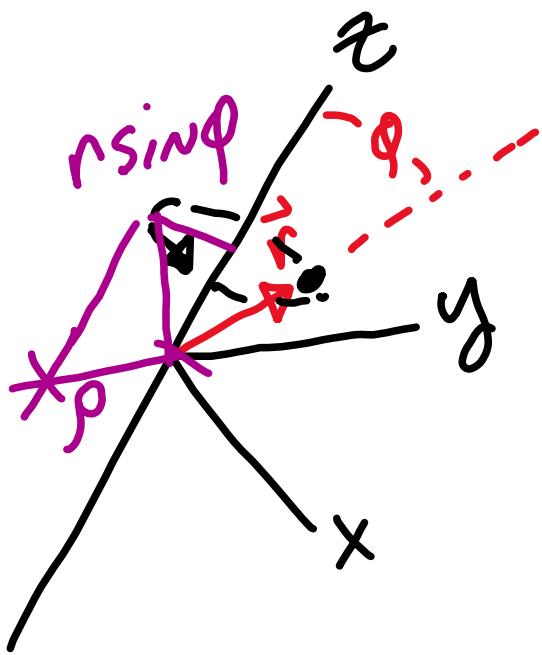
* Angle on xy-plane between \vec{r} & x-axis is θ

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* Take $\vec{u} \equiv \hat{k}$



For rotation about a fixed axis



* On a rigid body take a point

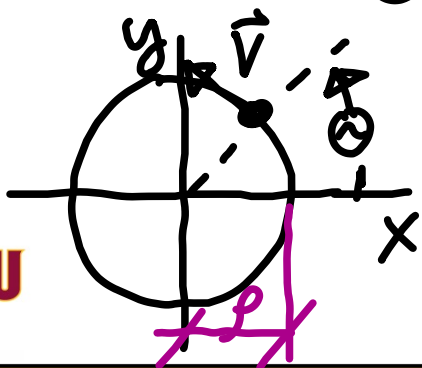
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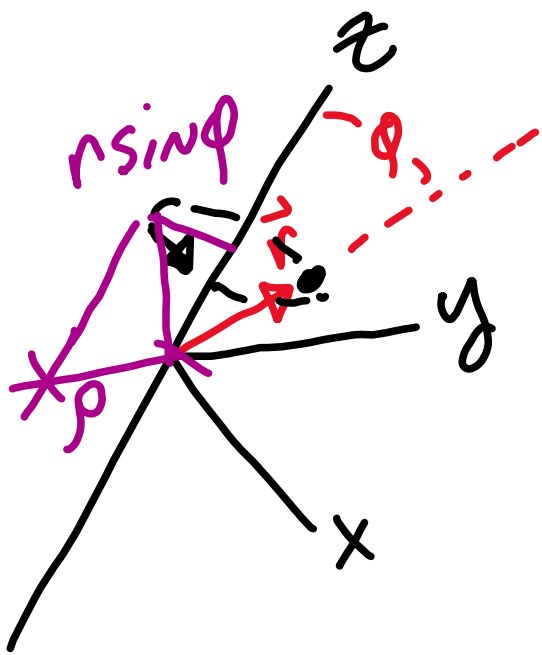
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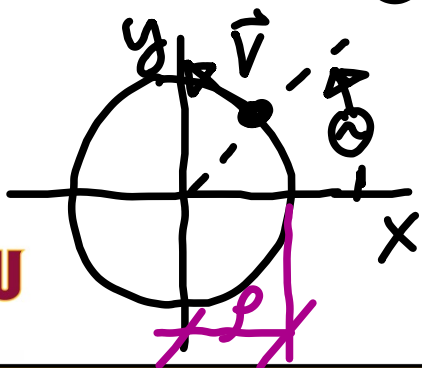
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 $\Rightarrow v = r \omega \sin \phi$



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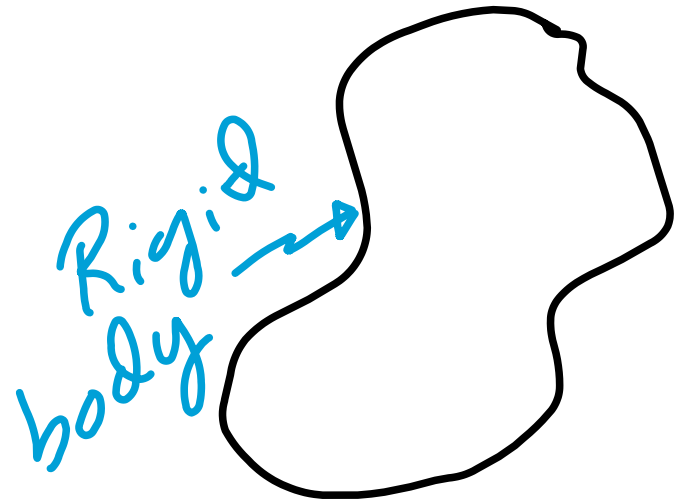
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Notation:

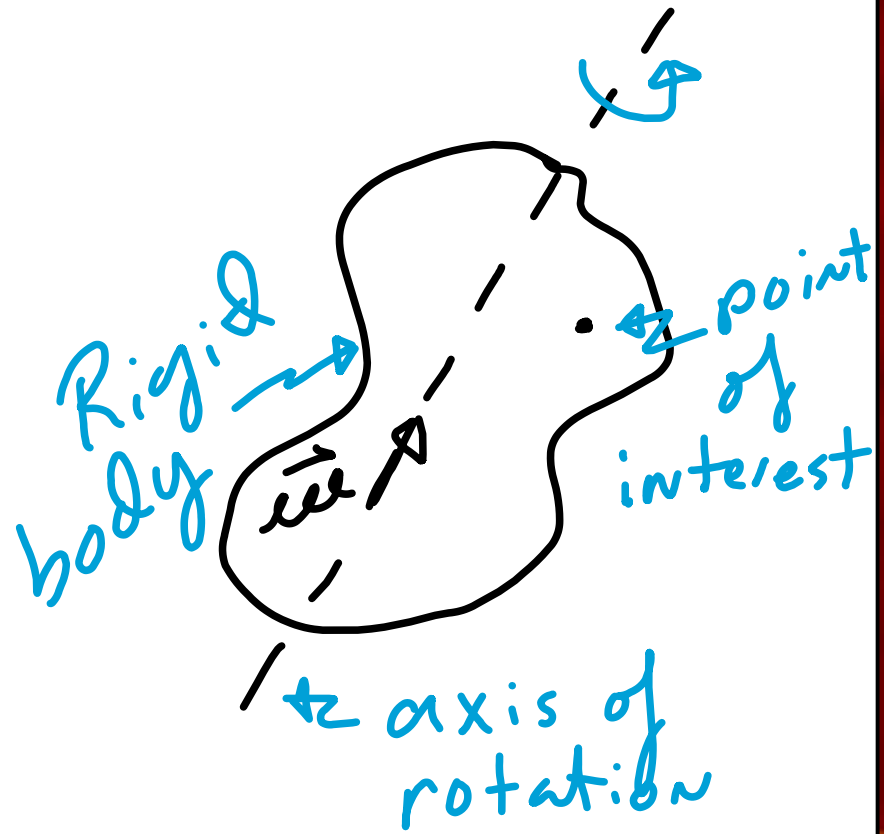
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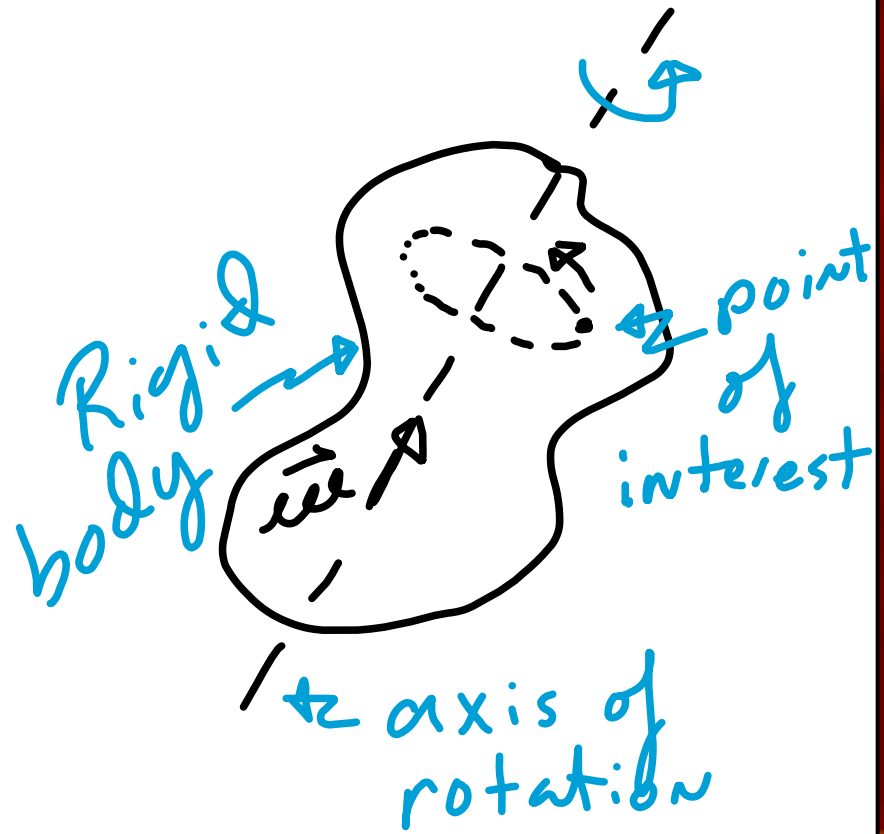


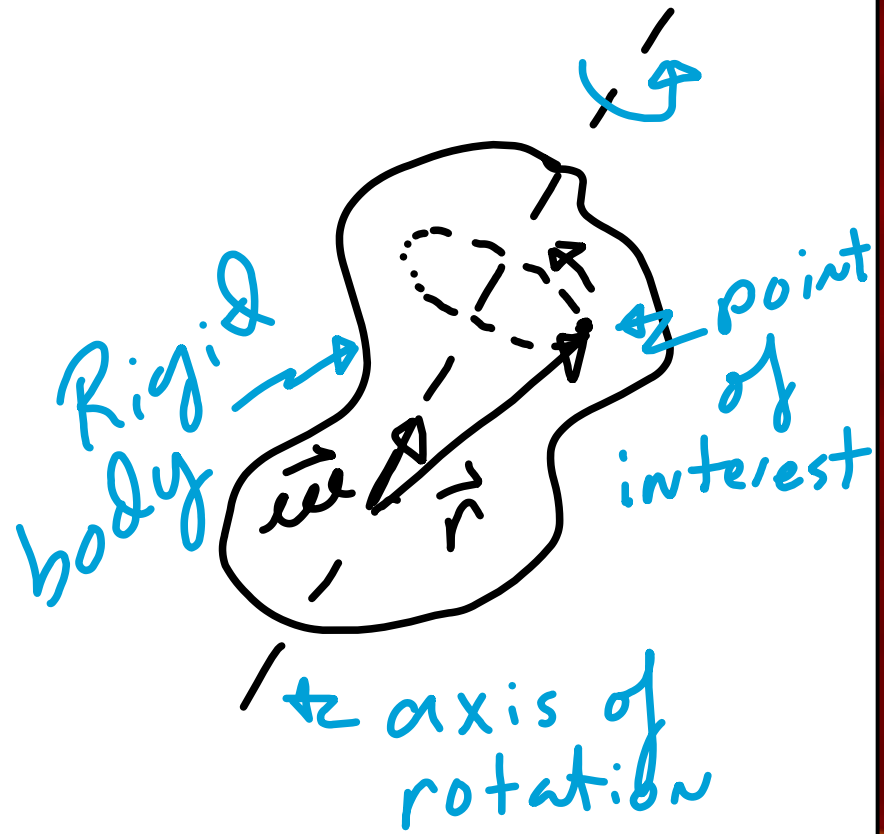




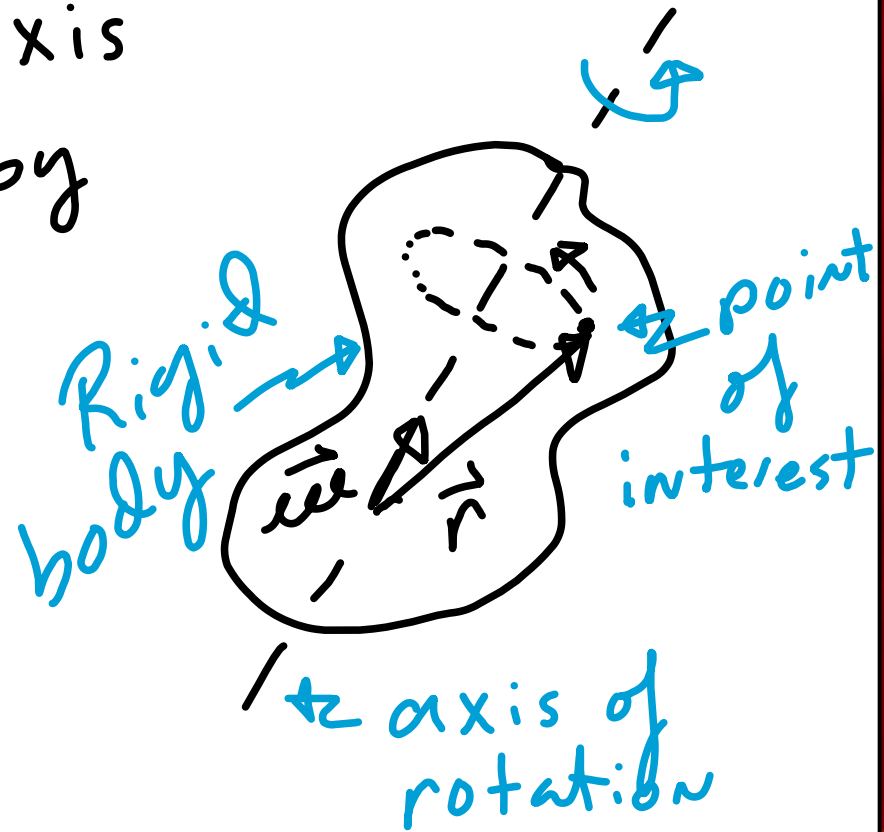




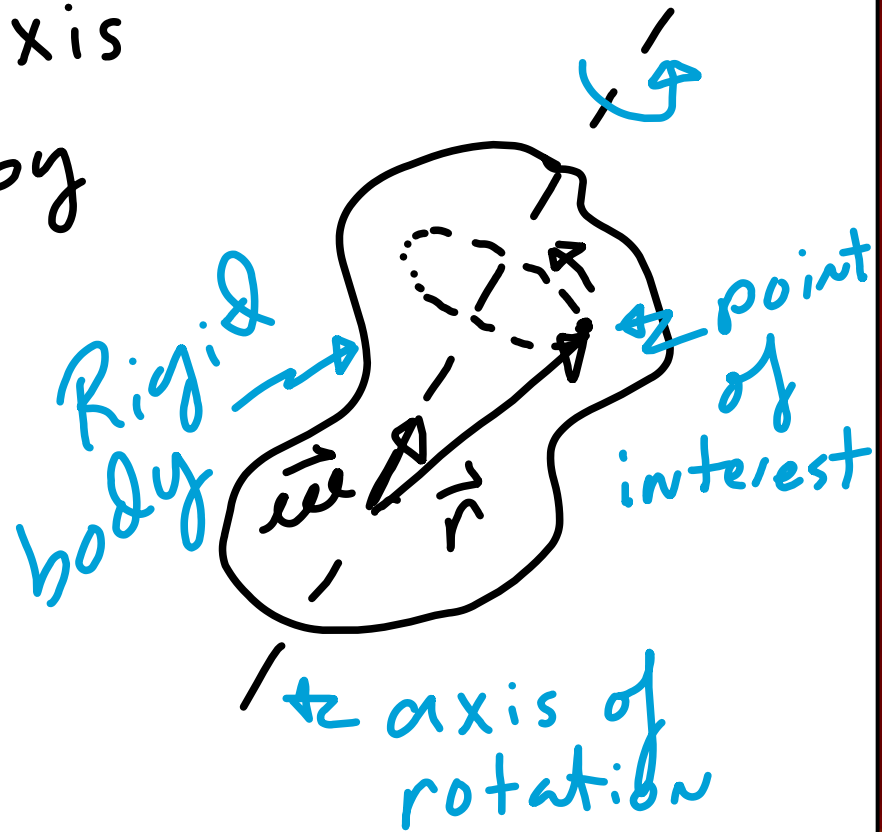




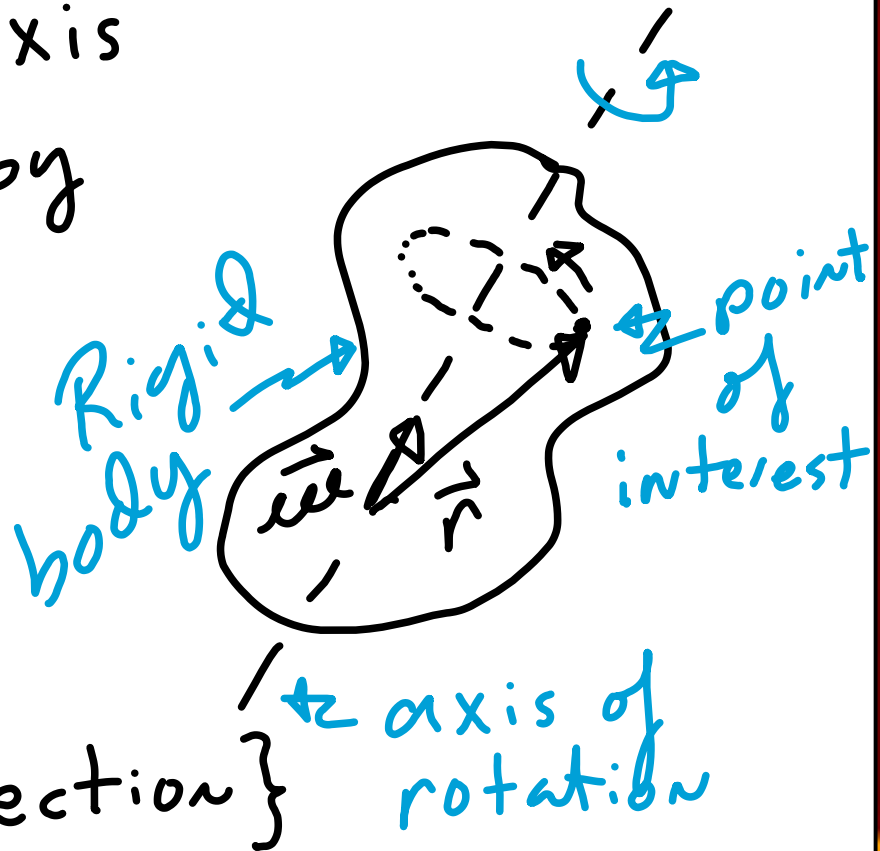
\vec{e} is along axis
of rotation defined by
the right hand rule



\vec{e}_e is along axis
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the right hand rule
{ grab axis of rotation
such that fingers wrap
in direction of rotation &



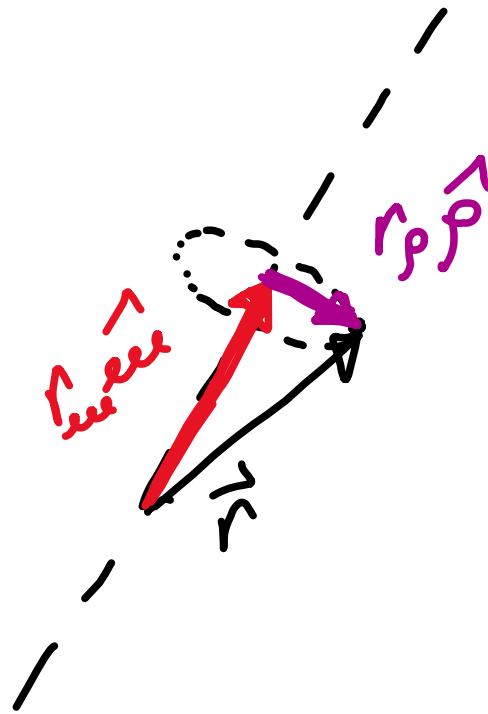
\vec{e} is along axis
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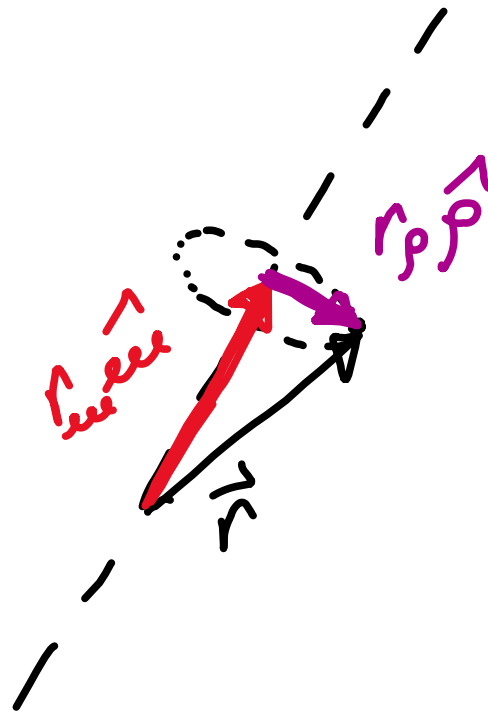
We can break up \vec{r} into
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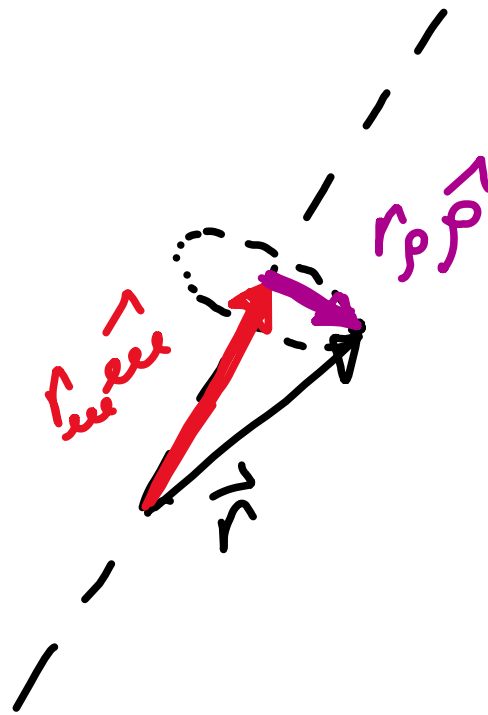
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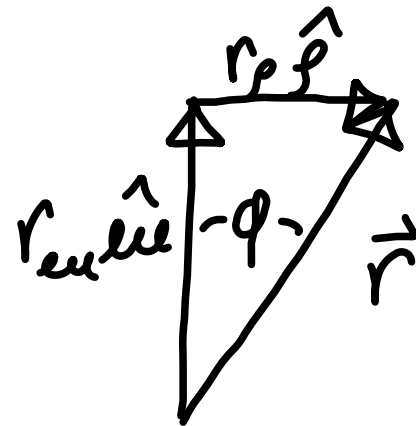
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& \hat{e} : $\vec{r} = r_{ee} \hat{e} + r_{\rho} \hat{\rho}$



We can break up \vec{r} into orthogonal directions of $\hat{\rho}$ & \hat{z} : $\vec{r} = r_{zz} \hat{z} + r_{\rho} \hat{\rho}$

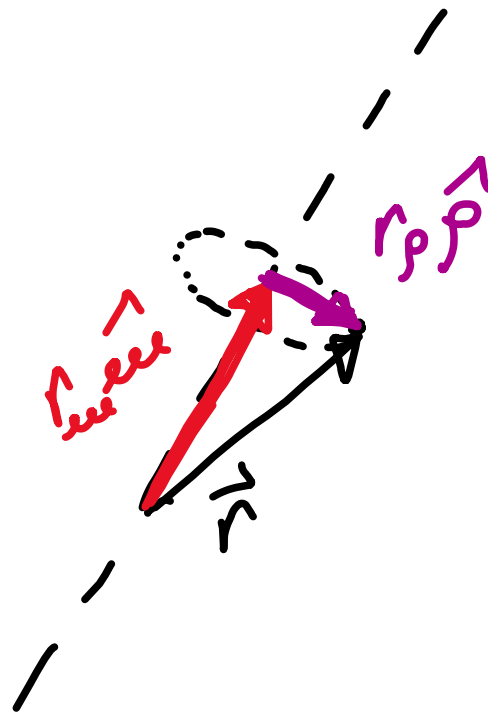


"side" view:

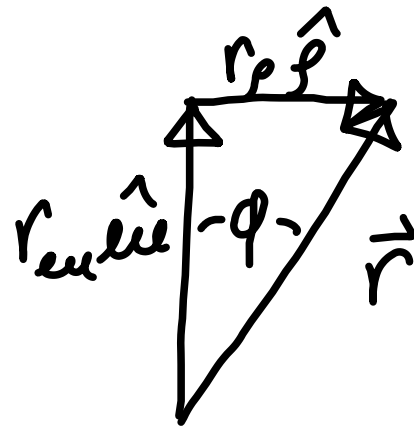


We can break up \vec{r} into orthogonal directions of $\hat{\rho}$ & \hat{u} : $\vec{r} = r_{uu} \hat{u} + r_{\rho} \hat{\rho}$

Now $\vec{r} = r \cos \phi \hat{u} + r \sin \phi \hat{\rho}$



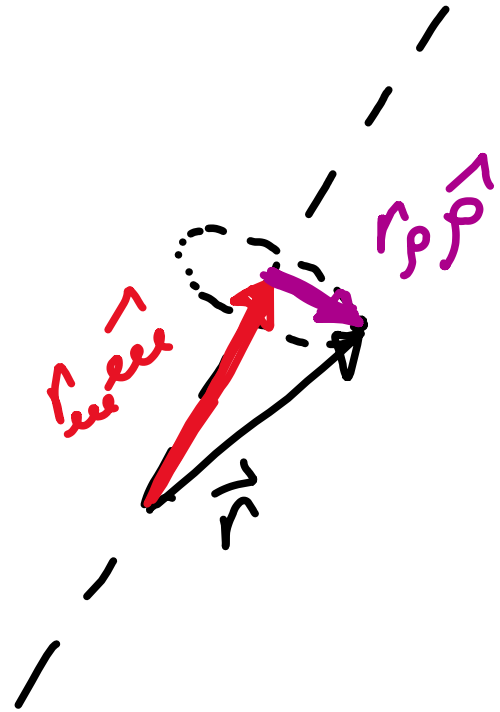
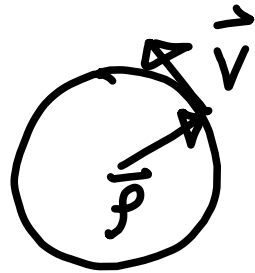
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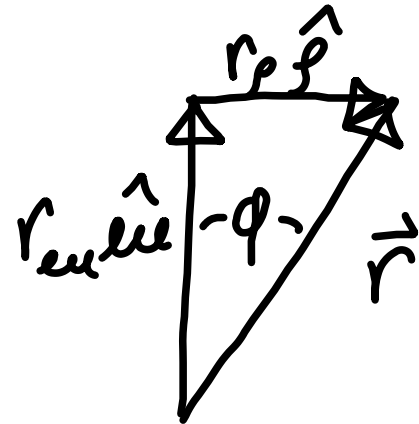
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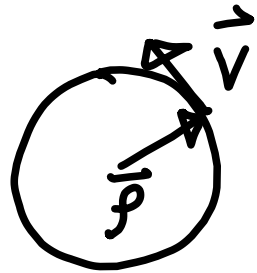
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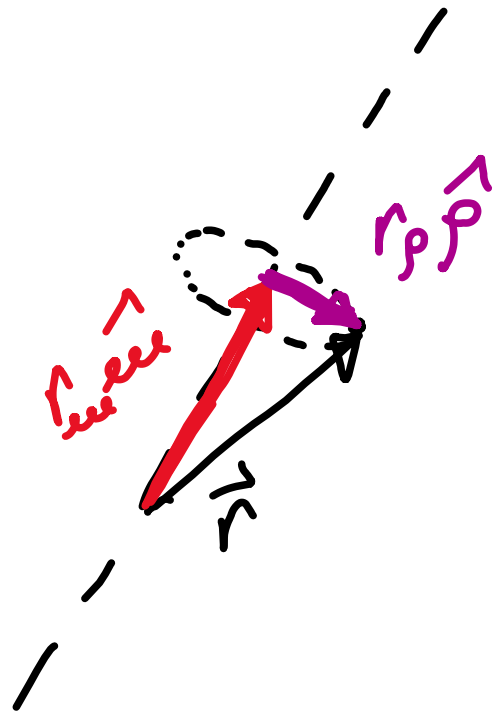
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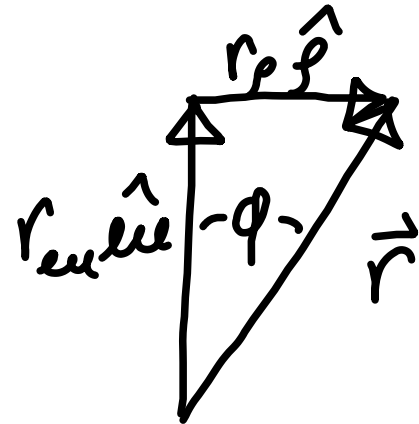


we know that

$$v = \rho \dot{\theta} = \rho u$$



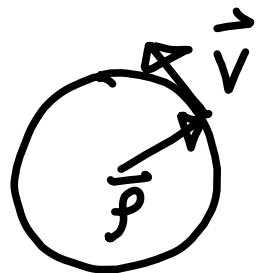
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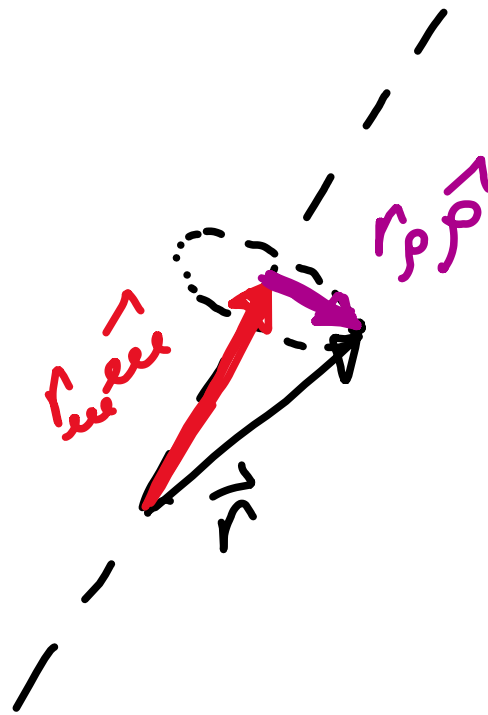
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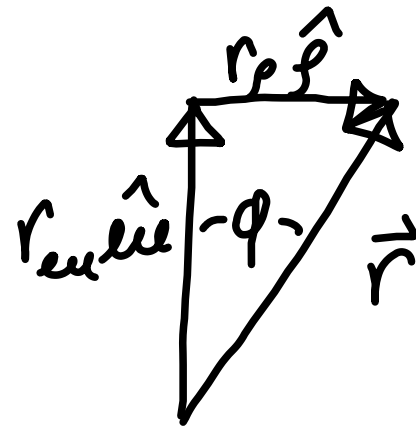


we know that

$v = \rho \dot{\theta} = \rho \omega$, but $\rho = r \sin \phi$



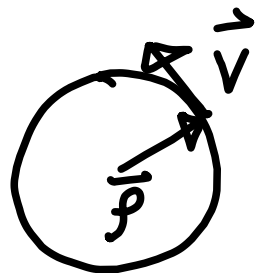
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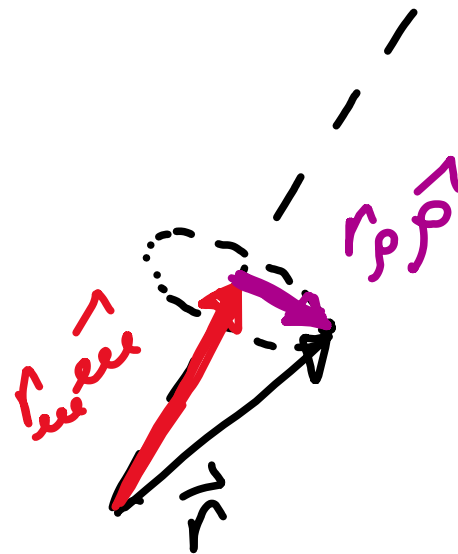
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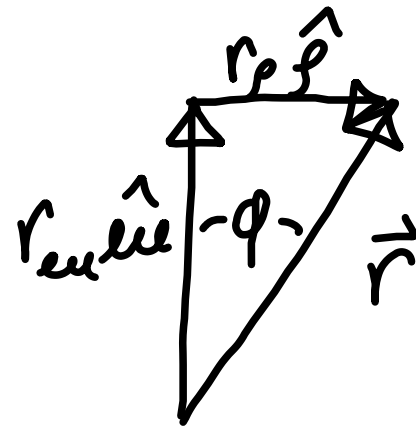
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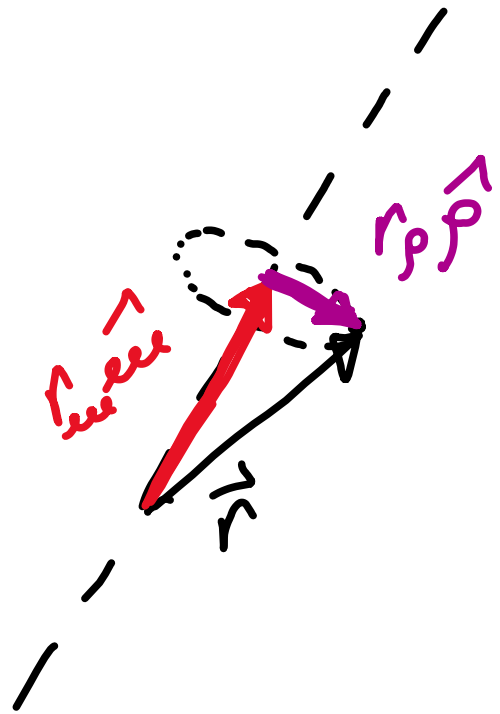
$\Rightarrow v = (r \sin \phi) u$



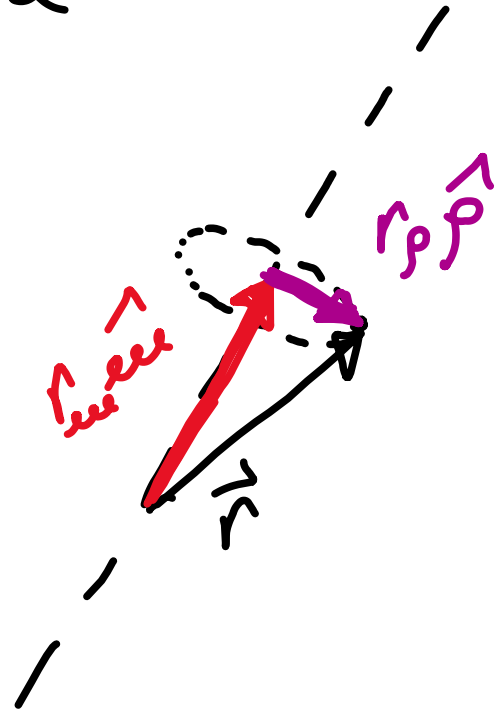
"Side" view:



We have $v = (r \sin \phi) \hat{e}_\phi$

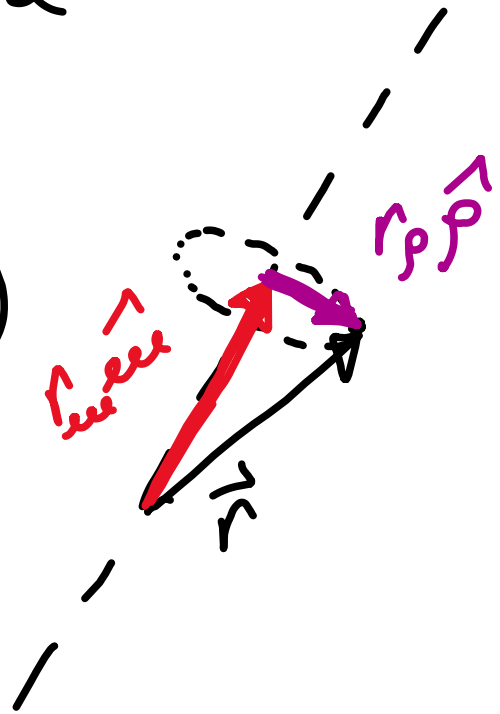


We have $v = (r \sin \phi) \hat{e}_\phi$ and
only need to obtain the
direction:



We have $v = (r \sin \phi) \omega \hat{e}_\phi$ and
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direction:

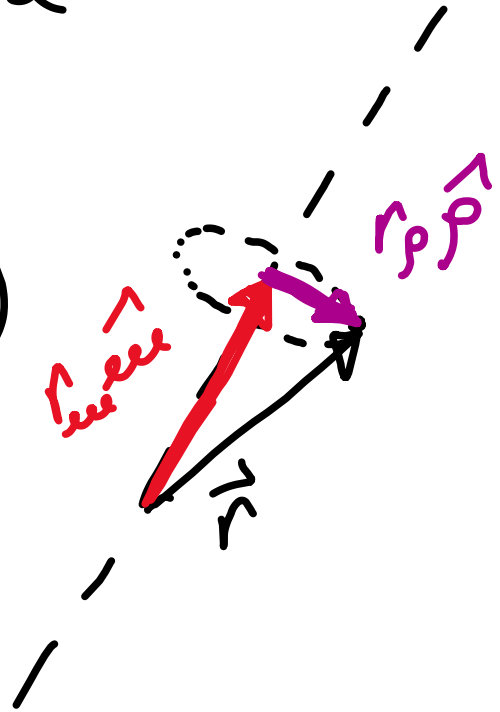
Note: $\hat{e}_\phi \times \vec{r} = \omega \hat{e}_\phi \times (r \hat{e}_r + \rho \hat{e}_\phi)$



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Note: $\hat{e}_\phi \times \hat{r} = \hat{e}_\theta \times (r \hat{e}_r + \rho \hat{\rho})$

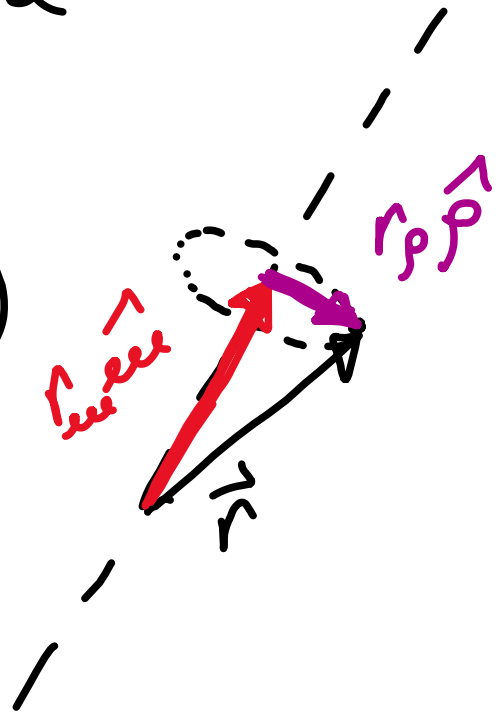
But $\rho = r_p = r \sin \phi$



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But $\mathbf{e}_\phi \times \mathbf{e}_r = \mathbf{0}$

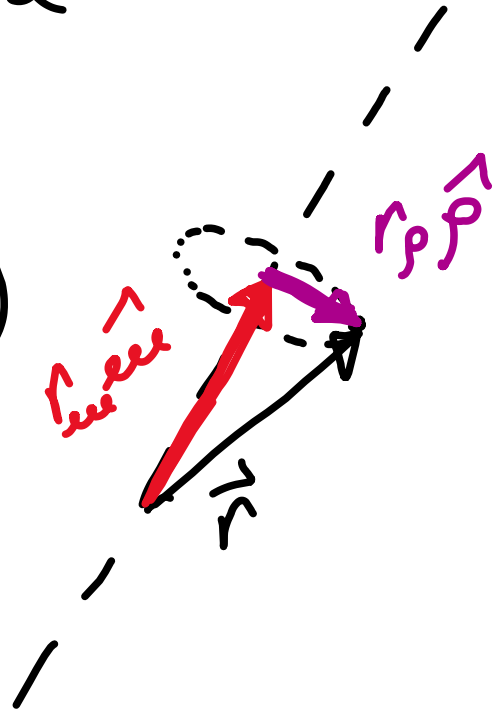


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But $\hat{e}_\phi \times \hat{e}_r = \hat{\theta}$ so

$$\hat{e}_\phi \times \vec{r} = \omega \rho \hat{e}_\phi \times \hat{\rho}$$

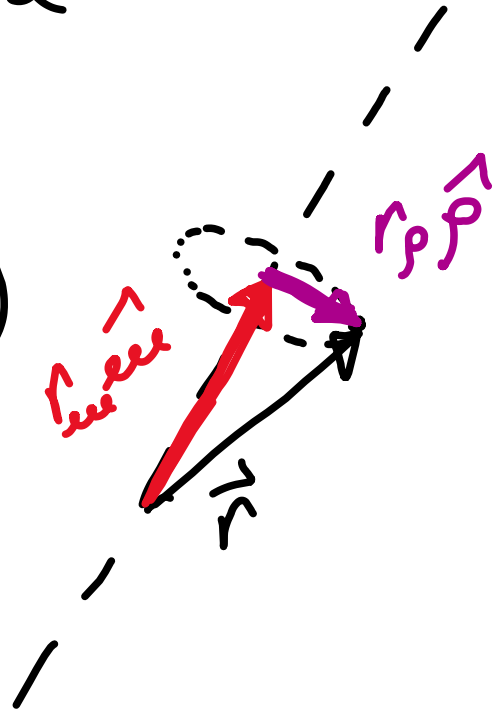


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But $\hat{e}_r \times \hat{e}_\phi = 0$ so

$\vec{e}_\phi \times \vec{r} = \omega \rho \hat{e}_\phi \times \hat{\rho}$ But
 $|\hat{e}_r \times \hat{\rho}| = 1$



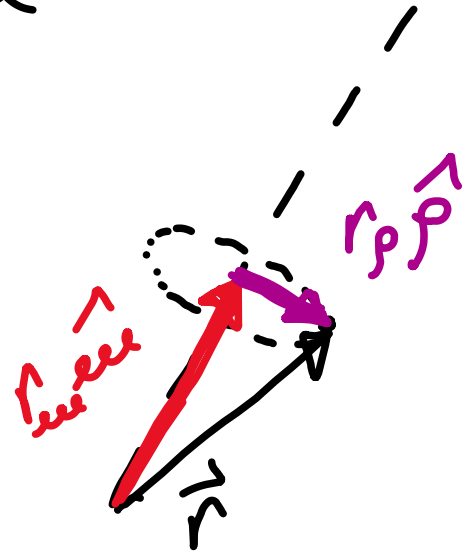
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But $\hat{e}_r \times \hat{e}_\phi = 0$ so

$\hat{e}_\phi \times \vec{r} = r \hat{e}_\phi \times \hat{p}$ But

$|\hat{e}_r \times \hat{p}| = 1$ [\hat{e}_r orthogonal to \hat{p}]



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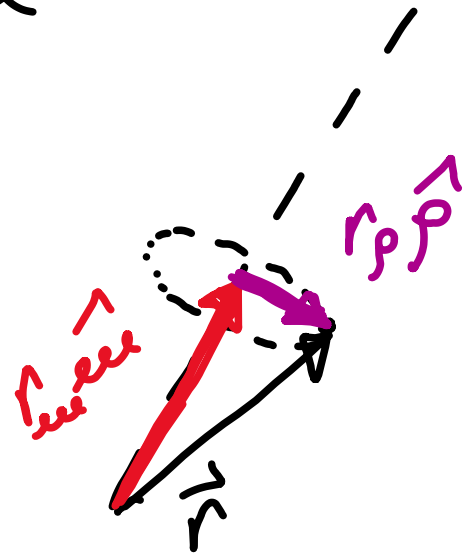
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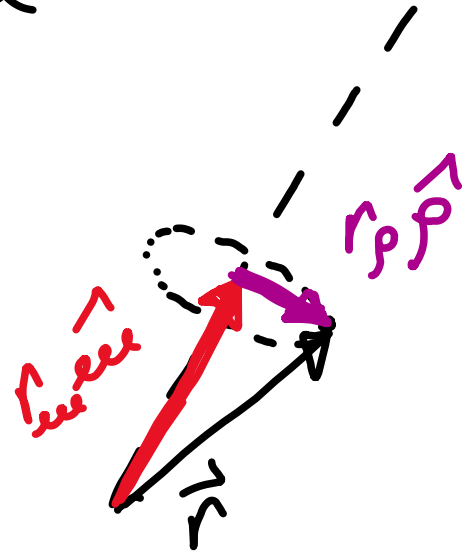
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Even better, $\hat{e}_\phi \times \hat{p} = \hat{v}$ [\hat{v} orthogonal to \hat{e}_r & \hat{p} points the correct way]



We have $v = (r \sin \phi) \hat{e}_\phi$ and only need to obtain the direction:

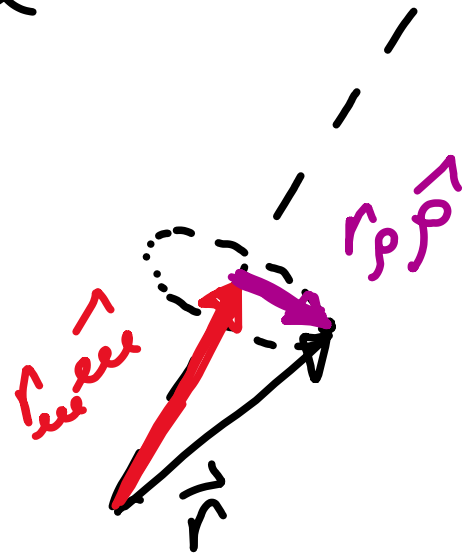
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Even better, $\hat{e}_\phi \times \hat{p} = \hat{v}$ [\hat{v} orthogonal to \hat{e}_r & \hat{p} points the correct way] so $\hat{e}_\phi \times \vec{r} = v \hat{v}$



We have $v = (r \sin \phi) \hat{e}_\phi$ and only need to obtain the direction:

Note: $\hat{e}_\theta \times \vec{r} = \hat{e}_\theta \times (r \hat{e}_r + \rho \hat{p})$

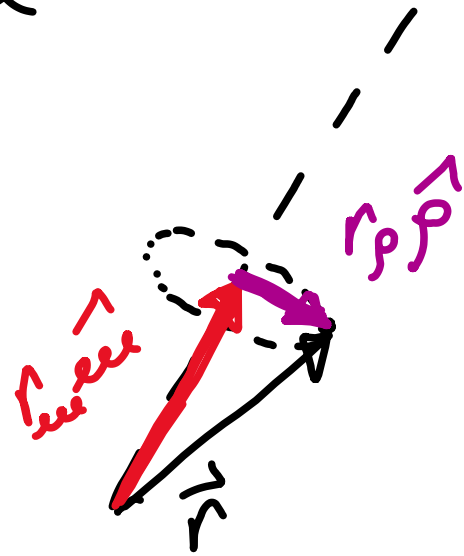
But $\hat{e}_\theta \times \hat{e}_\theta = \mathbf{0}$ so

$\hat{e}_\theta \times \vec{r} = r \hat{e}_\theta \times \hat{e}_r + \rho \hat{e}_\theta \times \hat{p}$ But

$|\hat{e}_\theta \times \hat{p}| = 1$ [\hat{e}_θ orthogonal to \hat{p}]

Even better, $\hat{e}_\theta \times \hat{p} = \hat{v}$ [\hat{v} orthogonal to \hat{e}_θ & \hat{p} points the correct way] so $\hat{e}_\theta \times \vec{r} = v \hat{v}$

$\Rightarrow \hat{v} = \hat{e}_\theta \times \vec{r}$



We have seen that $\vec{v} = \vec{e} \times \vec{r}$

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$$\dot{\vec{r}} = \vec{v} \quad \& \quad \text{we will define } \vec{\alpha} \equiv \frac{d\dot{\vec{e}}}{dt}$$

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$\dot{\vec{r}} = \vec{v}$ & we will define $\vec{\alpha} \equiv \frac{d\vec{e}}{dt}$ so

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{e} \times \vec{v}$$

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What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need
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$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} [\vec{e} \times \vec{r}] = \dot{\vec{e}} \times \vec{r} + \vec{e} \times \dot{\vec{r}}, \text{ but}$$

$\dot{\vec{r}} = \vec{v}$ & we will define $\vec{\alpha} \equiv \frac{d\vec{e}}{dt}$ so

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{e} \times \vec{v} \quad \& \text{ since } \vec{v} = \vec{e} \times \vec{r}$$

We have seen that $\vec{v} = \vec{e} \times \dot{\vec{r}}$
What about $\vec{a} = \frac{d\vec{v}}{dt}$? Just need
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$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} [\vec{e} \times \dot{\vec{r}}] = \dot{\vec{e}} \times \dot{\vec{r}} + \vec{e} \times \ddot{\vec{r}}, \text{ but}$$

$\dot{\vec{r}} = \vec{v}$ & we will define $\vec{\alpha} \equiv \frac{d\vec{e}}{dt}$ so

$$\vec{a} = \vec{\alpha} \times \dot{\vec{r}} + \vec{e} \times \vec{v} \quad \& \text{ since } \vec{v} = \vec{e} \times \dot{\vec{r}}$$

$$\text{then } \vec{a} = \vec{\alpha} \times \dot{\vec{r}} + \vec{e} \times (\vec{e} \times \dot{\vec{r}})$$

We have seen that $\vec{v} = \vec{e} \times \dot{\vec{r}}$
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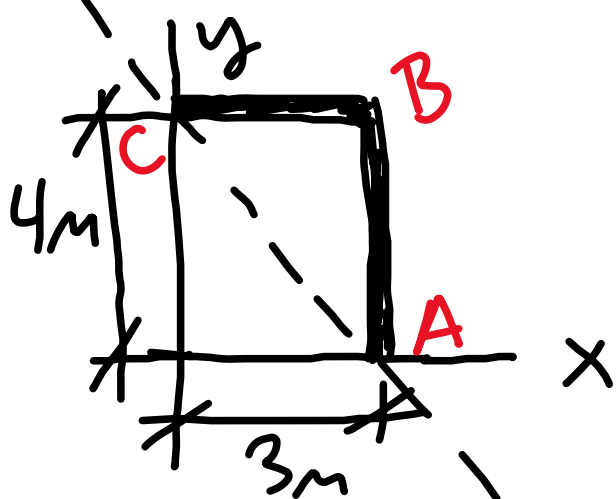
$\dot{\vec{r}} = \vec{v}$ & we will define $\vec{\alpha} \equiv \frac{d\vec{e}}{dt}$ so

$$\vec{a} = \vec{\alpha} \times \dot{\vec{r}} + \vec{e} \times \ddot{\vec{r}} \quad \& \text{ since } \vec{v} = \vec{e} \times \dot{\vec{r}}$$

then $\vec{a} = \vec{\alpha} \times \dot{\vec{r}} + \vec{e} \times (\vec{e} \times \ddot{\vec{r}})$

Example similar to problem 15.10

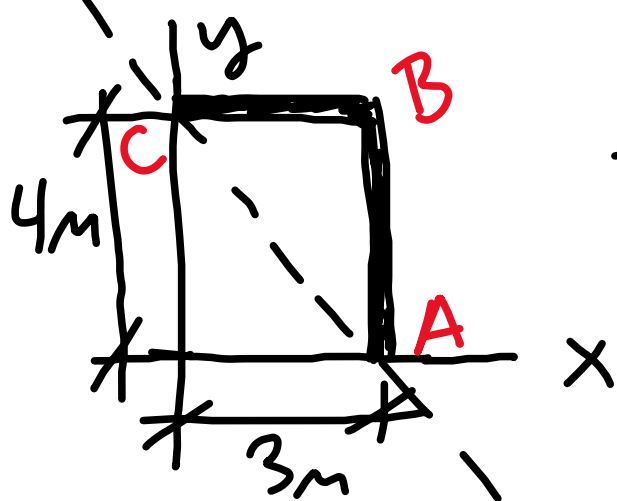
Example similar to problem 15.10



axis of rotation

Example similar to problem 15.10

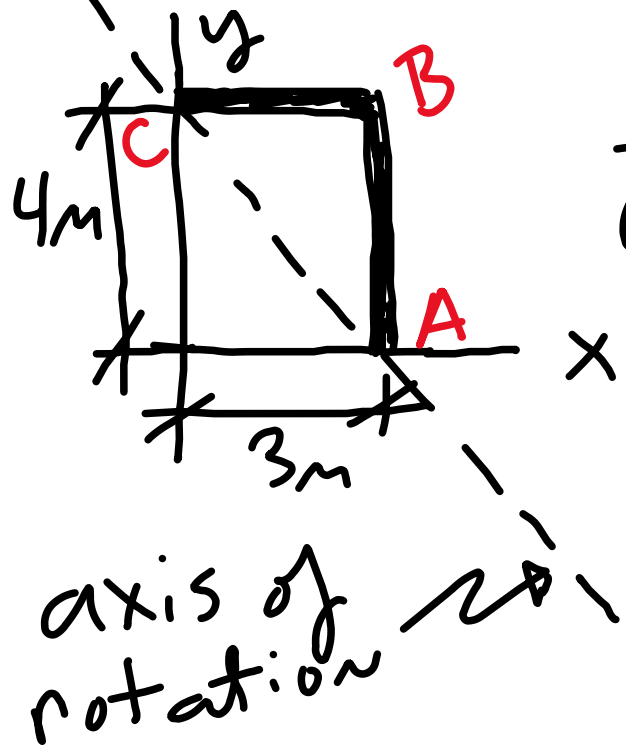
Rotation clockwise as viewed from point A.



axis of rotation

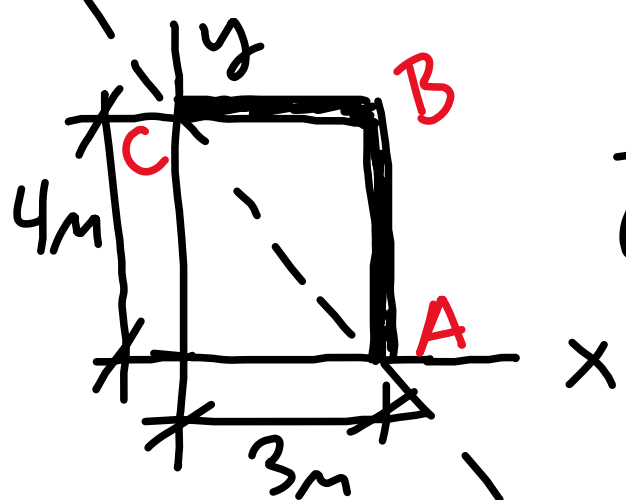
Example similar to problem 15.10

Rotation clockwise as viewed from point A. Find \vec{v}_B :



$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

Example similar to problem 15.10



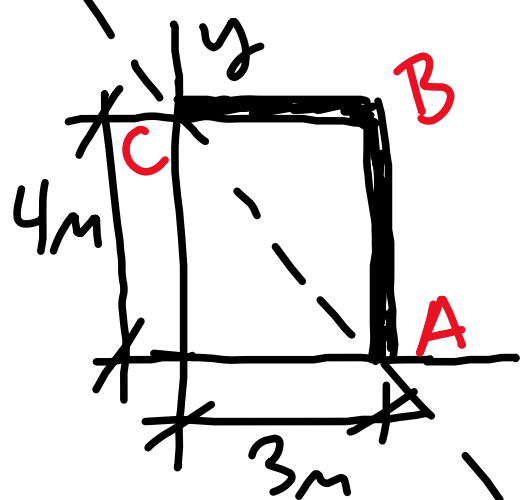
Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

Need to determine \vec{r} & $\vec{\omega}$.

axis of rotation

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

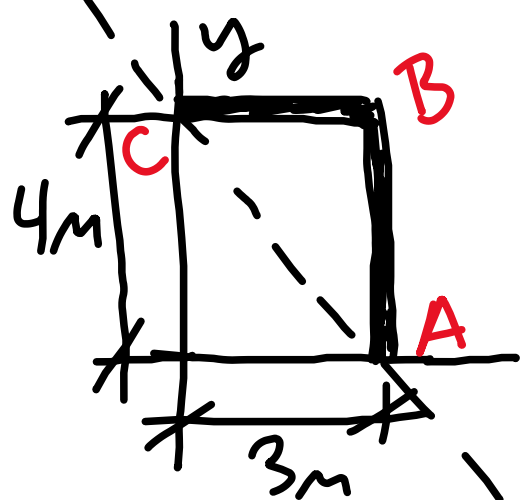
$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

Need to determine \vec{r} & $\vec{\omega}$.

axis of rotation

\vec{r} must start somewhere on the axis of rotation

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

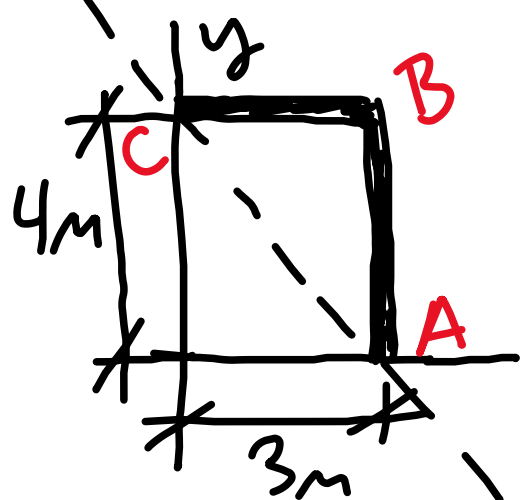
$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

Need to determine \vec{r} & $\vec{\omega}$.

axis of rotation

\vec{r} must start somewhere on the axis of rotation & end at point B.

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

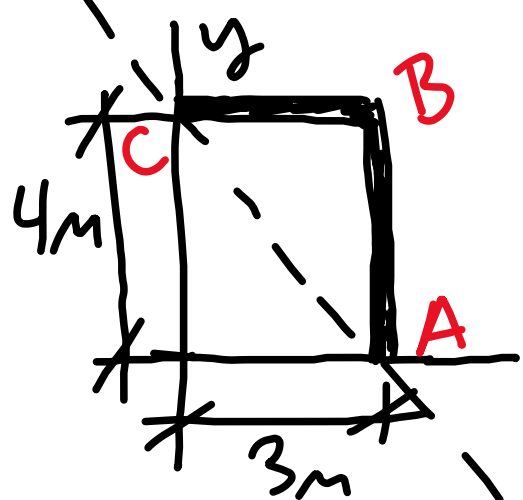
Need to determine \vec{r} & $\vec{\omega}$.

axis of rotation

\vec{r} must start somewhere

on the axis of rotation & end at point B. $\vec{\omega}$ must point along direction from point A to C.

Example similar to problem 15.10



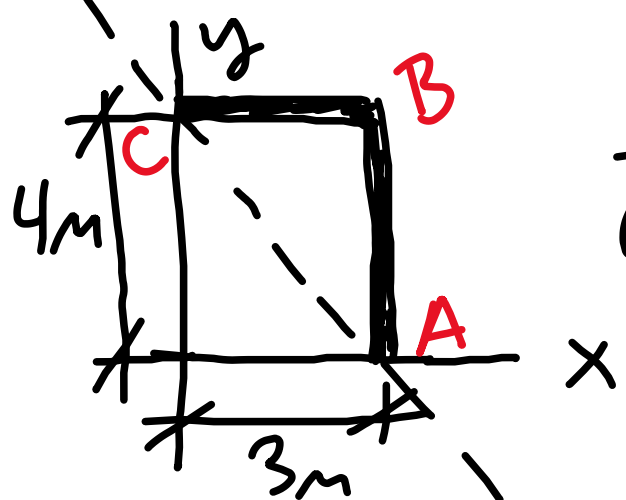
Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

ω -direction:

axis of rotation

Example similar to problem 15.10



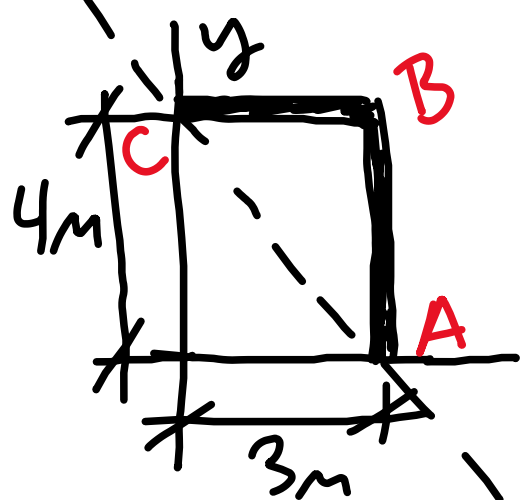
Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

$$\omega\text{-direction: } \hat{\omega} = \frac{\vec{r}_{CA}}{|\vec{r}_{CA}|}$$

axis of rotation \rightarrow

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

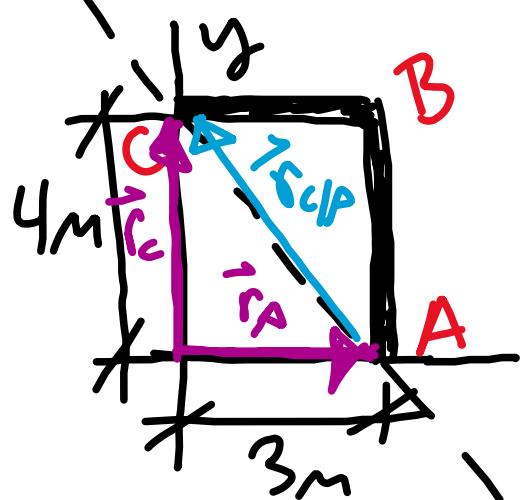
$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

ω -direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A$$

axis of rotation

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

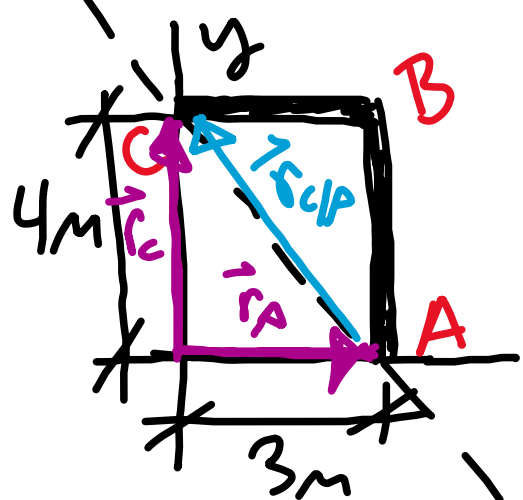
$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

ω -direction: $\hat{\omega} = \frac{\vec{r}_{c/A}}{|\vec{r}_{c/A}|}$, with

$$\vec{r}_{c/A} = \vec{r}_c - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

axis of rotation

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \vec{\omega} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

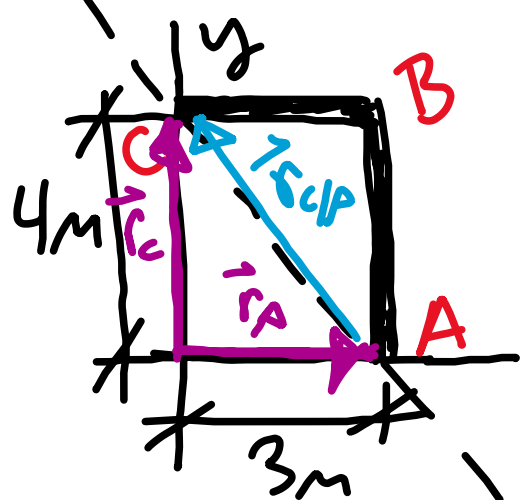
ω -direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

$$\Rightarrow |\vec{r}_{C/A}| = 5m$$

axis of rotation

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \underline{\underline{\hat{\omega}}} \times \vec{r}, \text{ with } \omega = 1 \frac{\text{rad}}{\text{s}}$$

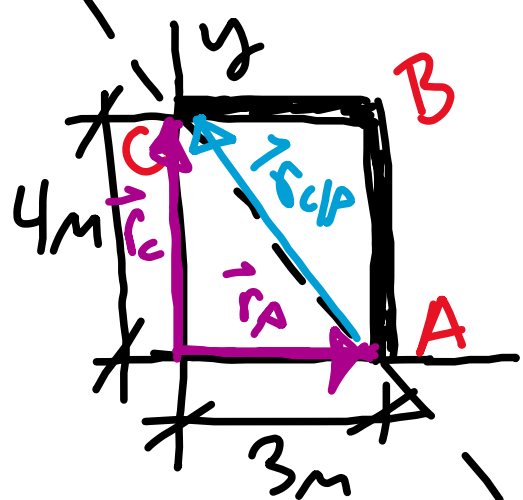
$\hat{\omega}$ -direction: $\hat{\omega} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

$$\Rightarrow |\vec{r}_{C/A}| = 5m \text{ Now } \hat{\omega} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

axis of rotation

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \underline{\underline{\hat{e}}} \times \vec{r}, \text{ with } \underline{\underline{e}} = 1 \frac{\text{rad}}{\text{s}}$$

$\underline{\underline{e}}$ -direction: $\hat{e} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

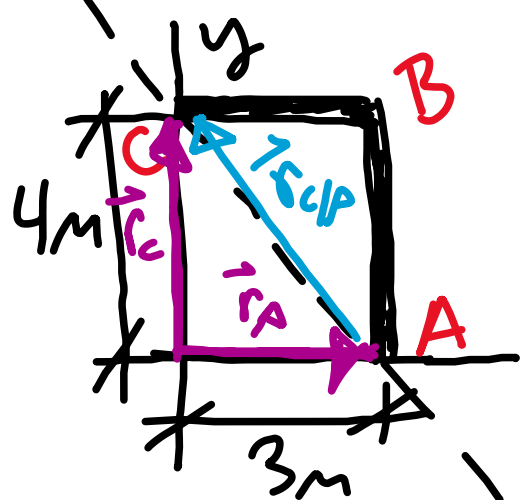
$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

$$\Rightarrow |\vec{r}_{C/A}| = 5m \text{ Now } \hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

axis of rotation

\vec{r} vector:

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \underline{\underline{\hat{e}_e}} \times \vec{r}, \text{ with } e = 1 \frac{\text{rad}}{\text{s}}$$

\hat{e}_e -direction: $\hat{e}_e = \frac{\vec{r}_{c/A}}{|\vec{r}_{c/A}|}$, with

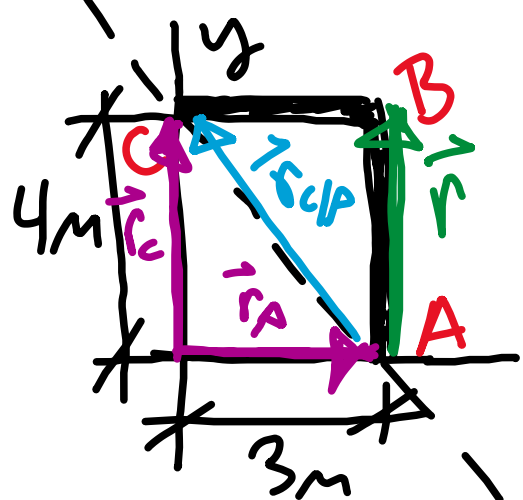
axis of rotation \curvearrowright

$$\vec{r}_{c/A} = \vec{r}_c - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

$$\Rightarrow |\vec{r}_{c/A}| = 5m \text{ Now } \hat{e}_e = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

\vec{r} vector: might as well start at point A

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \underline{\underline{\hat{e}_e}} \times \vec{r}, \text{ with } e = 1 \frac{\text{rad}}{\text{s}}$$

e -direction: $\hat{e}_e = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

axis of rotation \curvearrowright

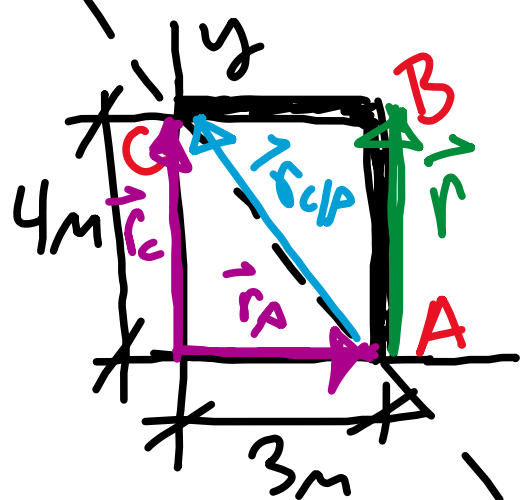
$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

$$\Rightarrow |\vec{r}_{C/A}| = 5m \text{ Now } \hat{e}_e = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

\vec{r} vector: might as well start at point A

$$\vec{r} = \vec{r}_{B/A}$$

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \underline{\underline{\hat{e}} \times \vec{r}}, \text{ with } \underline{\underline{e}} = 1 \frac{\text{rad}}{\text{s}}$$

$\underline{\underline{e}}$ -direction: $\hat{e} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

axis of rotation \curvearrowright

$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

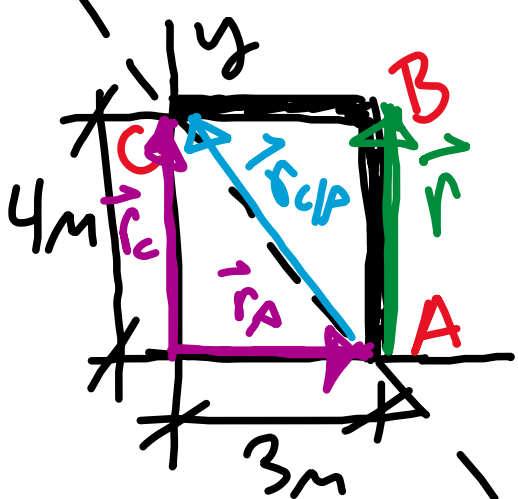
$$\Rightarrow |\vec{r}_{C/A}| = 5m \text{ Now } \hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

\vec{r} vector: might as well start at point A

$$\vec{r} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$$

Example similar to problem 15.10

Rotation clockwise as viewed from point A. Find \vec{v}_B :



$$\vec{v}_B = \underline{\underline{\hat{e}}} \times \vec{r}, \text{ with } \underline{\underline{\hat{e}}} = 1 \frac{\text{rad}}{\text{s}}$$

$\underline{\underline{\hat{e}}}$ -direction: $\underline{\underline{\hat{e}}} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

axis of rotation

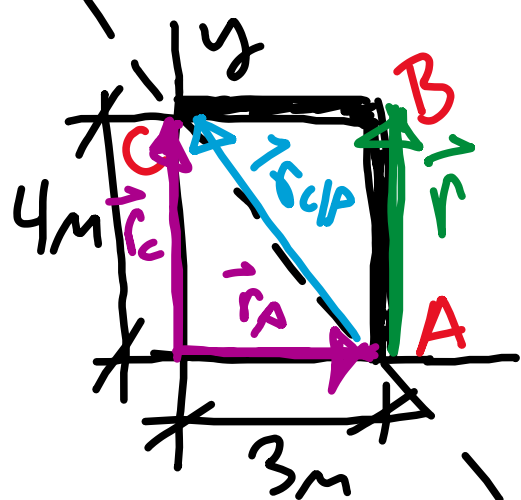
$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

$$\Rightarrow |\vec{r}_{C/A}| = 5m \text{ Now } \underline{\underline{\hat{e}}} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

\vec{r} vector: might as well start at point A

$$\vec{r} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = (3m\hat{x} + 4m\hat{y}) - 3m\hat{x}$$

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \underline{\underline{\hat{e}}} \times \vec{r}, \text{ with } \underline{\underline{\hat{e}}} = 1 \frac{\text{rad}}{\text{s}}$$

$\underline{\underline{\hat{e}}}$ -direction: $\underline{\underline{\hat{e}}} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

axis of rotation

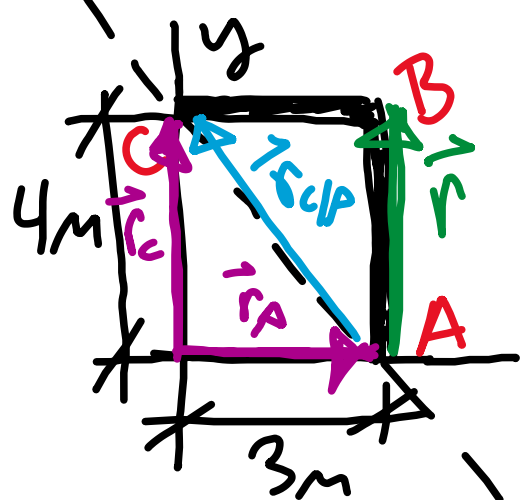
$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

$$\Rightarrow |\vec{r}_{C/A}| = 5m \text{ Now } \underline{\underline{\hat{e}}} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

\vec{r} vector: might as well start at point A

$$\vec{r} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = (3m\hat{x} + 4m\hat{y}) - 3m\hat{x} = 4m\hat{y}$$

Example similar to problem 15.10



Rotation clockwise as viewed from point A. Find \vec{v}_B :

$$\vec{v}_B = \underline{\underline{\hat{e}}} \times \vec{r}, \text{ with } \underline{\underline{\omega}} = 1 \frac{\text{rad}}{\text{s}}$$

$\underline{\underline{\omega}}$ -direction: $\hat{e} = \frac{\vec{r}_{C/A}}{|\vec{r}_{C/A}|}$, with

axis of rotation

$$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 4m\hat{y} - 3m\hat{x}$$

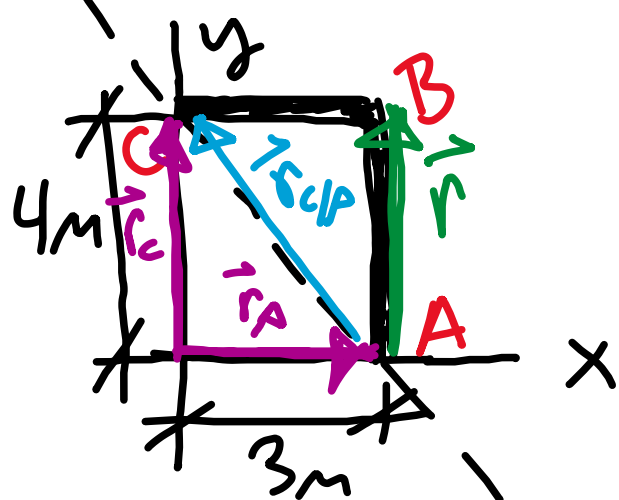
$$\Rightarrow |\vec{r}_{C/A}| = 5m \text{ Now } \hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

\vec{r} vector: might as well start at point A

$$\vec{r} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = (3m\hat{x} + 4m\hat{y}) - 3m\hat{x} = 4m\hat{y}$$

Could have just read off of diagram

Example similar to problem 15.10

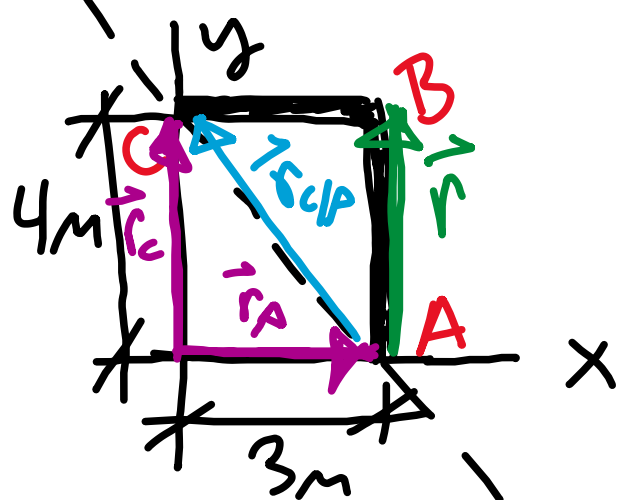


Now we have

$$\hat{e}_{ll} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x}$$

axis of rotation

Example similar to problem 15.10



Now we have

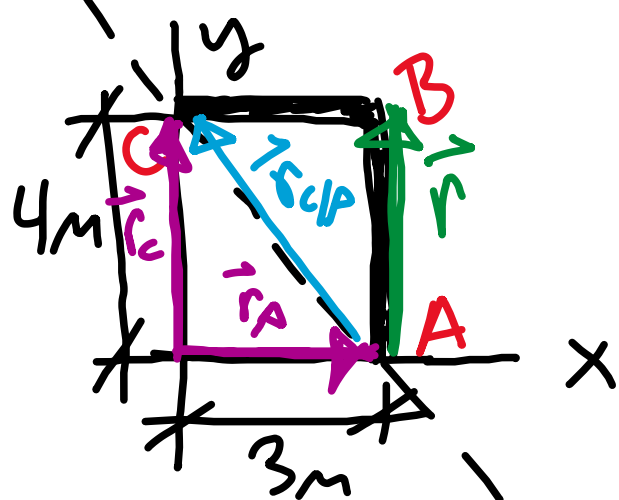
$$\hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \quad \& \quad \vec{r} = 4m\hat{y}$$

So

$$\vec{v}_B = \hat{e} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \right) e \right] \times 4m\hat{y}$$

axis of rotation

Example similar to problem 15.10



Now we have

$$\hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \quad \& \quad \vec{r} = 4m\hat{y}$$

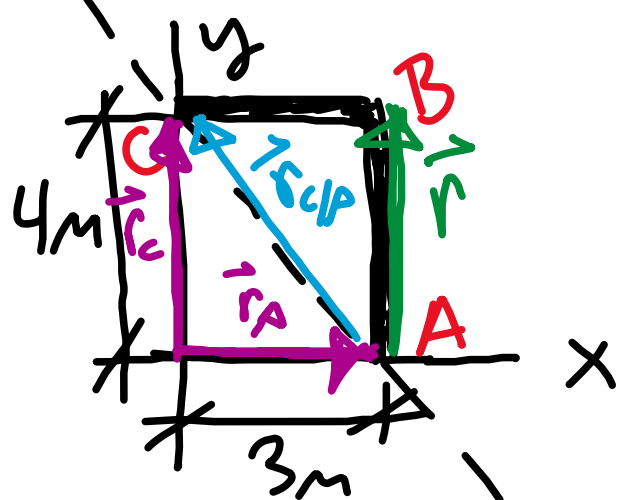
So

$$\vec{v}_B = \hat{e} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \right) e \right] \times 4m\hat{y}$$

$$\vec{v}_B = -\left(\frac{12}{5}\right) \frac{m}{s} \hat{z}$$

axis of rotation

Example similar to problem 15.10



Now we have

$$\hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \quad \& \quad \vec{r} = 4m\hat{y}$$

So

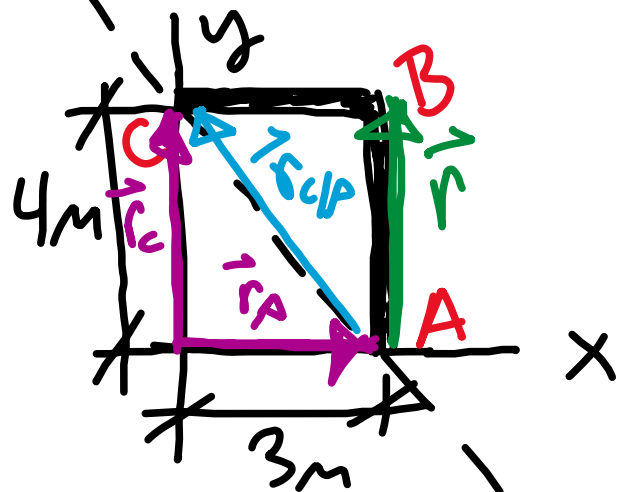
$$\vec{v}_B = \hat{e} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \right) e \right] \times 4m\hat{y}$$

$$\vec{v}_B = -\left(\frac{12}{5}\right) \frac{m}{s} \hat{z}$$

To find \vec{a}_B

axis of rotation

Example similar to problem 15.10



Now we have

$$\hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \quad \& \quad \vec{r} = 4m\hat{y}$$

So

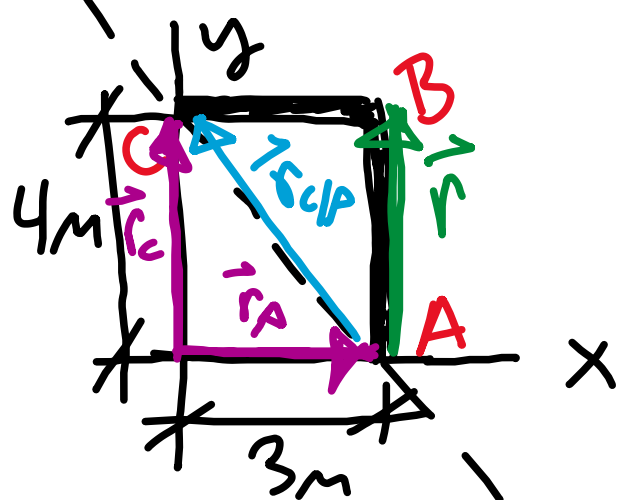
$$\vec{v}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \right) \omega \right] \times 4m\hat{y}$$

$$\vec{v}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\hat{z}$$

To find $\vec{a}_B = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

axis of rotation

Example similar to problem 15.10



Now we have

$$\hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \quad \& \quad \vec{r} = 4m\hat{y}$$

So

$$\vec{v}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \right) \omega \right] \times 4m\hat{y}$$

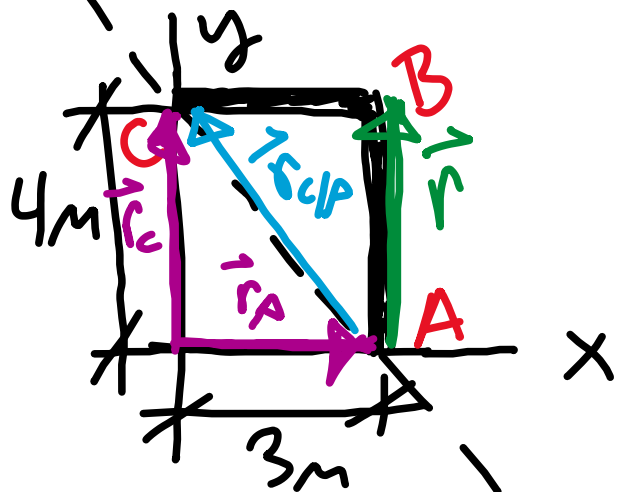
$$\vec{v}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\hat{z}$$

To find $\vec{a}_B = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Just notice that $\vec{\alpha} = 0$

axis of rotation \curvearrowright

Example similar to problem 15.10



Now we have

$$\hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \quad \& \quad \vec{r} = 4m\hat{y}$$

So

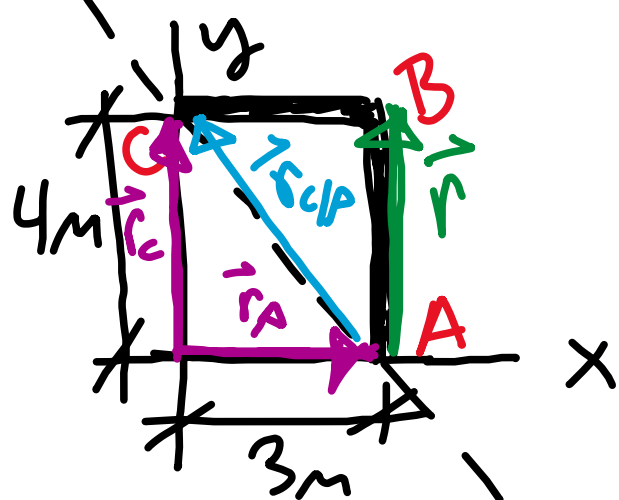
$$\vec{v}_B = \vec{\omega} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \right) \omega \right] \times 4m\hat{y}$$

$$\vec{v}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\hat{z}$$

To find $\vec{a}_B = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Just notice that $\vec{\alpha} = 0$ & $\vec{\omega} \times \vec{r} = \vec{v}_B$

Example similar to problem 15.10



Now we have

$$\hat{e} = \frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \quad \& \quad \vec{r} = 4m\hat{y}$$

So

$$\vec{v}_B = \vec{e} \times \vec{r} = \left[\left(\frac{4}{5}\hat{y} - \frac{3}{5}\hat{x} \right) e \right] \times 4m\hat{y}$$

$$\vec{v}_B = -\left(\frac{12}{5}\right)\frac{m}{s}\hat{z}$$

To find $\vec{a}_B = \vec{\alpha} \times \vec{r} + \vec{e} \times (\vec{e} \times \vec{r})$

Just notice that $\vec{\alpha} = 0$ & $\vec{e} \times \vec{r} = \vec{v}_B$

$$\Rightarrow \vec{a}_B = \vec{e} \times \vec{v}_B$$

Angular Kinematics

Angular Kinematics

Just as $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt}$

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Just as $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt}$ & $a = v \frac{dv}{dx}$
 $\omega = \frac{d\theta}{dt}$

Angular Kinematics

Just as $v = \frac{dx}{dt}$ & $a = \frac{dv}{dt}$ & $a = v \frac{dv}{dx}$
 $\omega = \frac{d\theta}{dt}$ & $\alpha = \frac{d\omega}{dt}$

Angular Kinematics

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$$\& \quad \frac{2\pi \text{ rad}}{360^\circ} = 1$$

