Today 15.2

L14

Today 15.2

L14

General plane motion, velocity

L14

Today 15.2

Tuesday 15.3

$\boxed{\text{L14}}$

Today 15.2

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Instantaneous center of rotation

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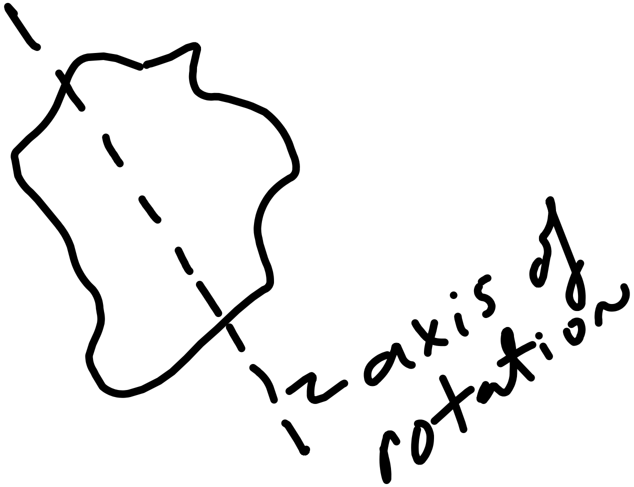

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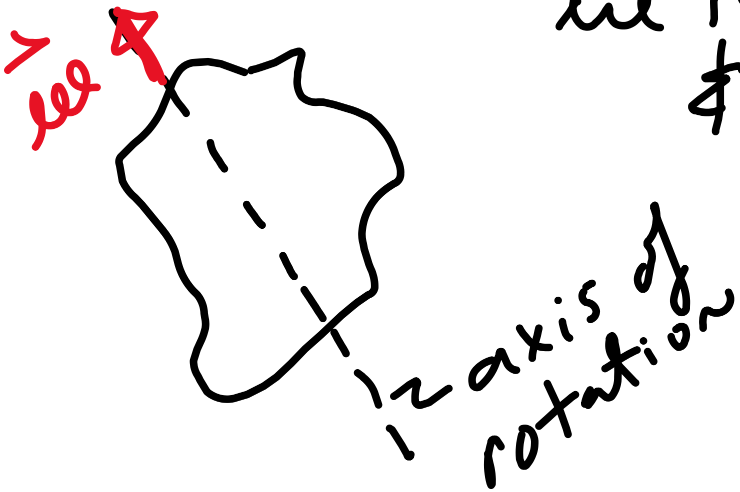

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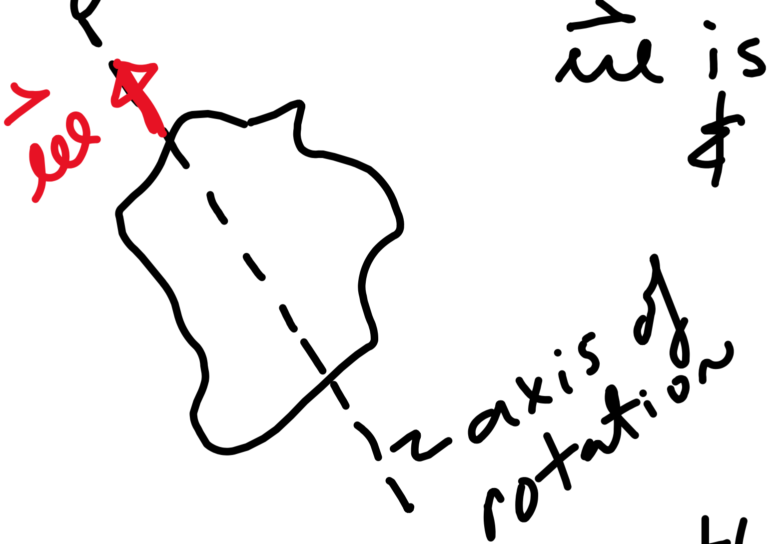

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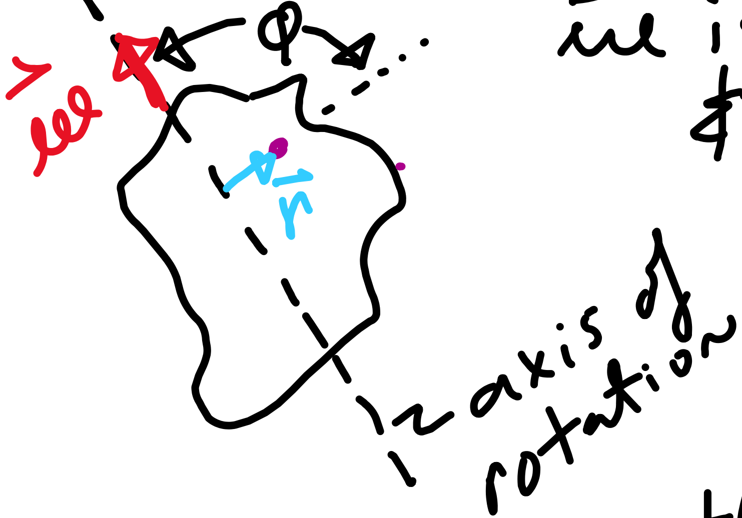

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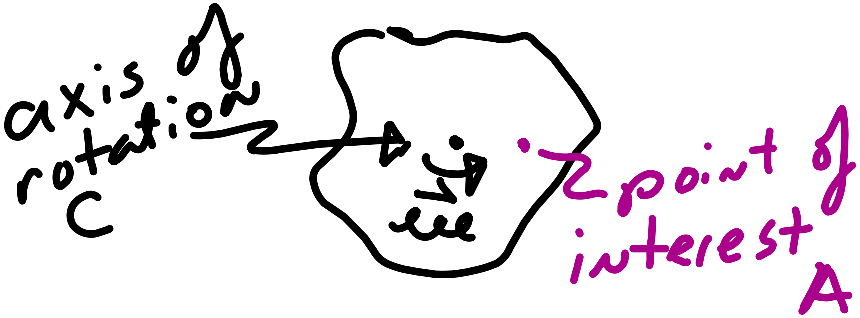

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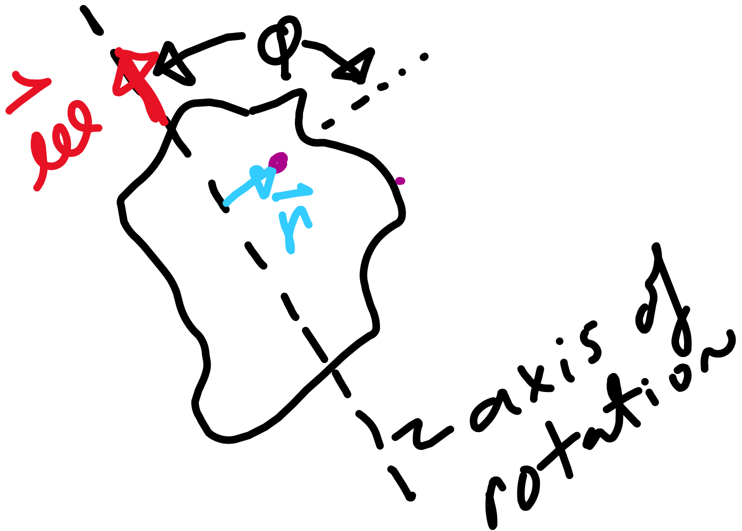

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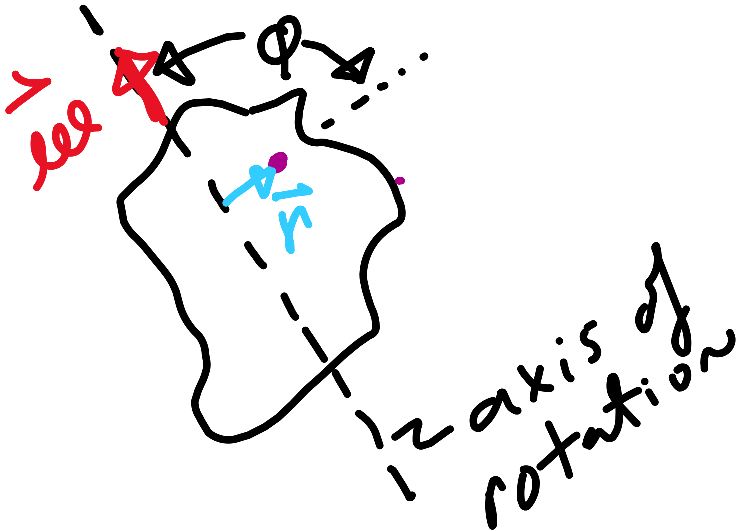

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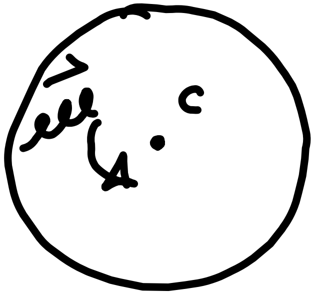
Fixed axis rotation of a wheel



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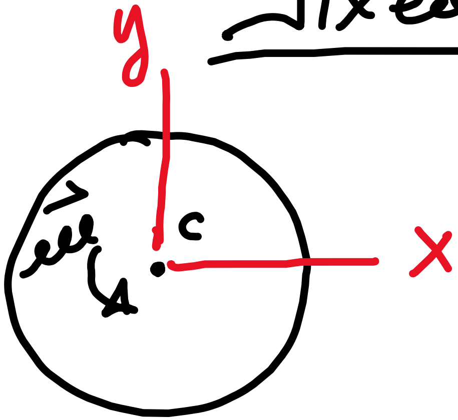

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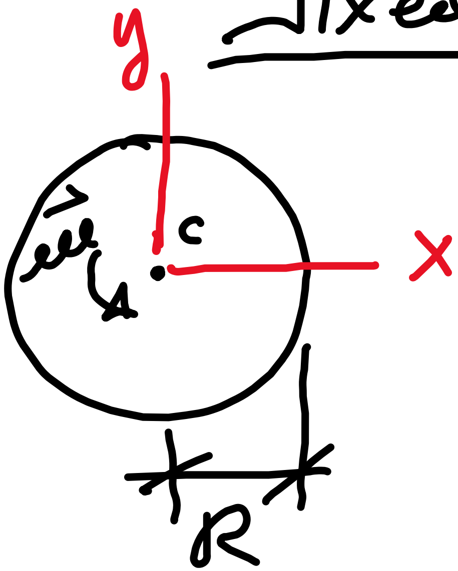

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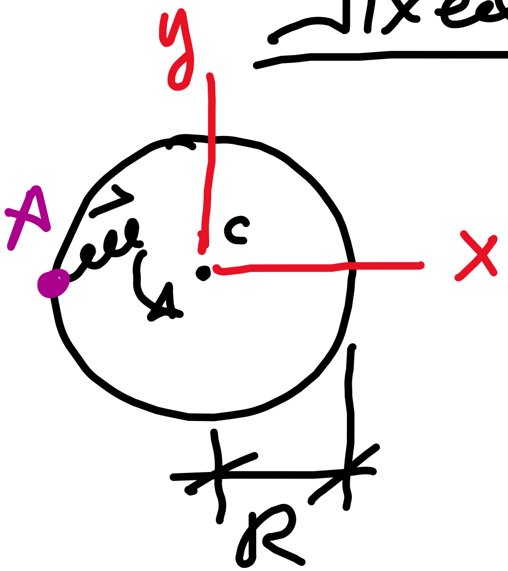

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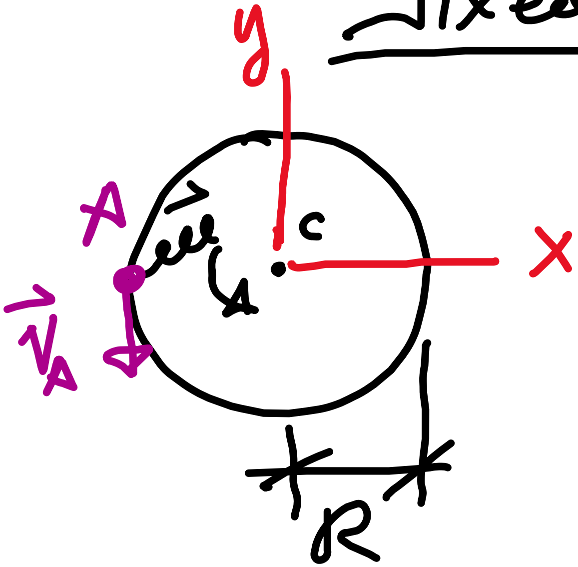

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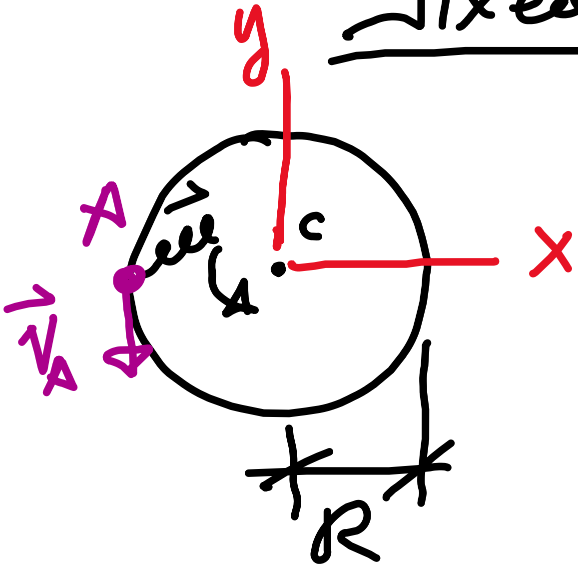

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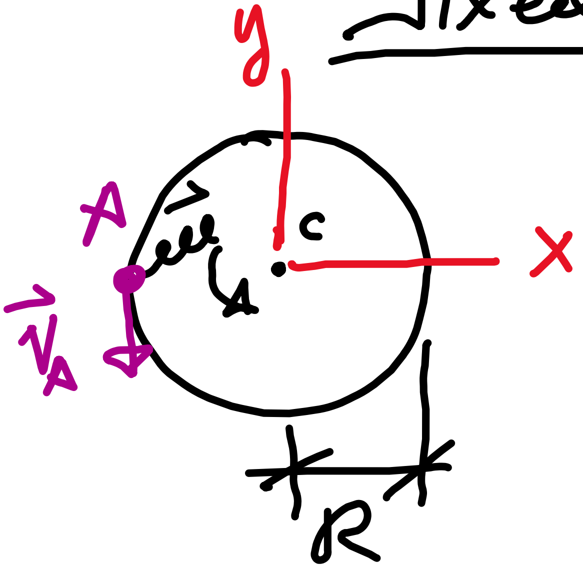

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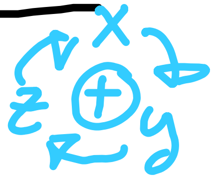

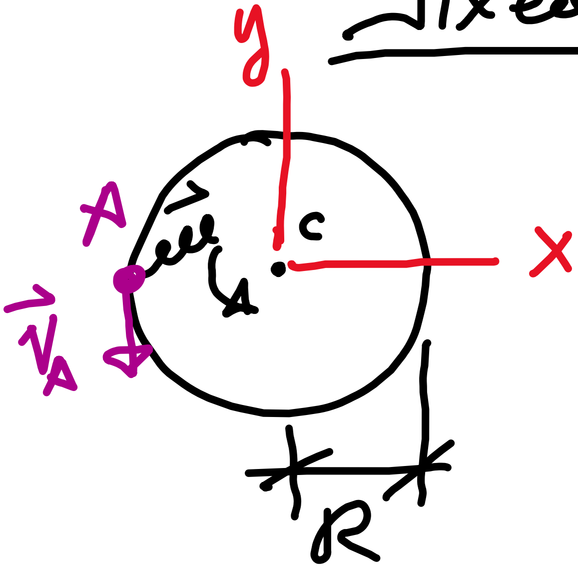

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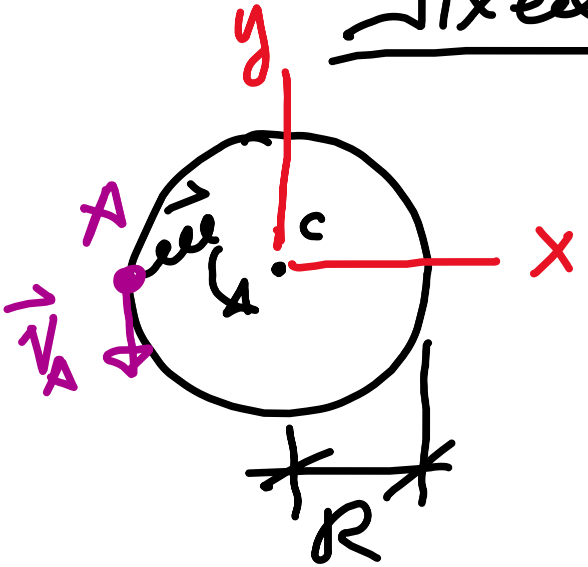

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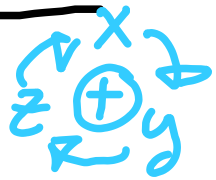

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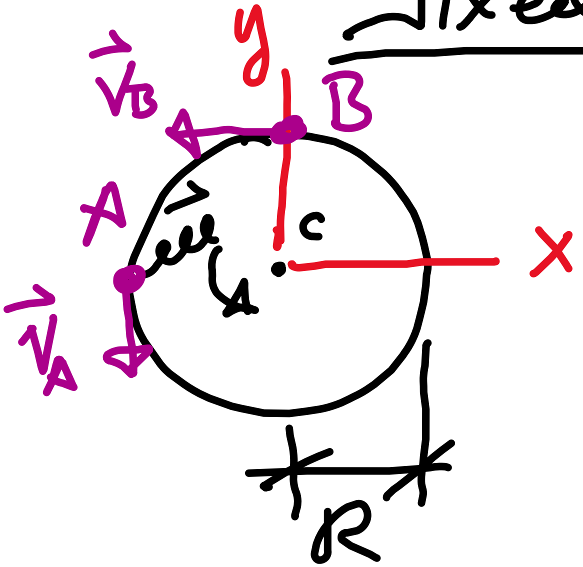

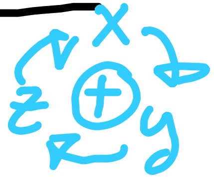

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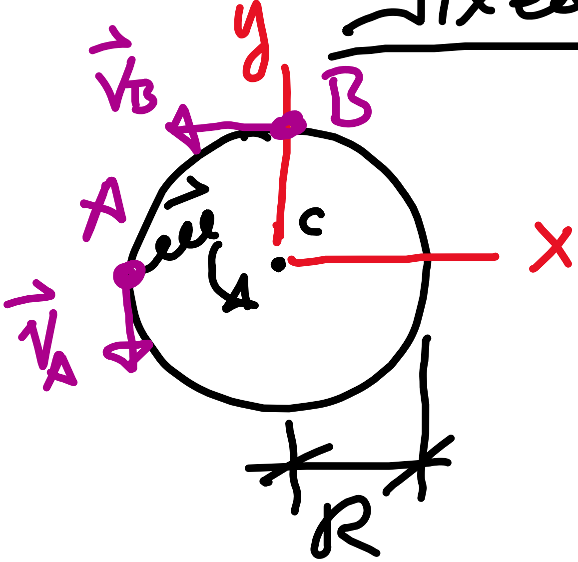

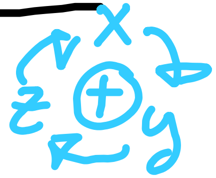

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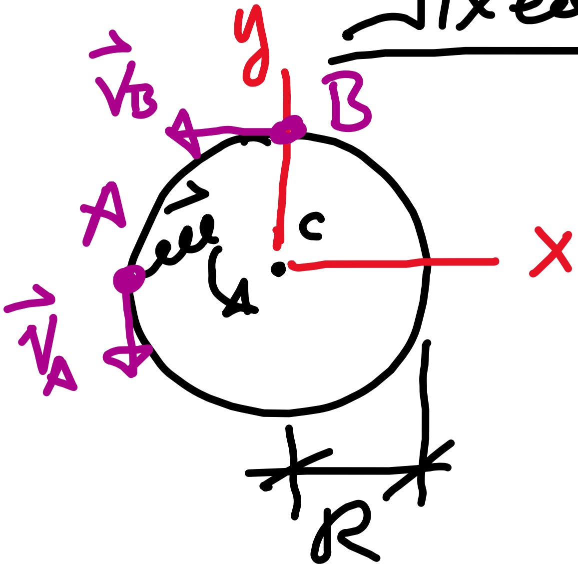

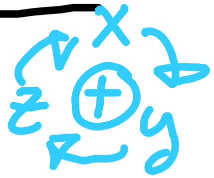

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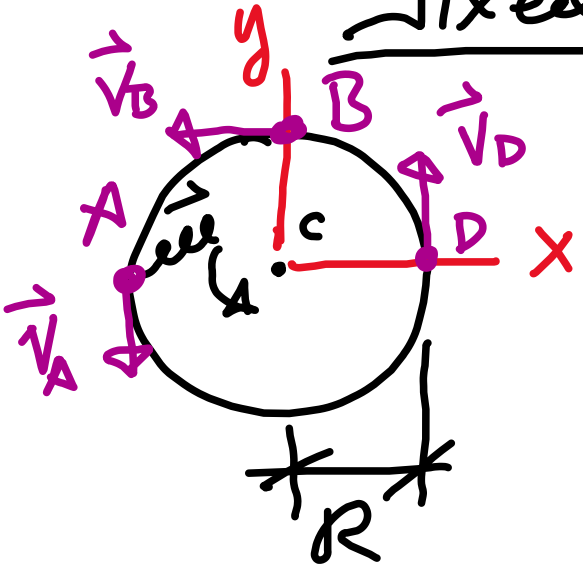

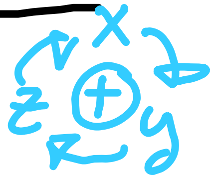

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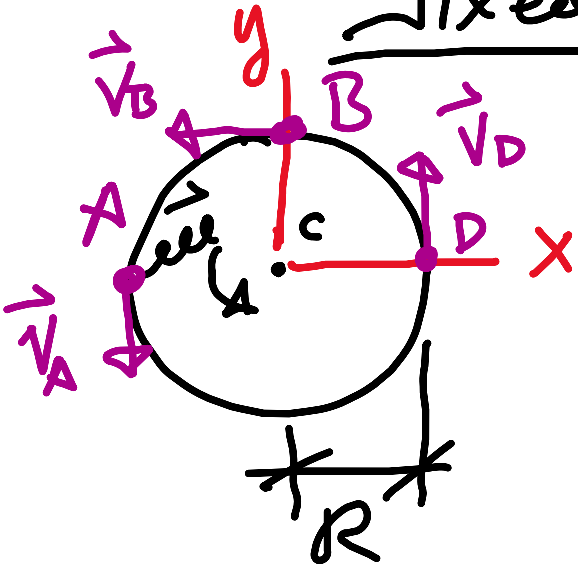

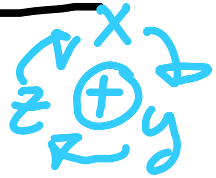

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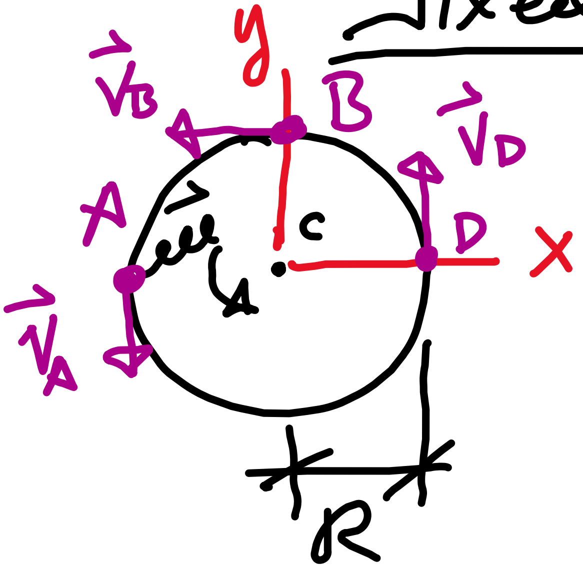

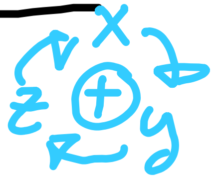

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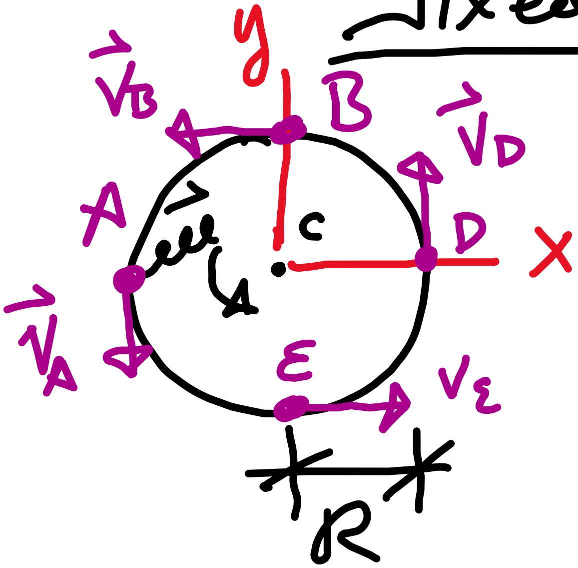

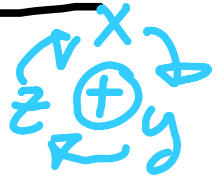

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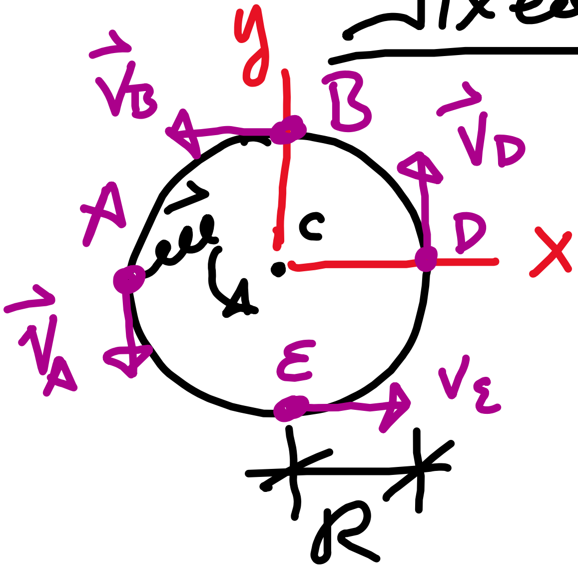

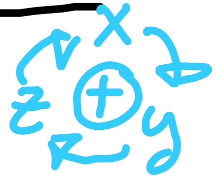

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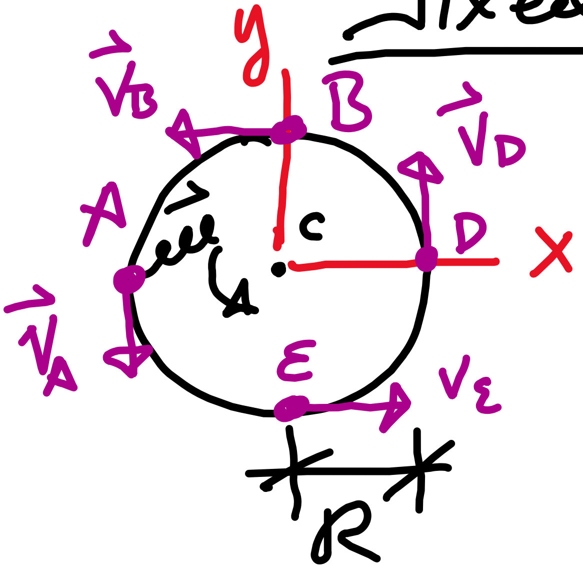

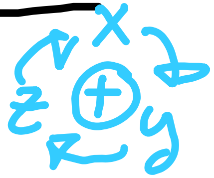

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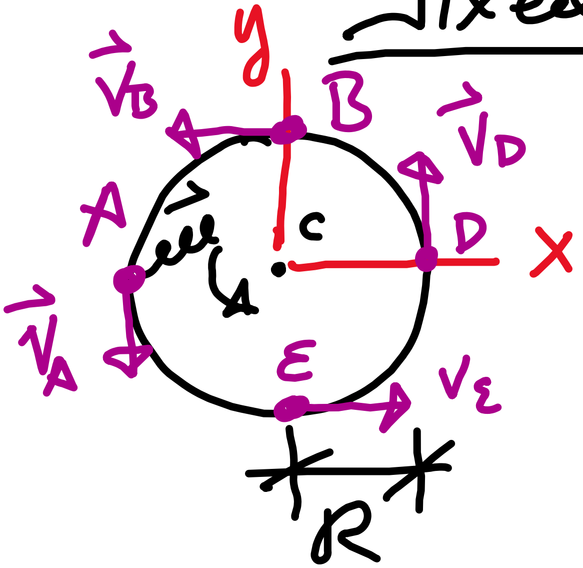

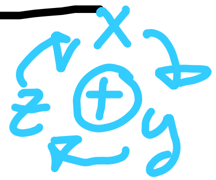

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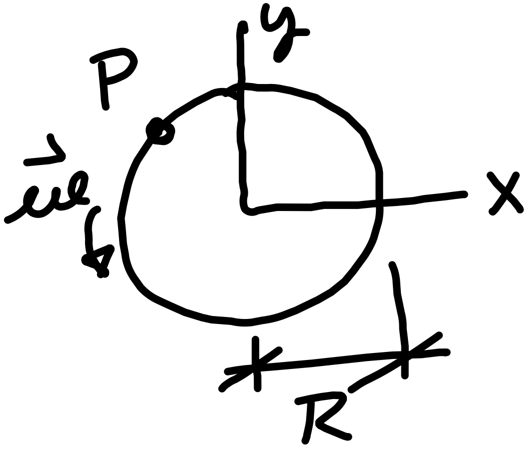
For some arbitrary point P:



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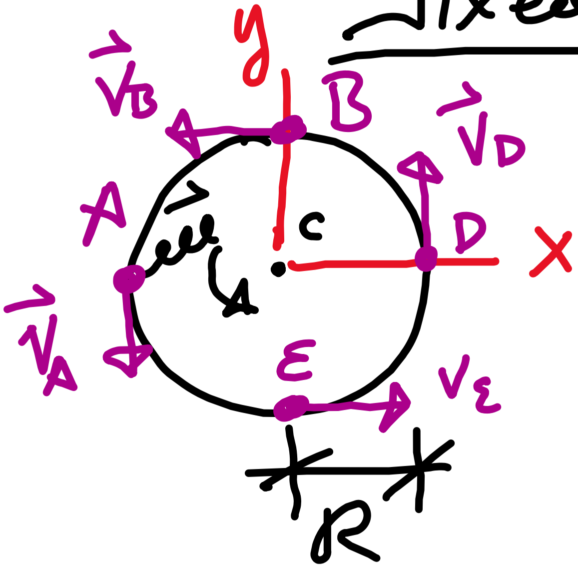

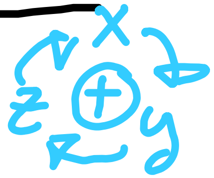

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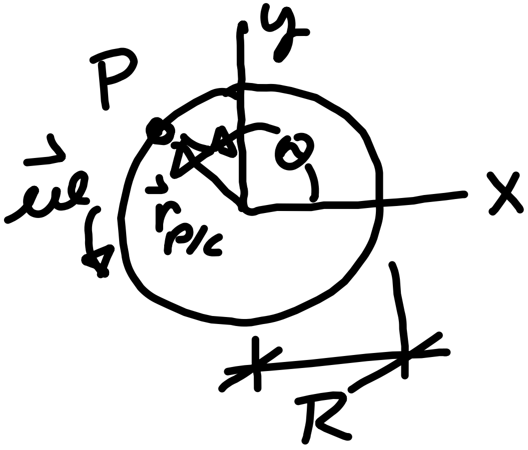
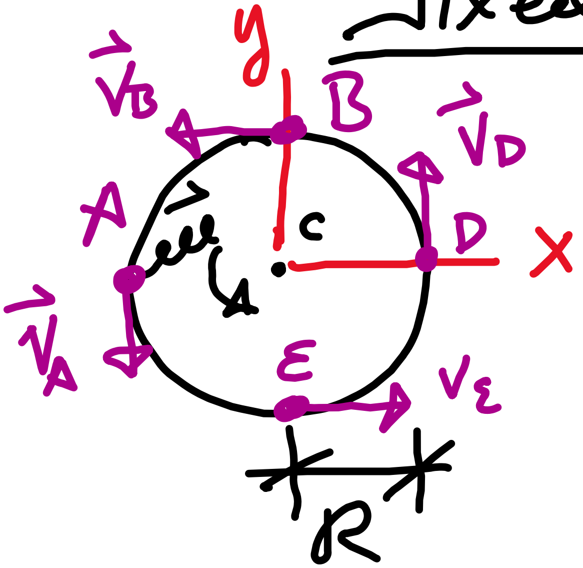
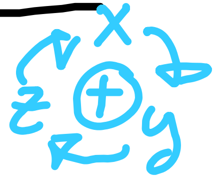


Fixed axis rotation of a wheel



Here $\vec{\omega} = \omega \hat{z}$



$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C} = \omega R \hat{z} \times (-\hat{x}) = \omega R(-\hat{y})$$

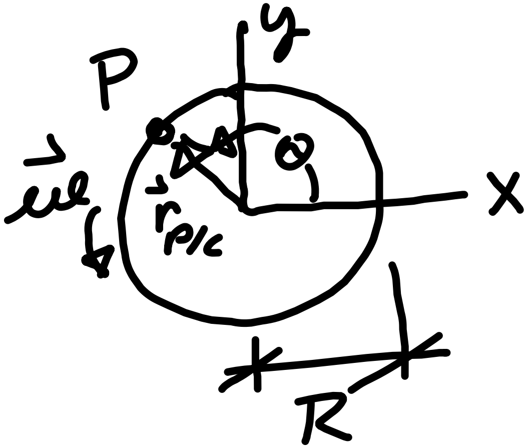
$$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/C} = \omega R \hat{z} \times \hat{y} = \omega R(-\hat{x})$$

$$\vec{V}_D = \vec{\omega} \times \vec{r}_{D/C} = \omega R \hat{z} \times \hat{x} = \omega R \hat{y}$$

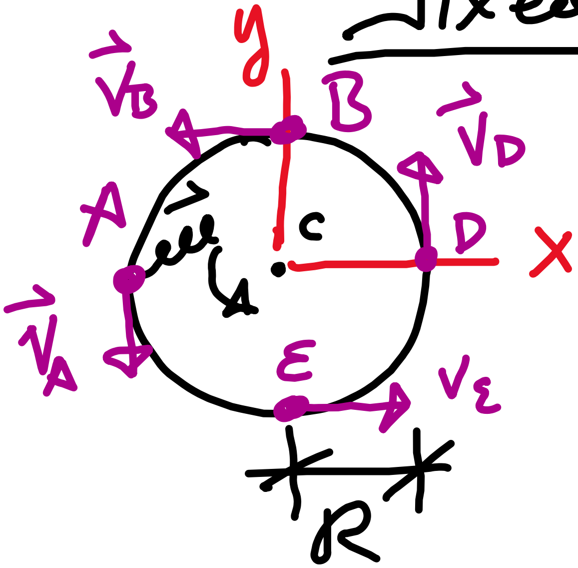
$$\vec{V}_E = \vec{\omega} \times \vec{r}_{E/C} = \omega R \hat{z} \times (-\hat{y}) = \omega R \hat{x}$$

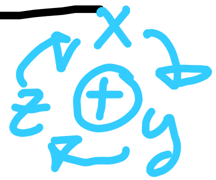
For some arbitrary point P:

$$\vec{V}_P = \vec{\omega} \times \vec{r}_{P/C}$$



Fixed axis rotation of a wheel





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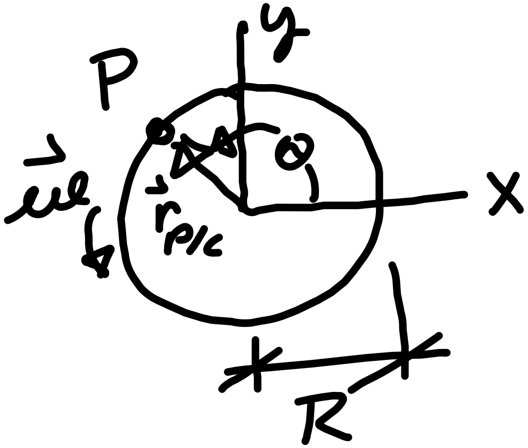

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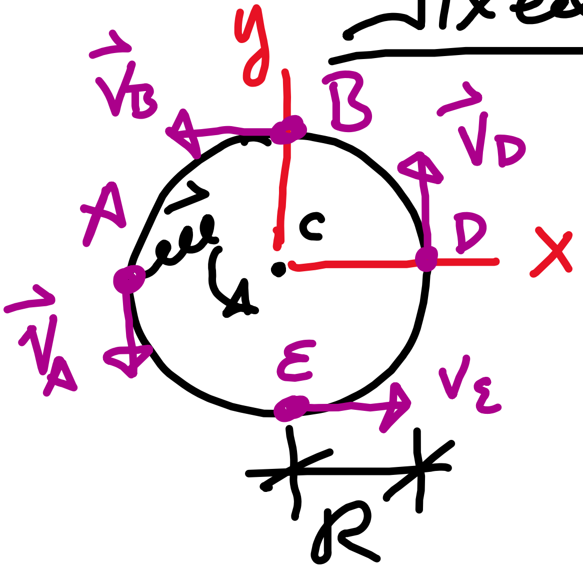

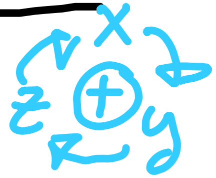

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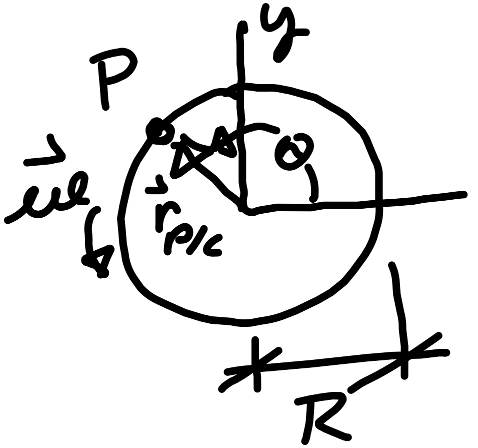

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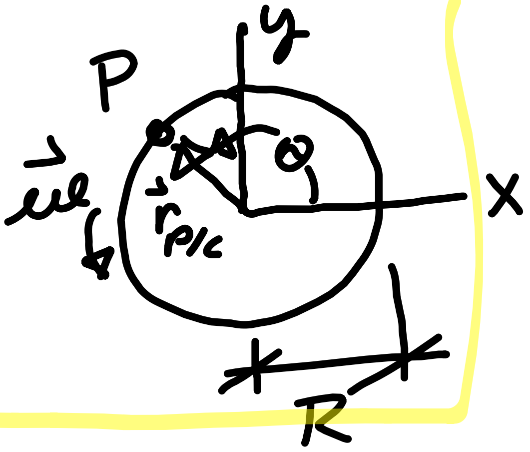

From previous slide

For some arbitrary point P:

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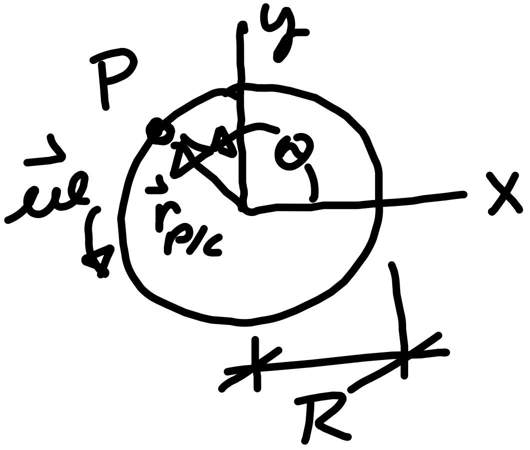
Lets check that the signs are correct for each Quadrant



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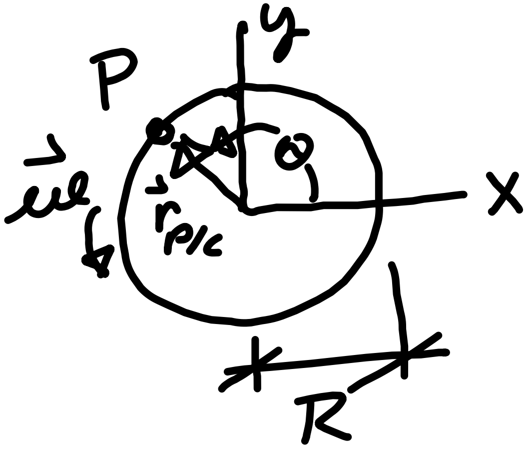
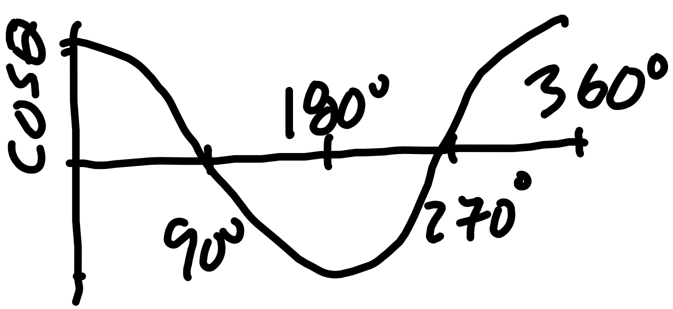
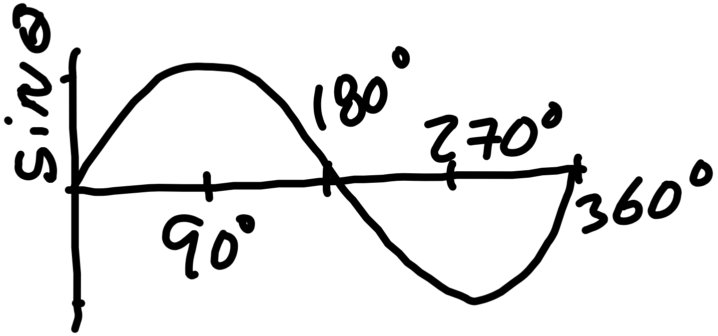
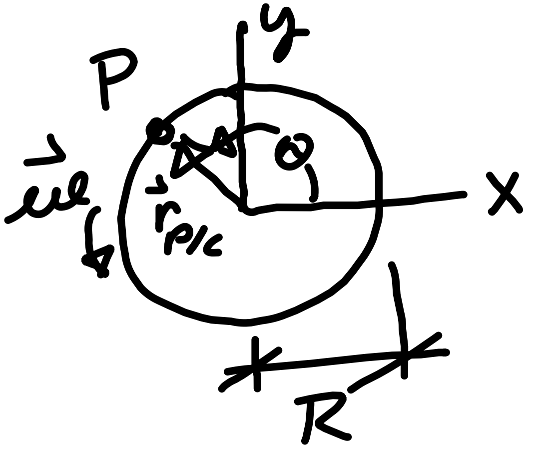






For some arbitrary point P:



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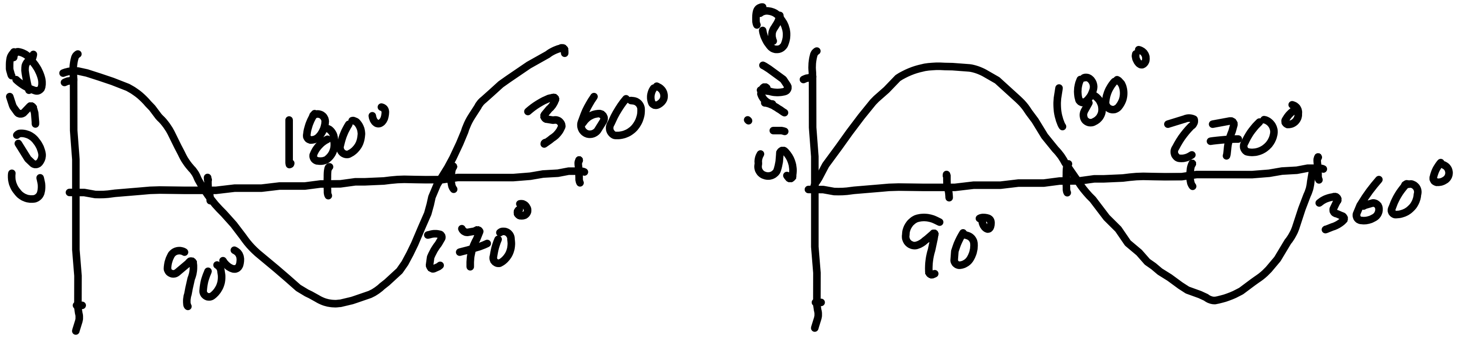

⇒ the signs will work out just fine.

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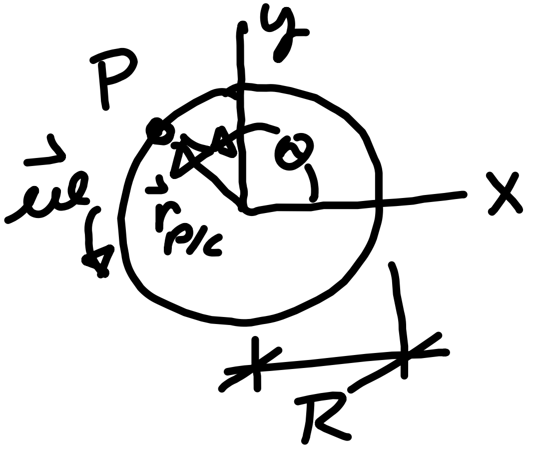

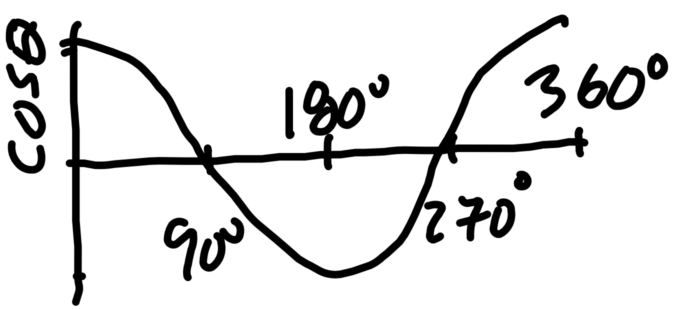

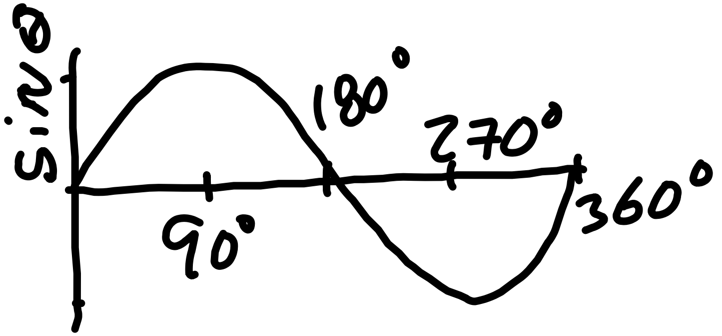

⇒ the signs will work out just fine.

cosθ is positive from 0°-90° & 270°-360° & negative from 90°-270°

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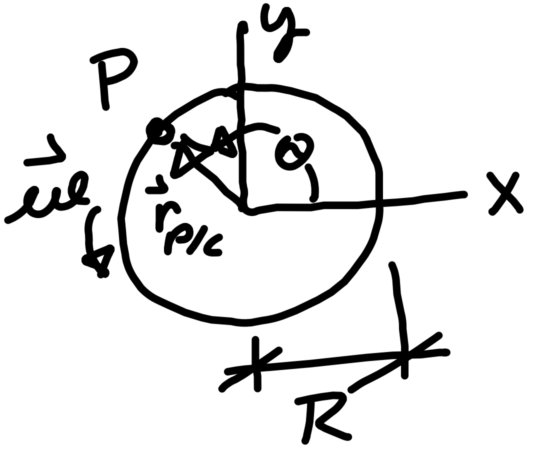

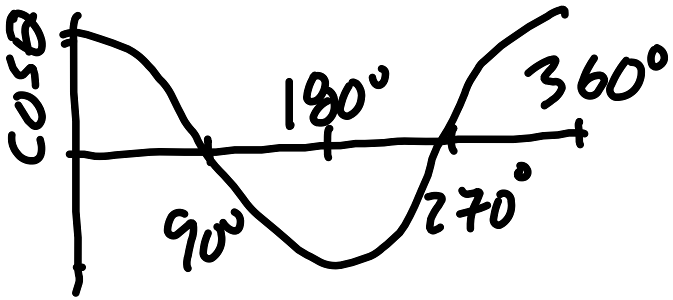

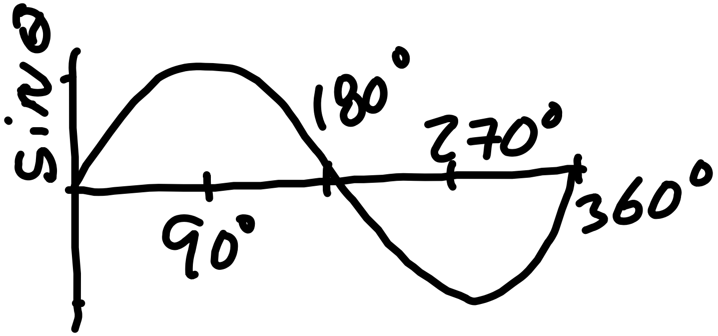

⇒

the signs will work out just fine.

cos θ is positive from 0°-90° & 270°-360° & negative from 90°-270° while sin θ is positive from 0°-180°

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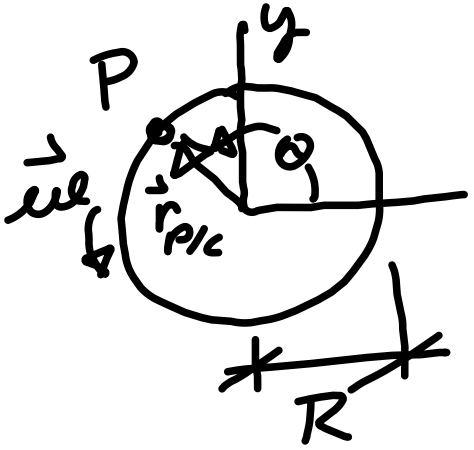
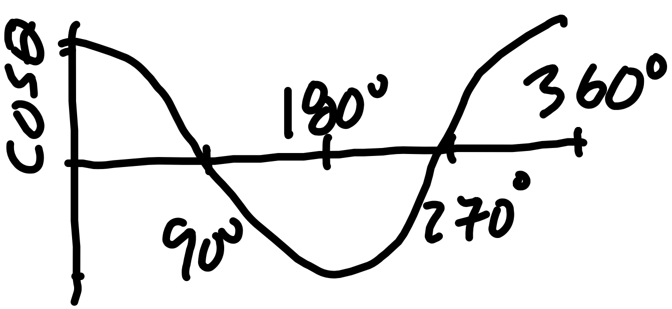
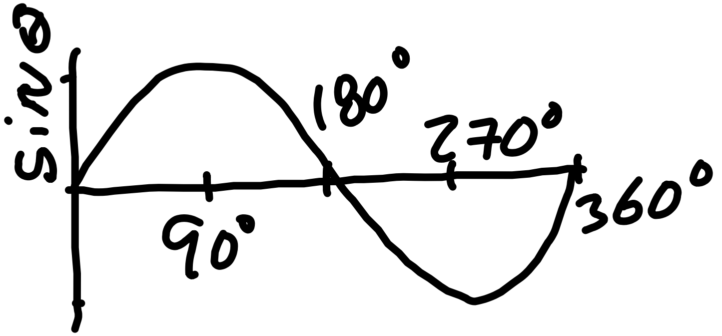


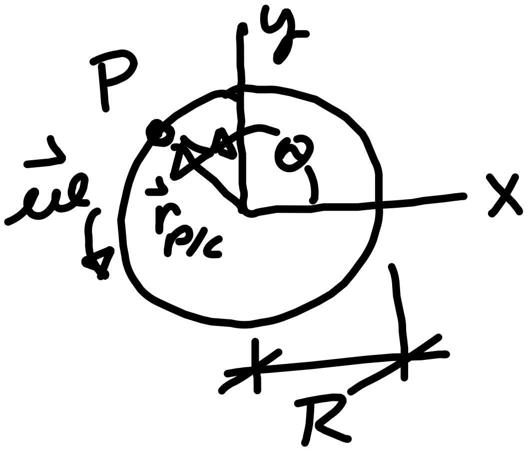




⇒ the signs will work out just fine.

cosθ is positive from 0°-90° & 270°-360° & negative from 90°-270° while sinθ is positive from 0°-180° & negative from 180°-360°

For some arbitrary point P:



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$$\vec{V}_P = (\omega R)\hat{z} \times [\hat{x}\cos\theta + y\sin\theta]$$

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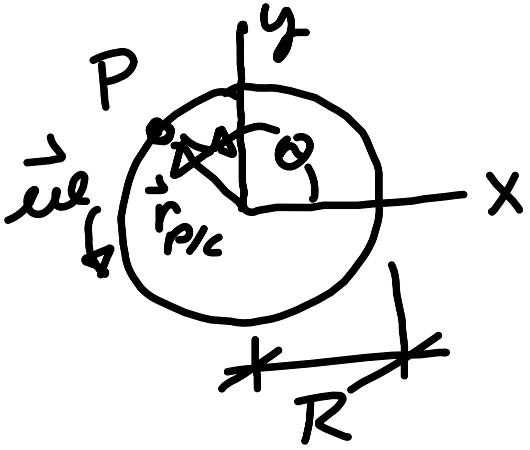

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$$\vec{V}_P = (\omega R)\hat{z} \times [\hat{x}\cos\theta + y\sin\theta]$$

$\Rightarrow$

$$\vec{V}_P = (\omega R)[\hat{y}\cos\theta - \hat{x}\sin\theta]$$

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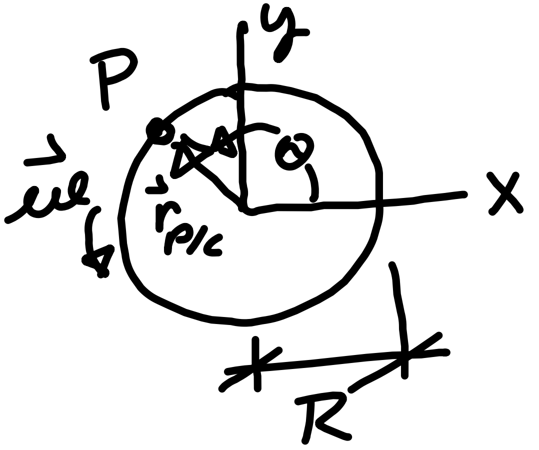

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$\Rightarrow$

$\vec{V}_P = (\omega R)[\hat{y}\cos\theta - \hat{x}\sin\theta]$, Assuming that

θ is measured from the x-axis!

Rolling wheel (General plane motion):

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If the axis of rotation is translating, we have to include that motion

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& $\vec{V}_c \rightarrow$ Translation,

Rolling wheel (General plane motion):

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Since $\vec{V}_P = \vec{V}_{P/C} + \vec{V}_C$ & $\vec{V}_{P/C} \rightarrow$ Rotation & $\vec{V}_C \rightarrow$ Translation, then the motion of P is a rotation plus a translation.

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Let a wheel roll without slipping for one full rotation:

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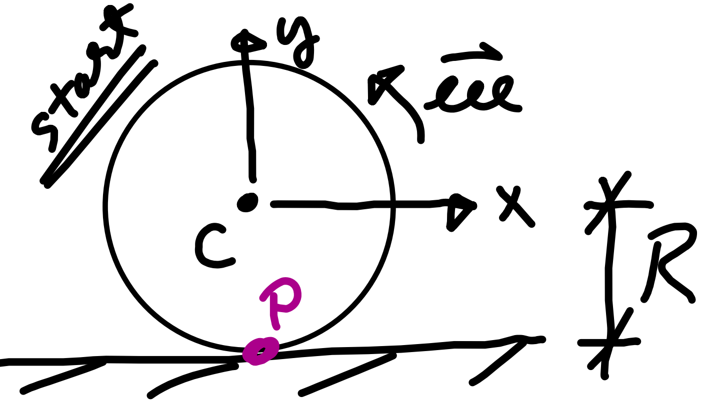

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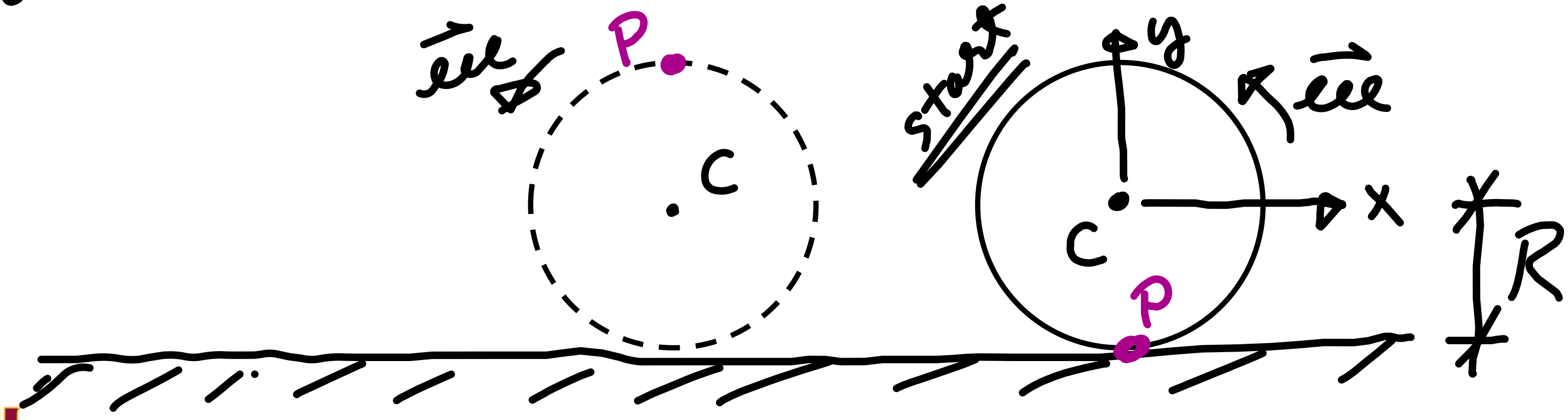

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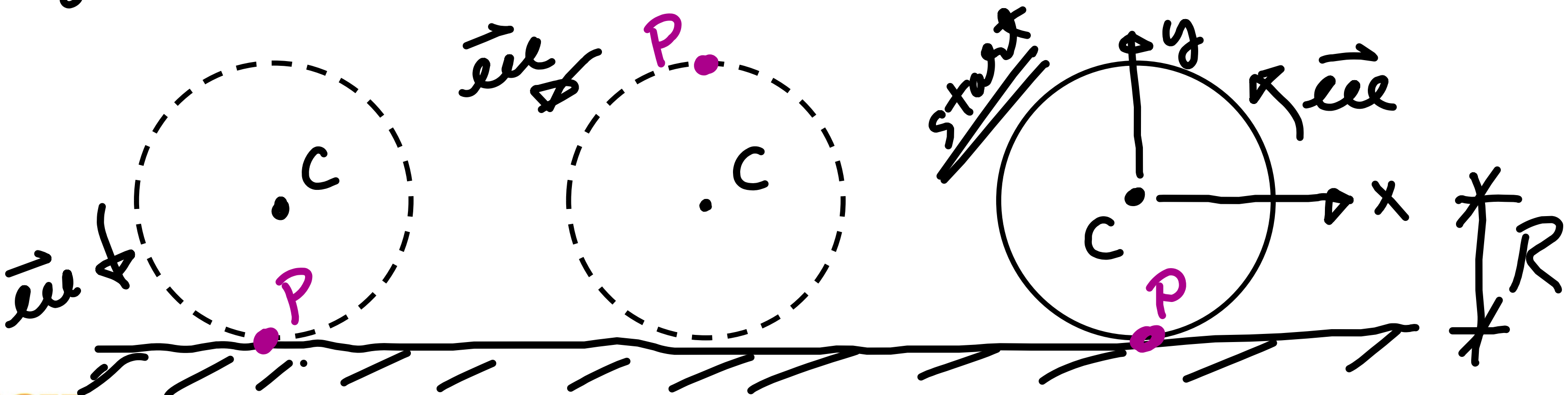

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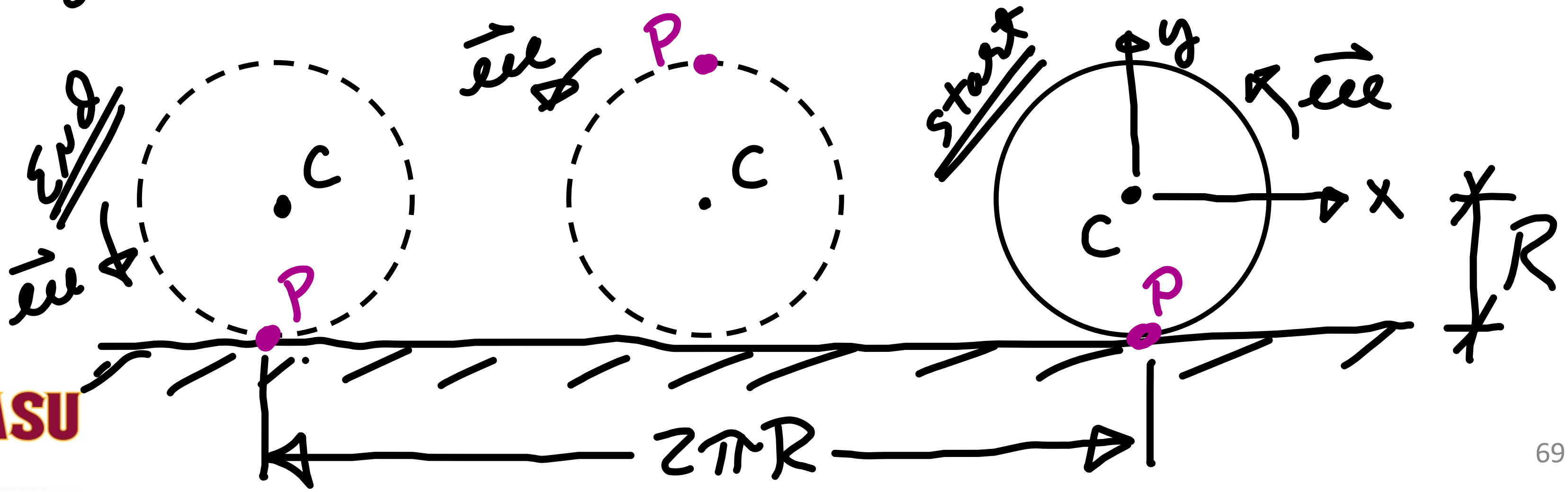
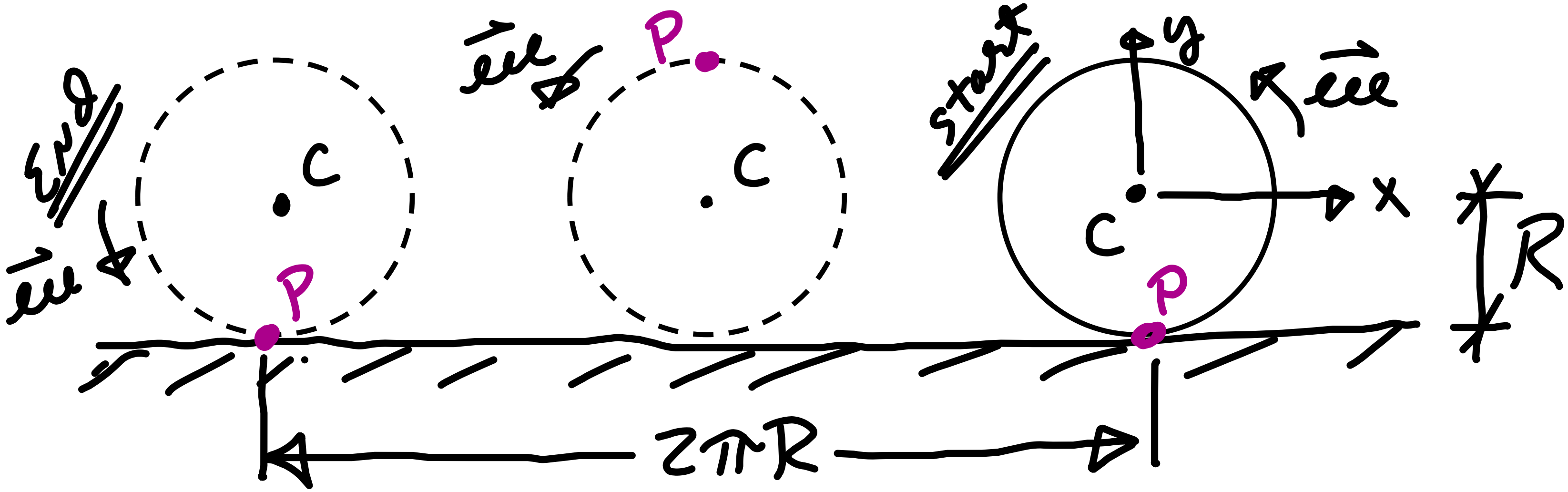
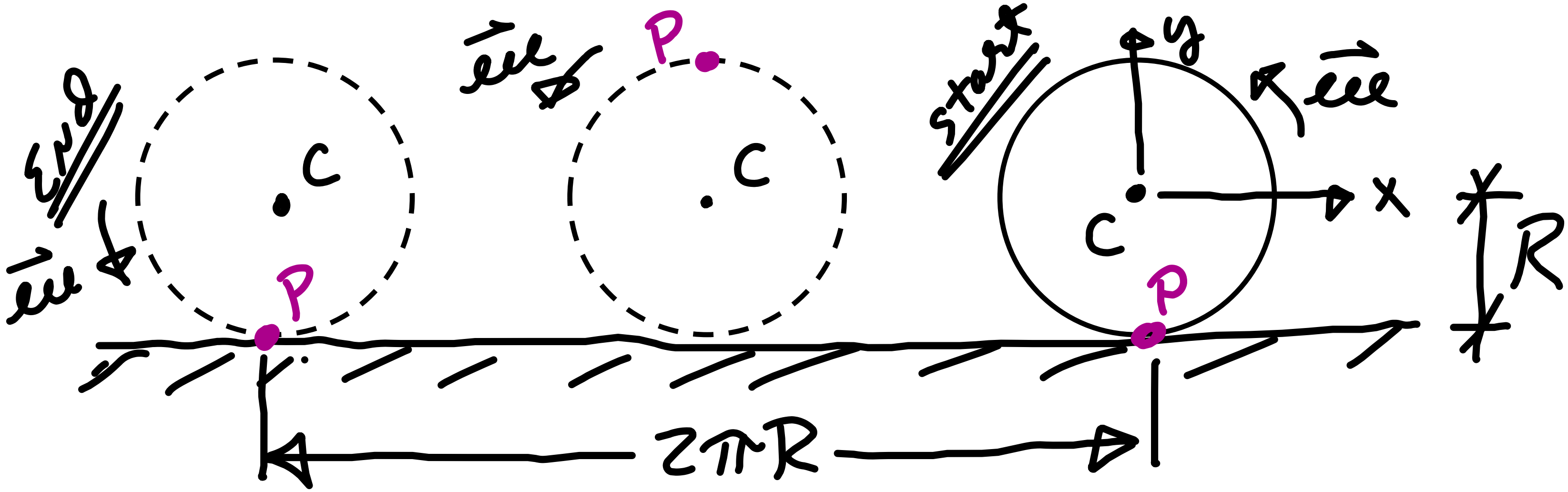
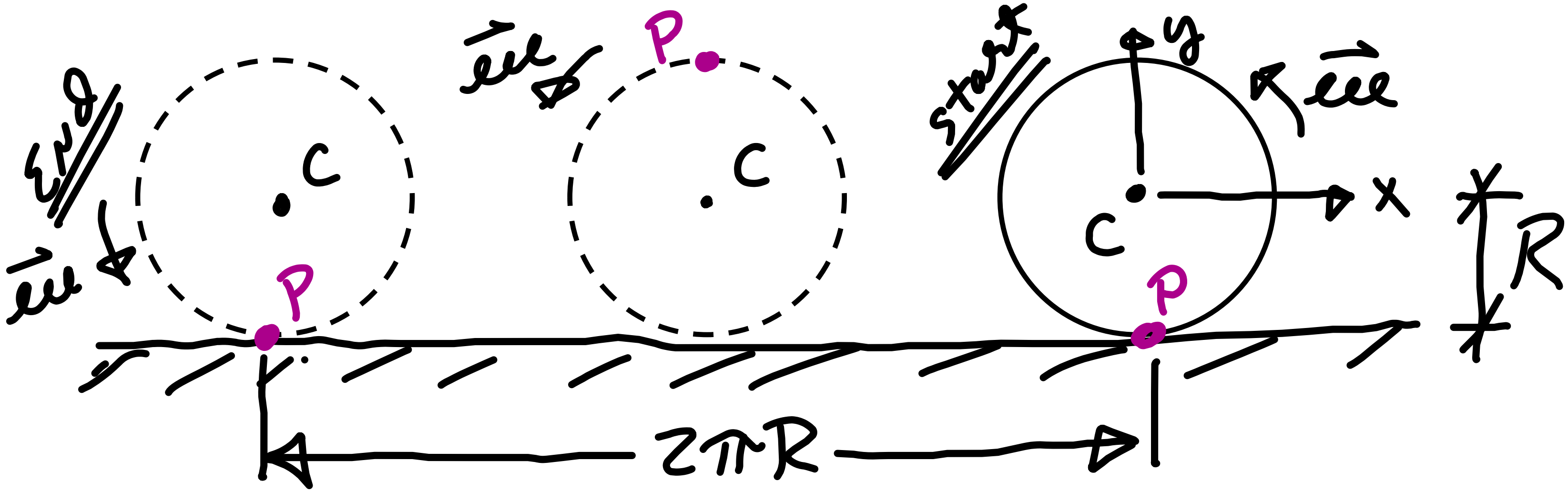


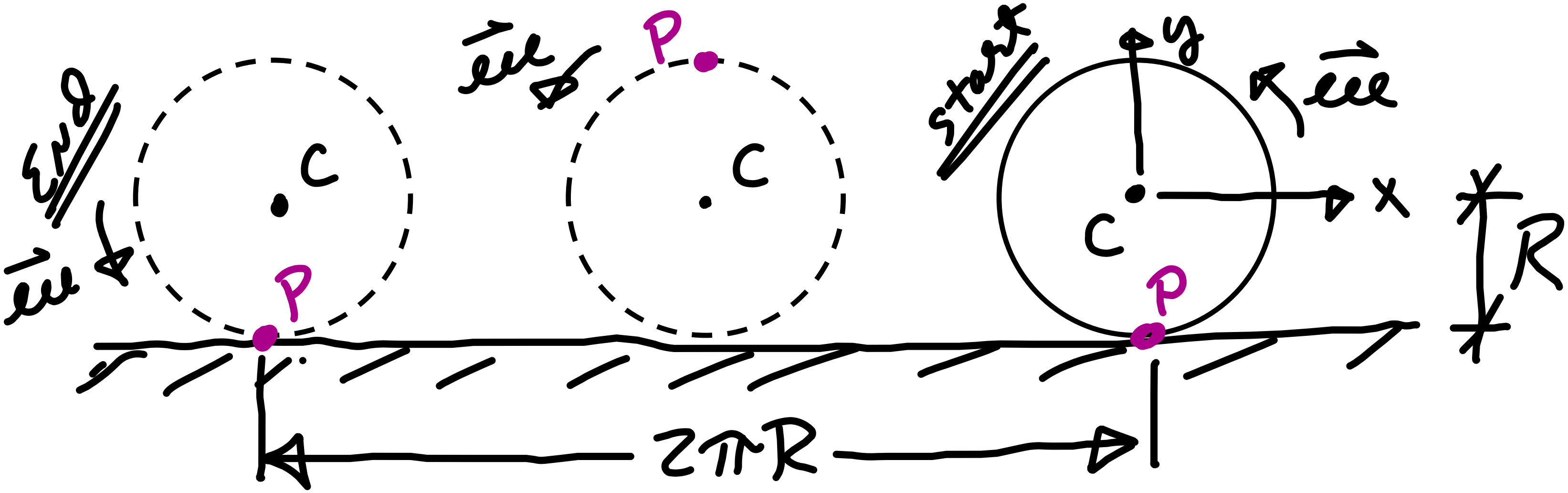

End
$\vec{\omega}$
P
C
$\vec{\omega}$
P
C
Start
y
$\vec{\omega}$
x
C
P
R
$2\pi R$



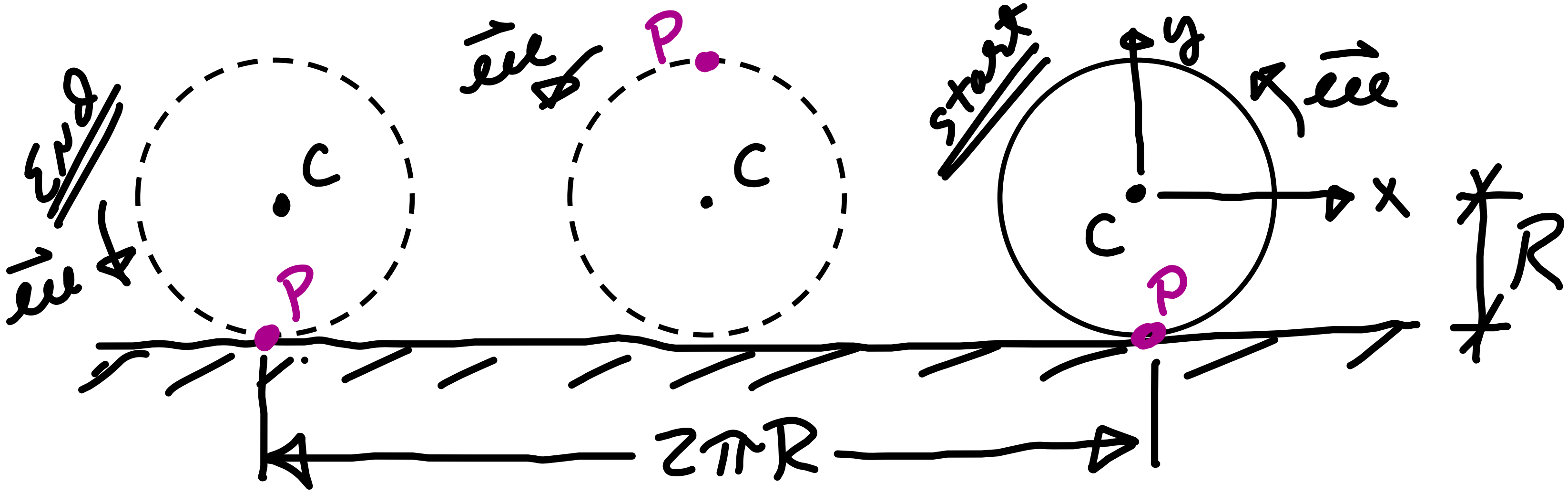
$$V_c = \frac{2\pi R}{\Delta t}$$



$$V_c = \frac{2\pi R}{\Delta t}(-\hat{x})$$



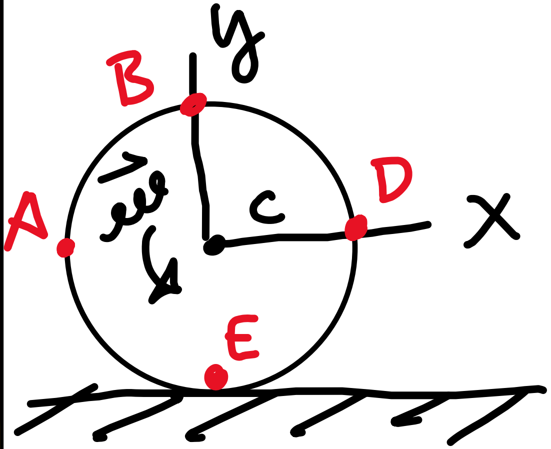
$$V_c = \frac{2\pi R}{\Delta t}(-\hat{x}) \quad \& \quad \omega = \frac{2\pi}{\Delta t}$$



$$V_c = \frac{2\pi R}{\Delta t}(-\hat{x}) \quad \& \quad \omega = \frac{2\pi}{\Delta t}$$

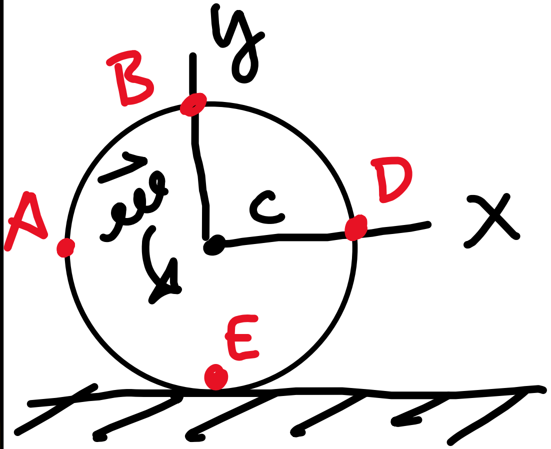
$$\Rightarrow \qquad \vec{V}_c = \omega R(-\hat{x})$$

For a wheel rotating <u>without slipping</u>

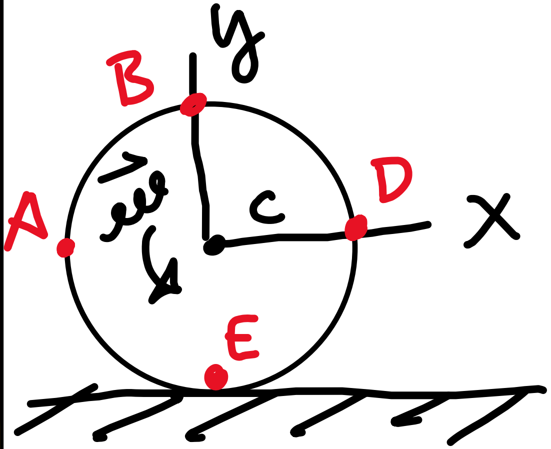


For a wheel rotating <u>without slipping</u>

$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C$$



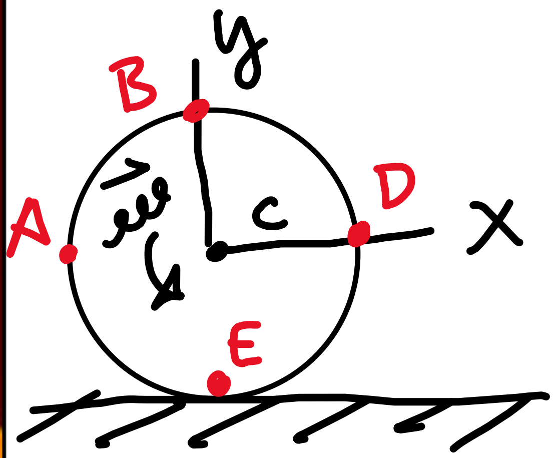
For a wheel rotating <u>without slipping</u>



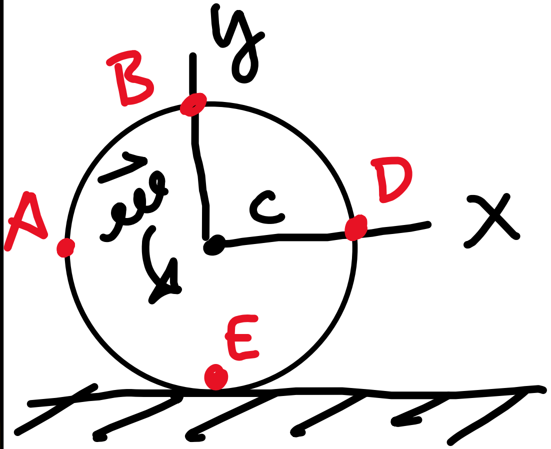
$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x}$$

For a wheel rotating <u>without slipping</u>

$$\vec{V}_A = \vec{V}_{A/C} + \vec{V}_C = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y} + \hat{x}]$$



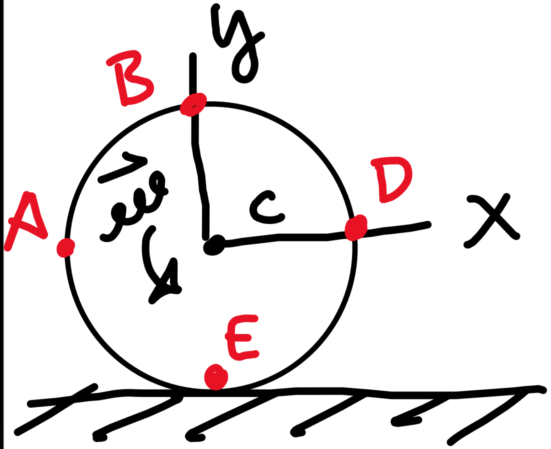
For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/c} + \vec{V}_c = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

$$\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c =$$

For a wheel rotating <u>without slipping</u>



$$\vec{V}_A = \vec{V}_{A/c} + \vec{V}_c = -\omega R\hat{y} - \omega R\hat{x} = -\omega R[\hat{y}+\hat{x}]$$

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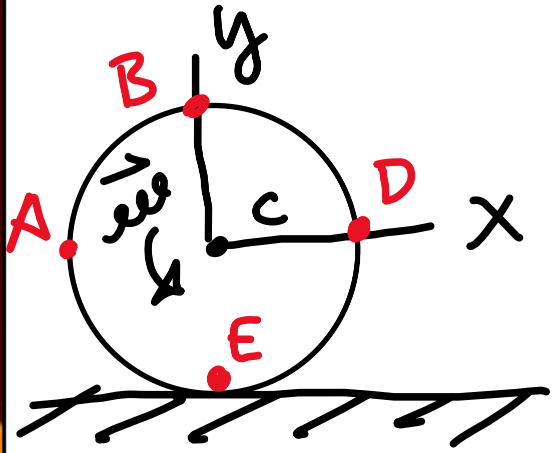
For a wheel rotating without slipping



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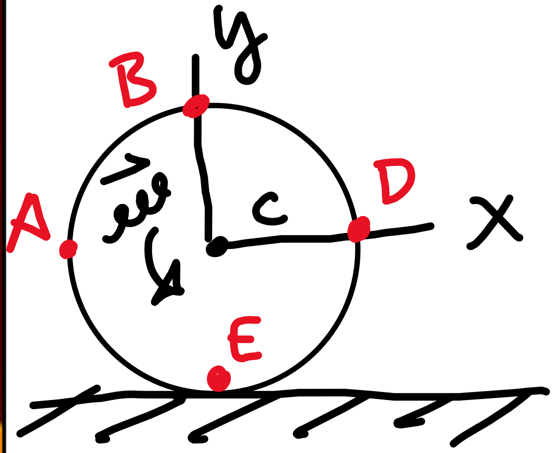

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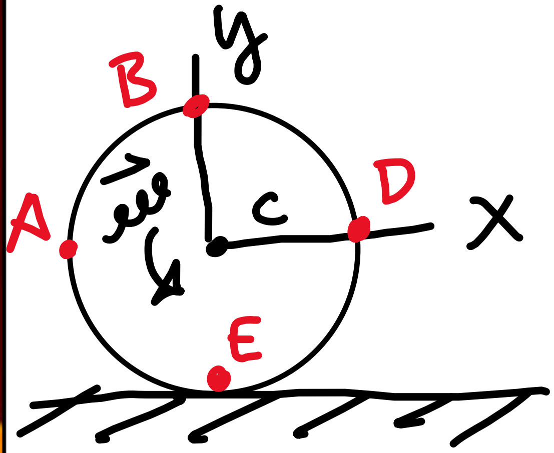

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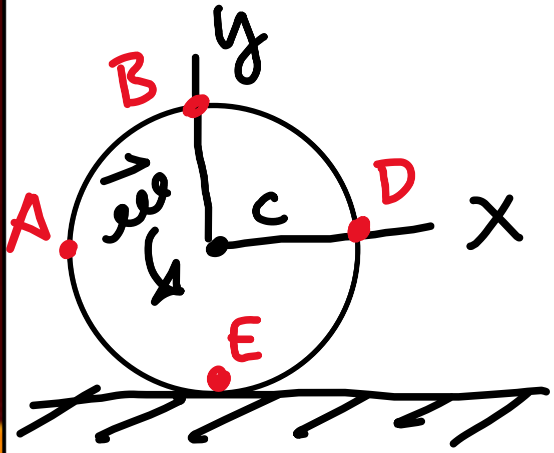

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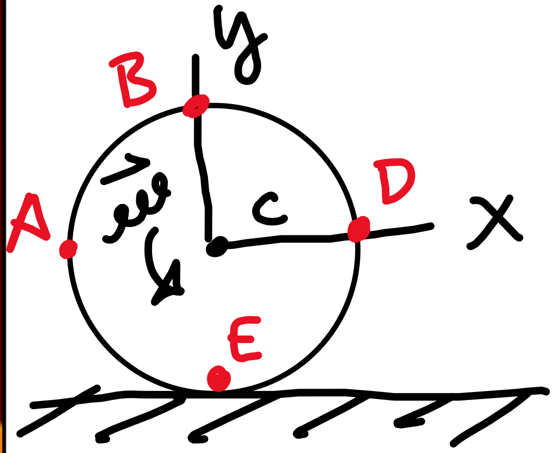

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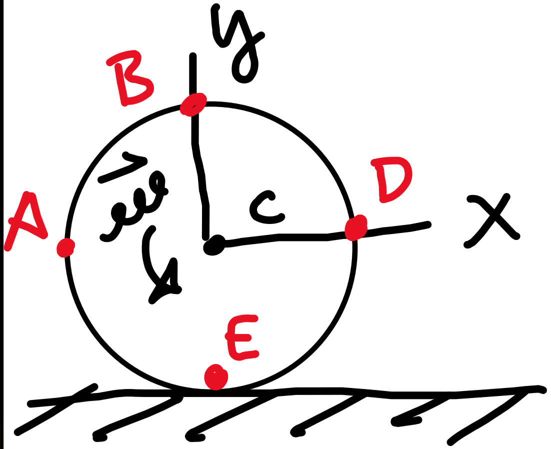
For a wheel rotating without slipping



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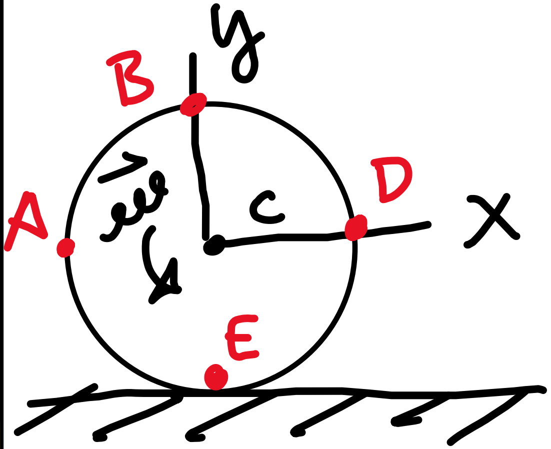

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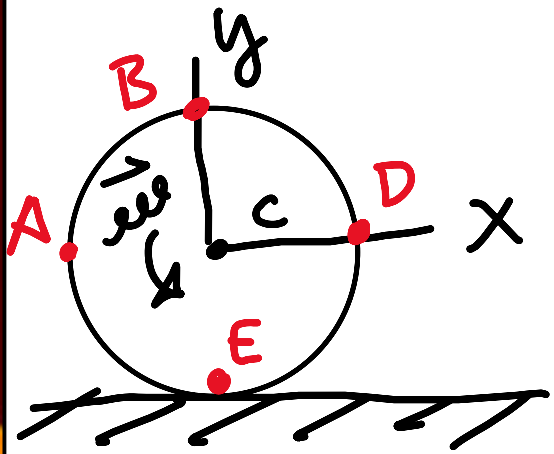

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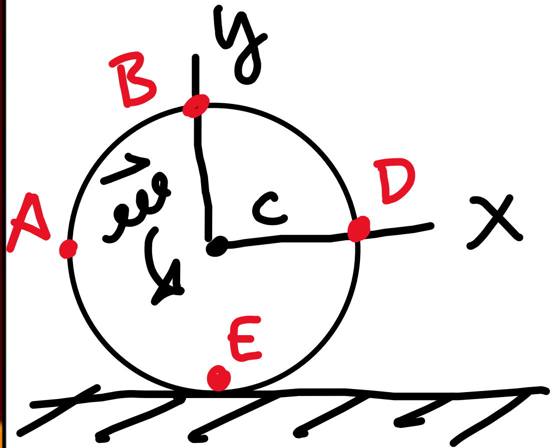
In General:

For a wheel rotating <u>without slipping</u>



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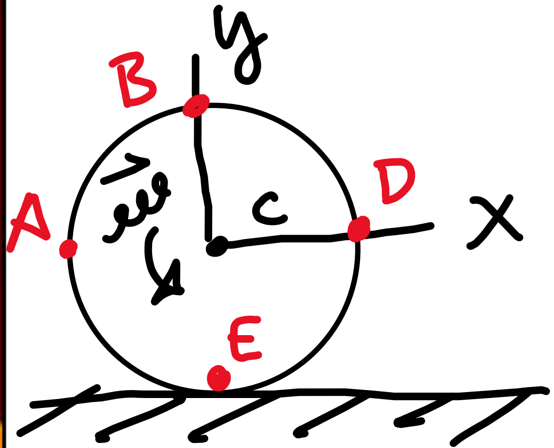
<u>In General</u>: If a body rotates <u>without</u> slipping against some object,

For a wheel rotating <u>without slipping</u>



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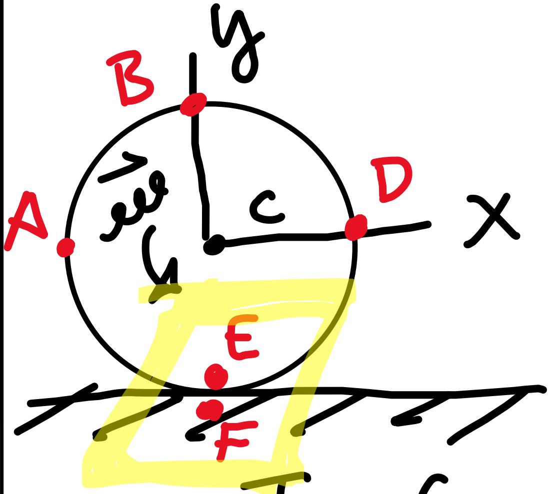
In General: If a body rotates without slipping against some object, the relative velocity between those two objects is zero.

For a wheel rotating without slipping



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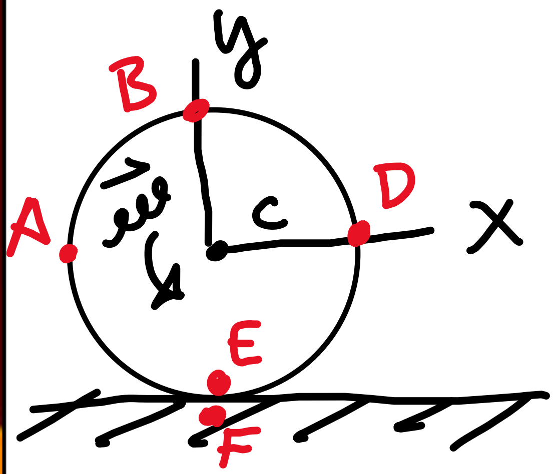
For a wheel rotating without slipping



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In General: If a body rotates without slipping against some object, the relative velocity between those two objects is zero. In the above case, where the wheel touches the road (point E) has the same velocity as the point on the road touching the wheel (point F). That is $\vec V_{E/F} = 0$

The intuition problem:

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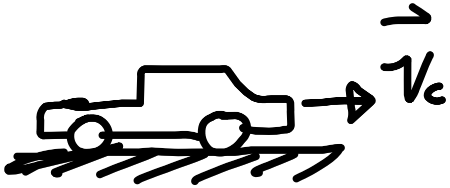
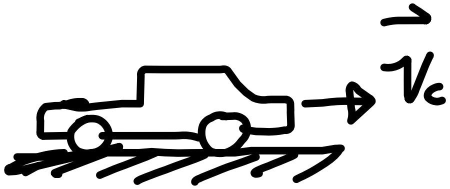


<u>The intuition problem</u>: If I see a car moving down the road, my mind wants to say that <u>all</u> of the car is moving down the road at the same speed.



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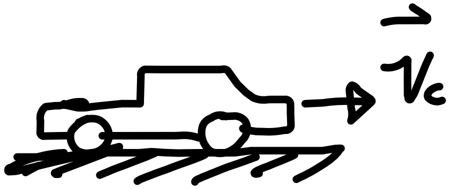

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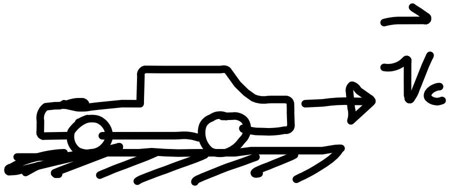

But my intuition is not compatible with the idea that there is a point on each wheel that has zero velocity.

You may not share this intuition problem with me, but many in this class will

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation.

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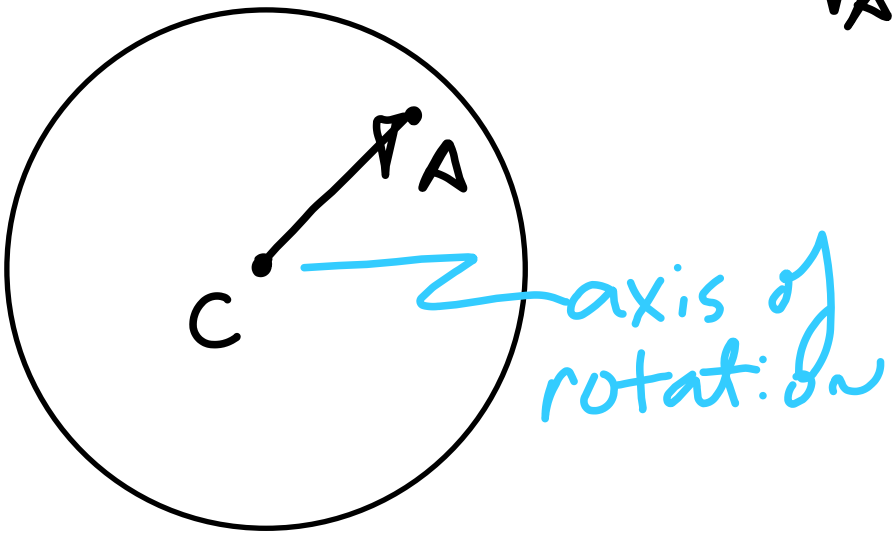
$$\vec{v}_A = \vec{\omega} \times \vec{r}_{A/C}$$



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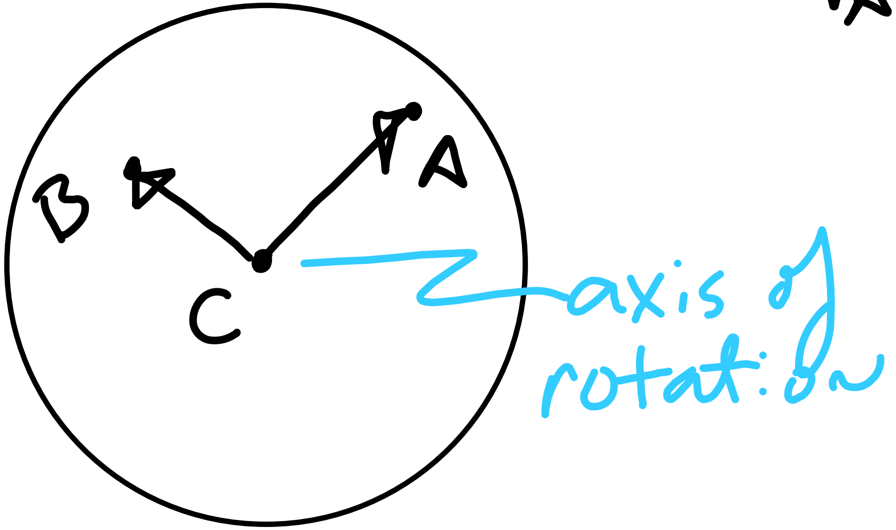
$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/C}$ & $\vec{V}_B = \vec{\omega} \times \vec{r}_{B/C}$



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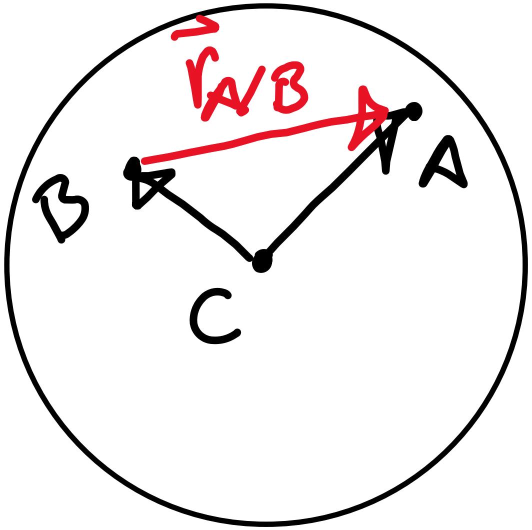

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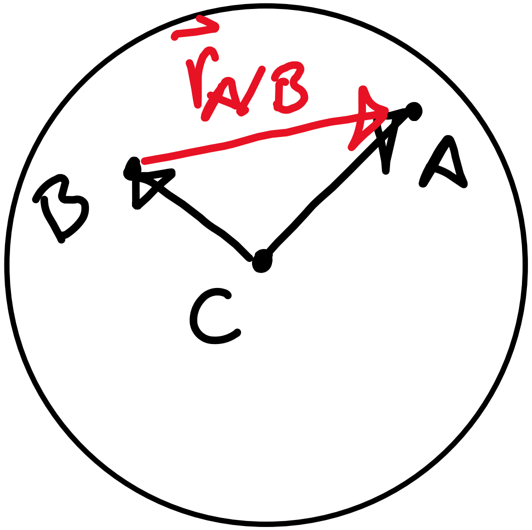

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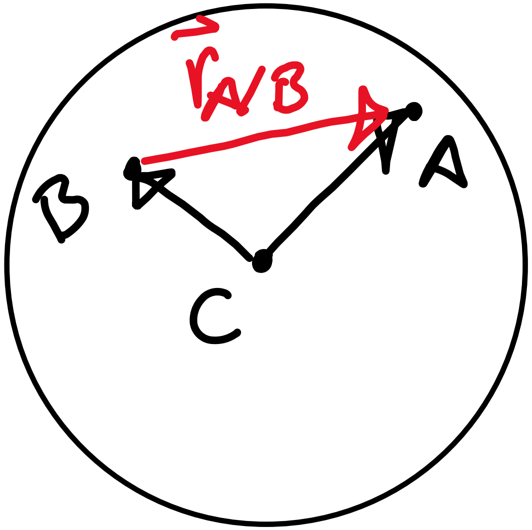

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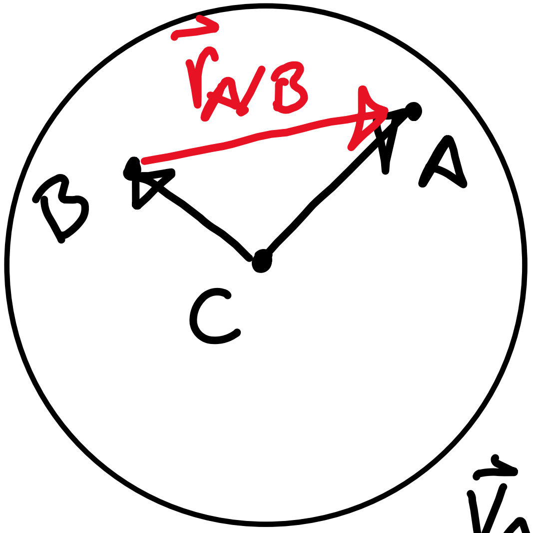

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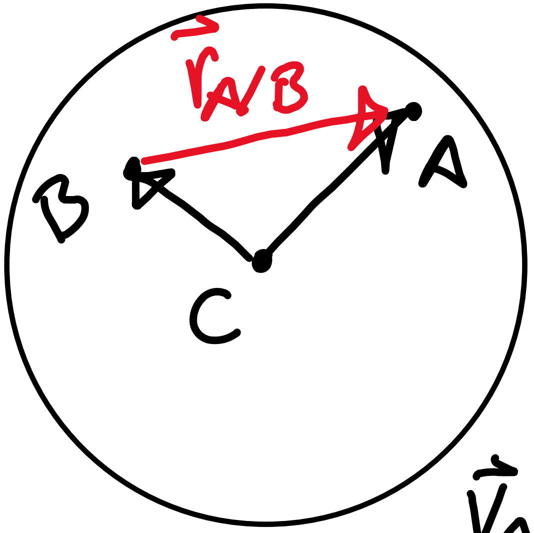

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$$\Rightarrow V_{A/B} = \vec{\omega} \times [\vec{r}_{A/C} - \vec{r}_{B/C}] \Rightarrow$$

$$\vec{V}_{A/B} = \vec{\omega} \times [(\vec{r}_A - \vec{r}_C) - (\vec{r}_B - \vec{r}_C)] = \vec{\omega} \times (\vec{r}_A - \vec{r}_B)$$

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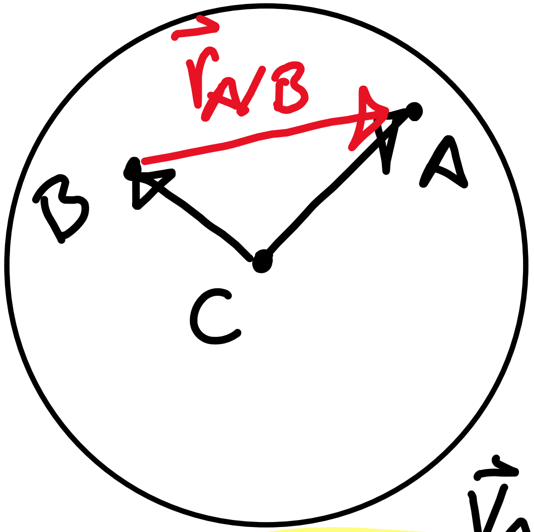

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$$\Rightarrow \boxed{\vec{V}_{A/B}=\vec{\omega}\times\vec{r}_{A/B}}$$

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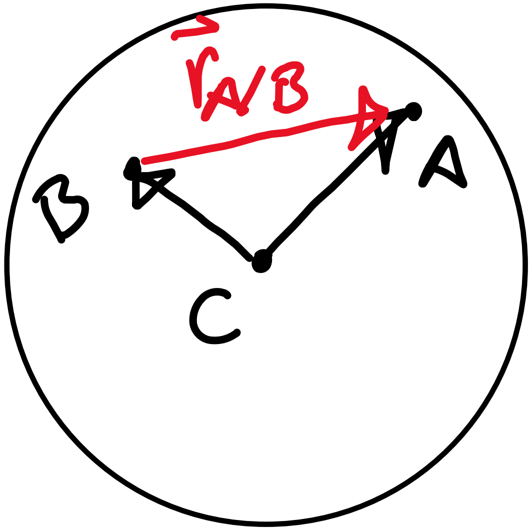
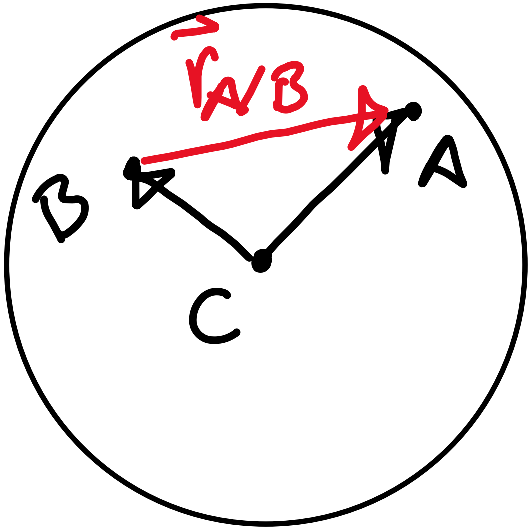


Motion of point A as seen standing at point C

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?



We have $\vec{V}_A = \vec{\omega} \times \vec{r}_A$

$$\& \quad \vec{V}_B = \underbrace{\vec{\omega} \times \vec{r}_B}_{\text{Motion of point B as seen standing at point C}}$$

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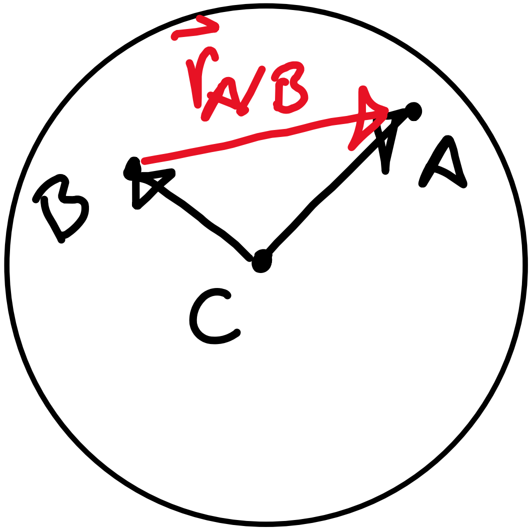

We have $\vec{V}_A = \vec{\omega} \times \vec{r}_A$

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& $$\underbrace{\vec{V}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}}_{\text{Motion of point A as seen from point B}}$$

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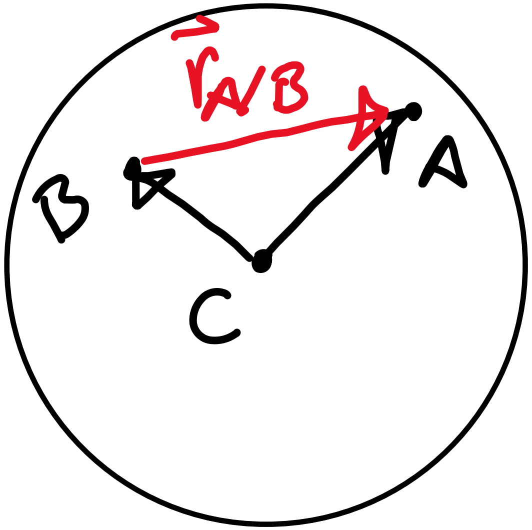


We have $\vec{v}_A=\vec{\omega}\times\vec{r}_A$

& $\vec{v}_B=\vec{\omega}\times\vec{r}_B$

& $\vec{v}_{A/B}=\vec{\omega}\times\vec{r}_{A/B}$

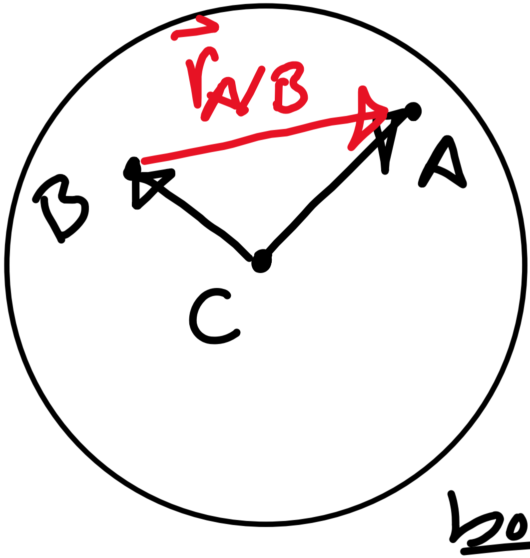
The $\vec{\omega}$ is identical, regardless of the reference point

So far, our derivation of $\vec{v} = \vec{\omega} \times \vec{r}$ has allowed us to talk about motion relative to the axis of rotation. What about other reference points?

We have $\vec{V}_A = \vec{\omega} \times \vec{r}_A$

& $\vec{V}_B = \vec{\omega} \times \vec{r}_B$

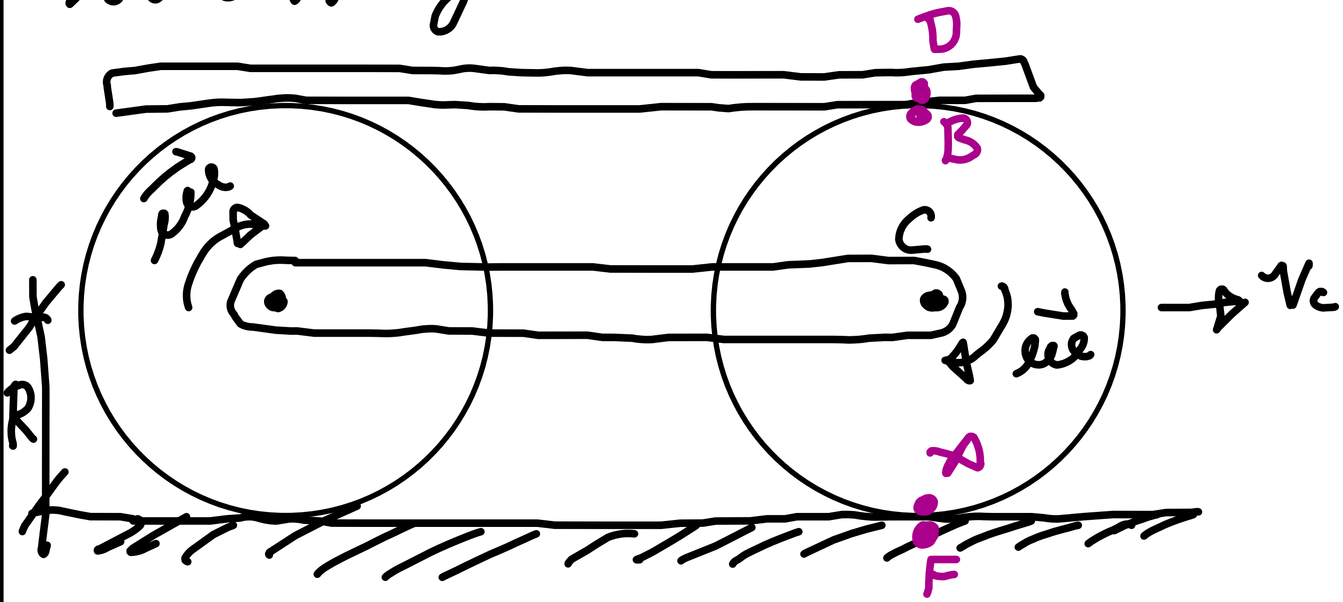
& $\vec{V}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}$



Angular velocity of a rigid body in plane motion is independent of the reference point

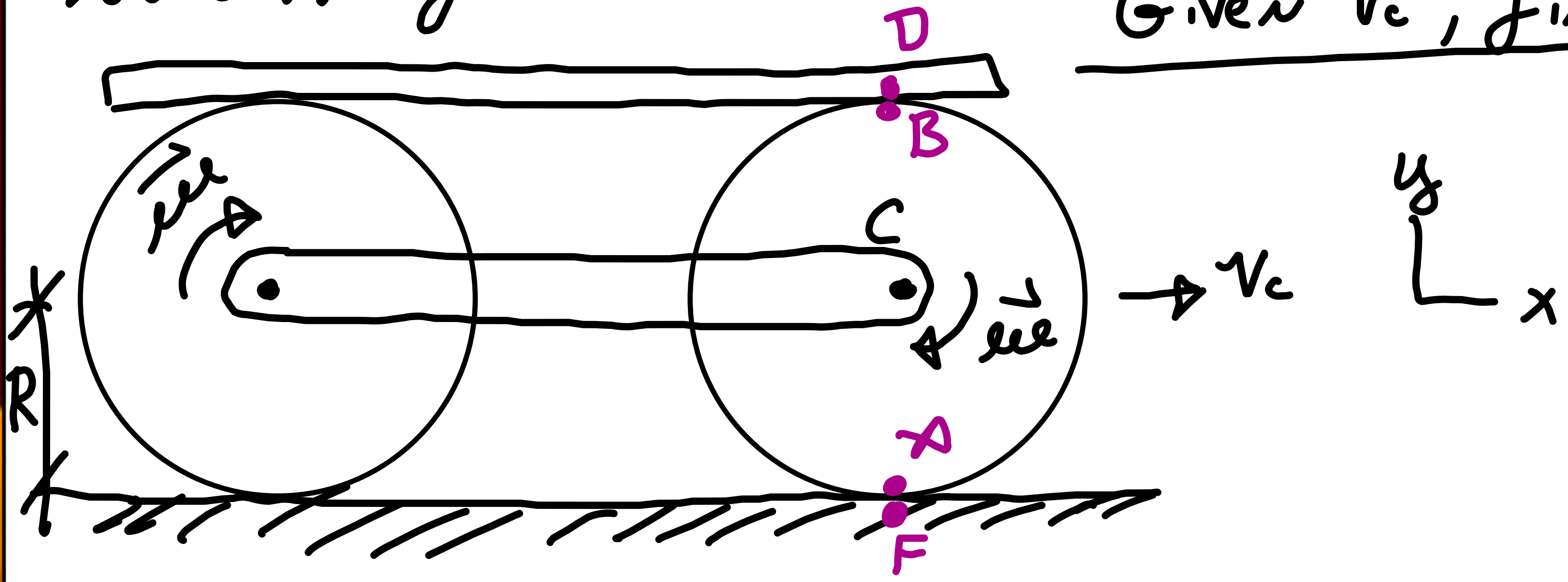
→ Not intuitive

Example: Board sitting on wheels. Rolling no slipping.



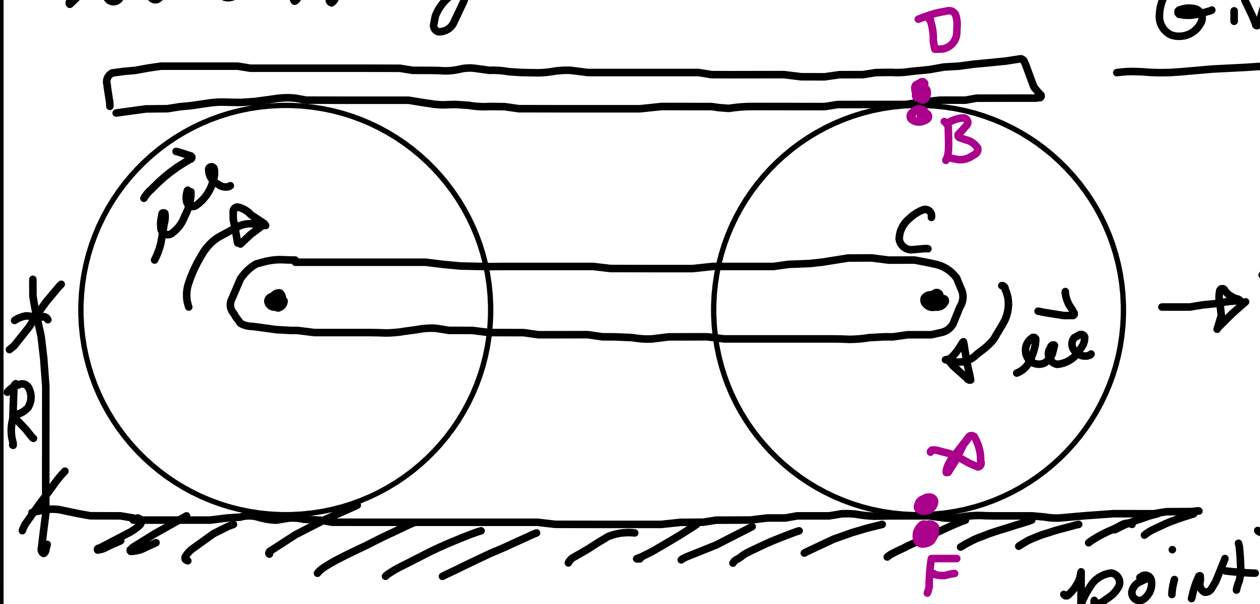
Example: Board sitting on wheels. Rolling no slipping.

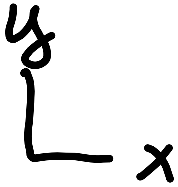
Given v_c, find v_D



Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

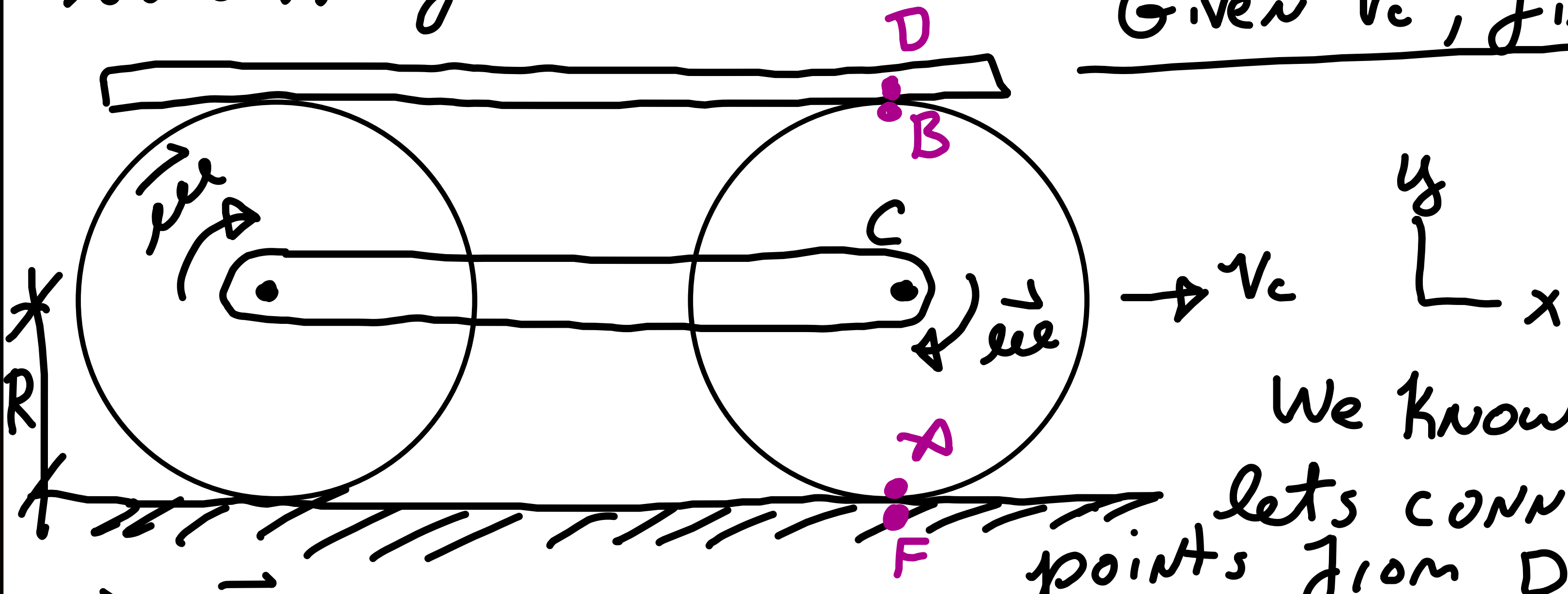




We know $\vec{V}_c$ so lets connect points from D to C

Example: Board sitting on wheels. Rolling no slipping.

Given V_C, find V_D

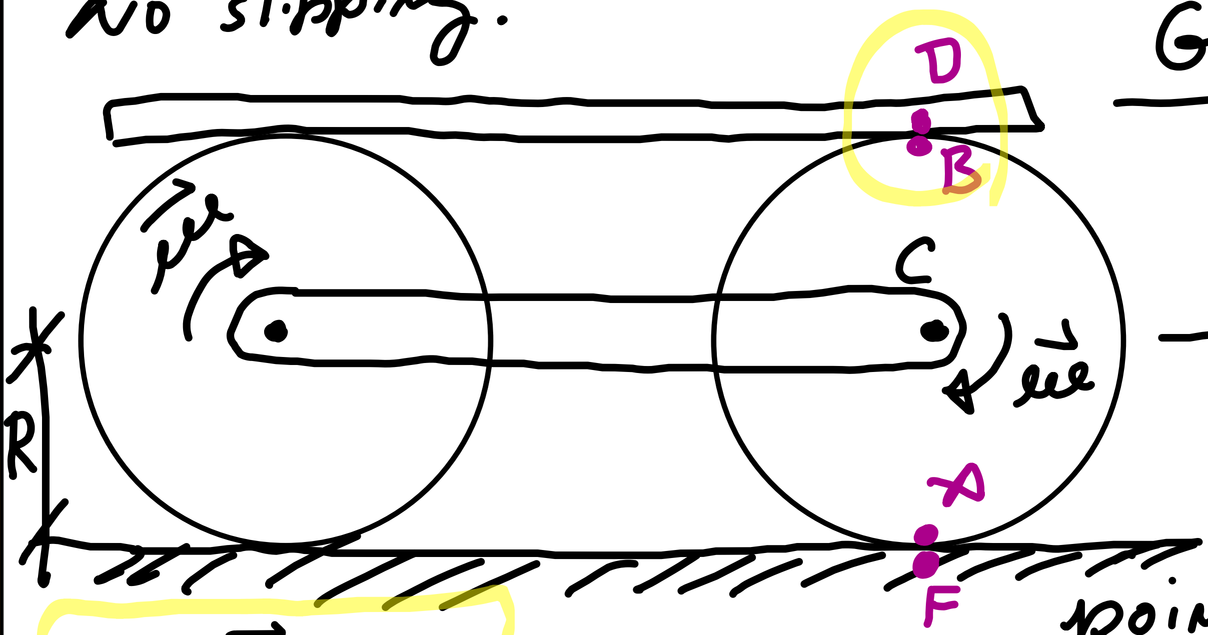


We know $\vec{V}_C$ so lets connect points from D to C

$$\vec{V}_D = \vec{V}_{D/B} + V_B$$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

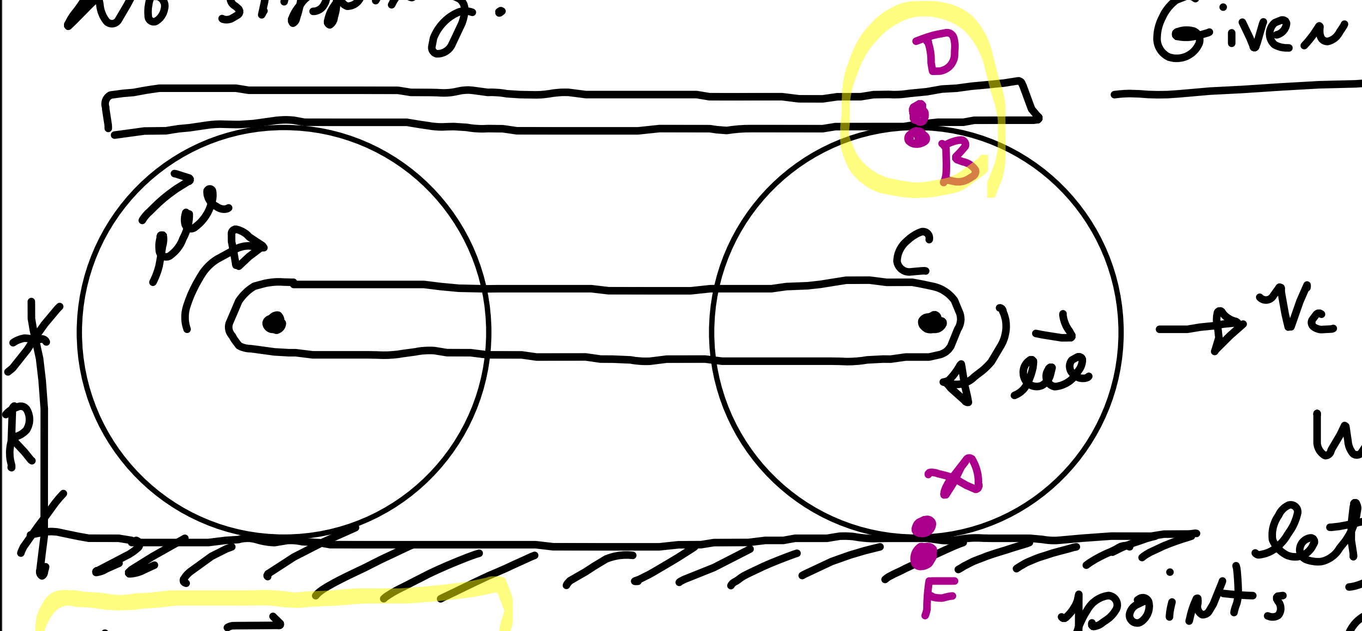


We know $\vec{V}_c$ so lets connect points from D to C

$$\boxed{\vec{V}_D = \vec{V}_{D/B} + V_B}$$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



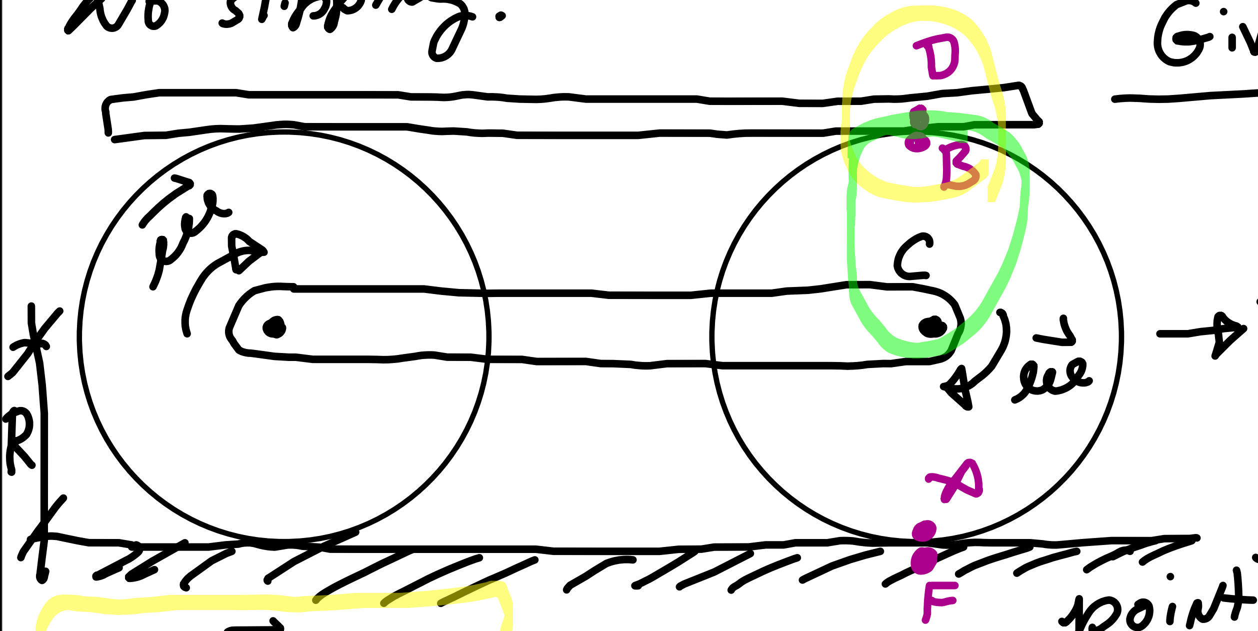
We know $\vec{V}_c$ so lets connect points from D to C

$$\boxed{\vec{V}_D = \vec{V}_{D/B} + V_B}$$

$$\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c$$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



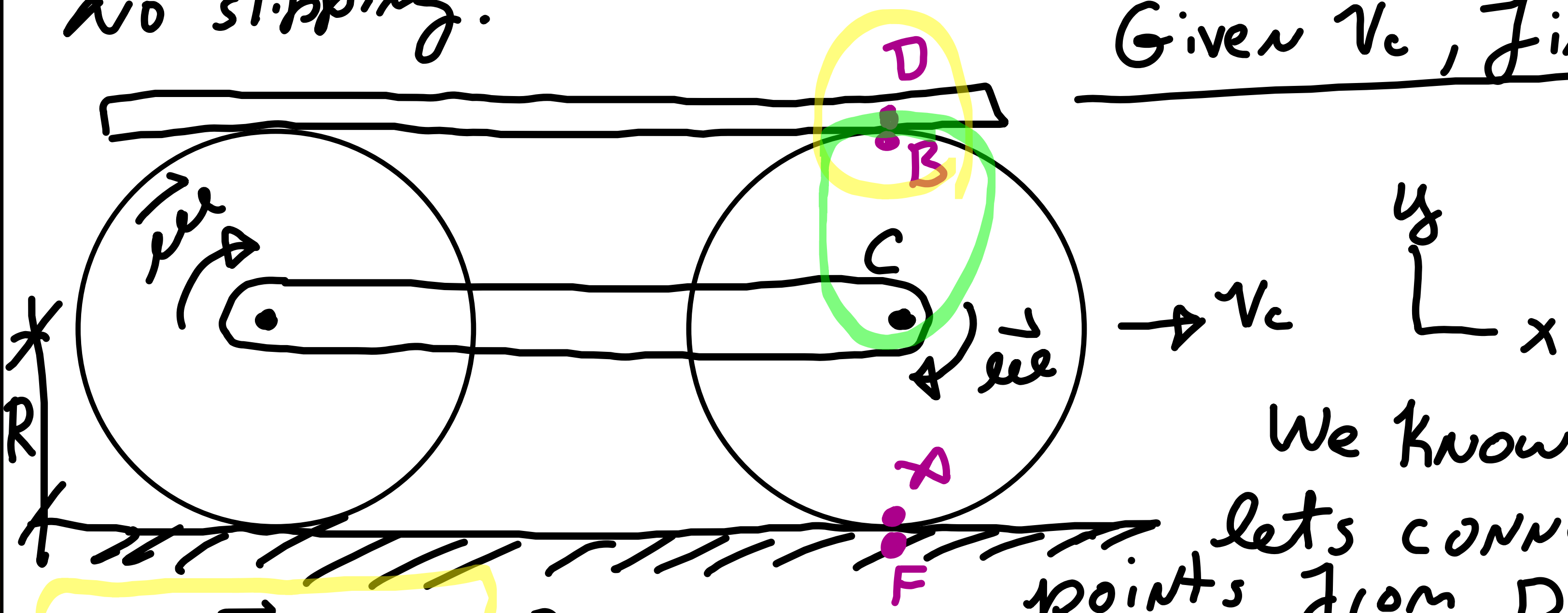
We know $\vec{V}_c$ so lets connect points from D to C

$$\vec{V}_D = \vec{V}_{D/B} + V_B$$

$$\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c$$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



We know $\vec{V}_c$ so lets connect points from D to C

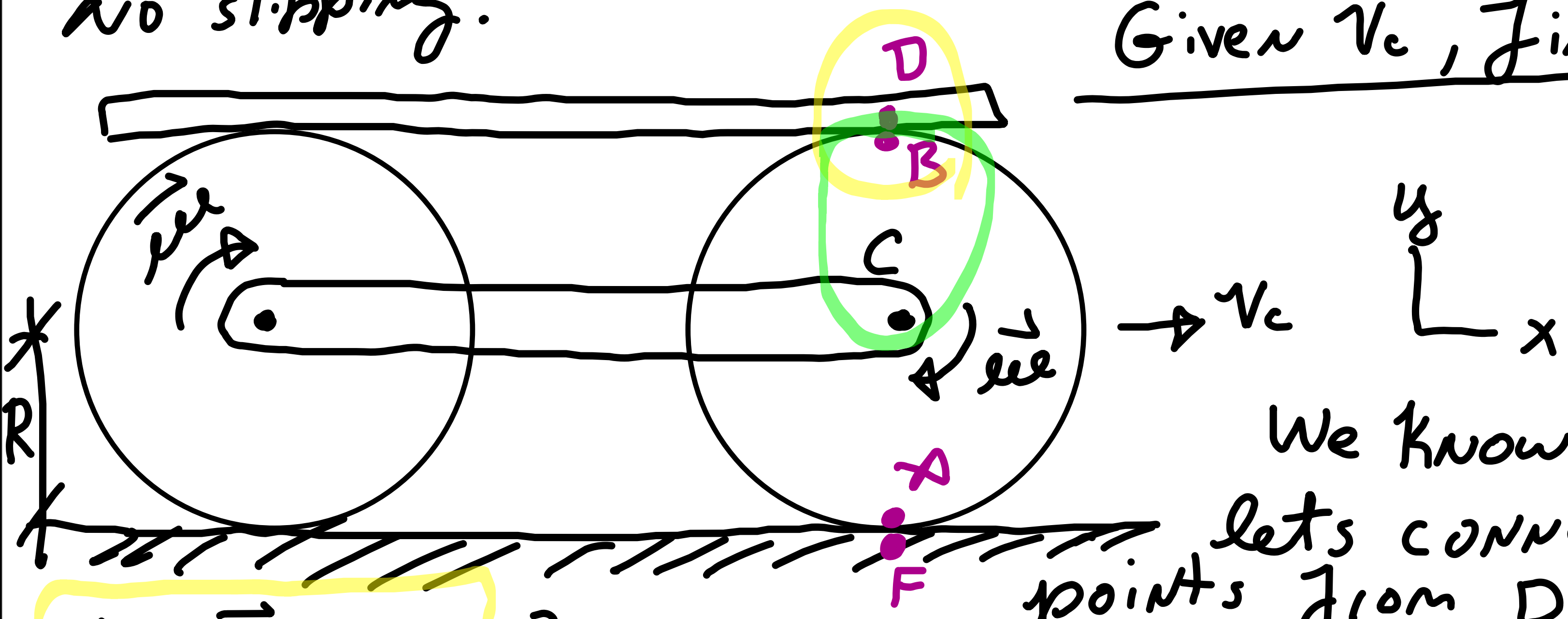
$$\vec{V}_D = \vec{V}_{D/B} + \vec{V}_B$$

$$\vec{V}_B = \vec{V}_{B/C} + \vec{V}_c$$

But roll no slip

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



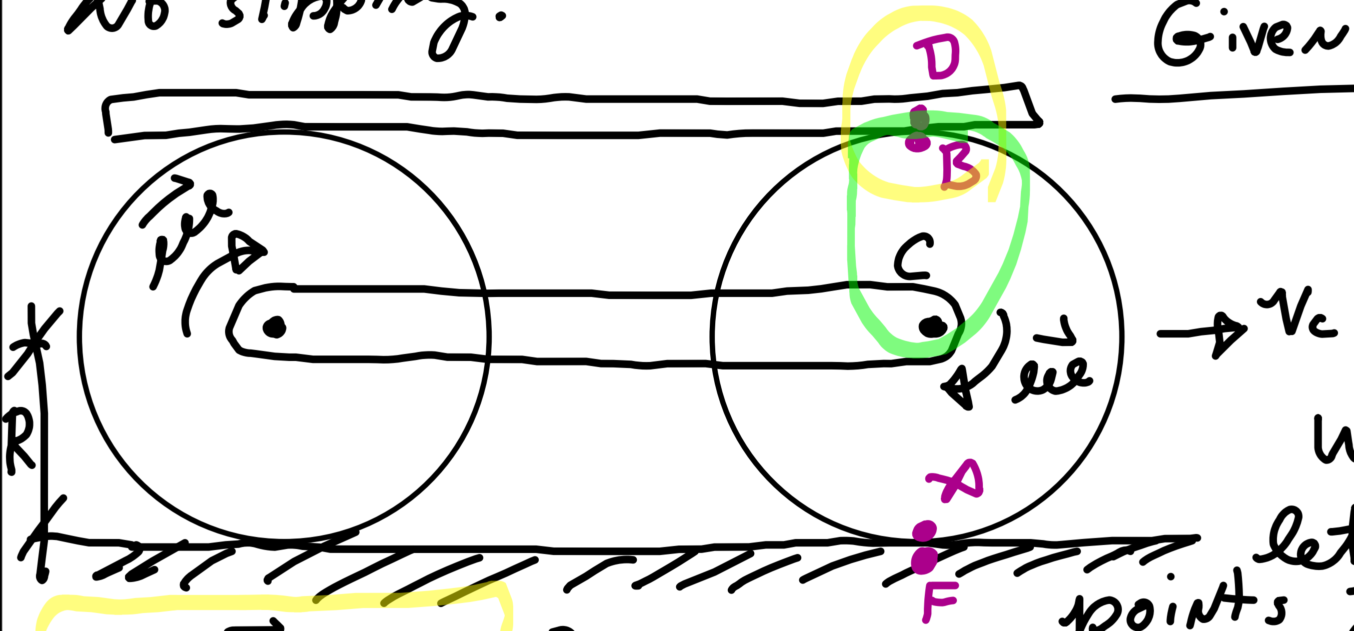
We know $\vec{V}_c$ so lets connect points from D to C

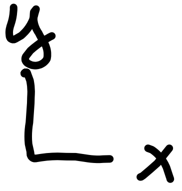
$$\left.\begin{array}{l}\vec{V}_D = \vec{V}_{D/B} + \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c\end{array}\right\}$$

But roll no slip $\Rightarrow \vec{V}_{D/B} = \theta$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



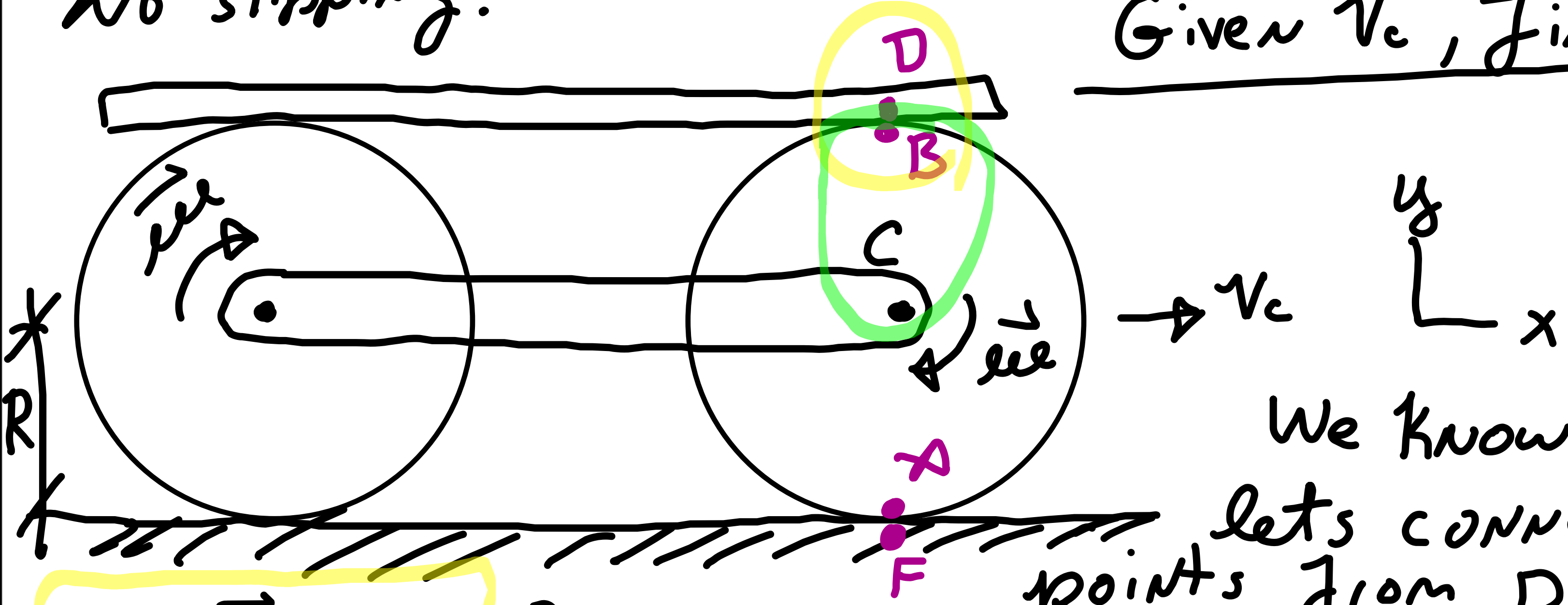


We know $\vec{V}_c$ so lets connect points from D to C

$$\left.\begin{aligned} \vec{V}_D &= \vec{V}_{D/B} + \vec{V}_B \\ \vec{V}_B &= \vec{V}_{B/C} + \vec{V}_C \end{aligned}\right\} \text{But roll no slip} \Rightarrow \vec{V}_{D/B} = 0 \text{ so } \begin{cases} \vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/C} + \vec{V}_C \end{cases}$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D



We know $\vec{v}_c$ so lets connect points from D to C

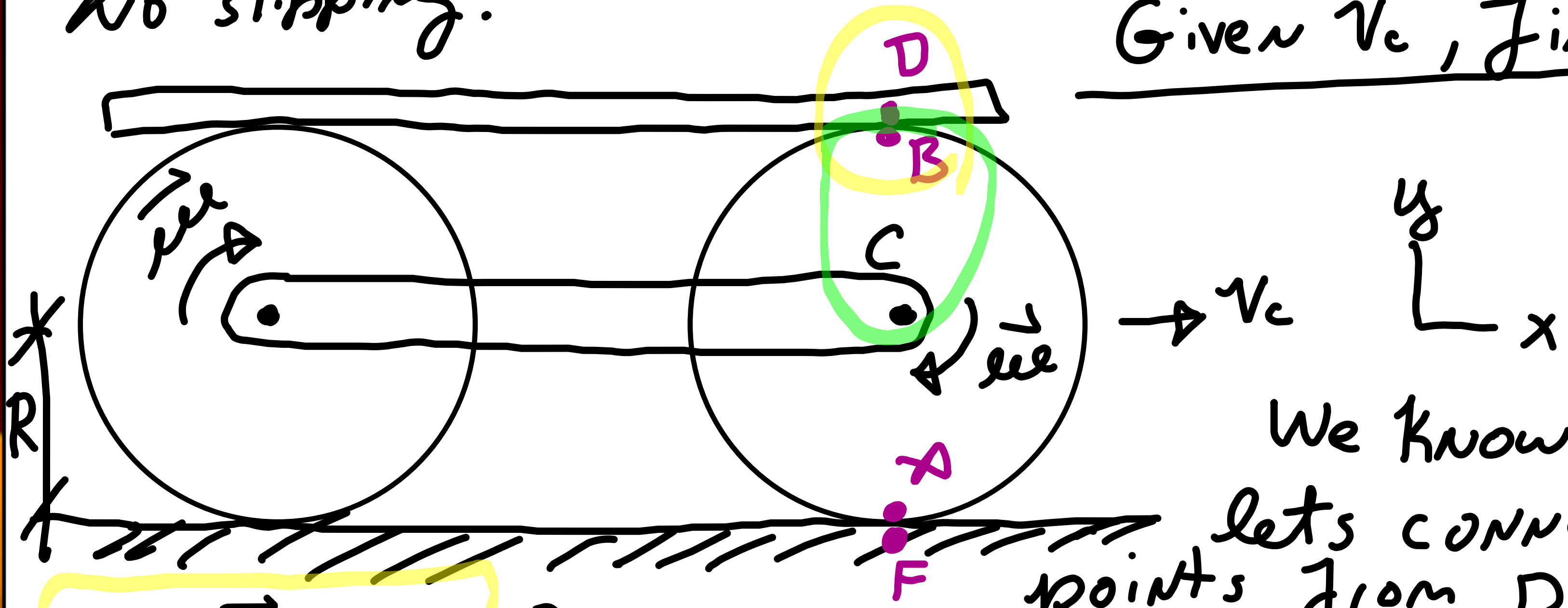
$$\left.\begin{array}{l}\vec{v}_D = \vec{v}_{D/B} + \vec{v}_B \\ \vec{v}_B = \vec{v}_{B/c} + \vec{v}_c\end{array}\right\}$$

But roll no slip $\Rightarrow \vec{v}_{D/B} = 0$ so $\left\{\begin{array}{l}\vec{v}_D = \vec{v}_B \\ \vec{v}_B = \vec{v}_{B/c} + \vec{v}_c\end{array}\right.$

$$\Rightarrow \vec{v}_D = \vec{v}_{B/c} + \vec{v}_c$$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



We know $\vec{V}_c$ so lets connect points from D to C

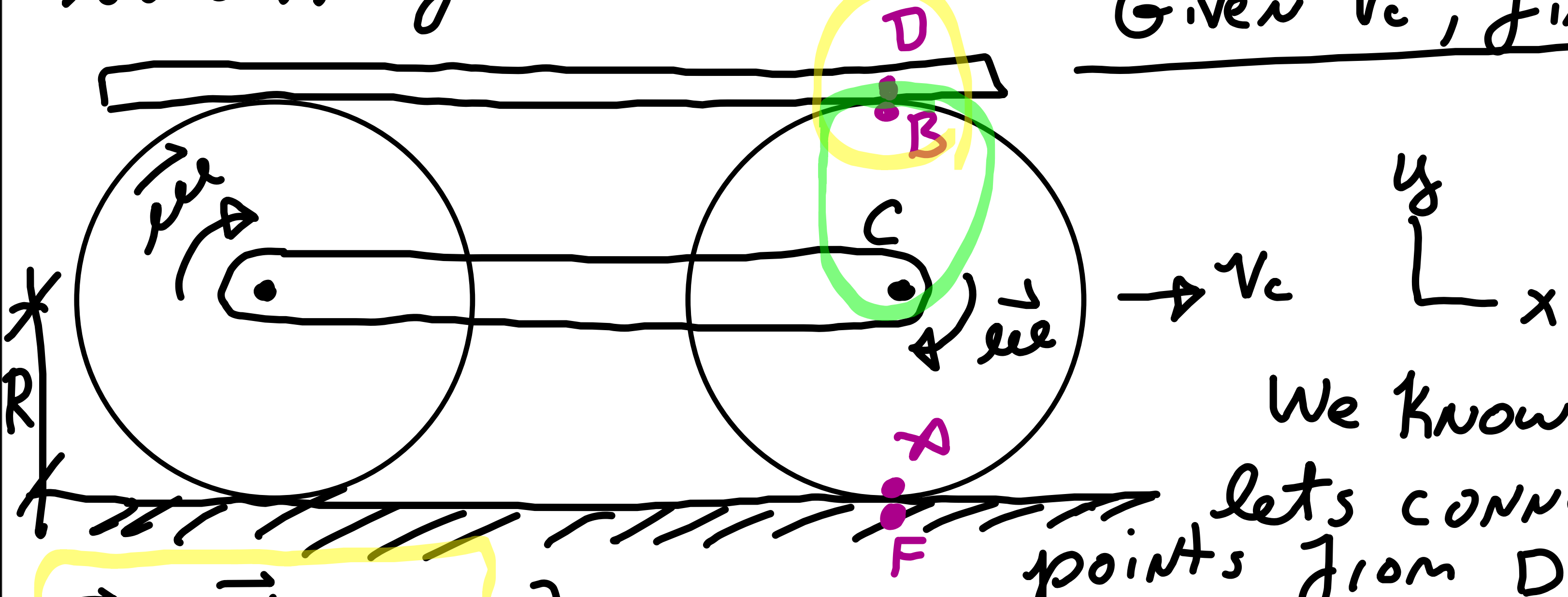
$$\left.\begin{aligned}\vec{V}_D &= \vec{V}_{D/B} + \vec{V}_B \\ \vec{V}_B &= \vec{V}_{B/c} + \vec{V}_c\end{aligned}\right\}$$

But roll no slip $\Rightarrow \vec{V}_{D/B} = 0$ so $\begin{cases}\vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c\end{cases}$

$\Rightarrow \vec{V}_D = \vec{V}_{B/c} + \vec{V}_c$ But $\vec{V}_c = R\omega\hat{x}$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



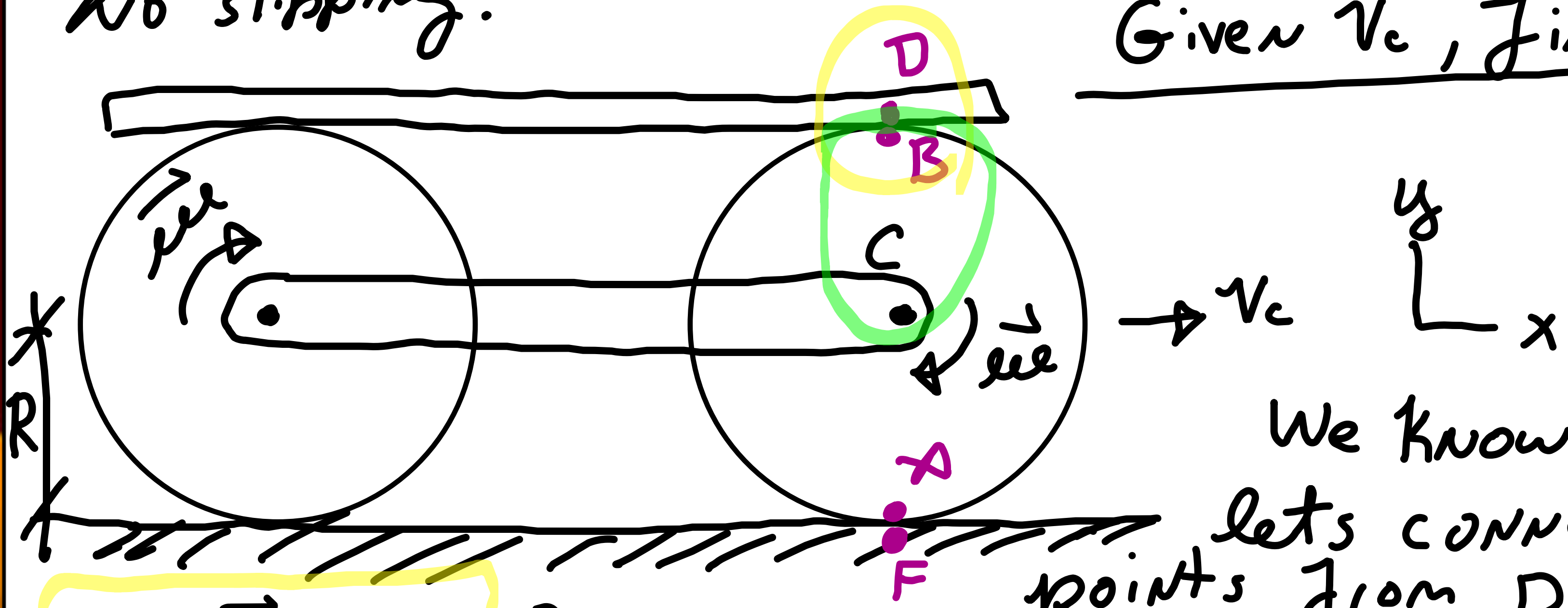
We know $\vec{V}_c$ so lets connect points from D to C

$$\left.\begin{array}{l}\vec{V}_D = \vec{V}_{D/B} + \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c\end{array}\right\}$$ But roll no slip $\Rightarrow \vec{V}_{D/B} = 0$ so $\left\{\begin{array}{l}\vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c\end{array}\right.$

$\Rightarrow \vec{V}_D = \vec{V}_{B/c} + \vec{V}_c$ But $\vec{V}_c = R\omega\hat{x}$ & $\vec{V}_{B/c} = R\omega\hat{x}$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



We know $\vec{V}_c$ so lets connect points from D to C

$$\left.\begin{aligned}\vec{V}_D &= \vec{V}_{D/B} + \vec{V}_B \\ \vec{V}_B &= \vec{V}_{B/C} + \vec{V}_C\end{aligned}\right\}$$

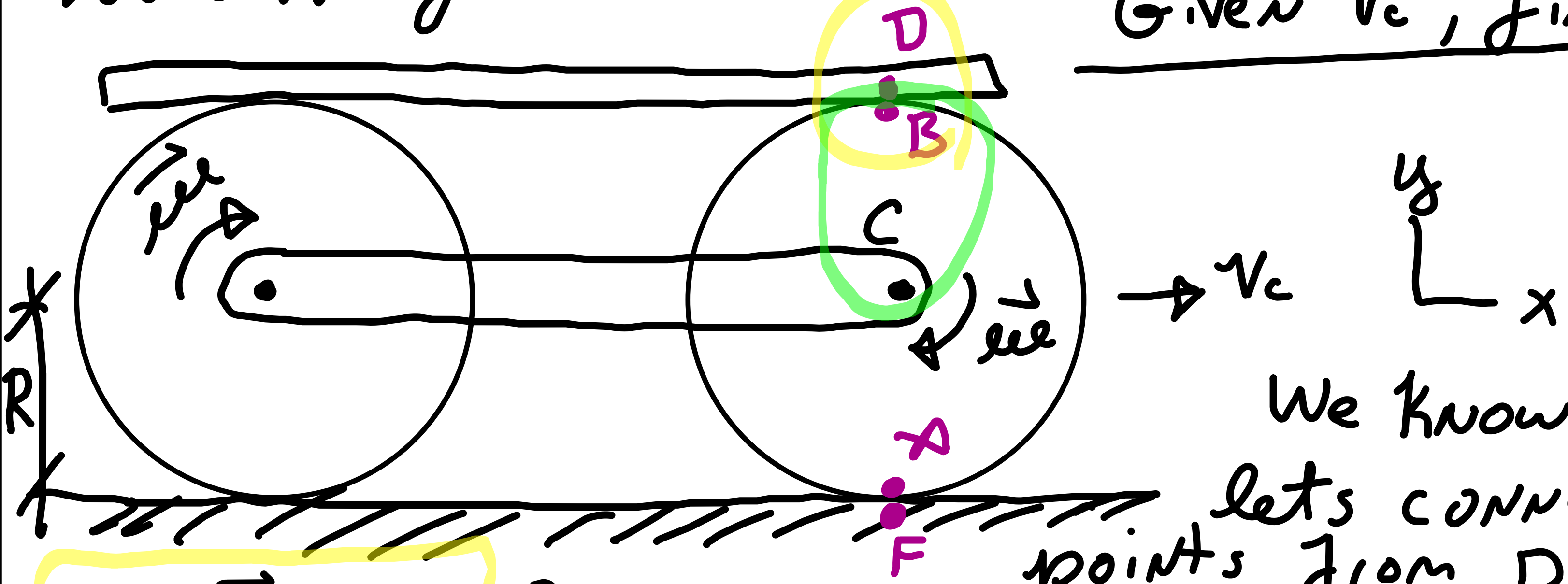
But roll no slip $\Rightarrow \vec{V}_{D/B} = 0$ so $\begin{cases}\vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/C} + \vec{V}_C\end{cases}$

$\Rightarrow \vec{V}_D = \vec{V}_{B/C} + \vec{V}_C$ But $\vec{V}_C = R\omega\hat{x}$ & $\vec{V}_{B/C} = R\omega\hat{x}$

so $\vec{V}_D = 2R\omega\hat{x}$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D



We know $\vec{V}_c$ so lets connect points from D to C

$$\left.\begin{array}{l}\vec{V}_D = \vec{V}_{D/B} + \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c\end{array}\right\}$$

But roll no slip $\Rightarrow \vec{V}_{D/B} = 0$ so $\left\{\begin{array}{l}\vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c\end{array}\right.$

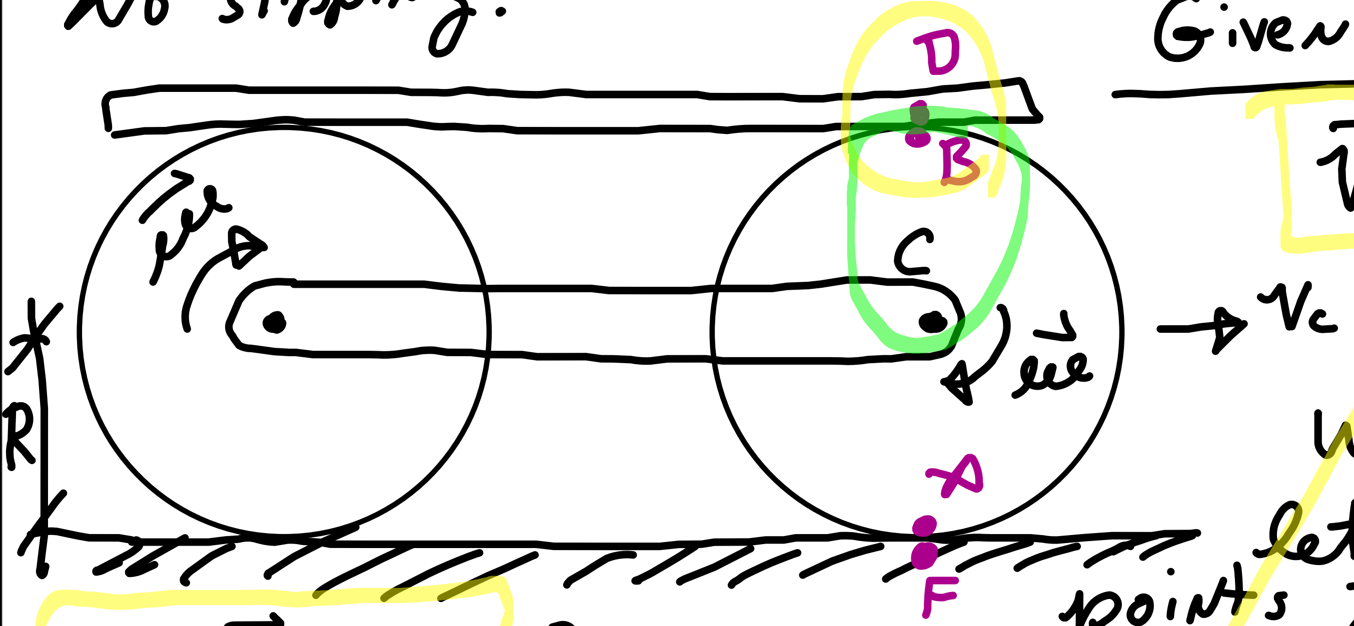
$\Rightarrow \vec{V}_D = \vec{V}_{B/c} + \vec{V}_c$ But $\vec{V}_c = R\omega\hat{x}$ & $\vec{V}_{B/c} = R\omega\hat{x}$

so $\vec{V}_D = 2R\omega\hat{x}$ or $\vec{V}_D = 2\vec{V}_c$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



We know $\vec{V}_c$ so lets connect points from D to C

$$\left.\begin{array}{l} \boxed{\vec{V}_D = \vec{V}_{D/B} + V_B} \\ \boxed{\vec{V}_B = \vec{V}_{B/c} + \vec{V}_c} \end{array}\right\} \text{But roll no slip} \Rightarrow \vec{V}_{D/B} = 0 \text{ so } \begin{cases} \vec{V}_D = \vec{V}_B \\ \vec{V}_B = \vec{V}_{B/c} + \vec{V}_c \end{cases}$$

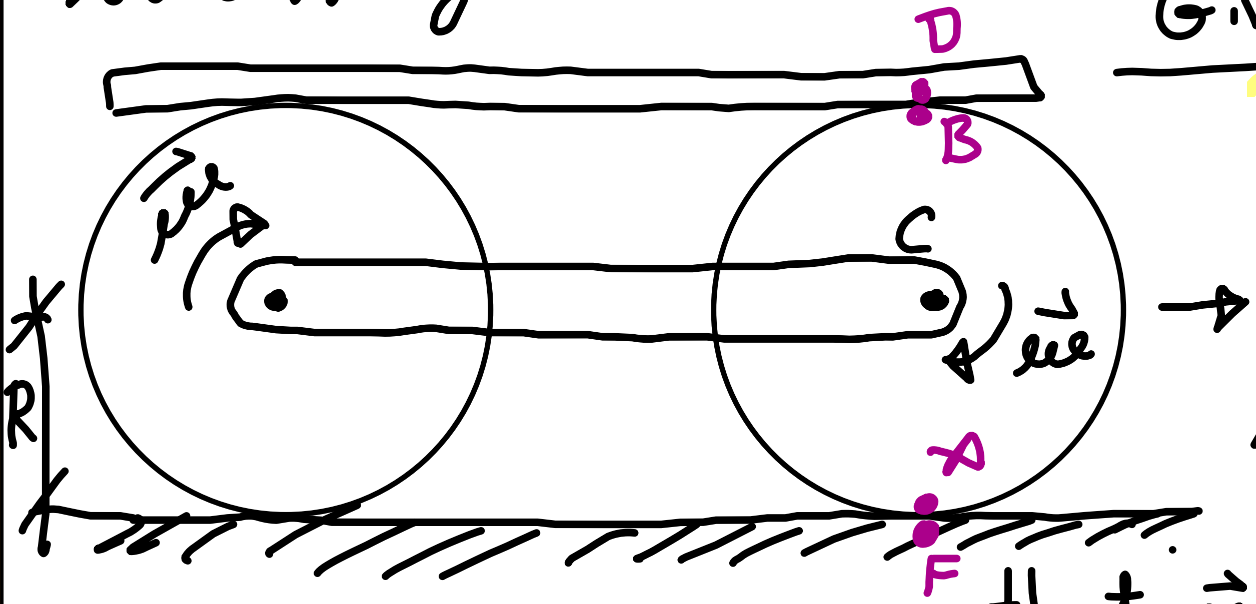
$\Rightarrow \vec{V}_D = \vec{V}_{B/c} + \vec{V}_c$ But $\vec{V}_c = R\omega\hat{x}$ & $\vec{V}_{B/c} = R\omega\hat{x}$

so $\vec{V}_D = 2R\omega\hat{x}$ or $\boxed{\vec{V}_D = 2\vec{V}_c}$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$

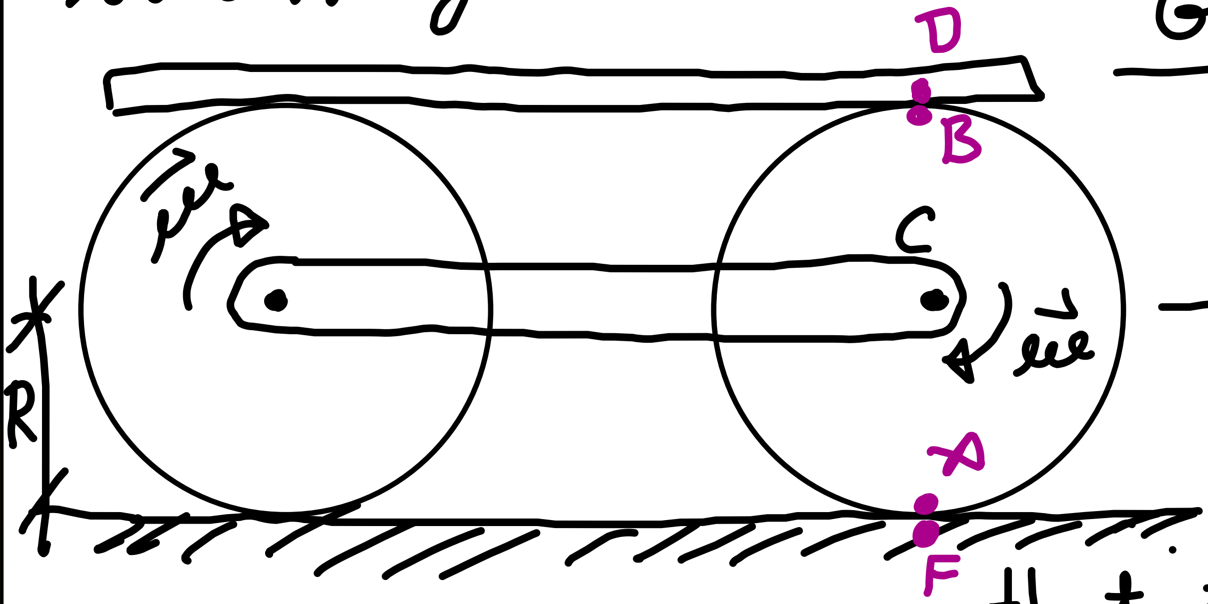


Another way is to notice that $\vec{\omega}$ is independent of the reference point

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$

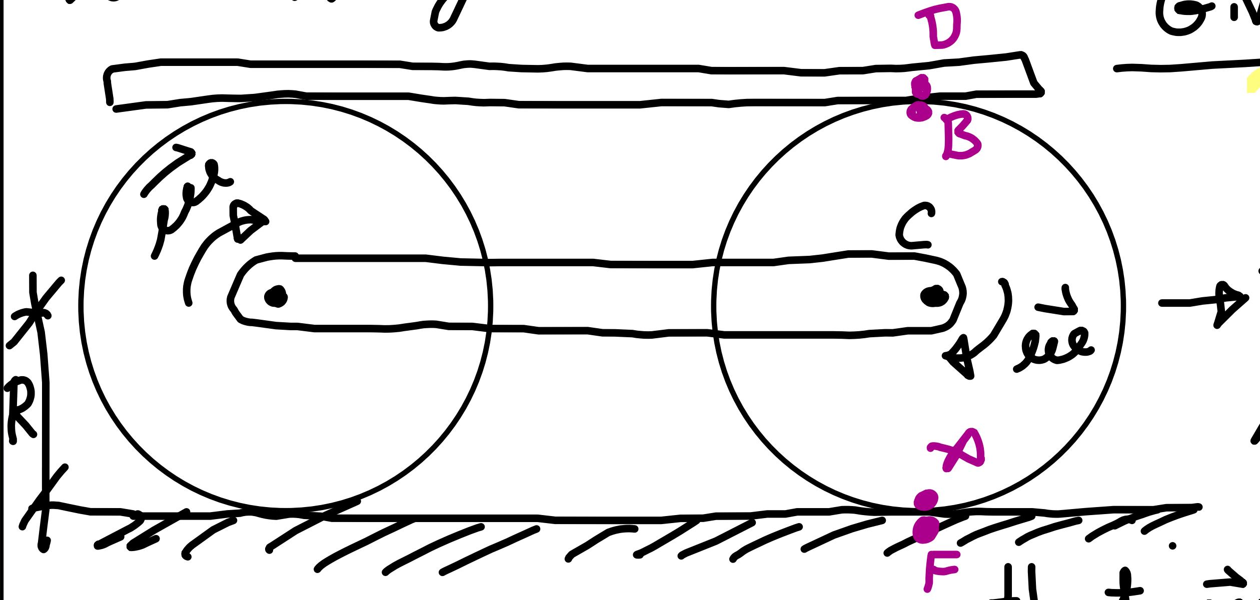


Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, Find V_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$

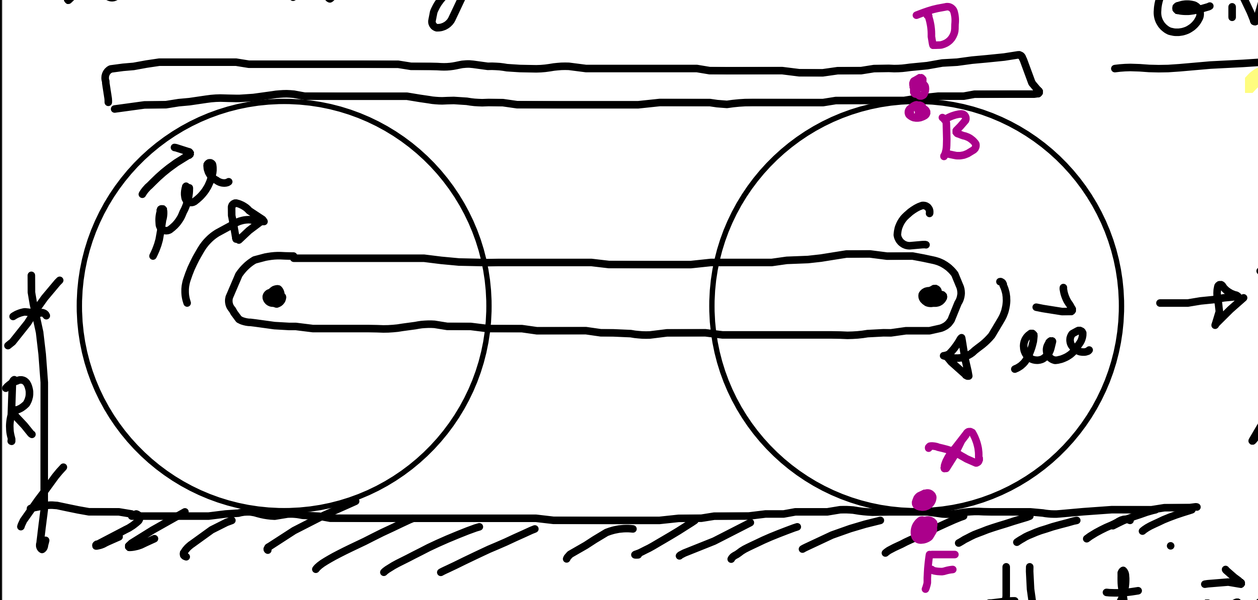


Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$ & since roll no slip $\vec{V}_A = \vec{V}_F$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



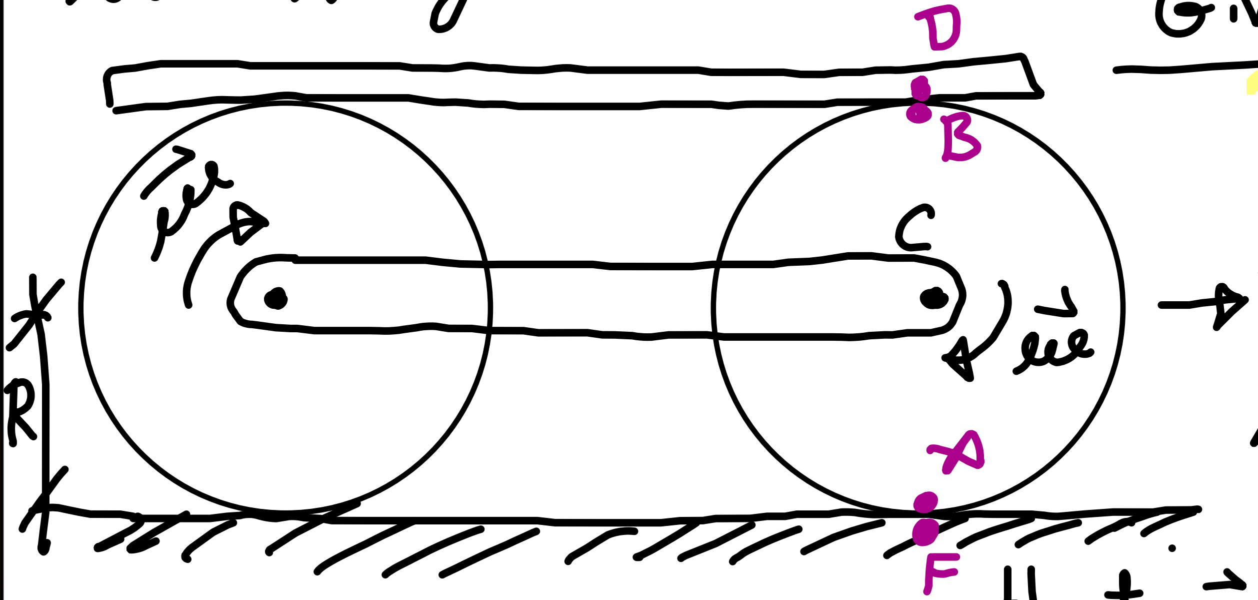
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F = \vec{0}$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



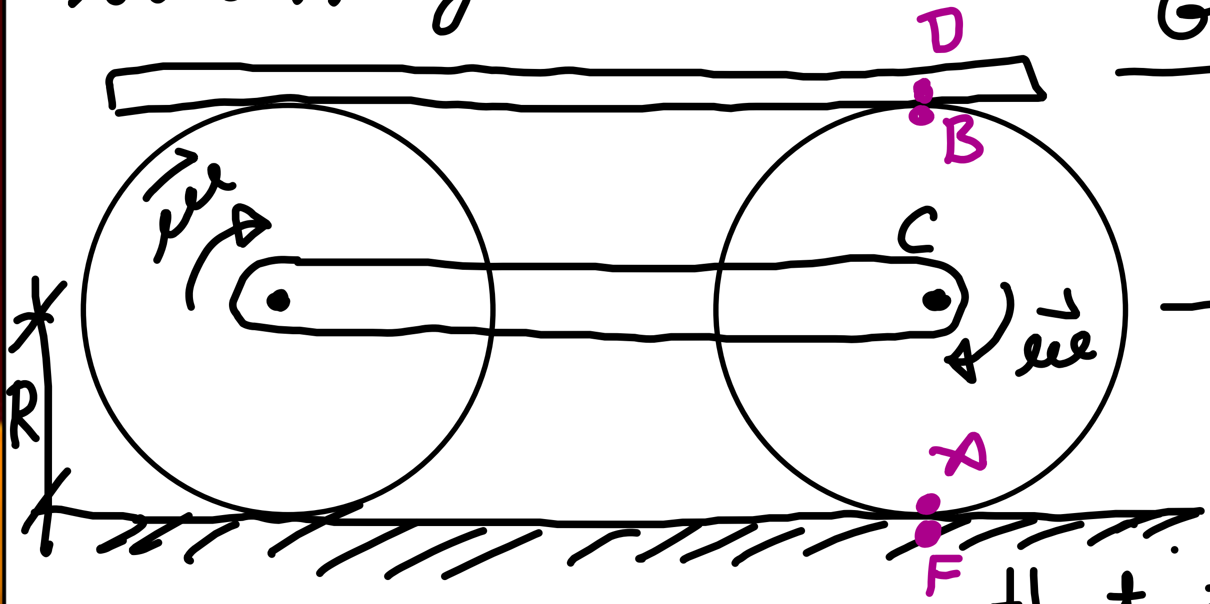
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



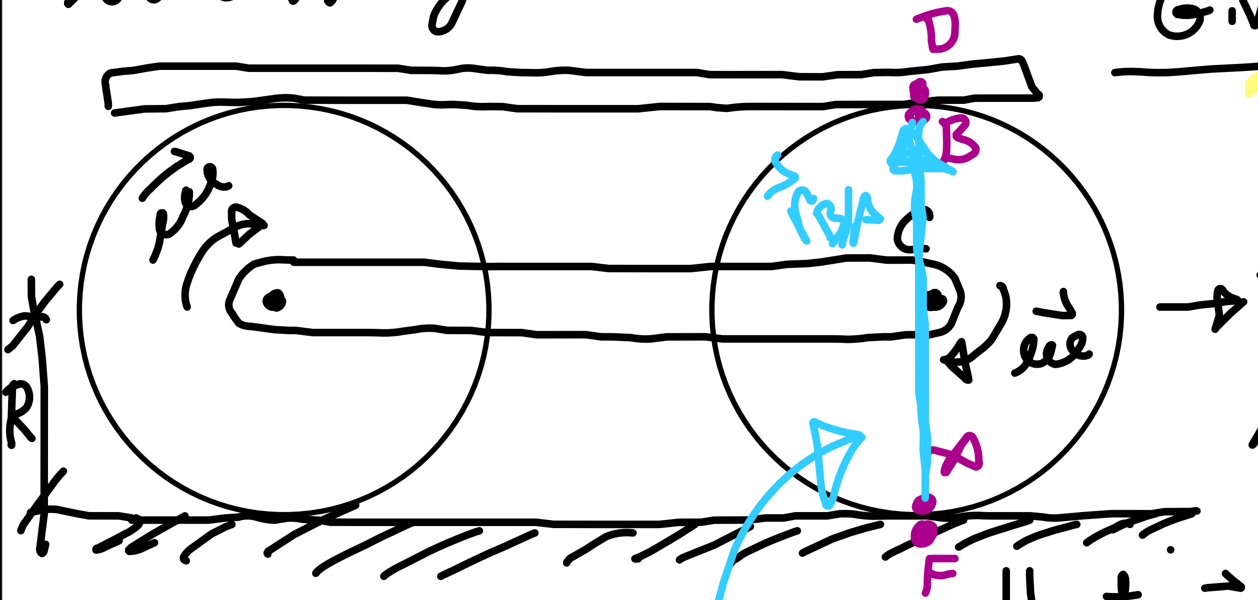
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$ & since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

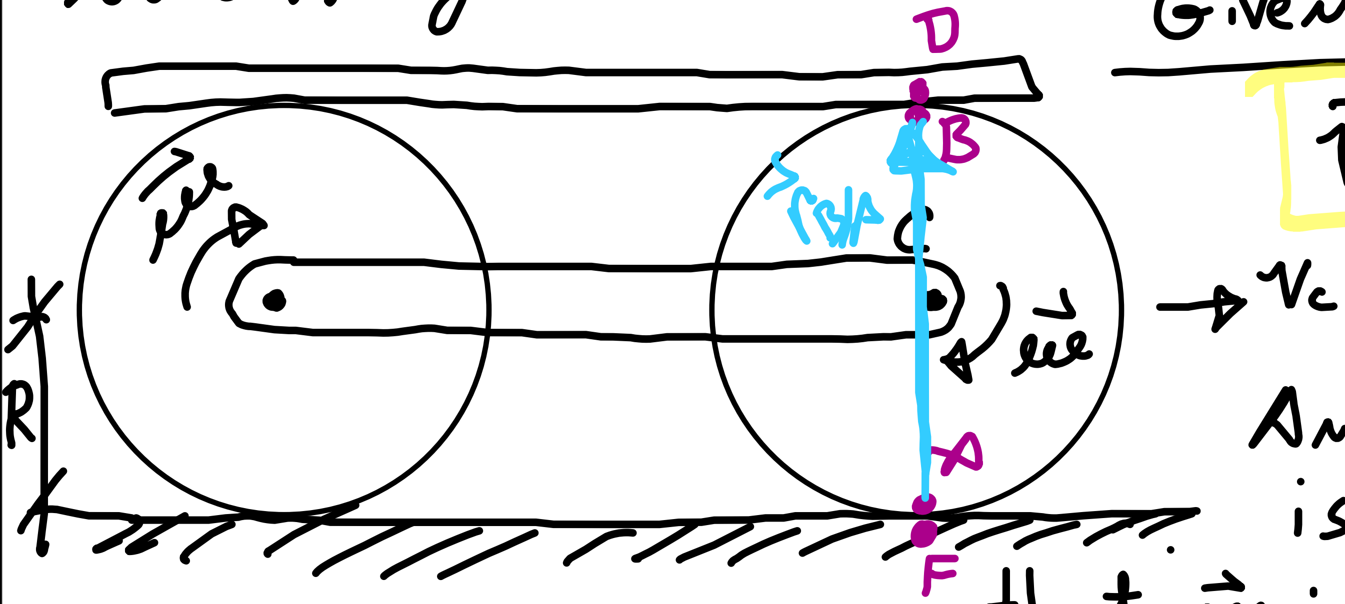
& since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



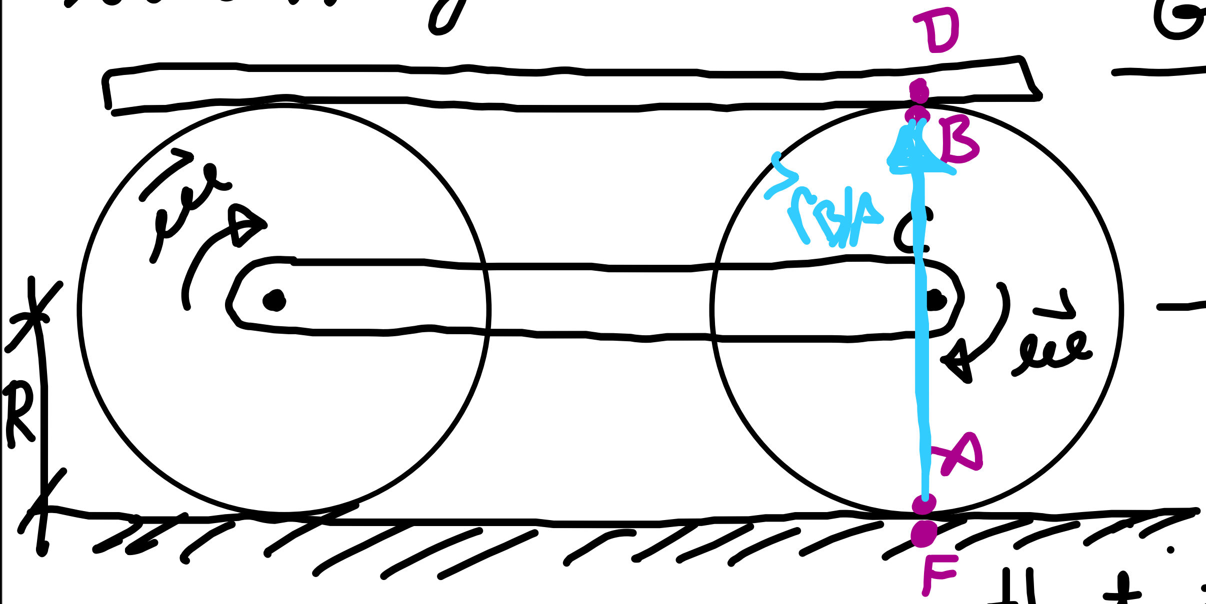
Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$ & since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = \vec{\omega} \times 2R\hat{y}$$

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{v}_D = 2\vec{v}_c}$$



Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$

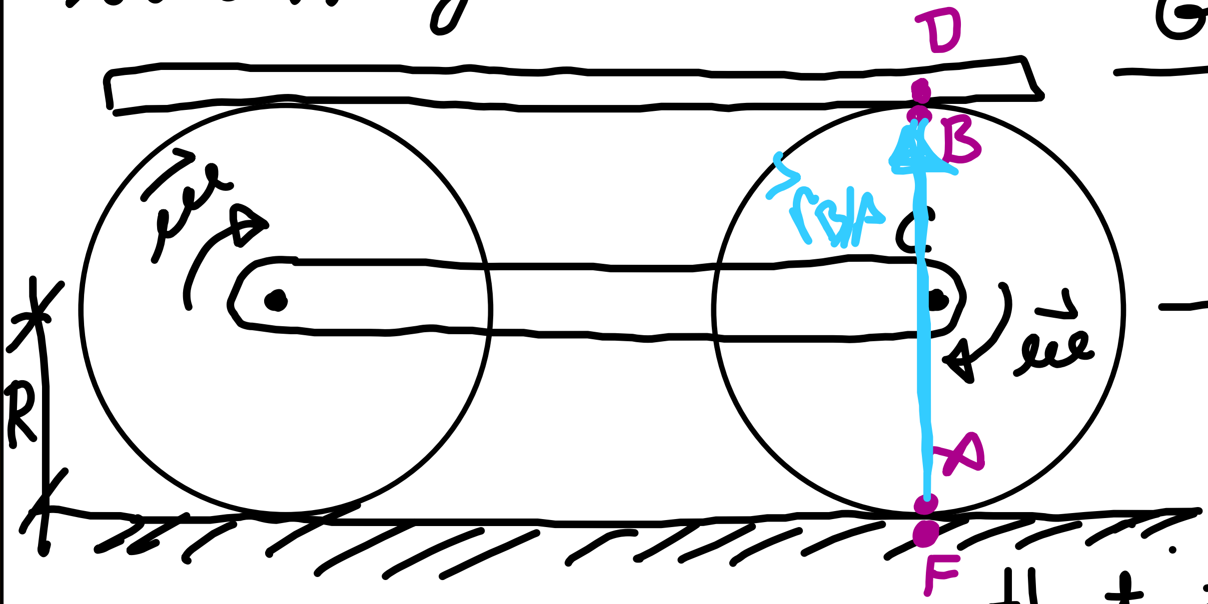
& since roll no slip $\vec{v}_A = \vec{v}_F = 0$ so $\vec{v}_B = \vec{v}_{B/A}$

$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = \vec{\omega} \times 2R\hat{y} = -2R\omega\hat{x}$$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$

& since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

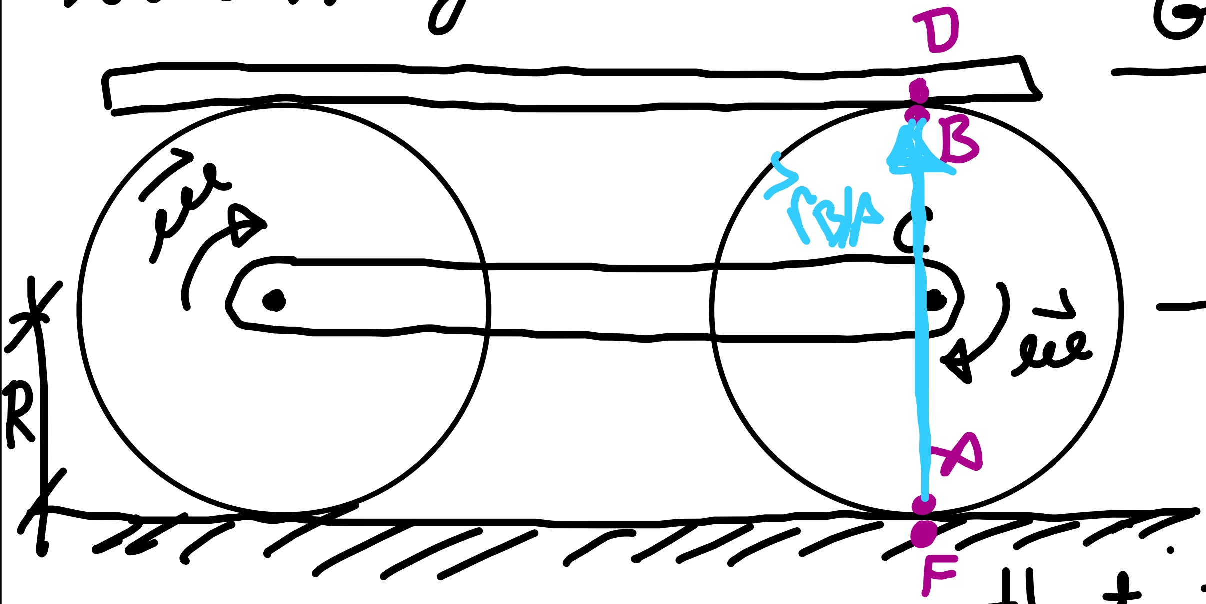
$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = \vec{\omega} \times 2R\hat{y} = -2R\omega\hat{x}$$

OR $\vec{V}_B = 2\vec{V}_c$

Example: Board sitting on wheels. Rolling no slipping.

Given V_c, find V_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$ & since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

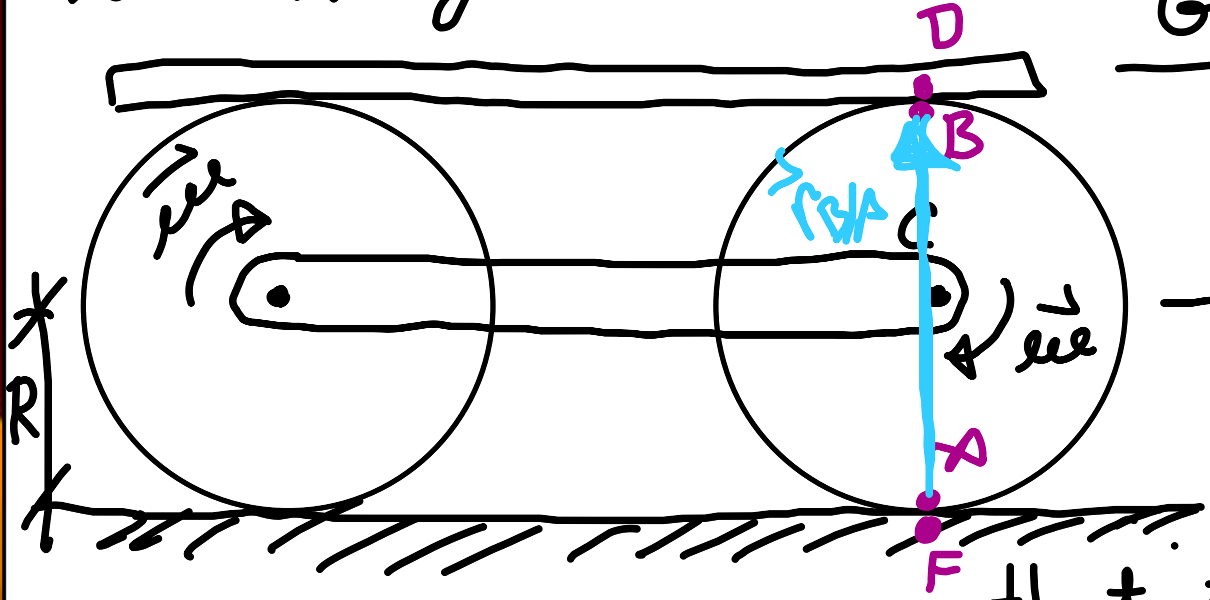
$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = \vec{\omega} \times 2R\hat{y} = -2R\omega\hat{x}$$

OR $\vec{V}_B = 2\vec{V}_c$ & since roll no slip

Example: Board sitting on wheels. Rolling no slipping.

Given v_c, find v_D

$$\boxed{\vec{V}_D = 2\vec{V}_c}$$



Another way is to notice that $\vec{\omega}$ is independent of the reference point & take $\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$ & since roll no slip $\vec{V}_A = \vec{V}_F = 0$ so $\vec{V}_B = \vec{V}_{B/A}$

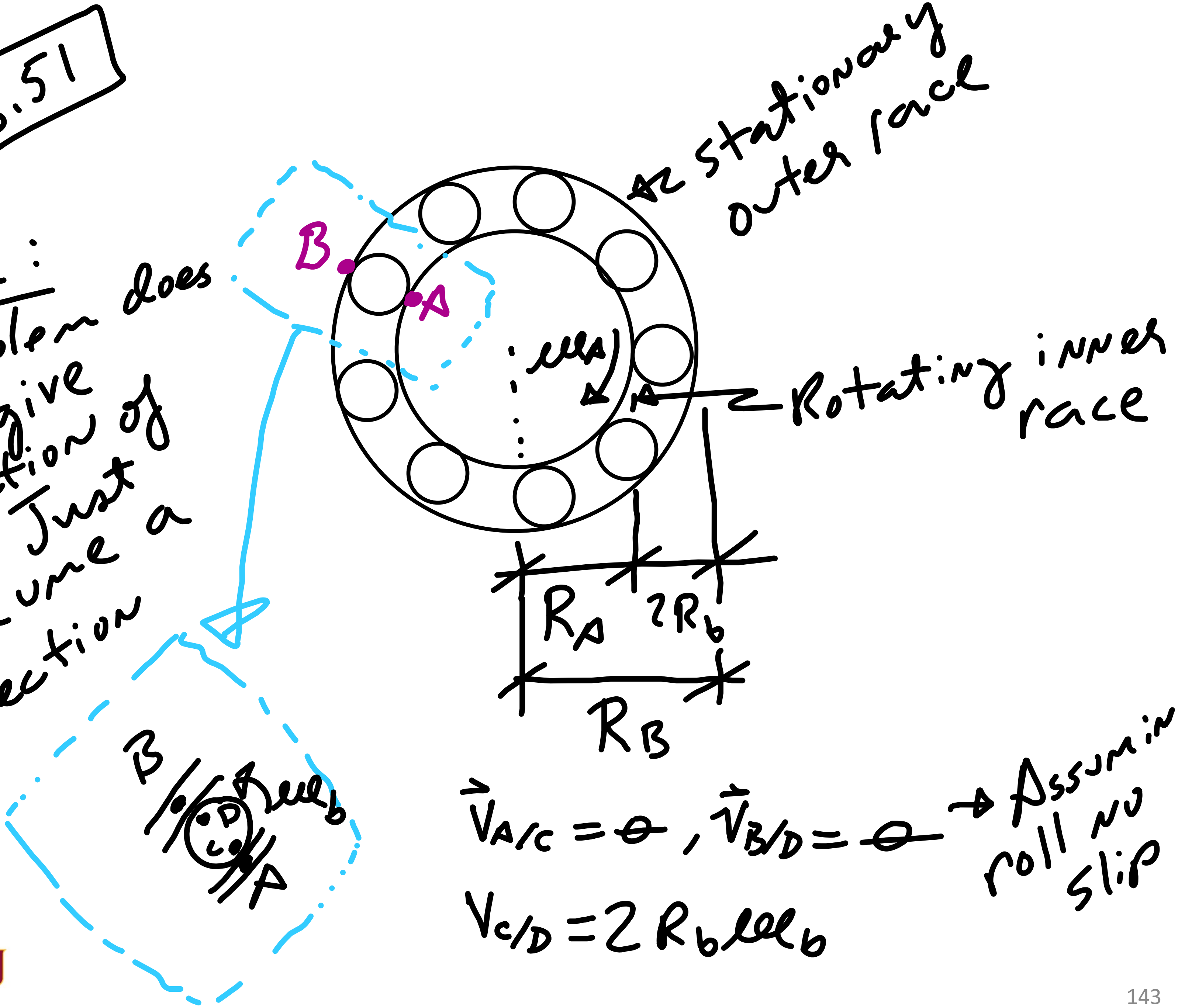
$$\Rightarrow \vec{V}_B = \vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = \vec{\omega} \times 2R\hat{y} = -2R\omega\hat{x}$$

OR $\vec{V}_B = 2\vec{V}_c$ & since roll no slip

$$\vec{V}_D = \vec{V}_B = 2\vec{V}_c$$

15.51

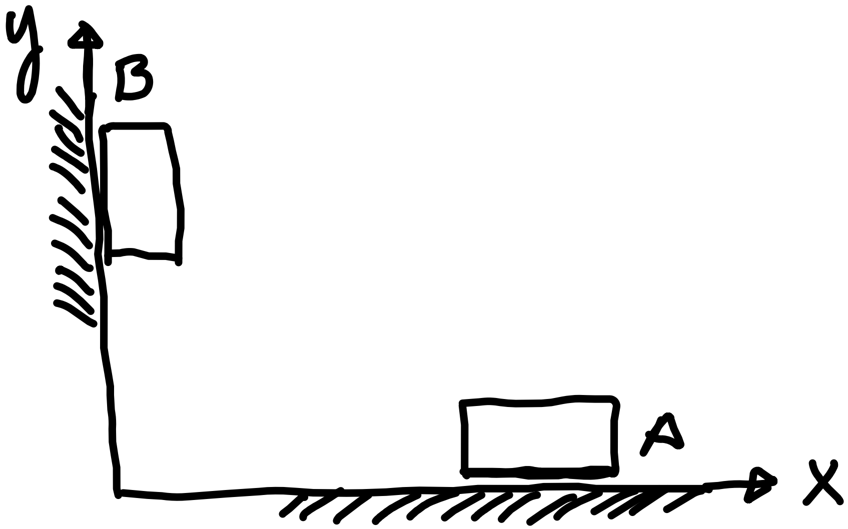
Note: problem does not give direction of ω_A. Just assume a direction

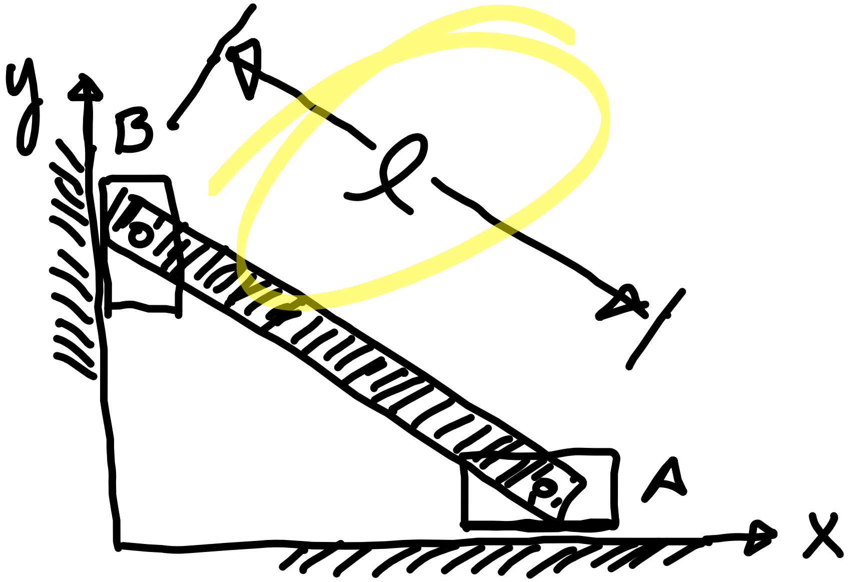


$\vec{V}_{A/C} = 0$, $\vec{V}_{B/D} = 0$ → Assuming roll no slip

$V_{C/D} = 2R_b\omega_b$

Example



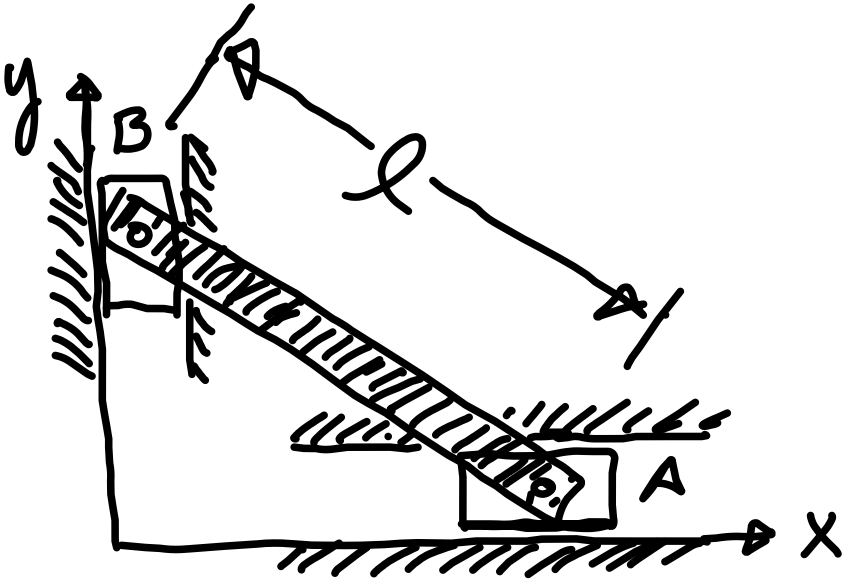


Example

Given l

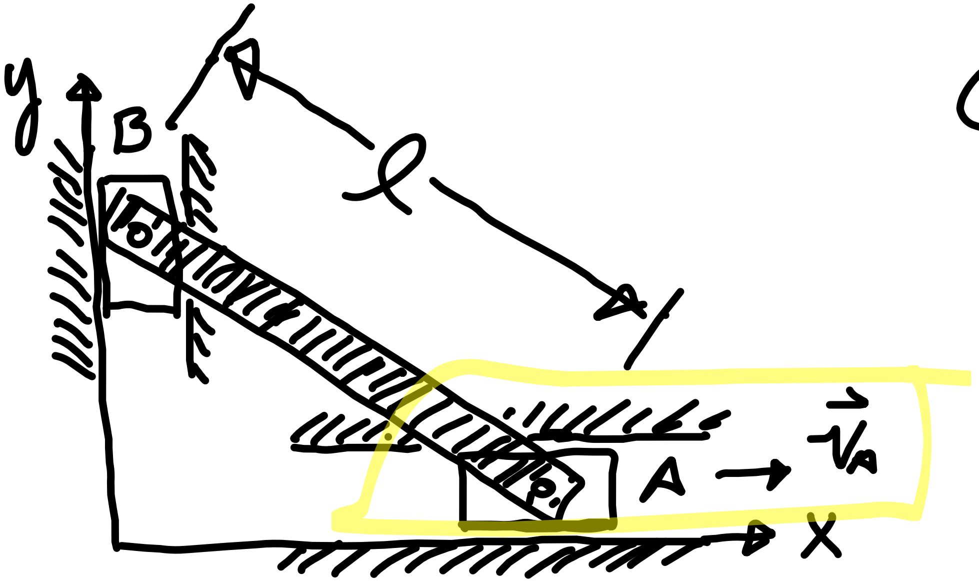
Example

Given l



Example

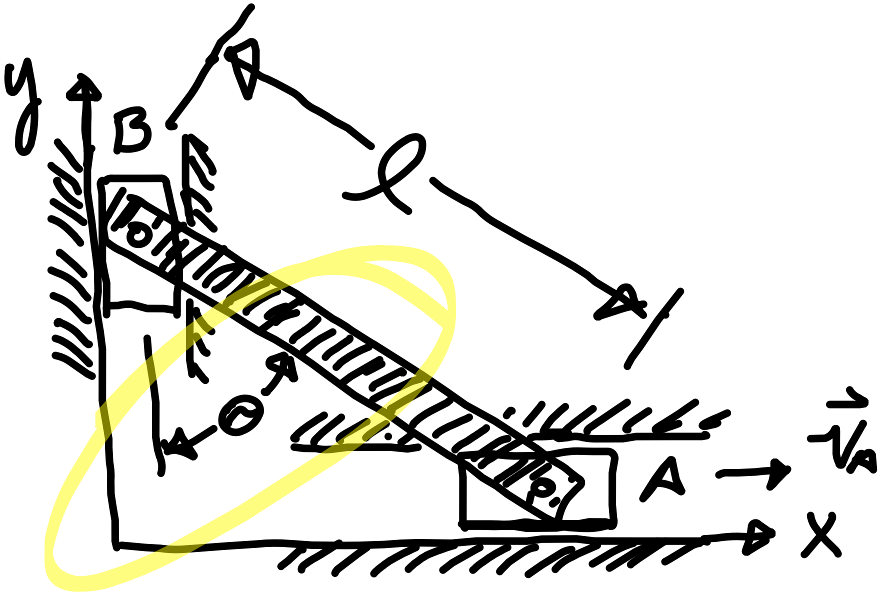
Given ℓ, $\vec{V}_A = V_A \hat{x}$

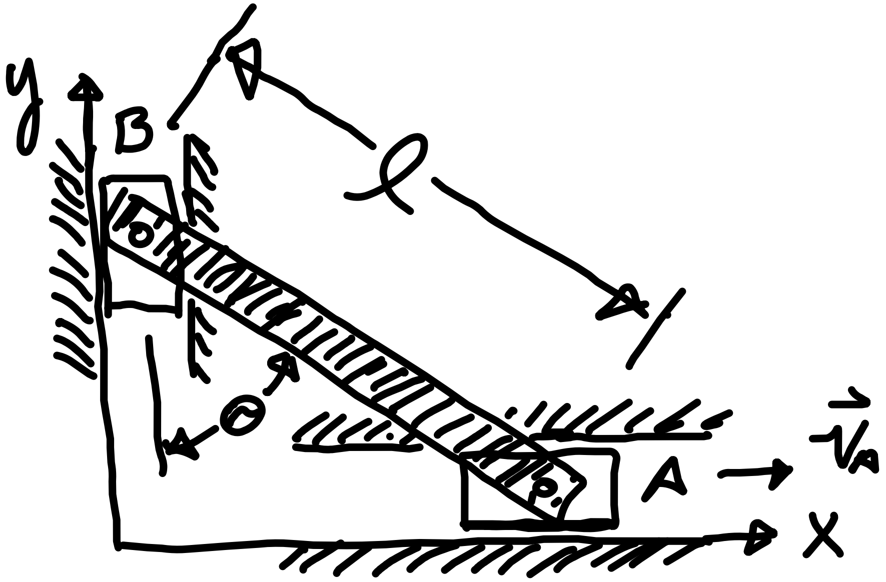


Example

Given l, $\vec{V}_A = V_A\hat{x}$ &

θ



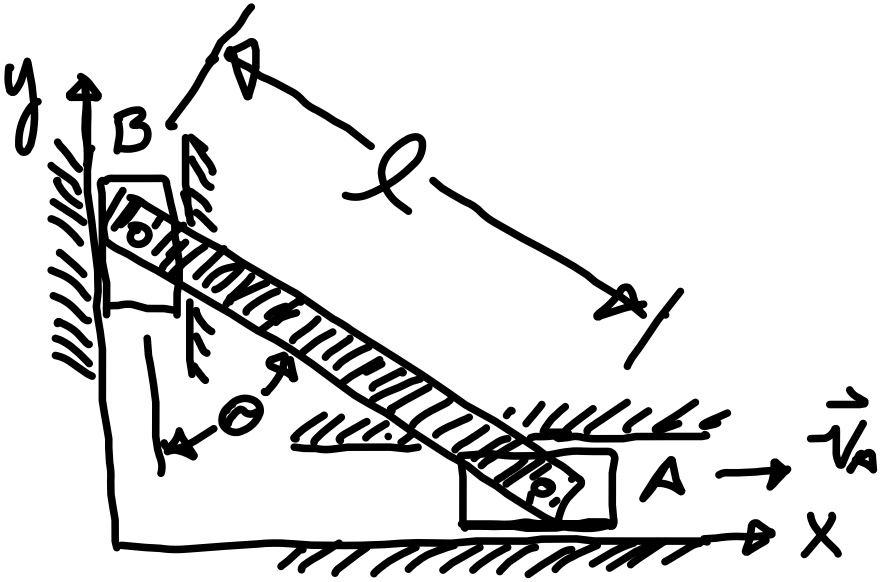


Example

Given ℓ, $\vec{v}_A = v_A\hat{x}$ &

θ

Find $\vec{v}_B$

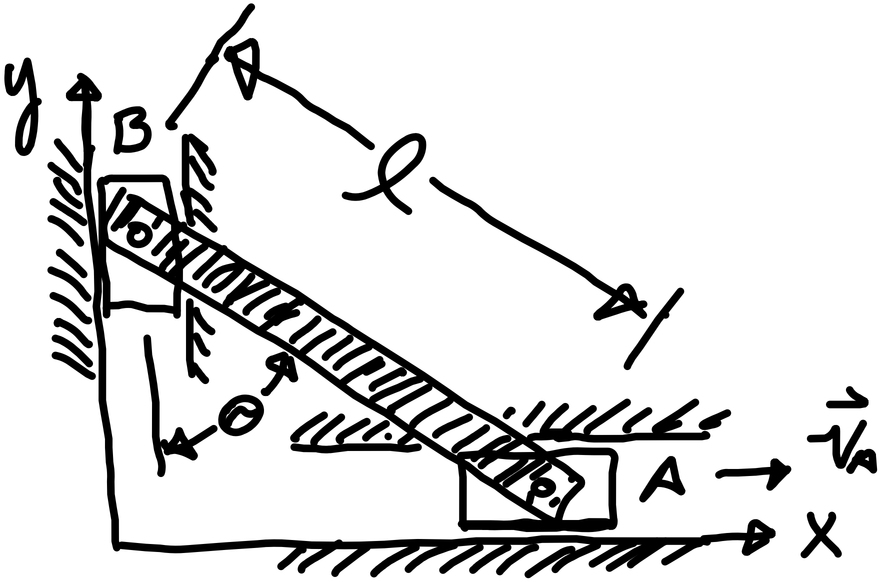


Example

Given l, $\vec{V}_A = V_A \hat{x}$ &

θ

Find $\vec{V}_B$ & ω_{ARM}:



Example

Given l, $\vec{V}_A = V_A \hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:

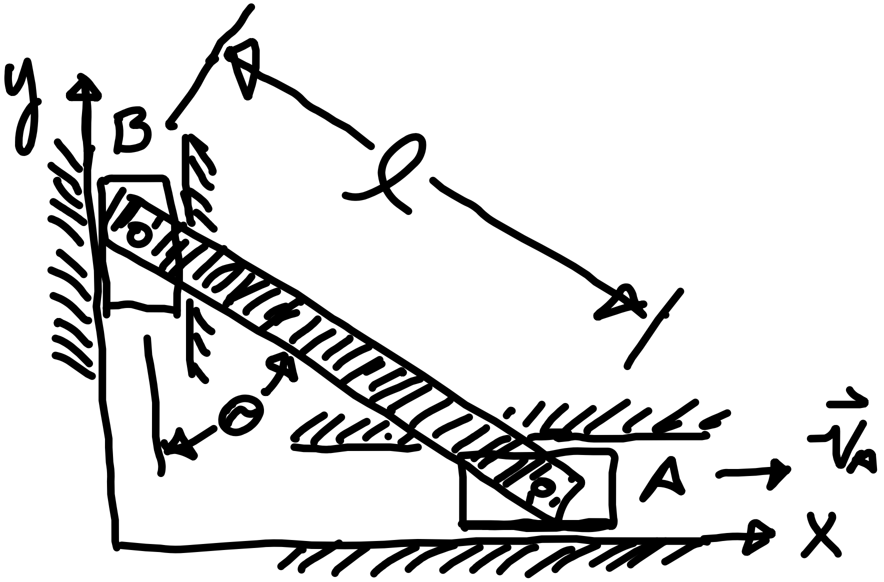
$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

Example

Given l, $\vec{V}_A = V_A \hat{x}$ &

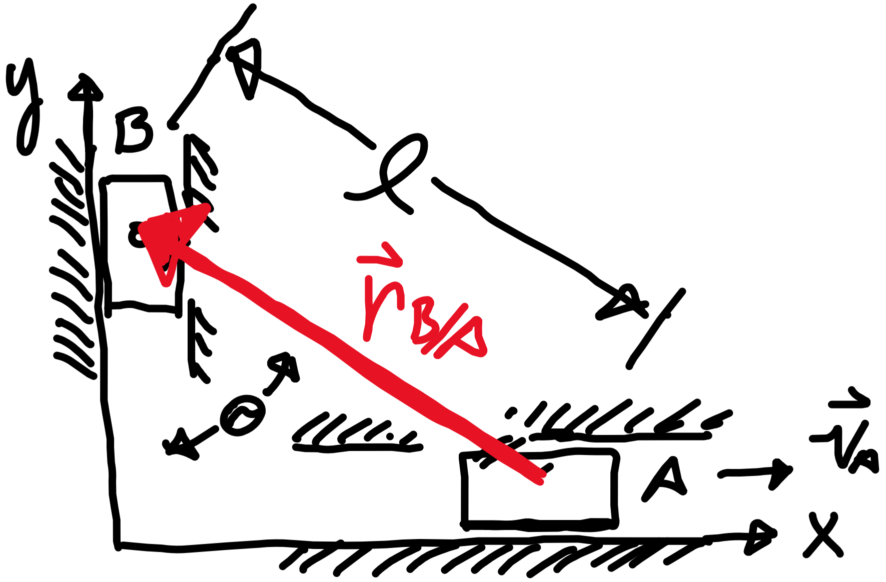
θ

Find $\vec{V}_B$ & ω_{ARM}:



$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

$$= \vec{\omega} \times \vec{r}_{B/A} + V_A \hat{x}$$



Example

Given ℓ, $\vec{V}_A = V_A\hat{x}$ &
θ

Find $\vec{V}_B$ & ω_{ARM}:

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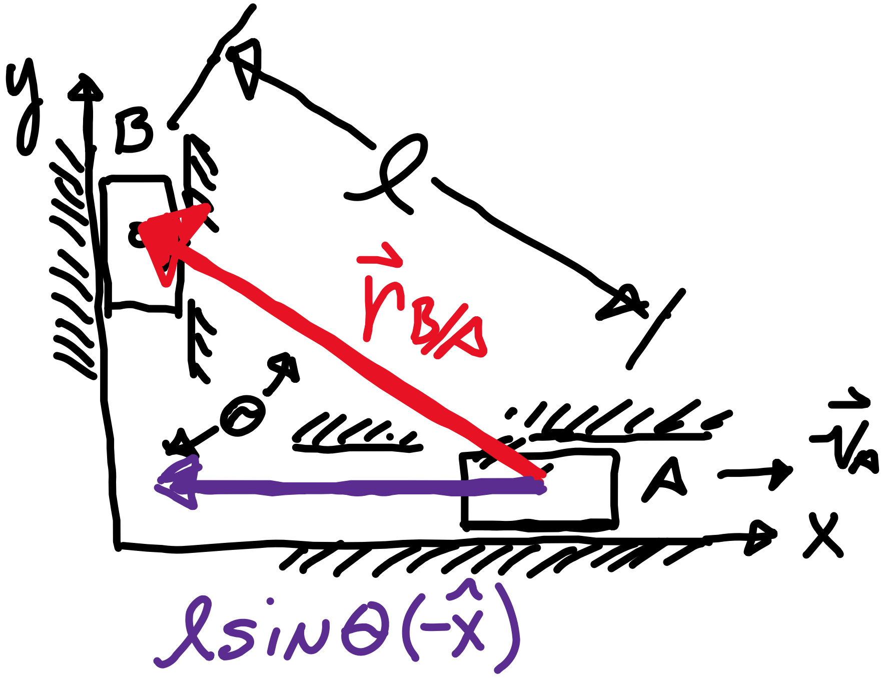

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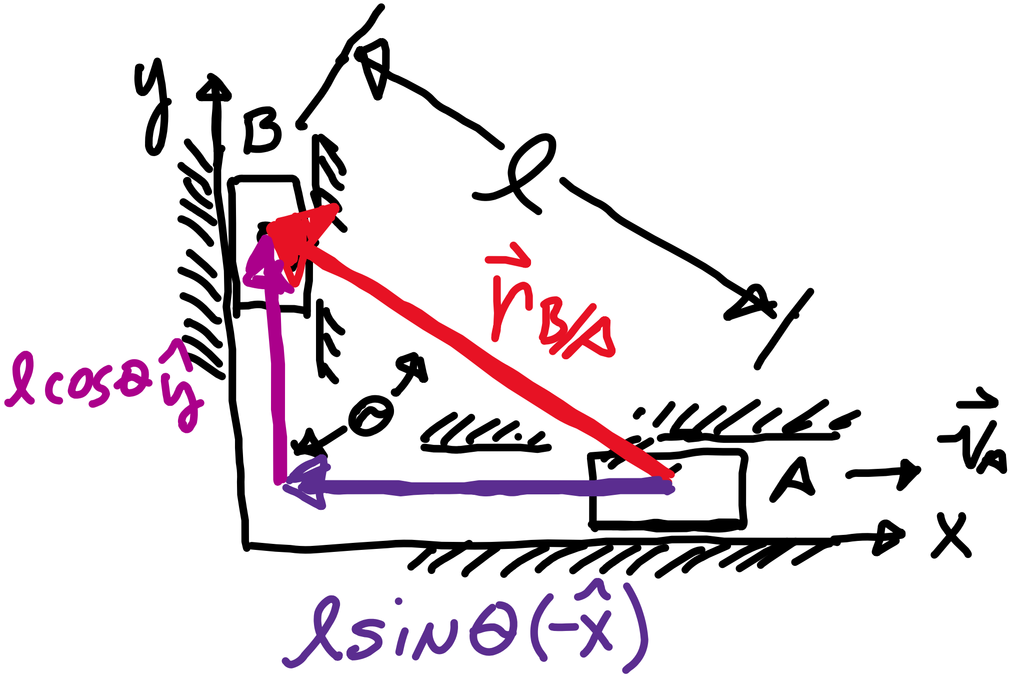

Example

Given l, $\vec{V}_A = V_A\hat{x}$ &

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Find $\vec{V}_B$ & ω_{ARM}:

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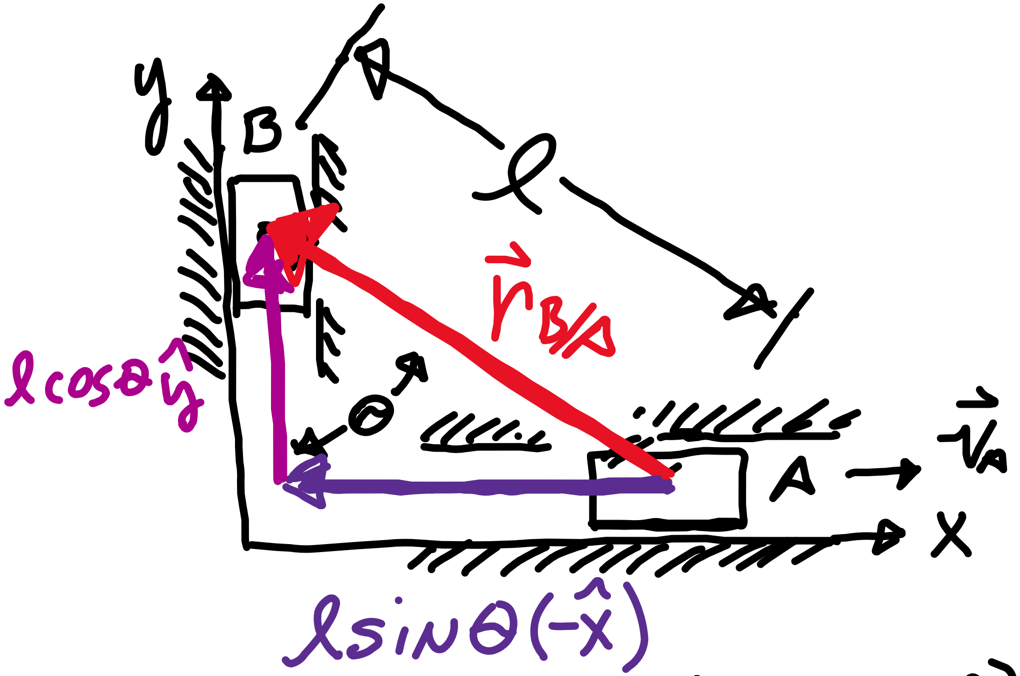

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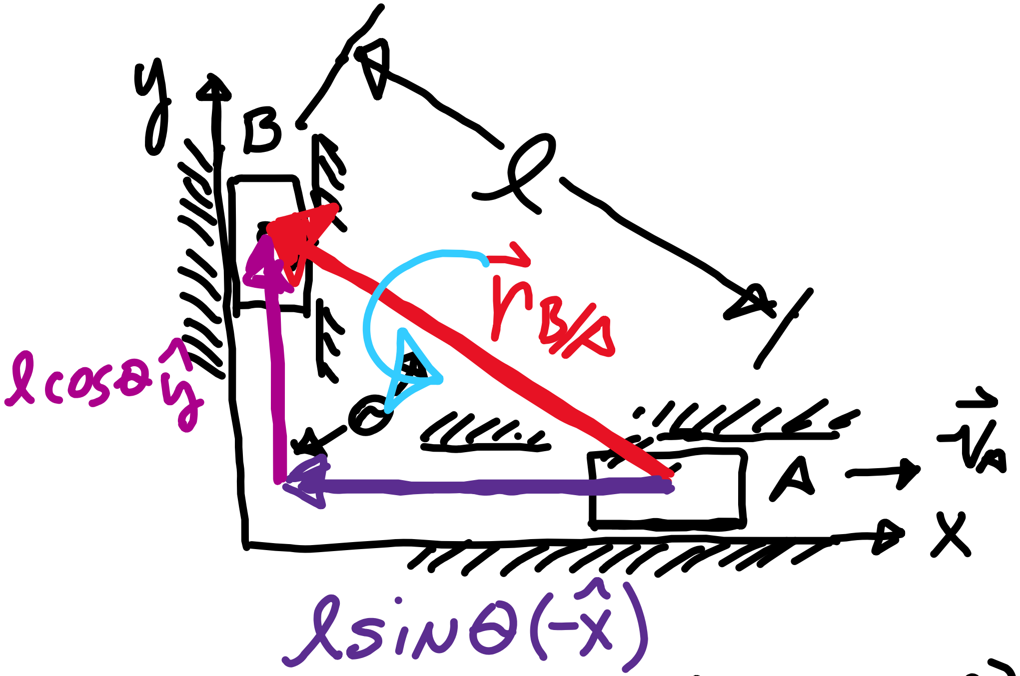
Find $\vec{V}_B$ & ω_{ARM}:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$
$$= \vec{\omega}\times\vec{r}_{B/A} + V_A\hat{x}$$

$$\Rightarrow \vec{r}_{B/A} = \ell[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$$



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Find $\vec{V}_B$ & ω_{ARM}:

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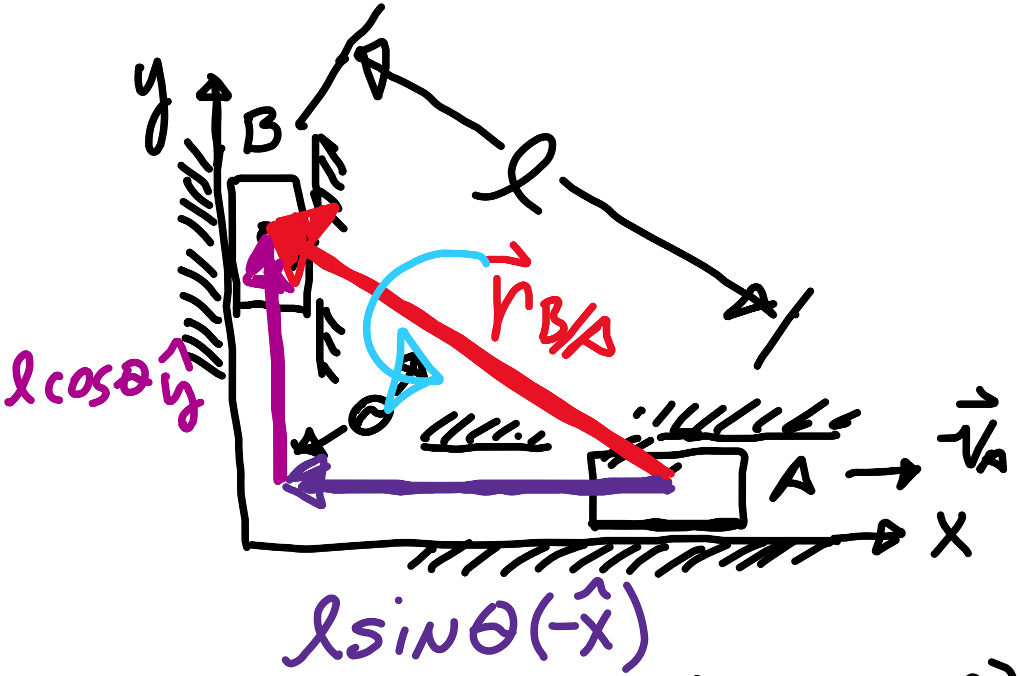


Example

Given l, $\vec{V}_A = V_A\hat{x}$ &

θ

Find $\vec{V}_B$ & ω_{ARM}:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

$$= \vec{\omega}\times\vec{r}_{B/A} + V_A\hat{x}$$

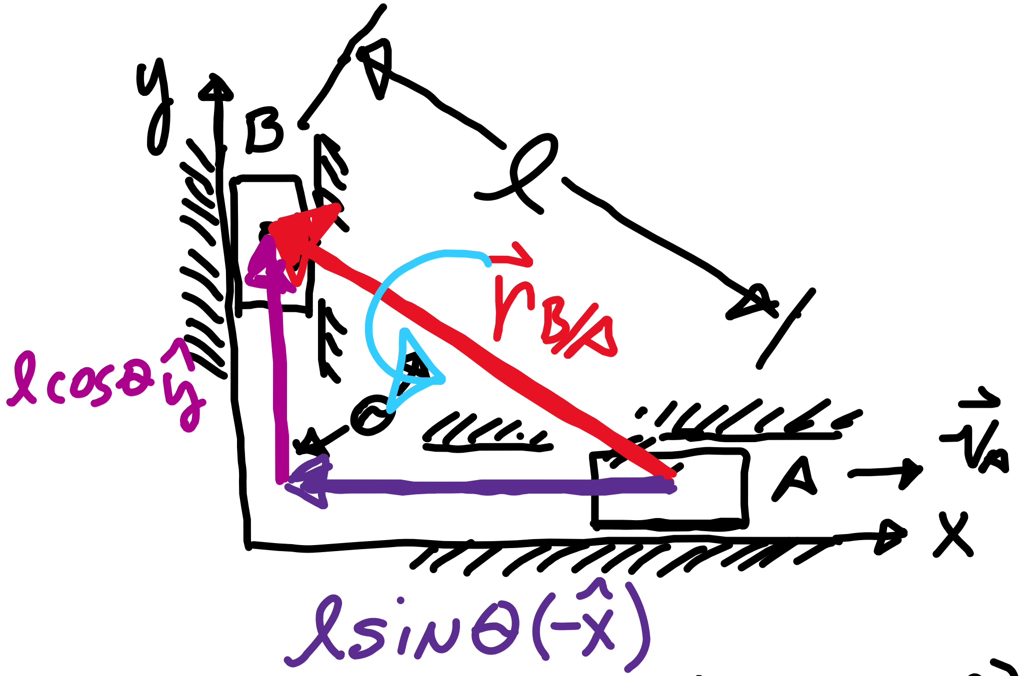
$\Rightarrow$ $\vec{r}_{B/A} = l[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

so $\vec{V}_B = \omega\,\hat{z}\times l[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}] + V_A\hat{x}$

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

$$= \vec{\omega}\times\vec{r}_{B/A} + V_A\hat{x}$$

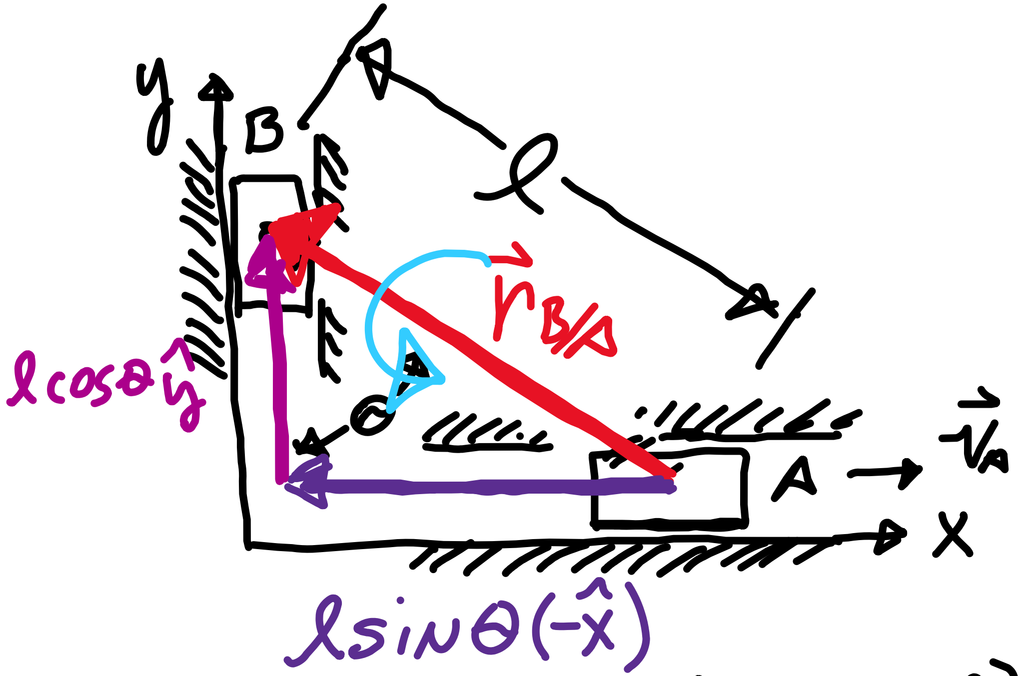
$\Rightarrow$ $\vec{r}_{B/A} = l[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

so $\vec{V}_B = \omega l[-\sin\theta(\hat{z}\times\hat{x}) + \cos\theta(\hat{z}\times\hat{y})] + V_A\hat{x}$

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



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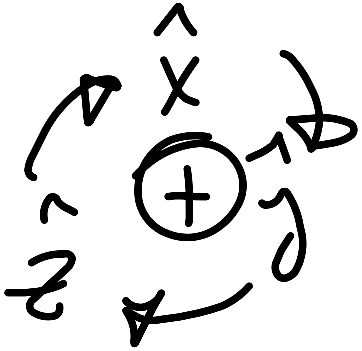
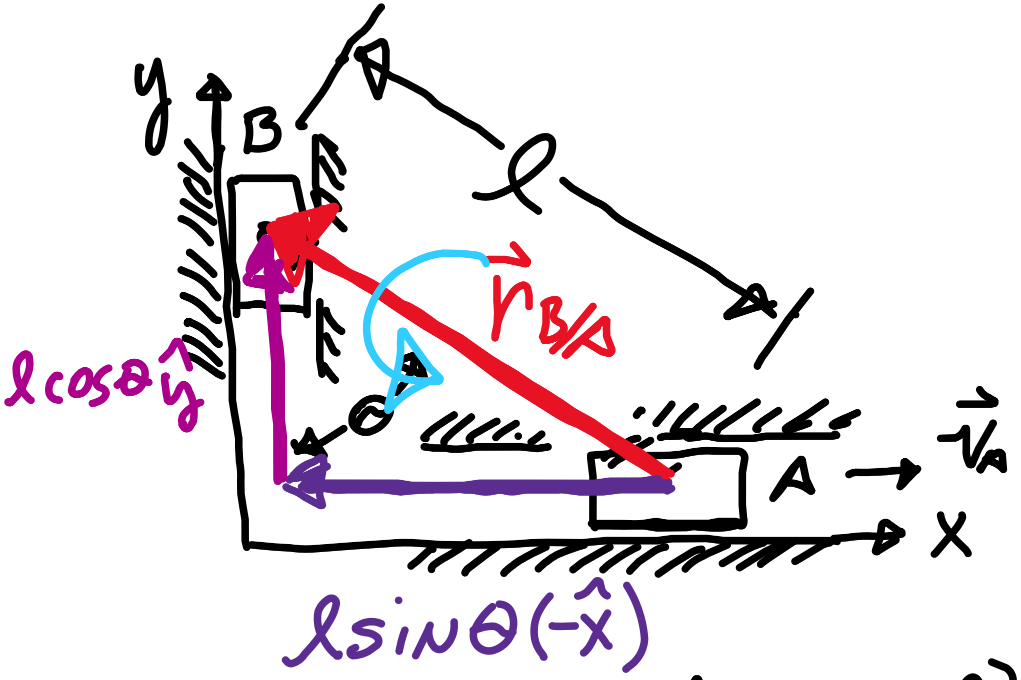




Example

Given ℓ, $\vec{V}_A = V_A\hat{x}$ &

θ

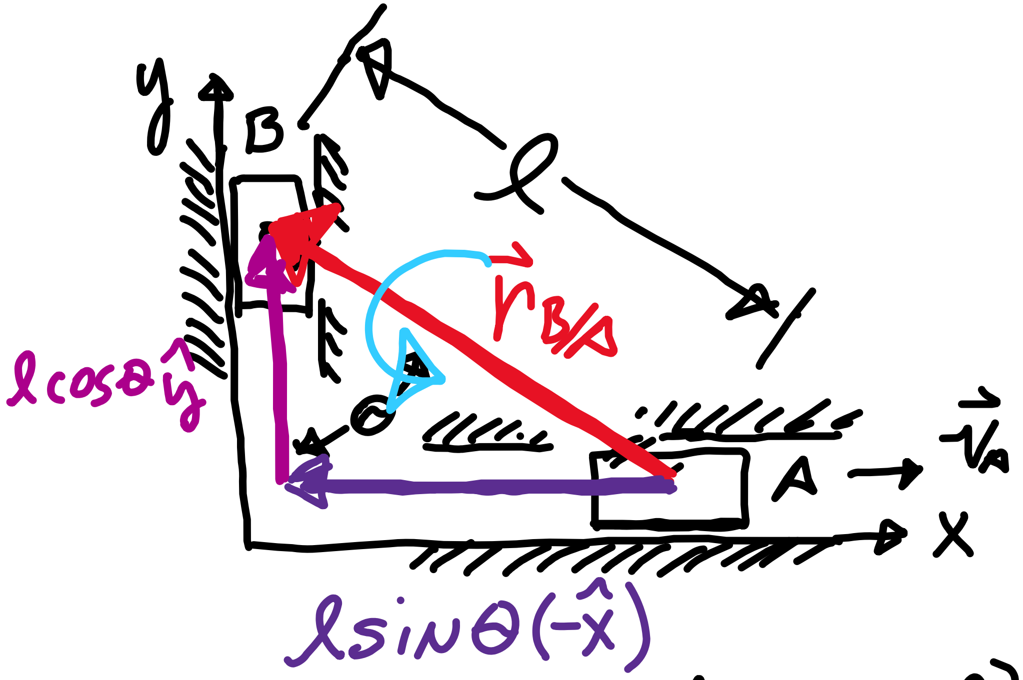
Find $\vec{V}_B$ & ω_{ARM}:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

$$= \vec{\omega} \times \vec{r}_{B/A} + V_A\hat{x}$$

$$\Rightarrow \quad \vec{r}_{B/A} = \ell[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}] \quad \& \quad \vec{\omega} = \omega\,\hat{z}$$

$$\text{so} \quad \vec{V}_B = \omega\ell[-\sin\theta\,\hat{y} + \cos\theta\,(-\hat{x})] + V_A\hat{x}$$



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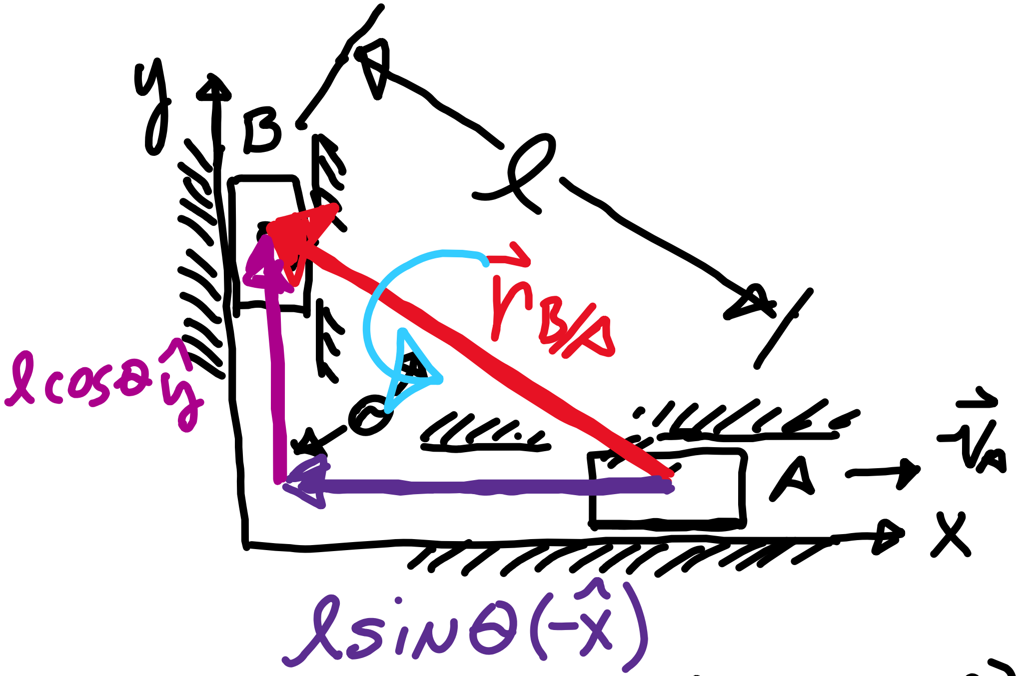
so $\vec{V}_B = \omega\ell[-\sin\theta\hat{y} + \cos\theta(-\hat{x})] + V_A\hat{x}$

2 unknowns but only single eqn.



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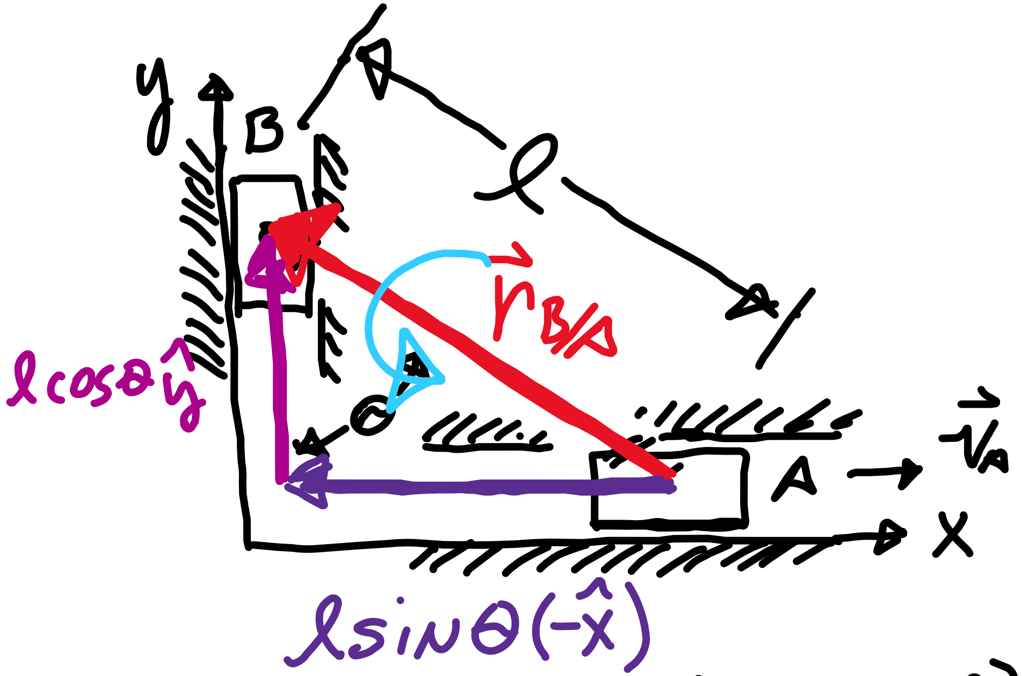
2 unknowns but only single eqn.

Need another equation!



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$$\text{so } \vec V_B = \omega l[-\sin\theta\,\hat y + \cos\theta(-\hat x)] + V_A\hat x$$

2 unknowns but only single eqn.

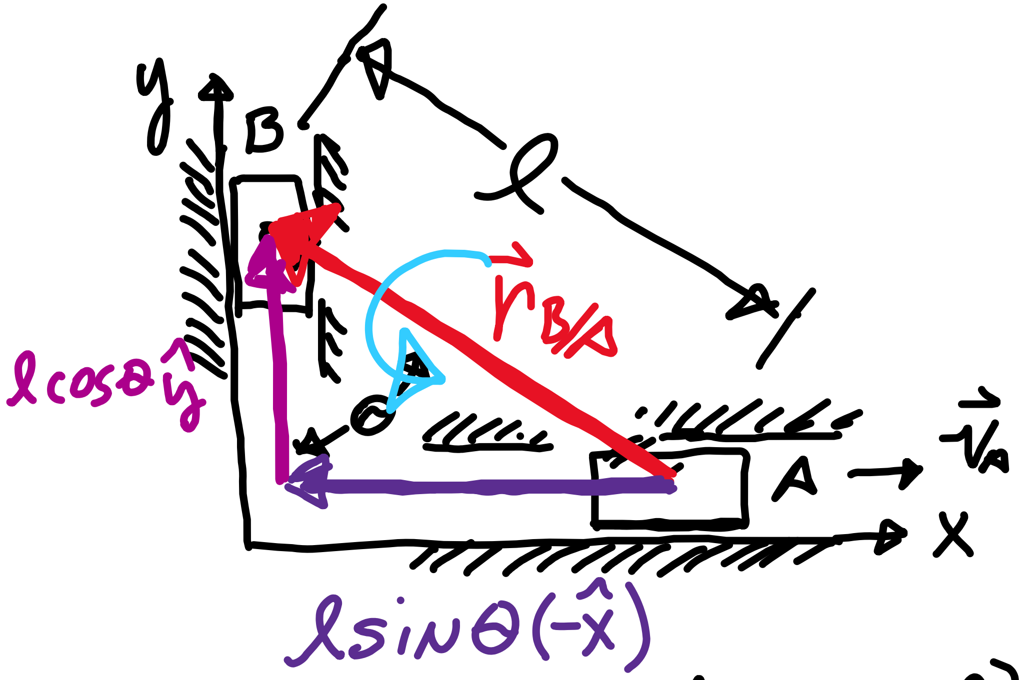
Need another equation!

Constraint: B can only move in y-direction

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$
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$\Rightarrow$ $\vec{r}_{B/A} = l[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

so $\vec{V}_B = \omega l[-\sin\theta\,\hat{y}$ $+ \cos\theta\,(-\hat{x})] + V_A\hat{X}$

MUST EQUAL ZERO

Example

Given ℓ, $\vec{V}_A = V_A\hat{x}$ &
θ

Find $\vec{V}_B$ & ω_{ARM}:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$
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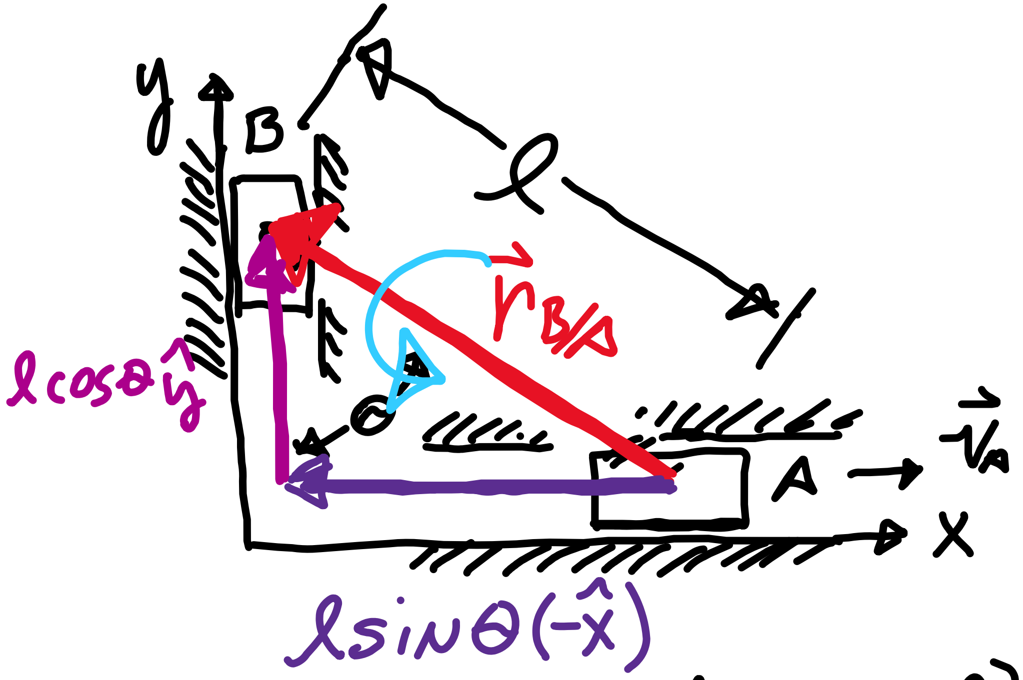


$\Rightarrow$ $\vec{r}_{B/A} = \ell[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

so $\vec{V}_B = \omega\ell[-\sin\theta\,\hat{y} + \cos\theta\,(-\hat{x})] + V_A\hat{x}$

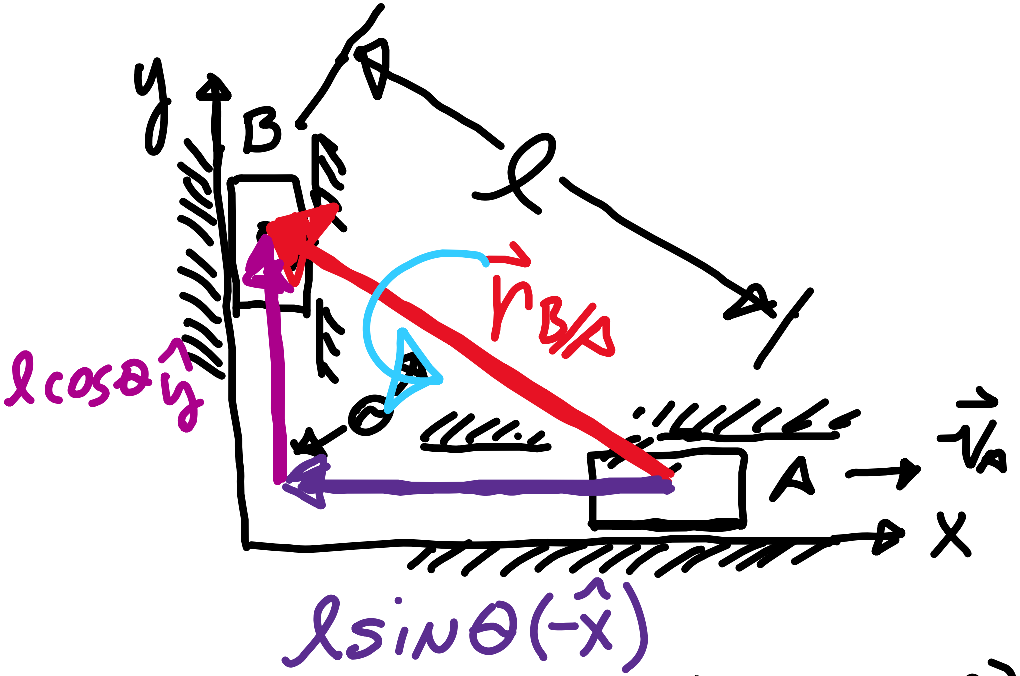
MUST EQUAL ZERO

so $\vec{V}_B = \omega\ell\sin\theta(-\hat{y})$

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

$$= \vec{\omega}\times\vec{r}_{B/A} + V_A\hat{x}$$

$\Rightarrow$ $\vec{r}_{B/A} = l[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

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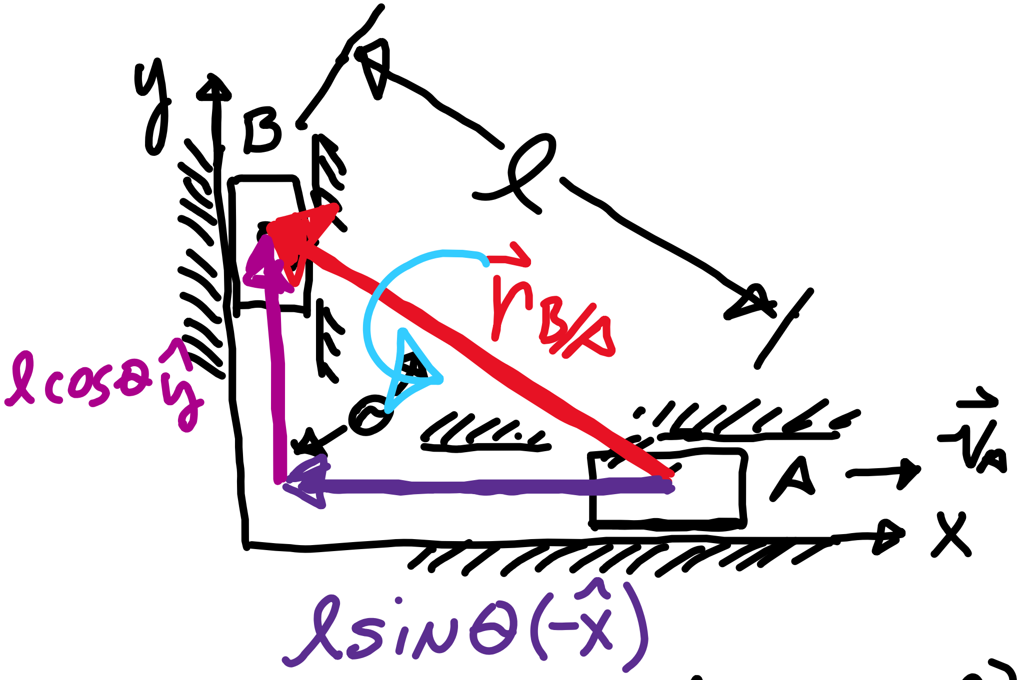
so $\vec{V}_B = \omega l\sin\theta(-\hat{y})$ & $V_A = \omega l\cos\theta$

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



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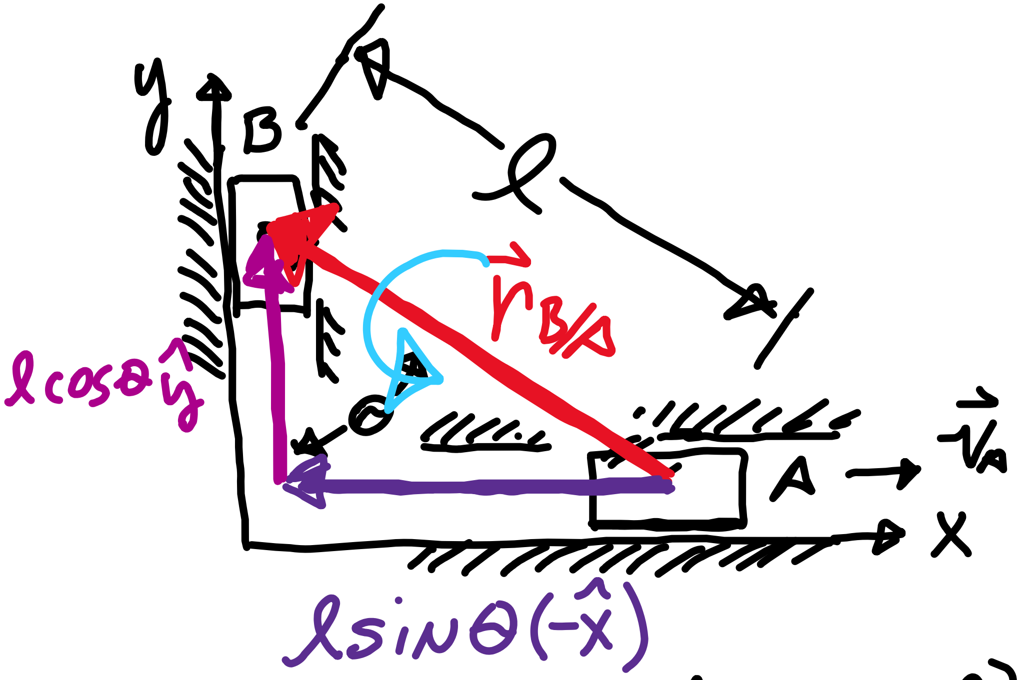
Now we have 2 equations & 2 unknowns



Example

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Example

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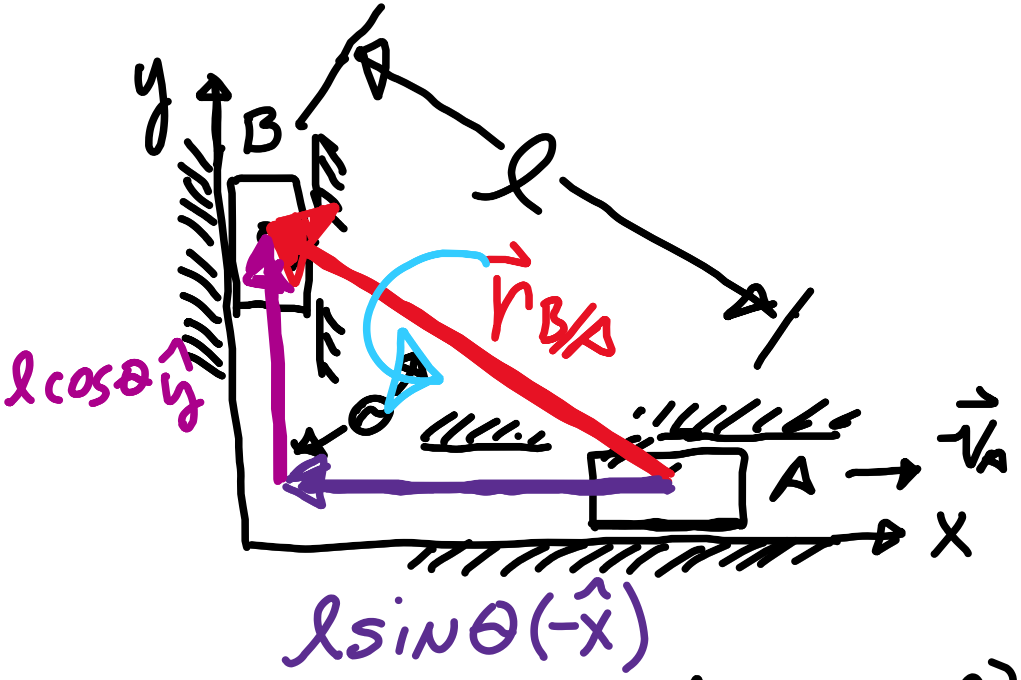

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

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so $\vec{V}_B = \omega l \sin\theta(-\hat{y})^{(1)}$ & $V_A = \omega l \cos\theta^{(2)}$

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Given l, $\vec{V}_A = V_A\hat{x}$ & θ

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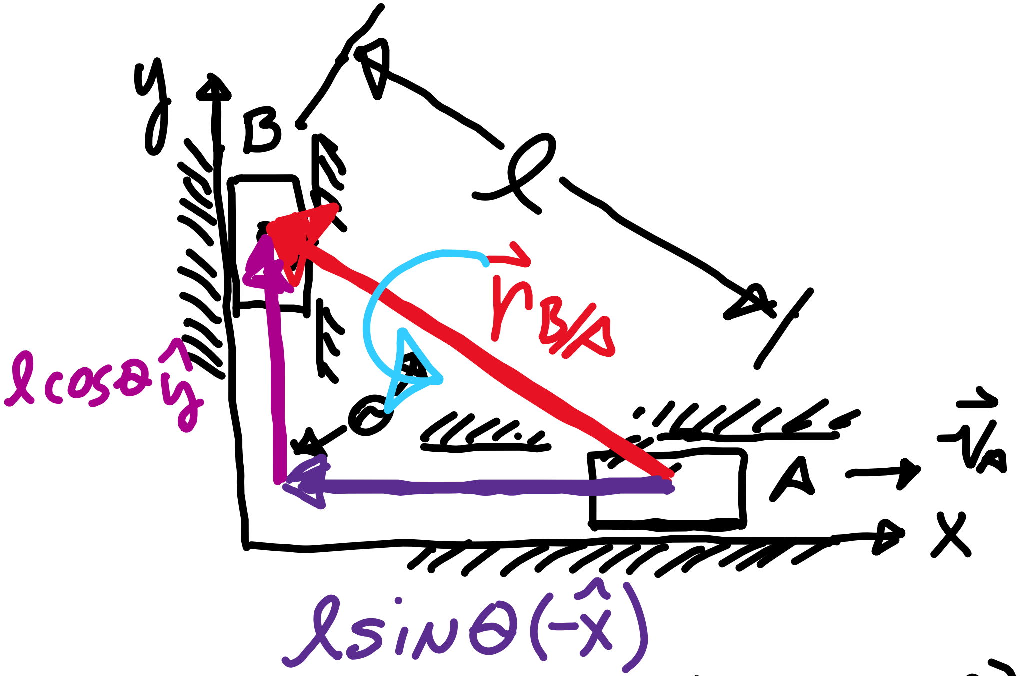

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$
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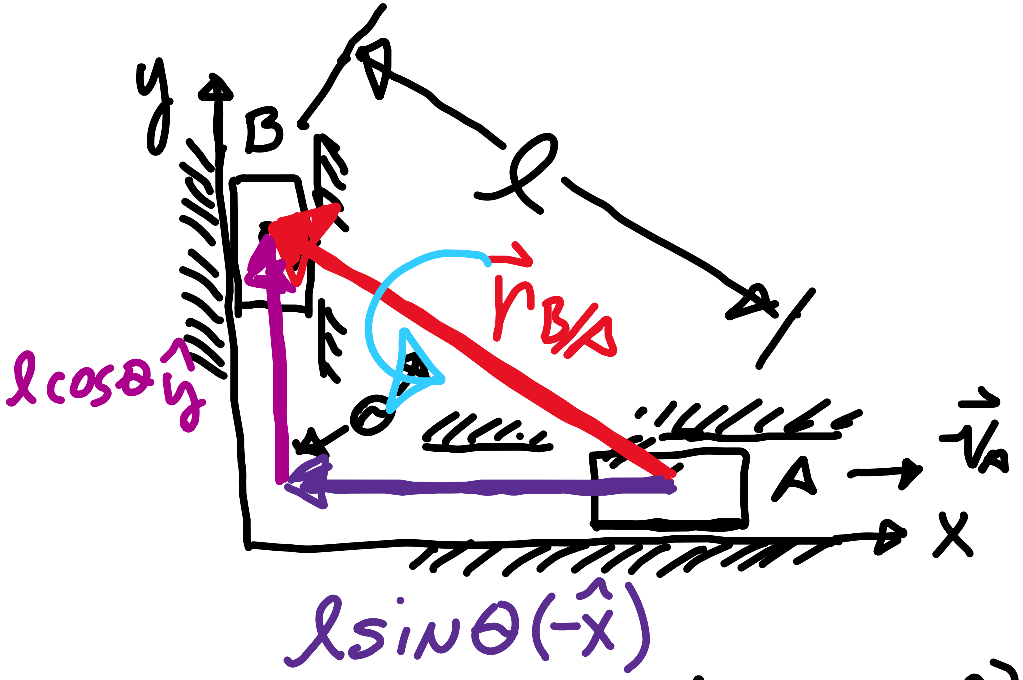
so $\vec{V}_B = \omega l\sin\theta(-\hat{y})^{(1)}$ & $V_A = \omega l\cos\theta^{(2)}$

EQN 2 $\Rightarrow$ $\boxed{\omega = V_A/(l\cos\theta)}$



Example

Given ℓ, $\vec{v}_A = v_A\hat{x}$ & θ

Find $\vec{v}_B$ & ω_{ARM}:

$$\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$

$$= \vec{\omega}\times\vec{r}_{B/A} + v_A\hat{x}$$

$\Rightarrow$ $\vec{r}_{B/A} = \ell[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

so $\vec{v}_B = \omega\ell[-\sin\theta\,\hat{y} + \cos\theta\,(-\hat{x})] + v_A\hat{x}$

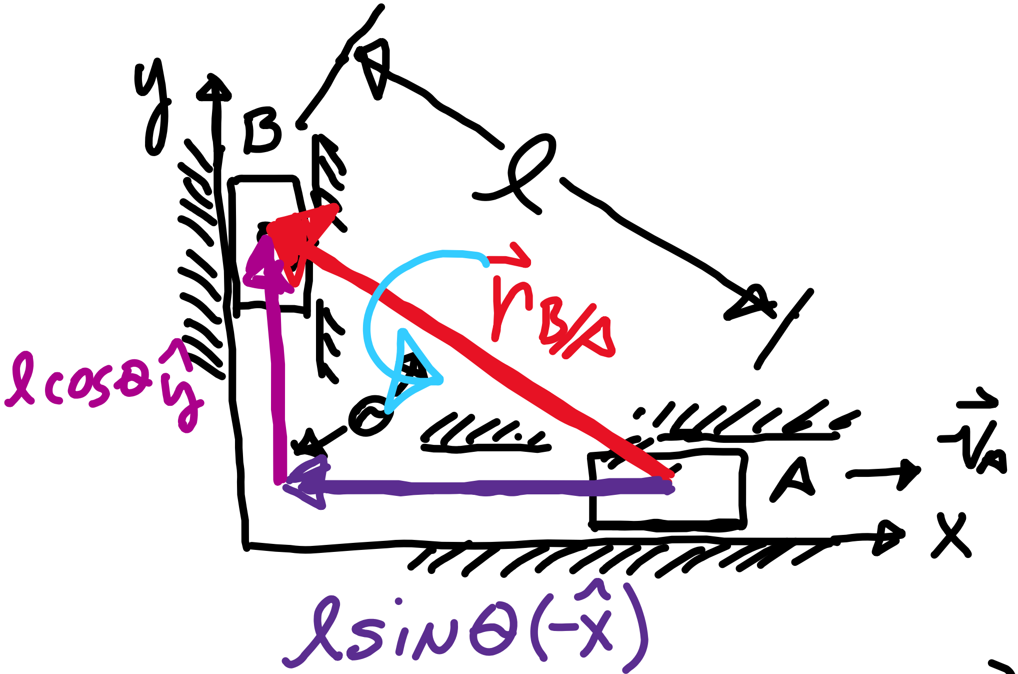
so $\vec{v}_B = \omega\ell\sin\theta(-\hat{y})$ (1) & $v_A = \omega\ell\cos\theta$ (2)

EQN 2 $\Rightarrow$ $\boxed{\omega = v_A/(\ell\cos\theta)}$ (3)

Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A$$

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$\Rightarrow \vec{r}_{B/A} = l[-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}]$ & $\vec{\omega} = \omega\,\hat{z}$

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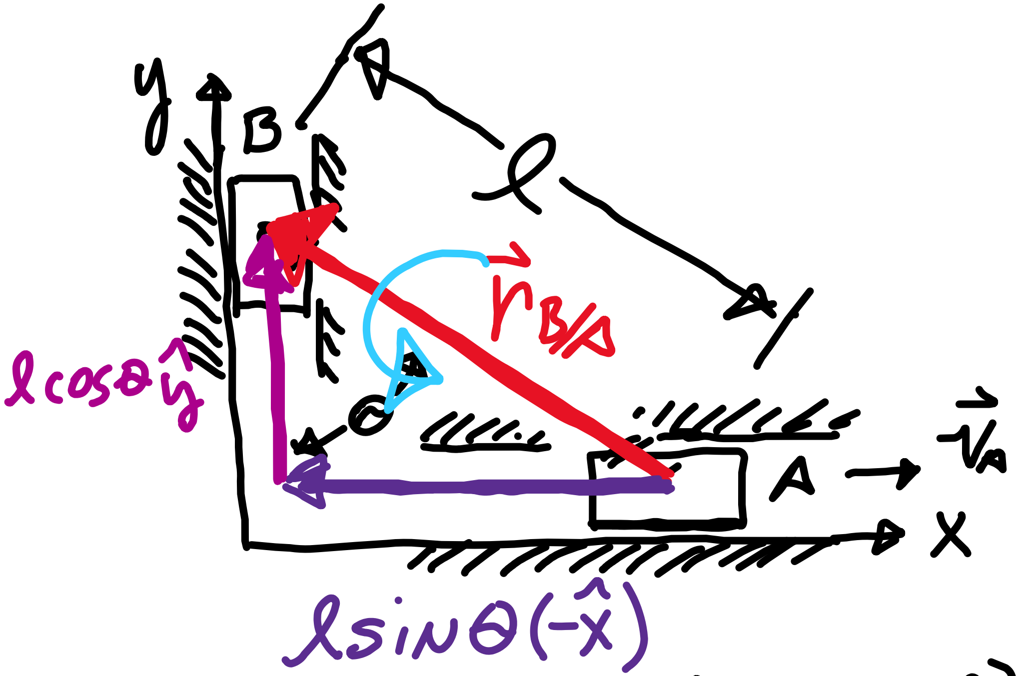
so $\vec{V}_B = \omega l\sin\theta(-\hat{y})^{(1)}$ & $V_A = \omega l\cos\theta^{(2)}$

EQN 2 ⇒ $\boxed{\omega = V_A/(l\cos\theta)}^{(3)}$ EQN 1&3 ⇒

$$\vec{V}_B = \left(\frac{lV_A}{l\cos\theta}\right)\sin\theta(-\hat{y})$$



Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:

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$\Rightarrow$ $\vec{r}_{B/A} = l[-\sin\theta\hat{x} + \cos\theta\hat{y}]$ & $\vec{\omega} = \omega\hat{z}$

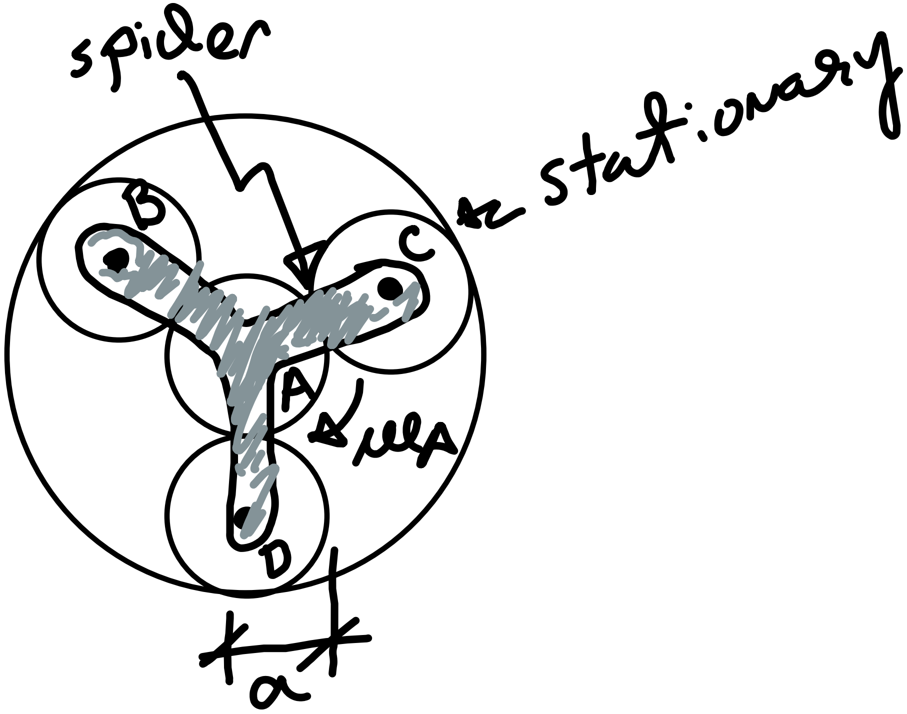
so $\vec{V}_B = \omega l[-\sin\theta\hat{y} + \cos\theta(-\hat{x})] + V_A\hat{x}$

so $\vec{V}_B = \omega l\sin\theta(-\hat{y})$ (1) & $V_A = \omega l\cos\theta$ (2)

EQN 2 $\Rightarrow$ $\boxed{\omega = V_A/(l\cos\theta)}$ (3) EQN 1&3 $\Rightarrow$

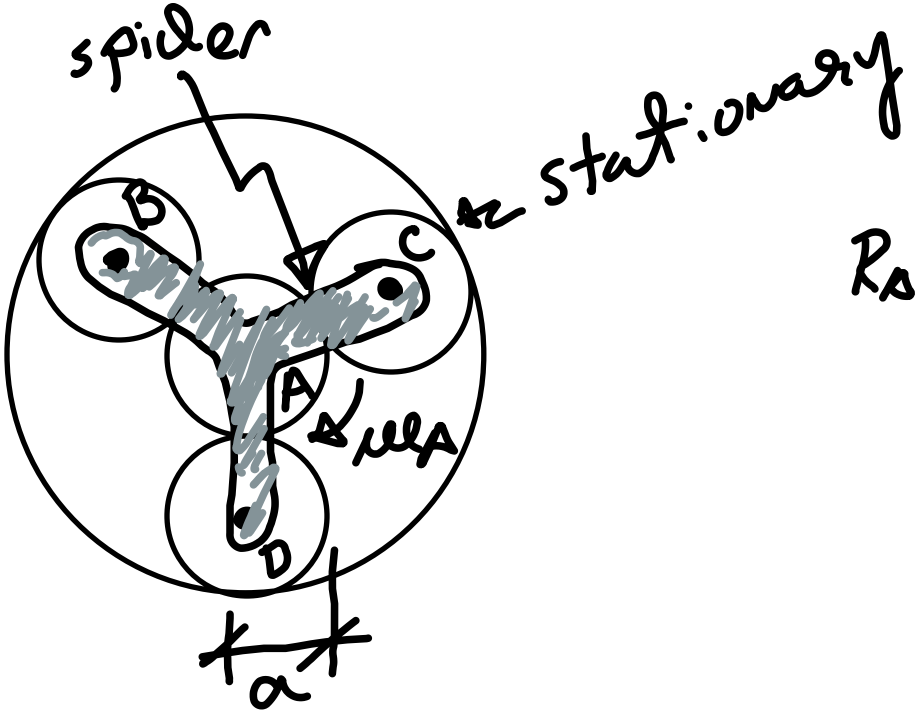
$\vec{V}_B = \left(\frac{lV_A}{l\cos\theta}\right)\sin\theta(-\hat{y})$ $\Rightarrow$ $\boxed{\vec{V}_B = (V_A\tan\theta)(-\hat{y})}$

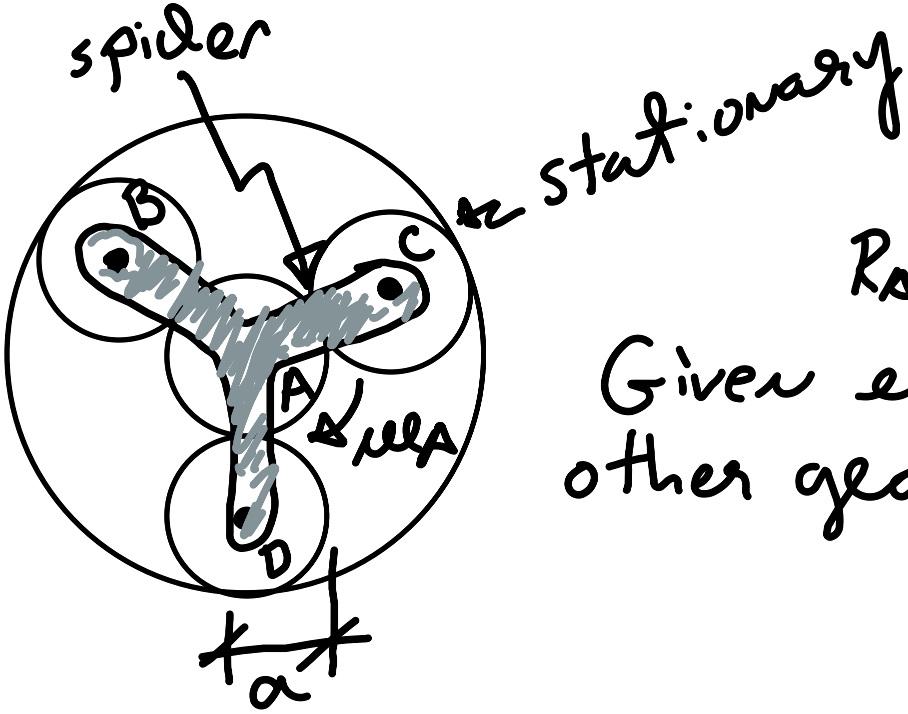
<u>Example</u>: Planetry gear system



Example: Planetry gear system

$R_A = R_B = R_C = R_D = a$

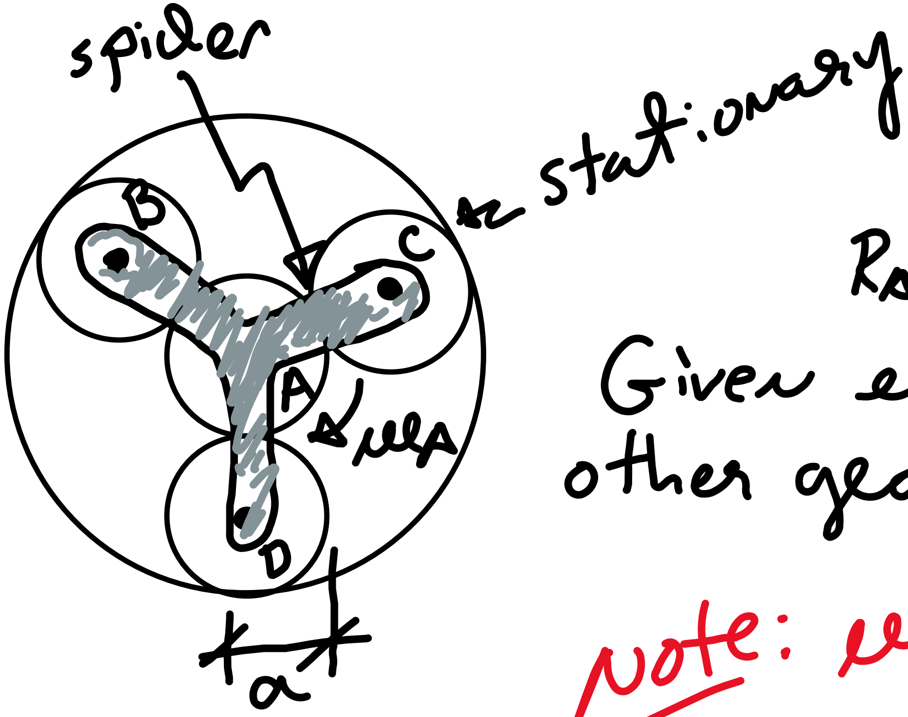




Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.



<u>Example</u>: Planetry gear system

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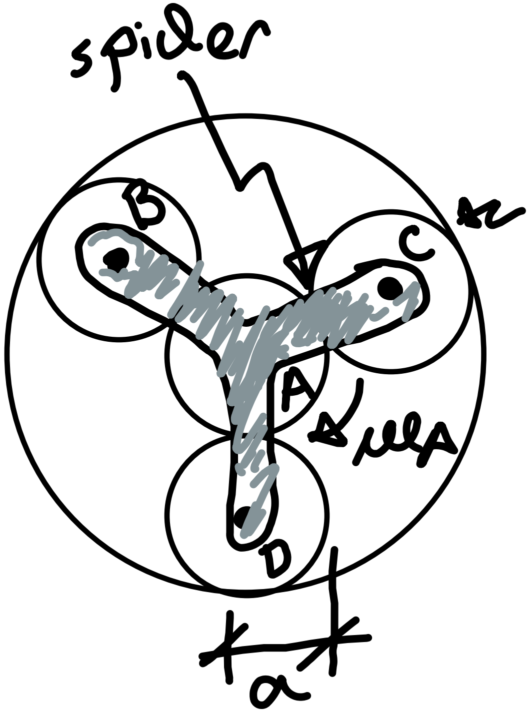
<u>Note</u>: $\omega_B = \omega_C = \omega_D$



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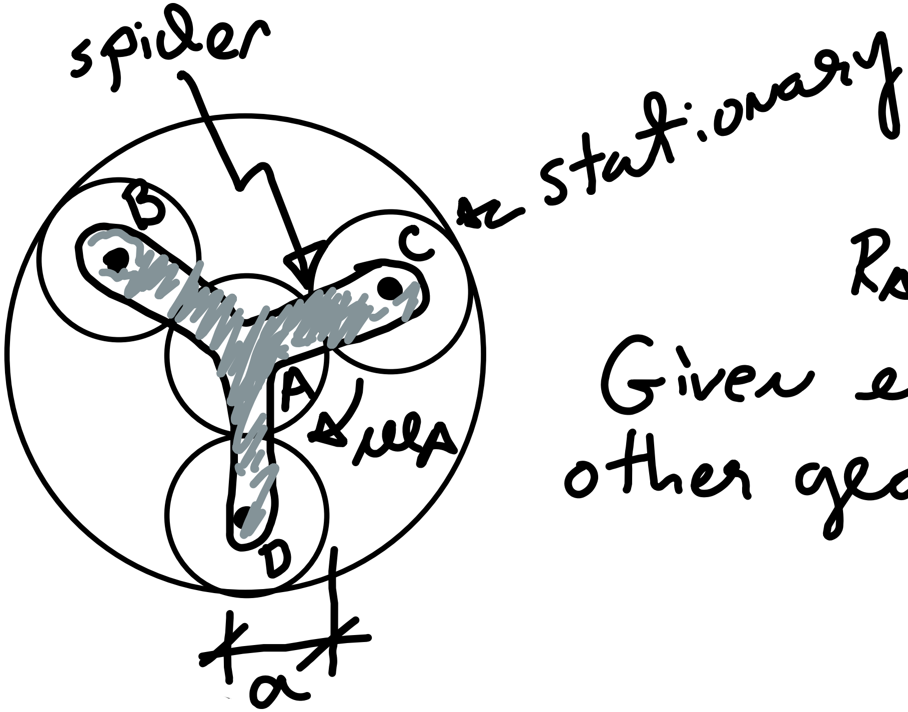
Just need to look at one of these



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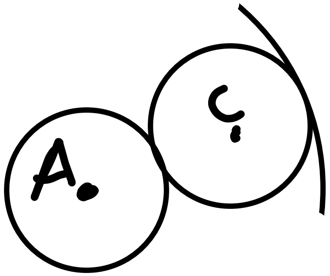

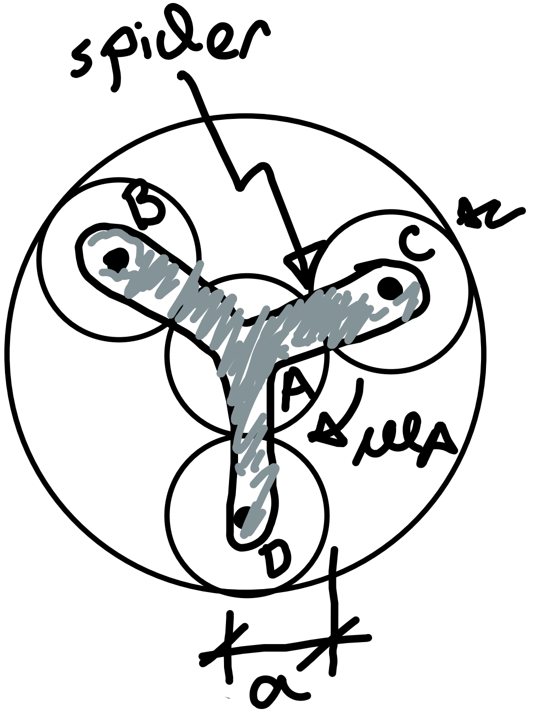

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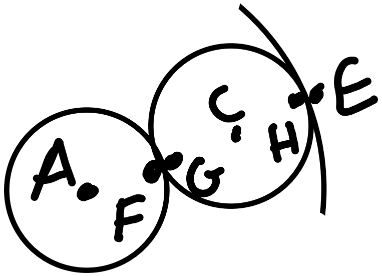

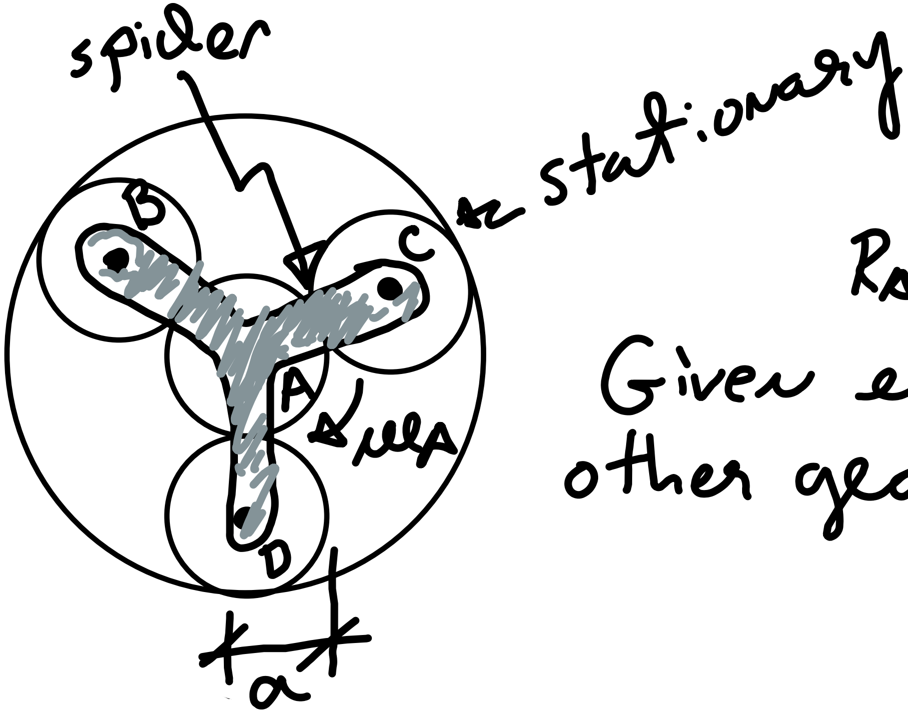

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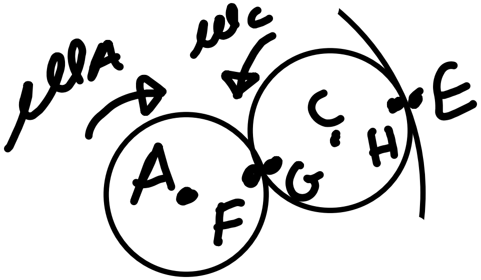
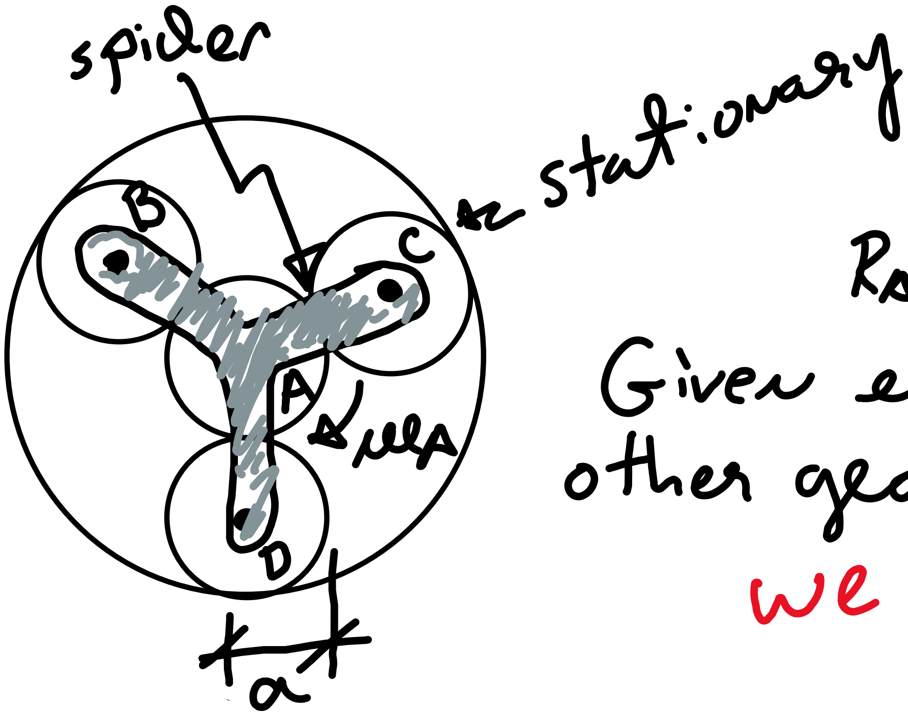


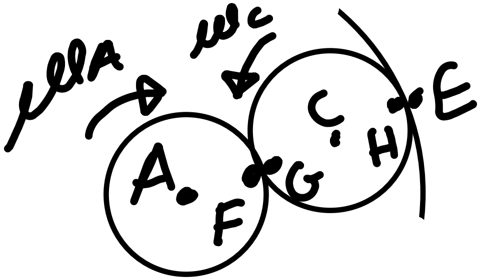
Example: Planetry gear system

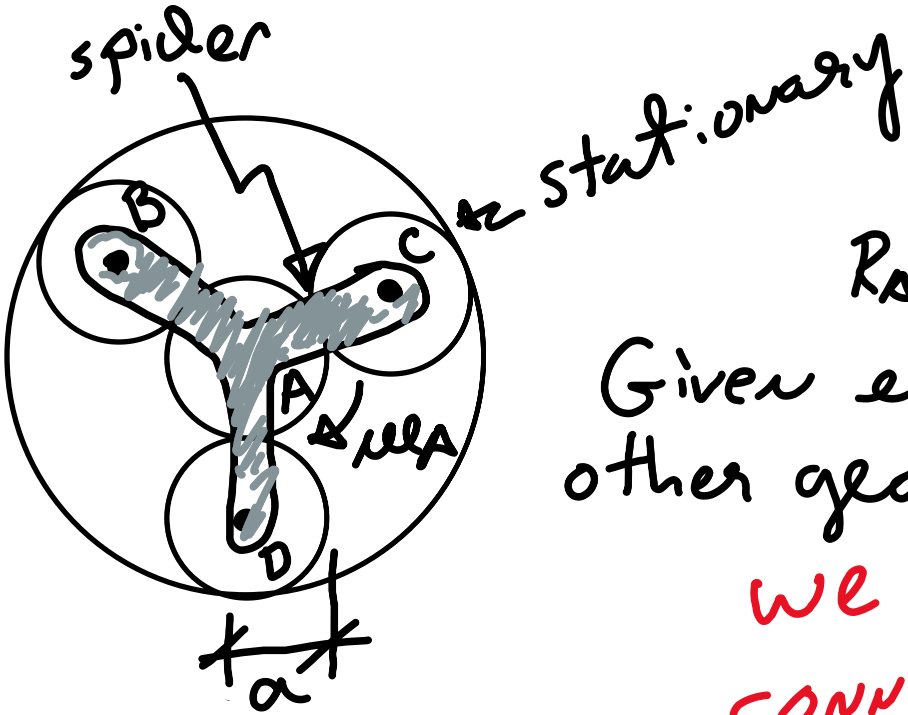


$R_A = R_B = R_C = R_D = a$

Given ω_A, find ω of the other gears.

we know V_E & V_F



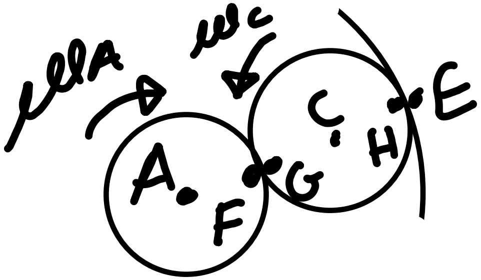


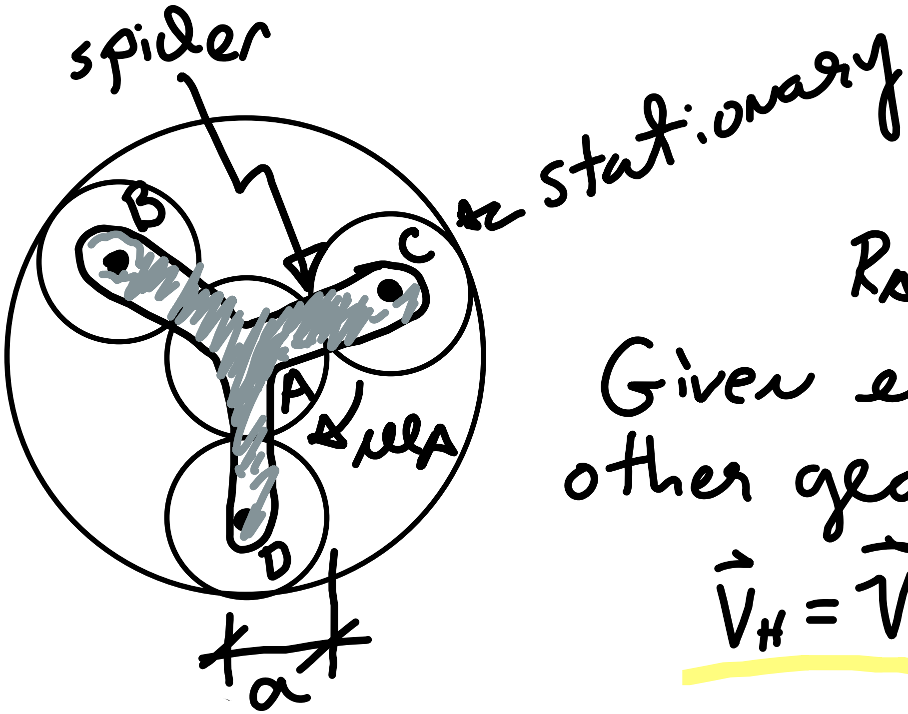
Example: Planetry gear system

$R_A = R_B = R_C = R_D = a$

Given ω_A, find ω of the other gears.

we know V_E & V_F so connect the points!



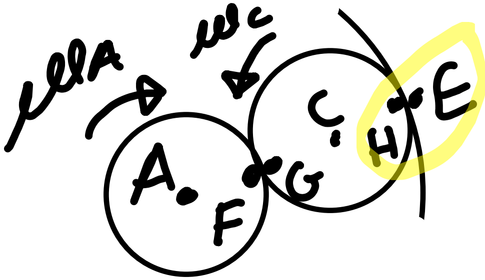


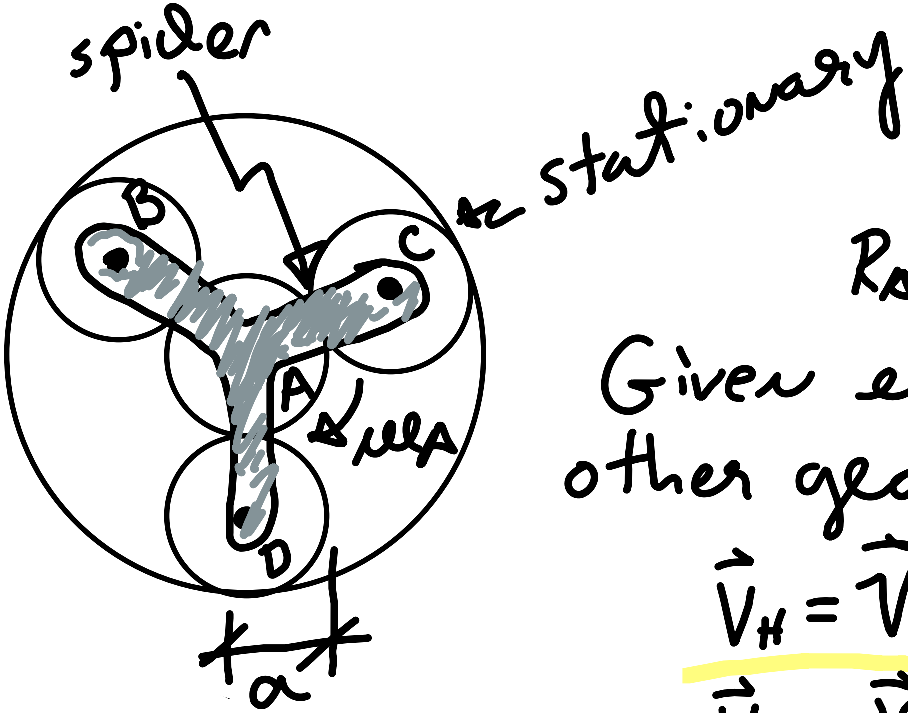
Example: Planetry gear system

$R_A = R_B = R_C = R_D = a$

Given ω_A, find ω of the other gears.

$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$





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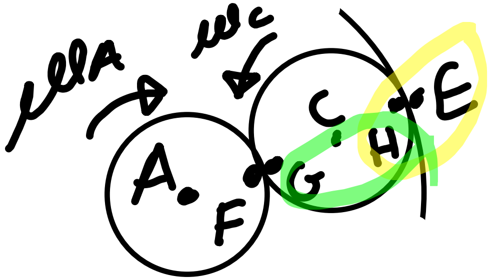
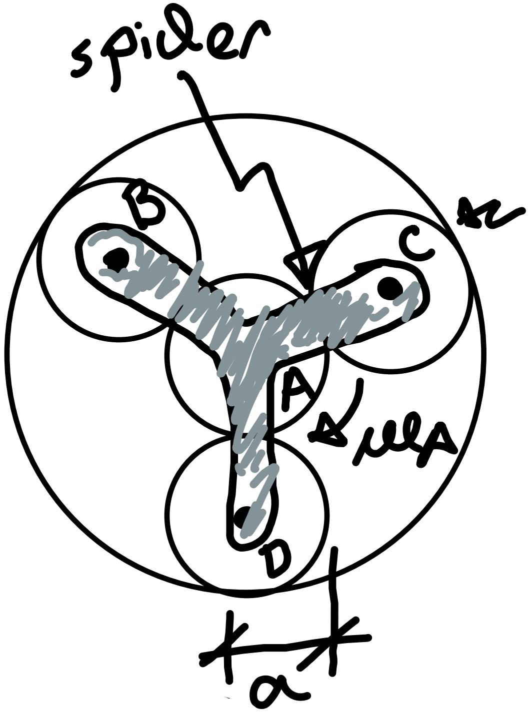


$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H$$



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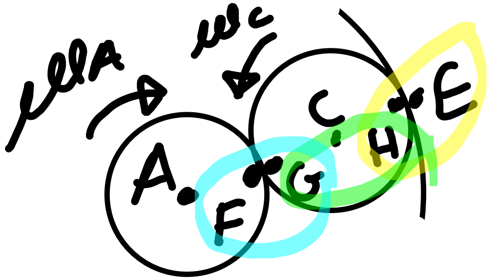

$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

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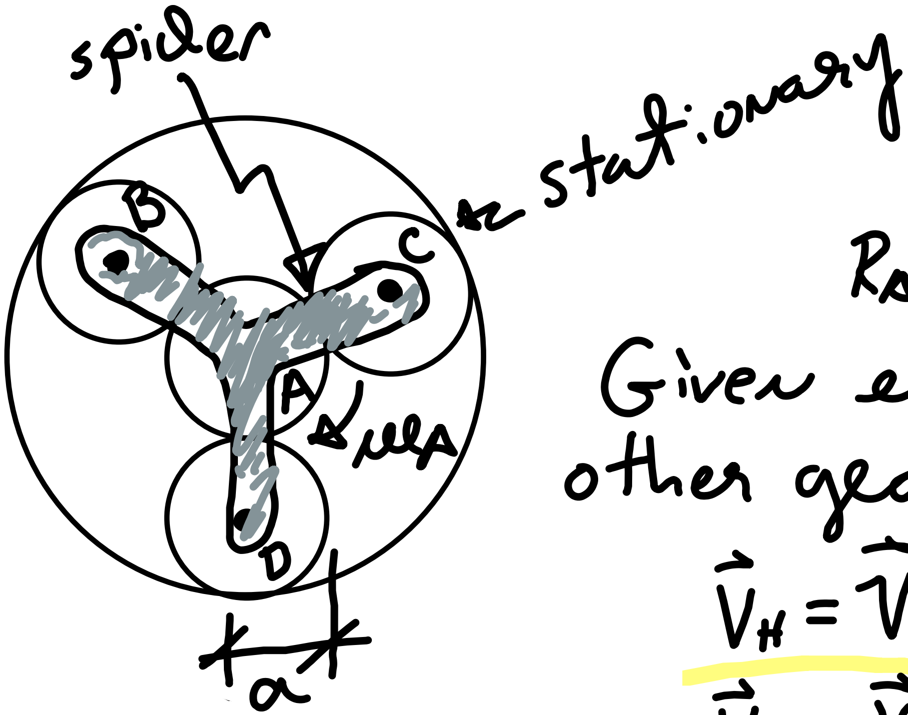

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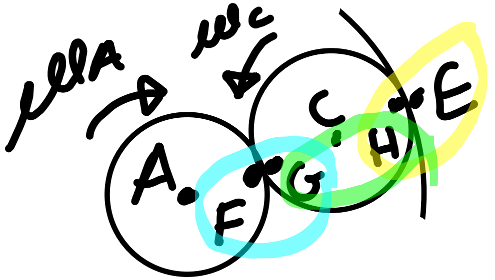
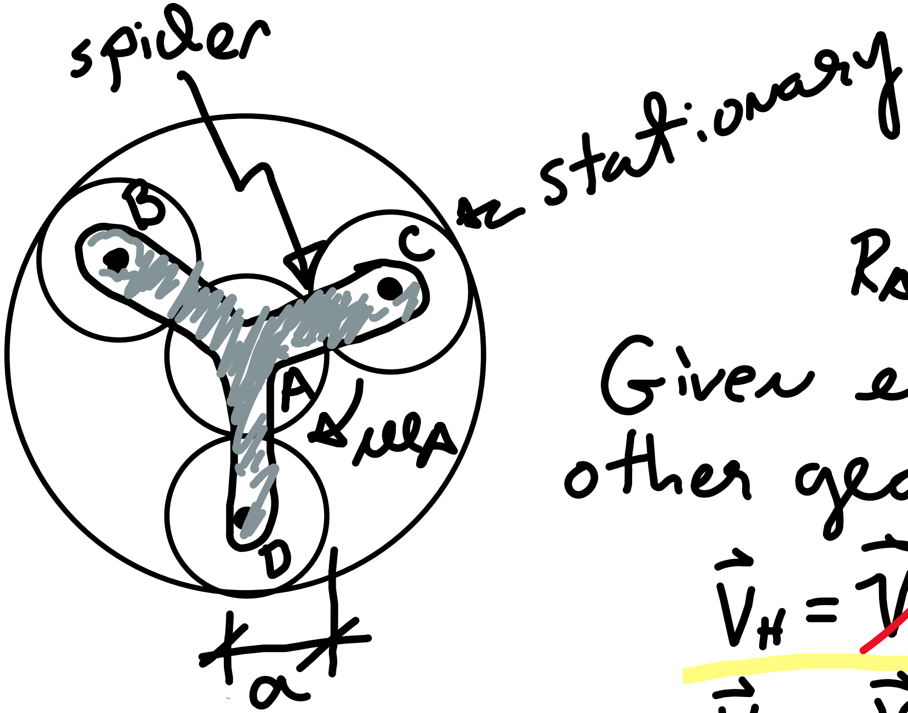


$$\vec{V}_H = \vec{V}_{H/E} + \cancelto{0}{\vec{V}_E}$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H$$

$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F$$



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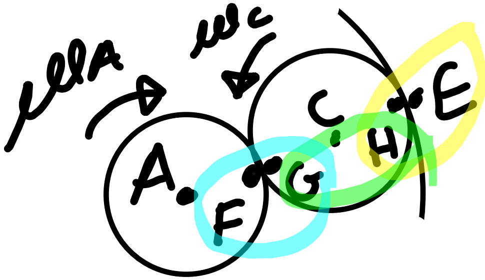

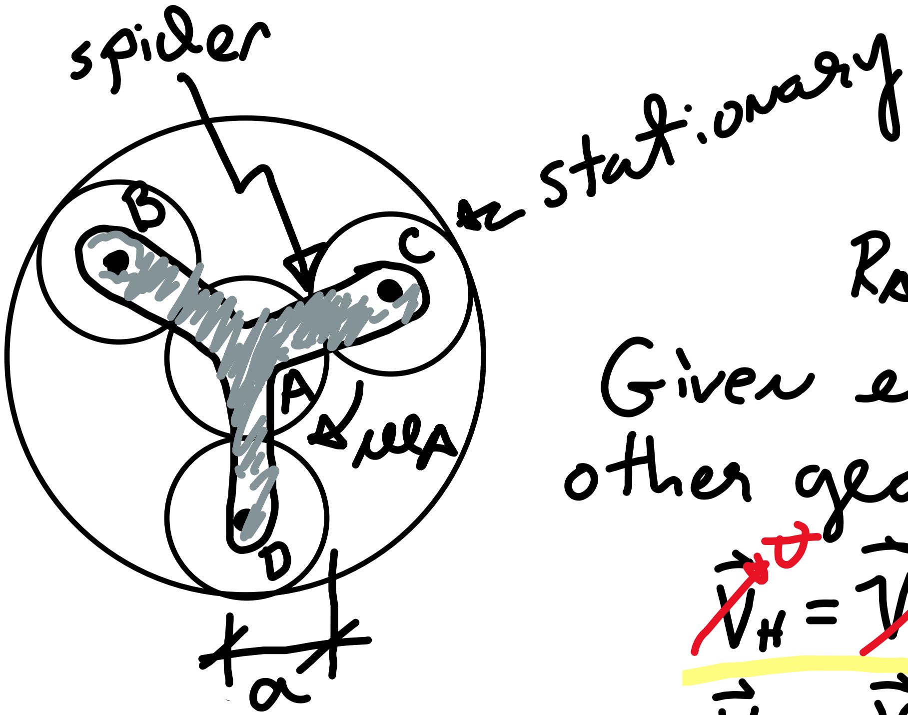

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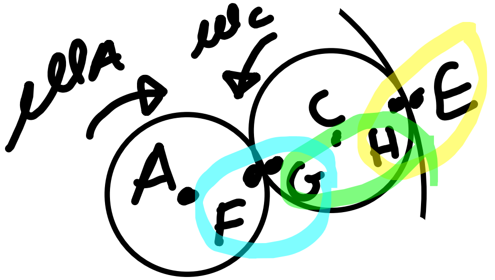

$$\overset{0}{\cancel{\vec{V}_H}} = \overset{0}{\cancel{\vec{V}_{H/E}}} + \overset{0}{\cancel{\vec{V}_E}}$$

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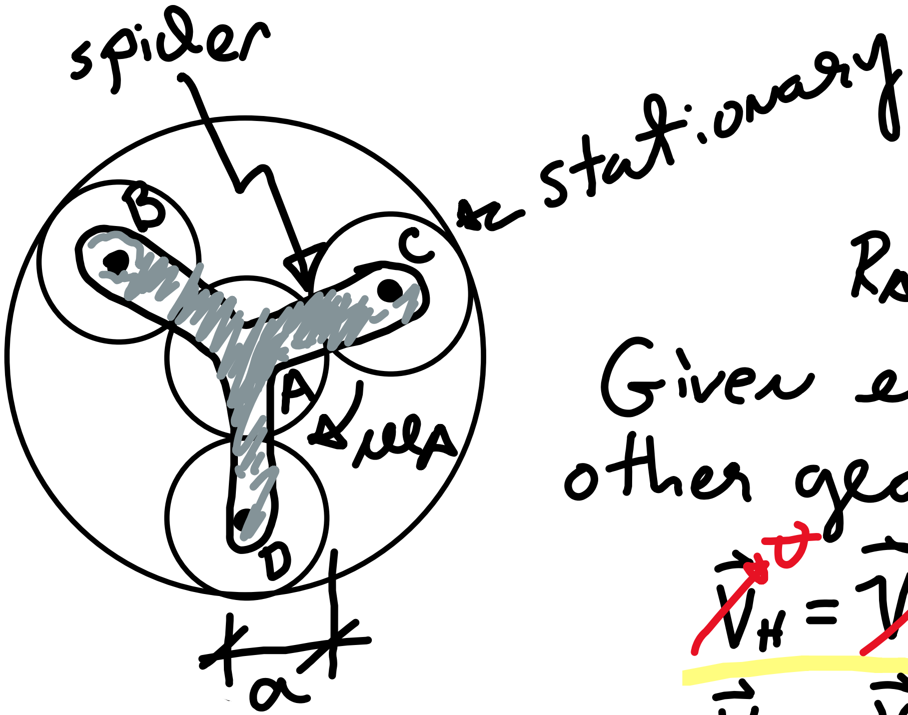

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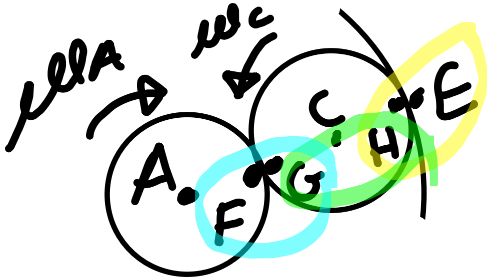

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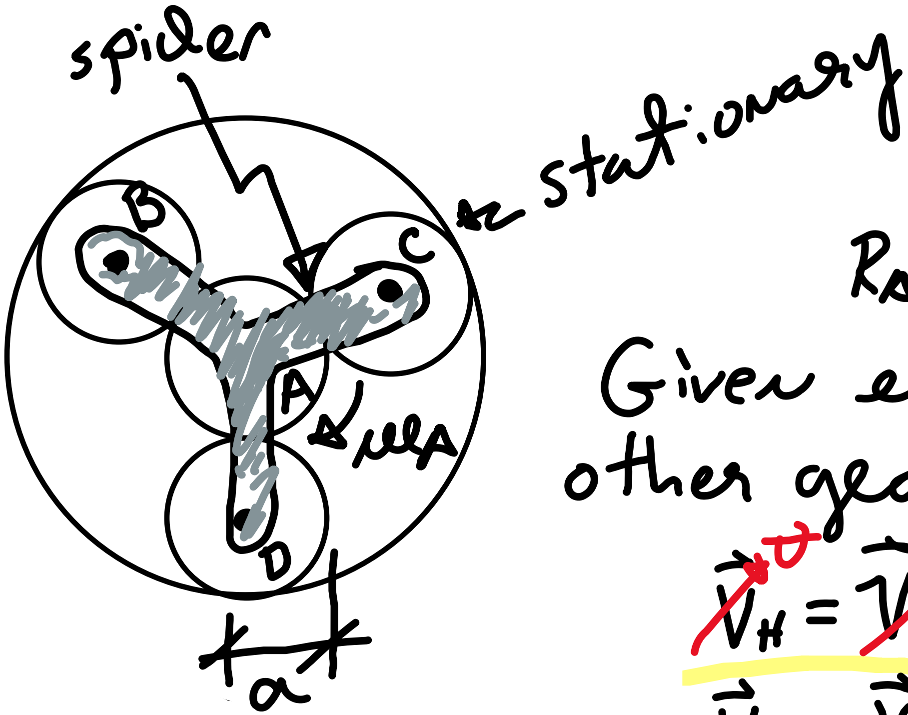

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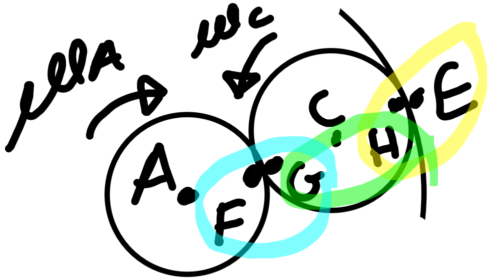

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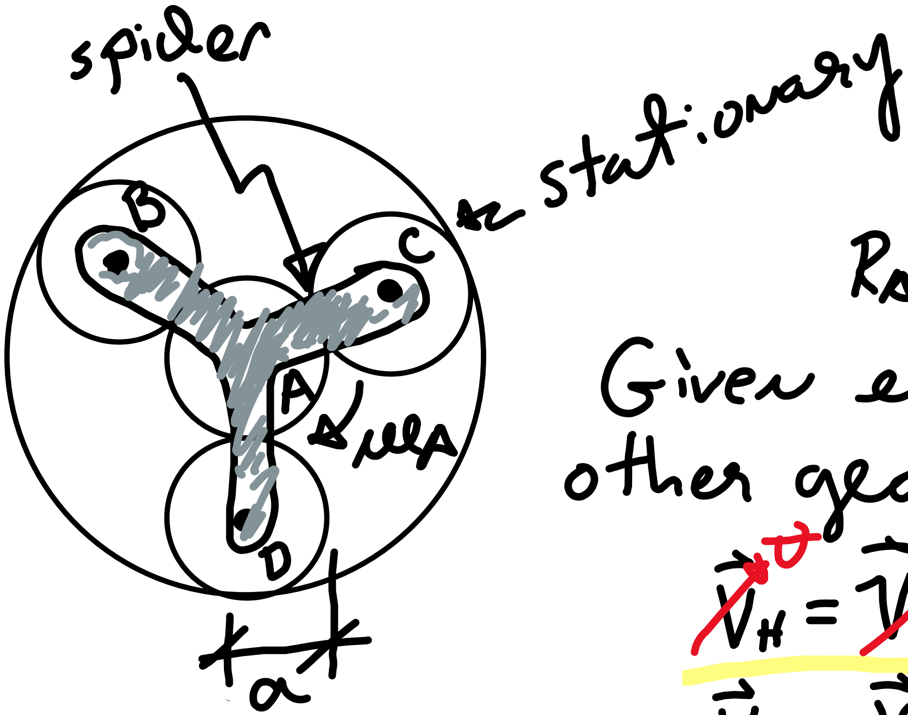

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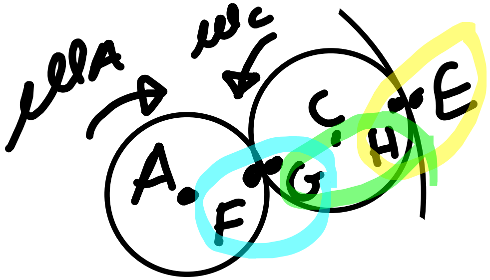

$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

$$\vec{V}_G = \vec{V}_{G/H} + \vec{V}_H = 2a\omega_C$$

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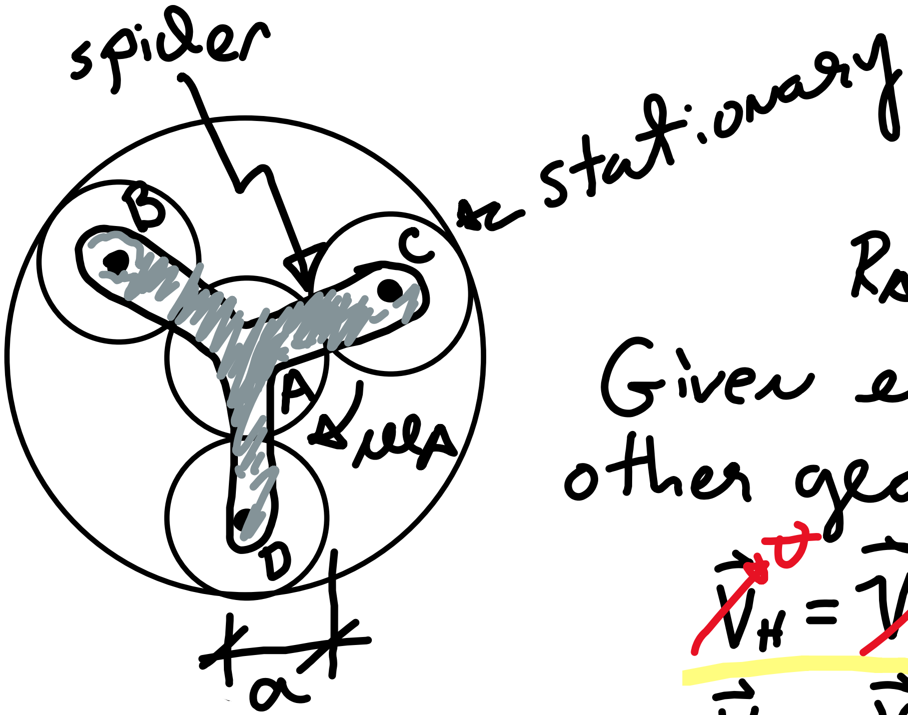

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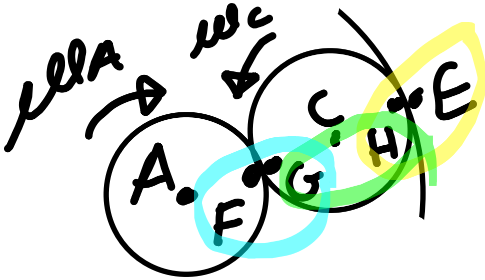

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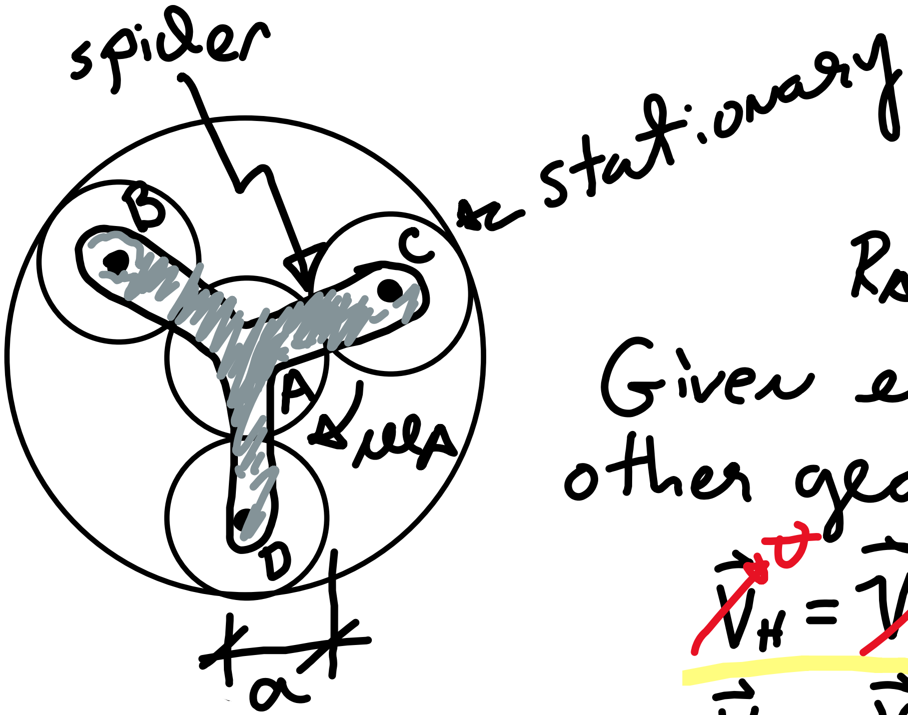
So $2a\omega_C = a\omega_A$



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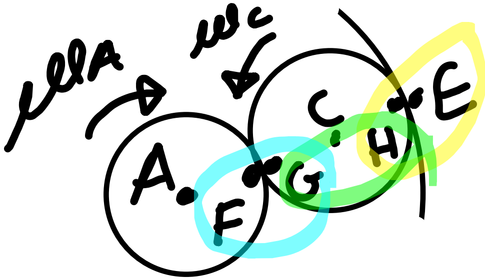

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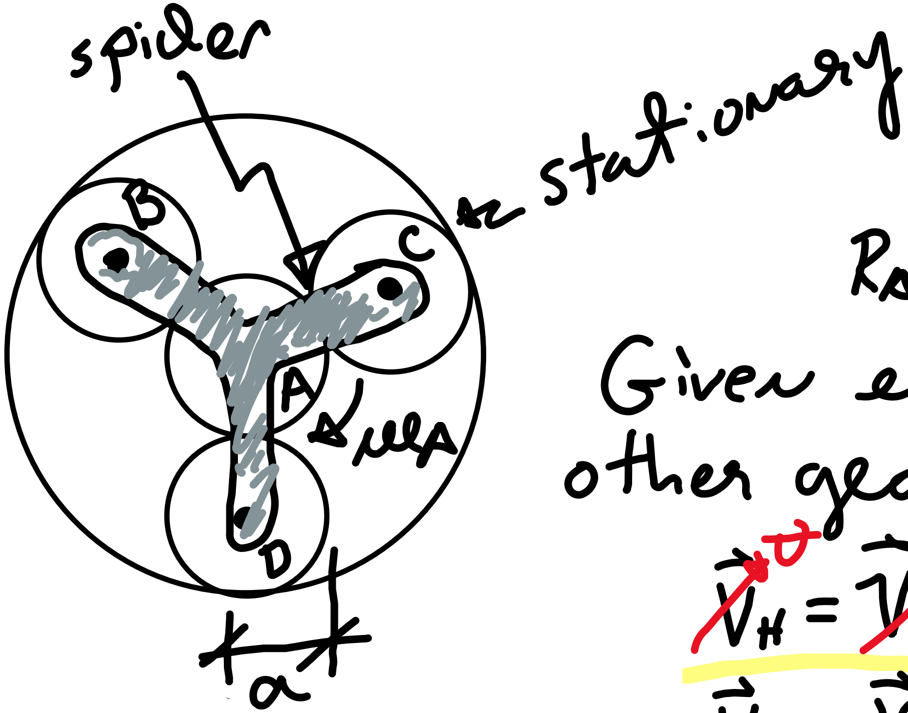
$$\vec{V}_G = \cancelto{0}{\vec{V}_{G/F}} + \vec{V}_F = a\omega_A$$

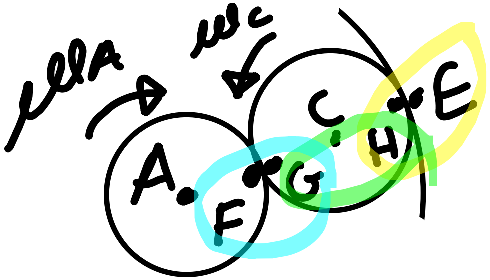
So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Example: Planetry gear system



$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.



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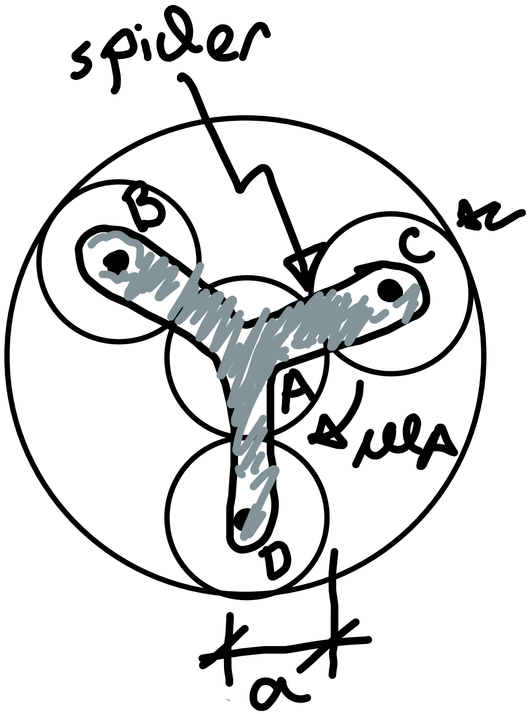
So $2a\omega_c = a\omega_A \Rightarrow \boxed{\omega_c = \frac{\omega_A}{2}}$

Find ω_{spider}:



Example: Planetry gear system

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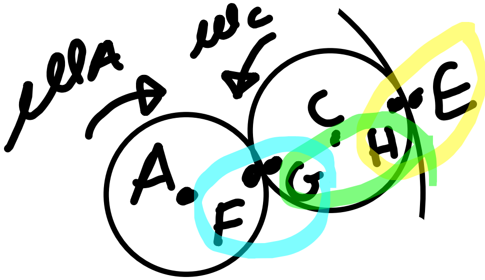
Given ω_A, find ω of the other gears.



$$\vec V_H = \vec V_{H/E} + \vec V_E$$

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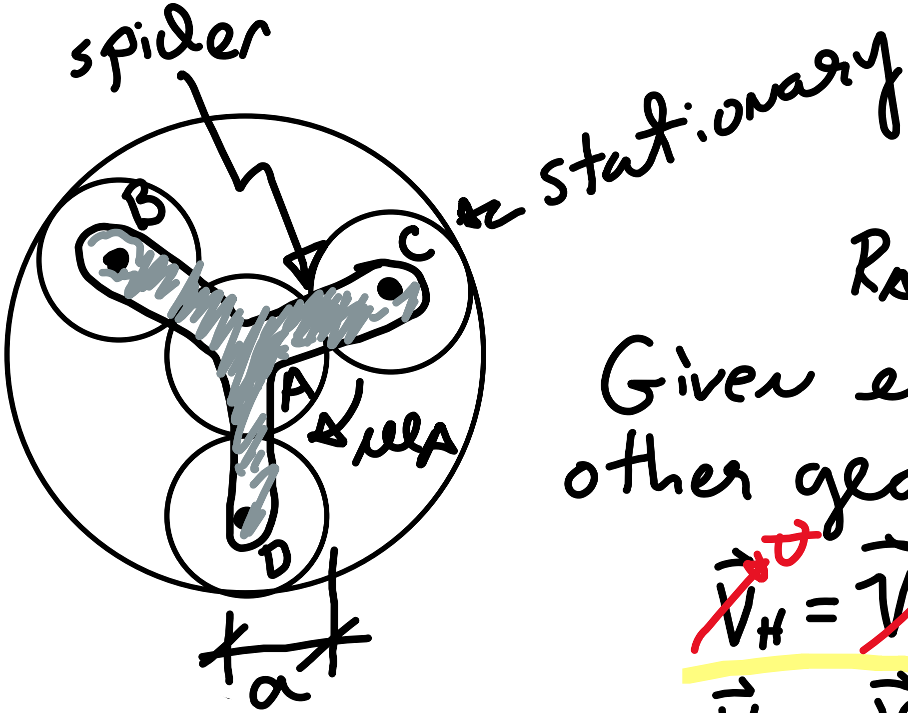
$$\vec V_G = \vec V_{G/F} + \vec V_F = a\omega_A$$

So $2a\omega_c = a\omega_A \Rightarrow \boxed{\omega_c = \frac{\omega_A}{2}}$

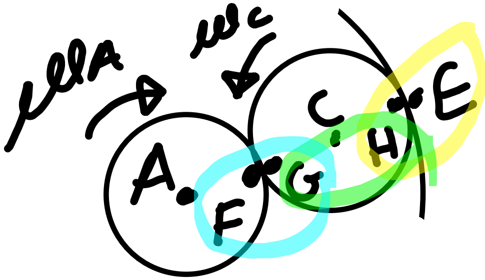
Find ω_{spider}: $\vec V_c = \vec V_{c/A} + \vec V_A$



Example: Planetry gear system

$R_A = R_B = R_C = R_D = a$

Given ω_A, find ω of the other gears.



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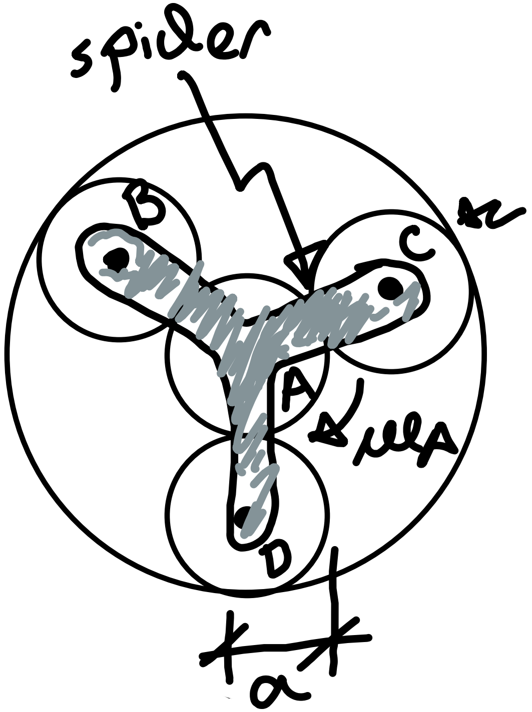
$$\vec{V}_G = \vec{V}_{G/F} + \vec{V}_F = a\omega_A$$

So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_C = \vec{V}_{C/A} + \vec{V}_A$



Example: Planetry gear system

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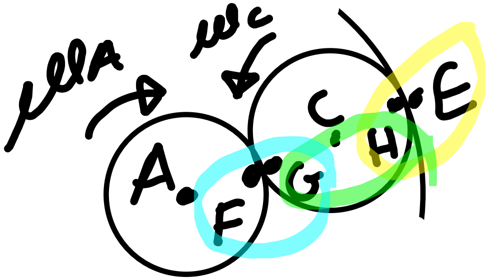

$$\cancelto{0}{\vec{V}_H} = \cancelto{0}{\vec{V}_{H/E}} + \cancelto{0}{\vec{V}_E}$$

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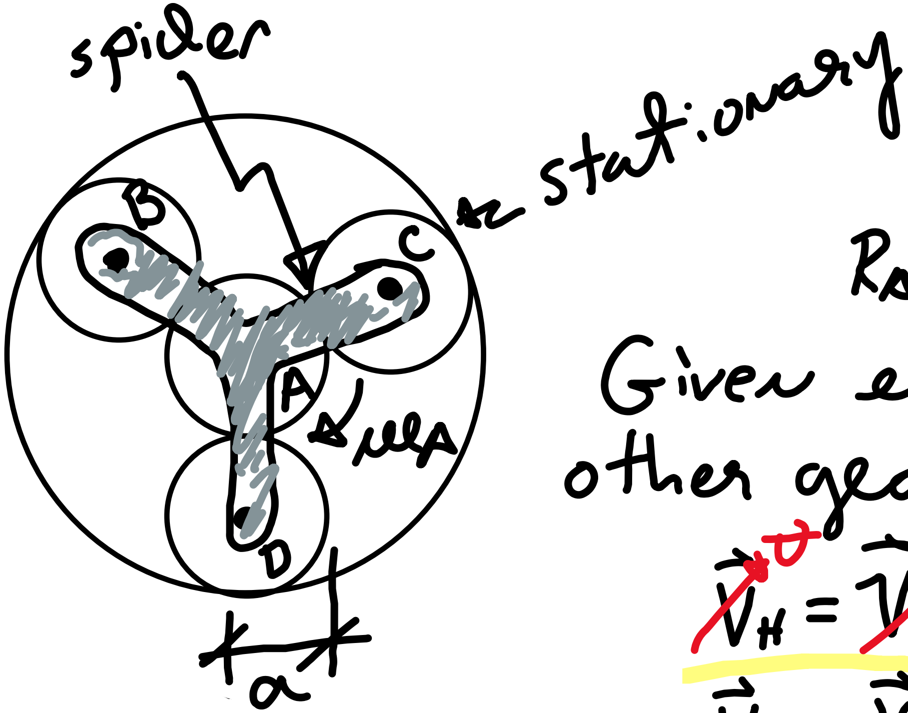
$$\vec{V}_G = \cancelto{0}{\vec{V}_{G/F}} + \vec{V}_F = a\omega_A$$

So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_c = \vec{V}_{c/A} + \cancelto{0}{\vec{V}_A} = 2a\omega_{spider}$



Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.

$$\cancelto{0}{\vec{V}_H} = \cancelto{0}{\vec{V}_{H/E}} + \cancelto{0}{\vec{V}_E}$$

$$\vec{V}_G = \vec{V}_{G/H} + \cancelto{0}{\vec{V}_H} = 2a\omega_C$$

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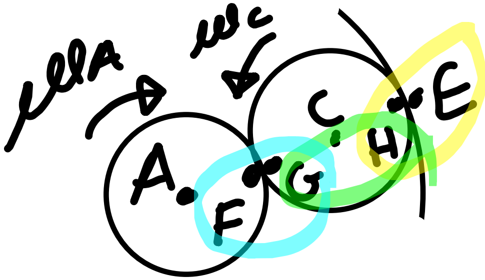
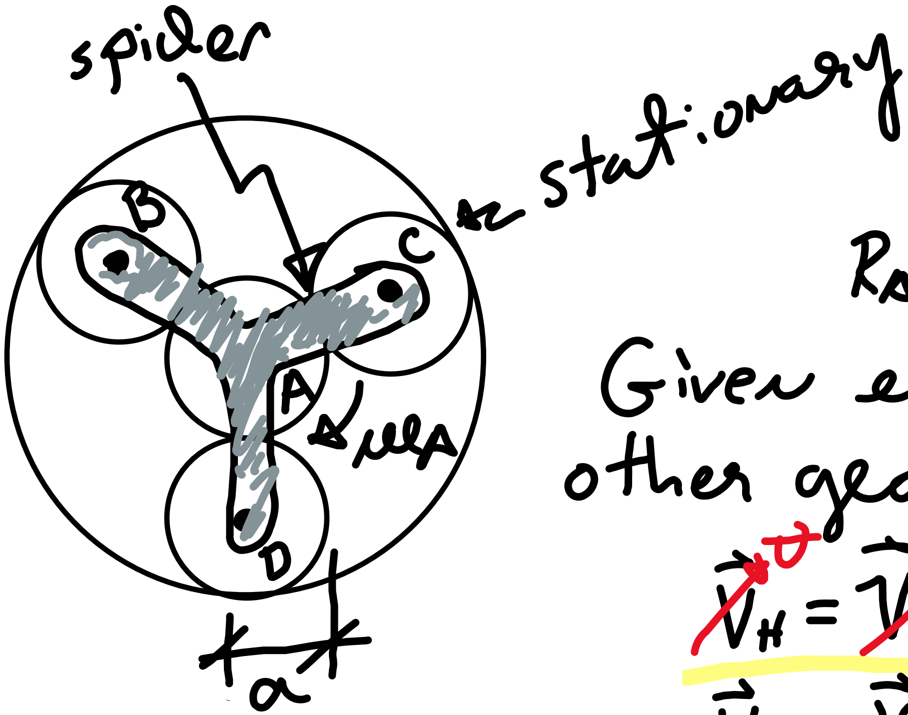


So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_C = \vec{V}_{C/A} + \cancelto{0}{\vec{V}_A} = 2a\omega_{spider}$

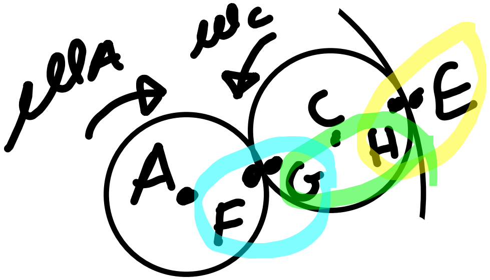
Also $\vec{V}_C = \vec{V}_{C/E} + \vec{V}_E$



Example: Planetry gear system

$$R_A = R_B = R_C = R_D = a$$

Given ω_A, find ω of the other gears.



$$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$$

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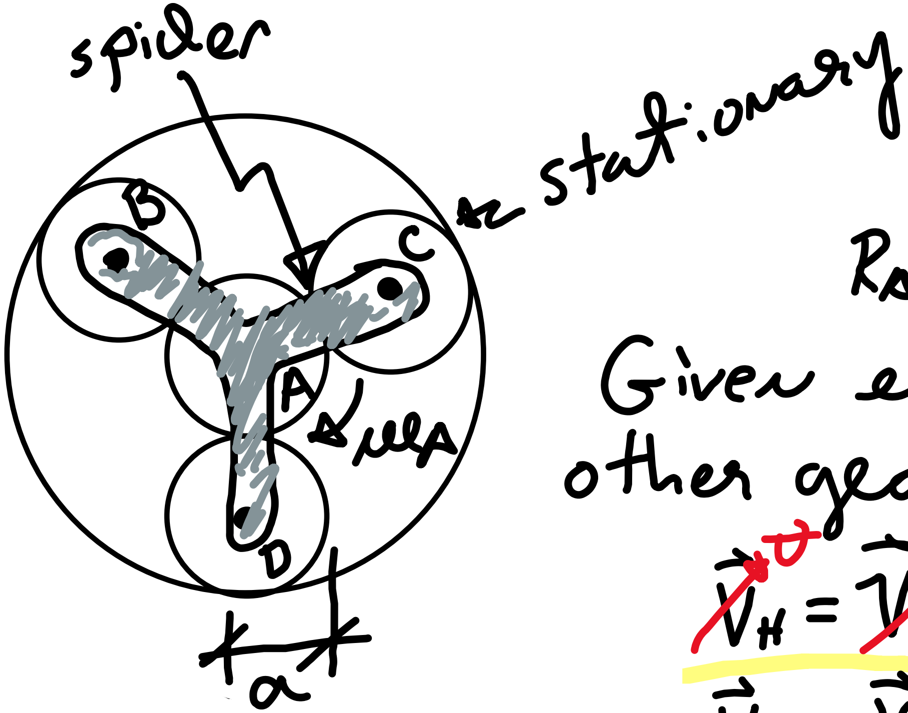
So $2a\omega_c = a\omega_A \Rightarrow \boxed{\omega_c = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_c = \vec{V}_{c/A} + \vec{V}_A = 2a\omega_{spider}$

Also $\vec{V}_c = \vec{V}_{c/E} + \vec{V}_E$



Example: Planetry gear system

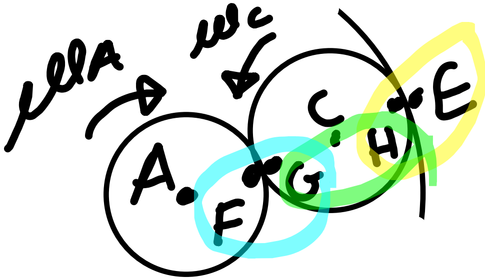
$R_A = R_B = R_C = R_D = a$

Given ω_A, find ω of the other gears.

$$\cancelto{0}{\vec V_H} = \cancelto{0}{\vec V_{H/E}} + \cancelto{0}{\vec V_E}$$

$$\vec V_G = \vec V_{G/H} + \cancelto{0}{\vec V_H} = 2a\omega_C$$

$$\vec V_G = \cancelto{0}{\vec V_{G/F}} + \vec V_F = a\omega_A$$

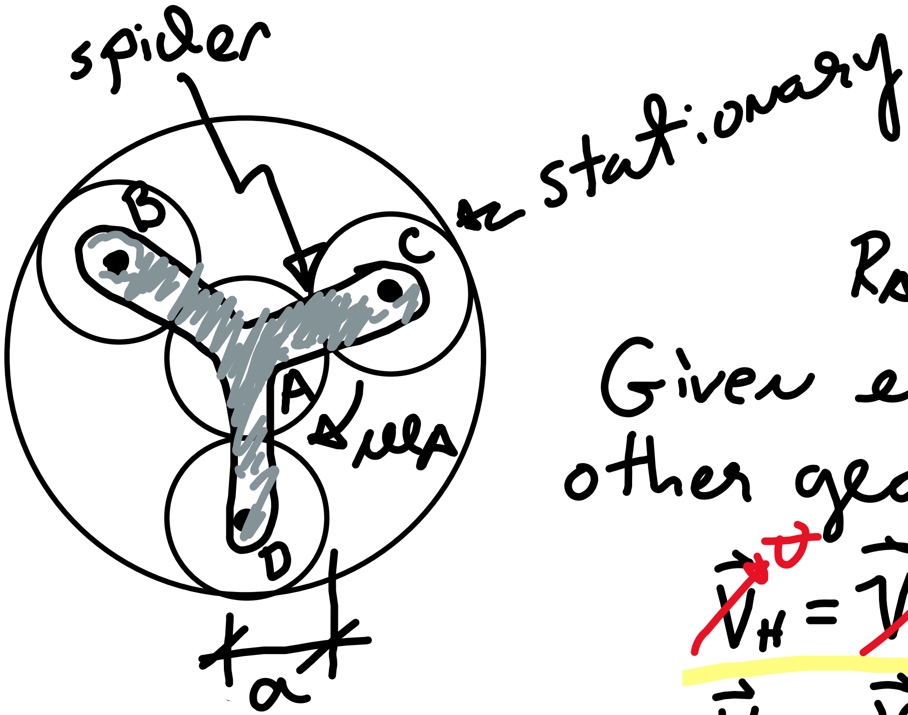


So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec V_C = \vec V_{C/A} + \cancelto{0}{\vec V_A} = 2a\omega_{spider}$

Also $\vec V_C = \vec V_{C/E} + \cancelto{0}{\vec V_E} = a\omega_C$

Example: Planetry gear system



$R_A = R_B = R_C = R_D = a$

Given ω_A, find ω of the other gears.

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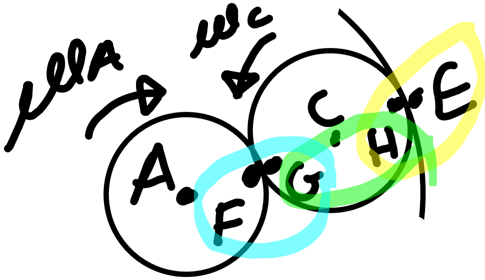
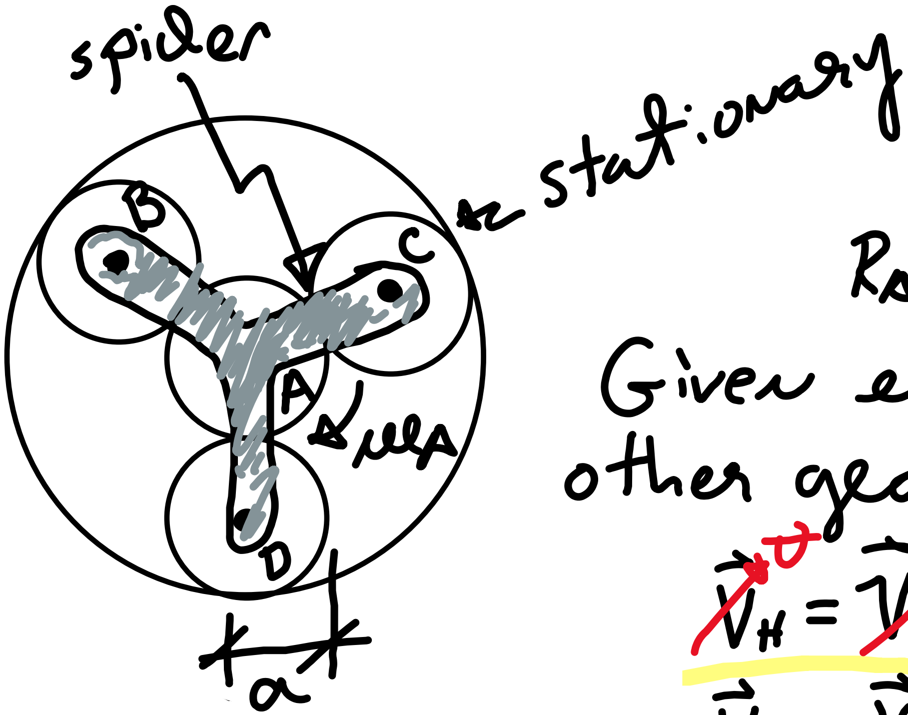
$$\vec{V}_G = \cancelto{0}{\vec{V}_{G/F}} + \vec{V}_F = a\omega_A$$



So $2a\omega_c = a\omega_A \Rightarrow \boxed{\omega_c = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_c = \vec{V}_{c/A} + \cancelto{0}{\vec{V}_A} = 2a\omega_{spider}$

Also $\vec{V}_c = \vec{V}_{c/E} + \cancelto{0}{\vec{V}_E} = a\omega_c = a\omega_A/2$



Example: Planetry gear system

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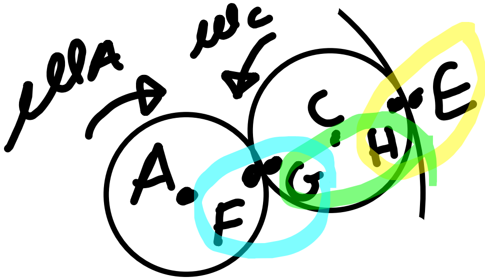

$$\cancelto{0}{\vec V_H} = \cancelto{0}{\vec V_{H/E}} + \cancelto{0}{\vec V_E}$$

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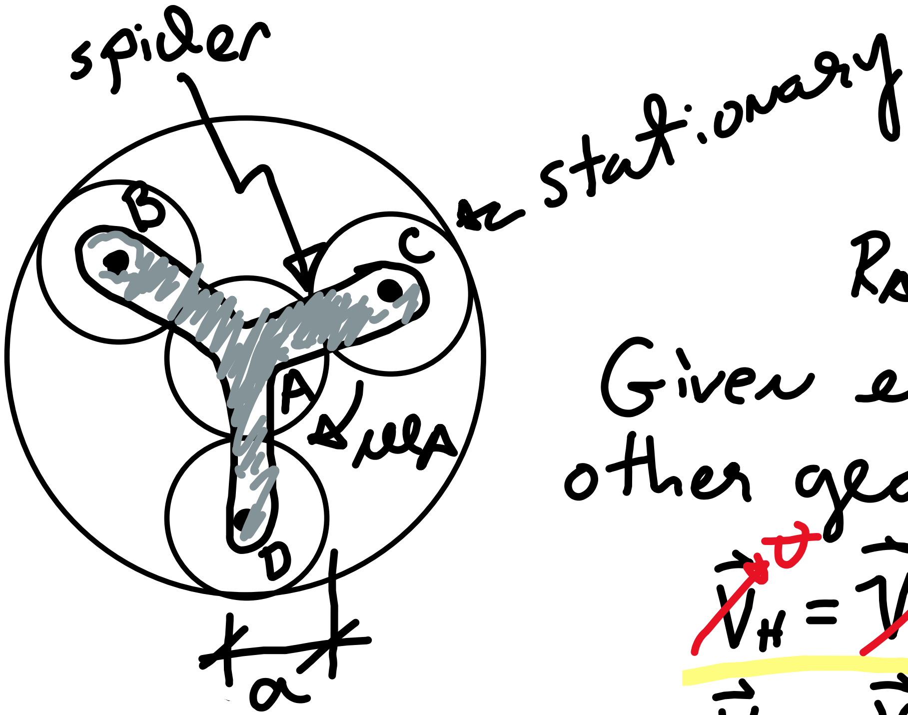
$$\vec V_G = \cancelto{0}{\vec V_{G/F}} + \vec V_F = a\omega_A$$

So $2a\omega_C = a\omega_A \Rightarrow \boxed{\omega_C = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec V_C = \vec V_{C/A} + \cancelto{0}{\vec V_A} = 2a\omega_{spider}$

Also $\vec V_C = \vec V_{C/E} + \cancelto{0}{\vec V_E} = a\omega_C = a\omega_A/2$ so

$$a\omega_A/2 = 2a\omega_{spider}$$



Example: Planetry gear system

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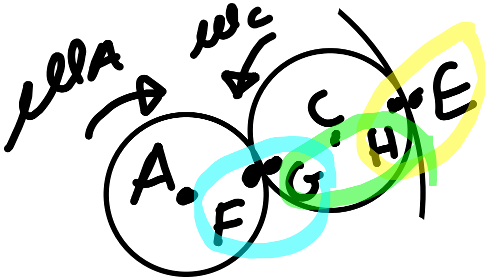
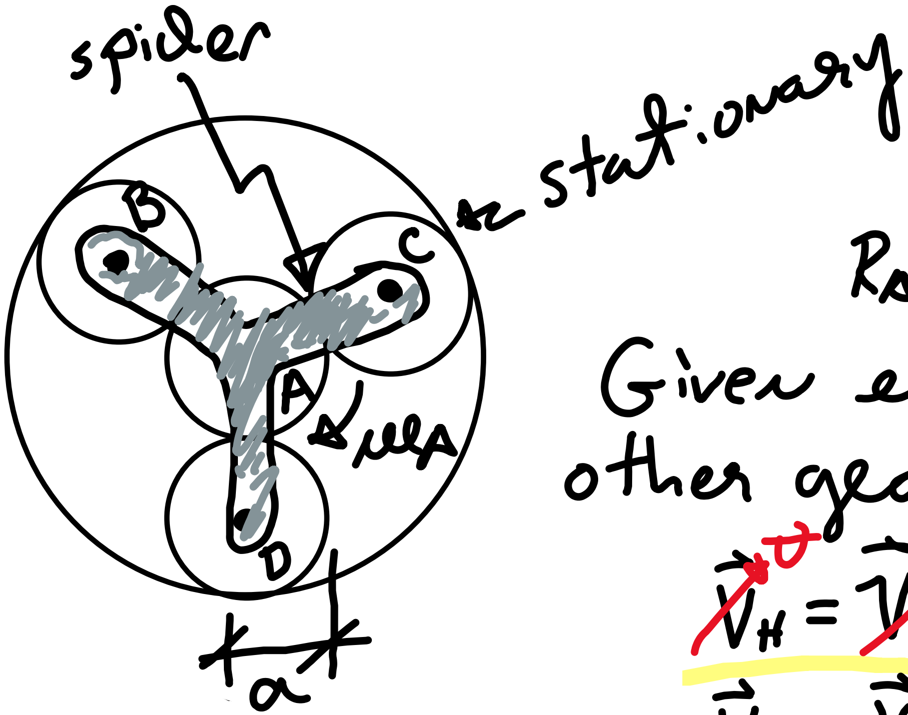


So $2a\omega_c = a\omega_A \Rightarrow \boxed{\omega_c = \frac{\omega_A}{2}}$

Find ω_{spider}: $\vec{V}_c = \vec{V}_{c/A} + \cancelto{0}{\vec{V}_A} = 2a\omega_{spider}$

Also $\vec{V}_c = \vec{V}_{c/E} + \cancelto{0}{\vec{V}_E} = a\omega_c = a\omega_A/2$ so

$$a\omega_A/2 = 2a\omega_{spider} \Rightarrow \omega_{spider} = \frac{\omega_A}{4}$$



Example: Planetry gear system

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Given ω_A, find ω of the other gears.

$\vec{V}_H = \vec{V}_{H/E} + \vec{V}_E$

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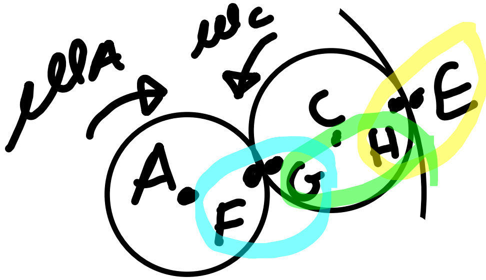

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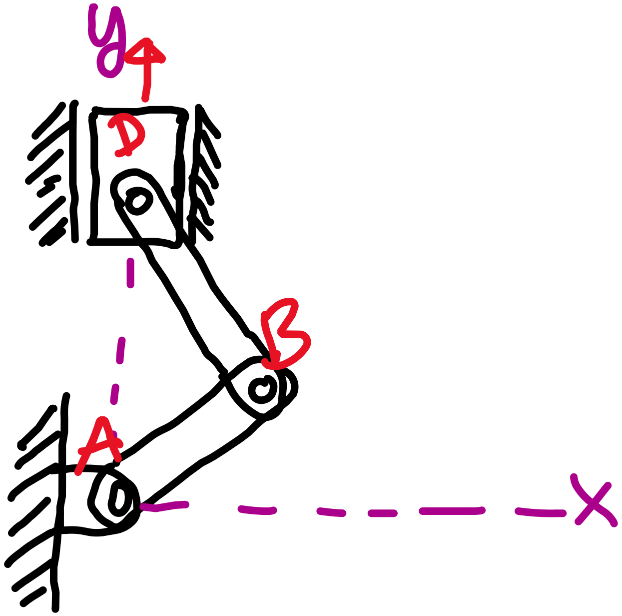
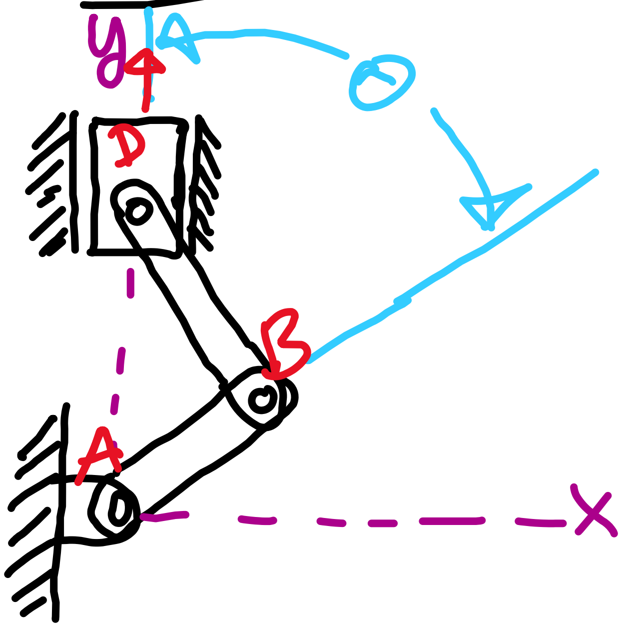
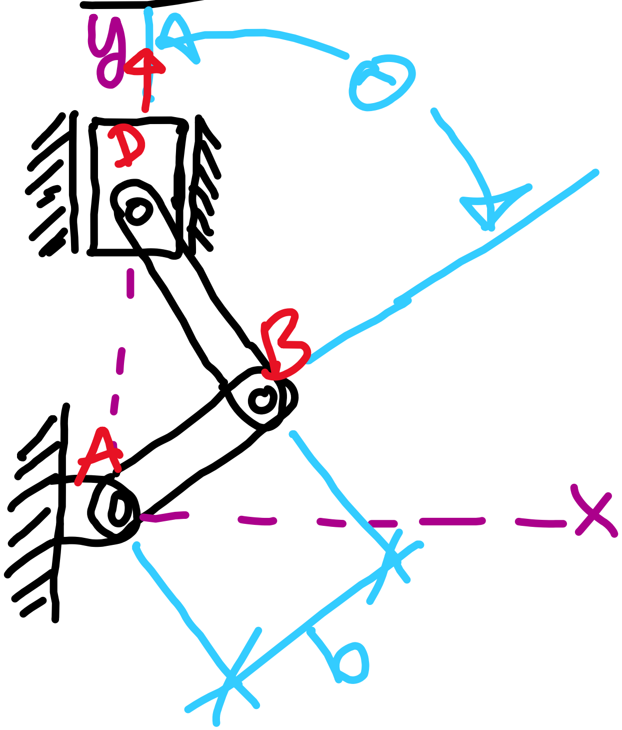
Notes on 15.62 :



Notes on 15.62 : Given $\theta = 60°$



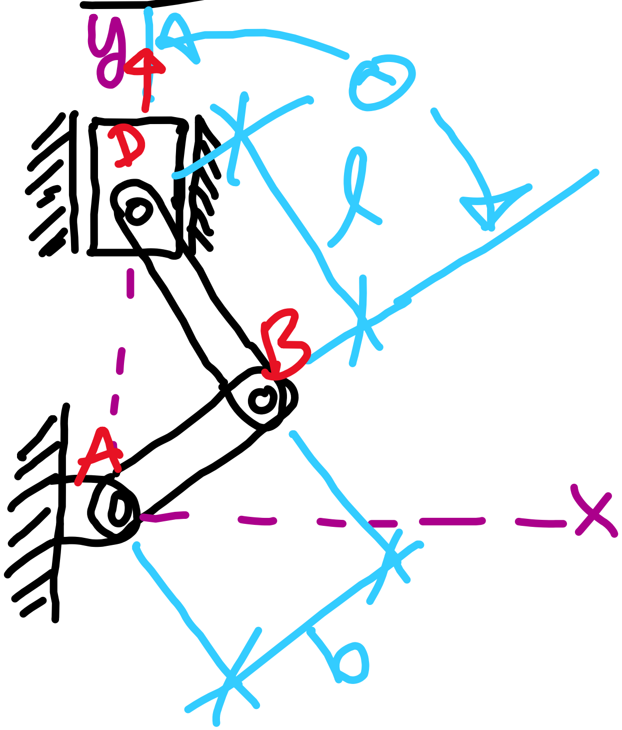
Notes on 15.62: Given $\theta = 60°$, $b = 60\text{mm}$



Notes on 15.62:

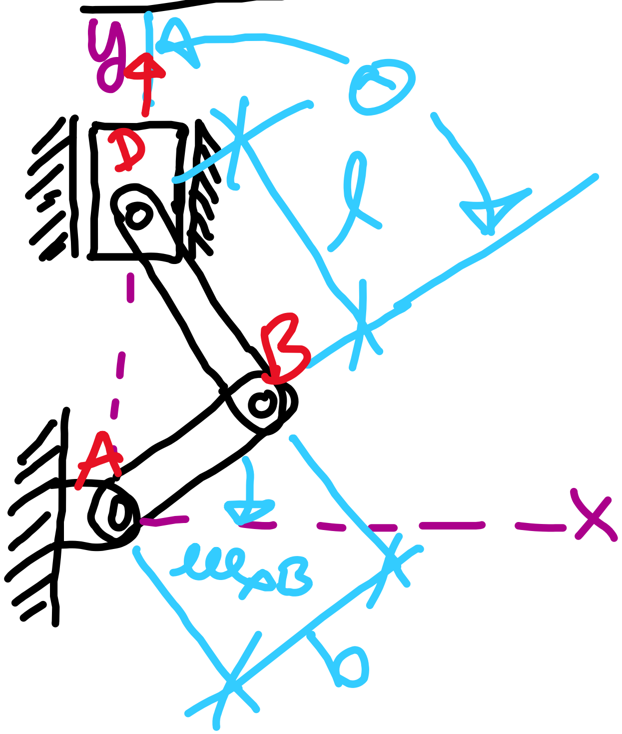
Given $\theta = 60°$, $b = 60mm$

$l = 120mm$



Notes on 15.62:

Given $\theta = 60°$, $b = 60mm$
$l = 120mm$ & $\omega_{AB} = 1000 rpm$



Notes on 15.62:

Given $\theta = 60°$, $b = 60\,mm$

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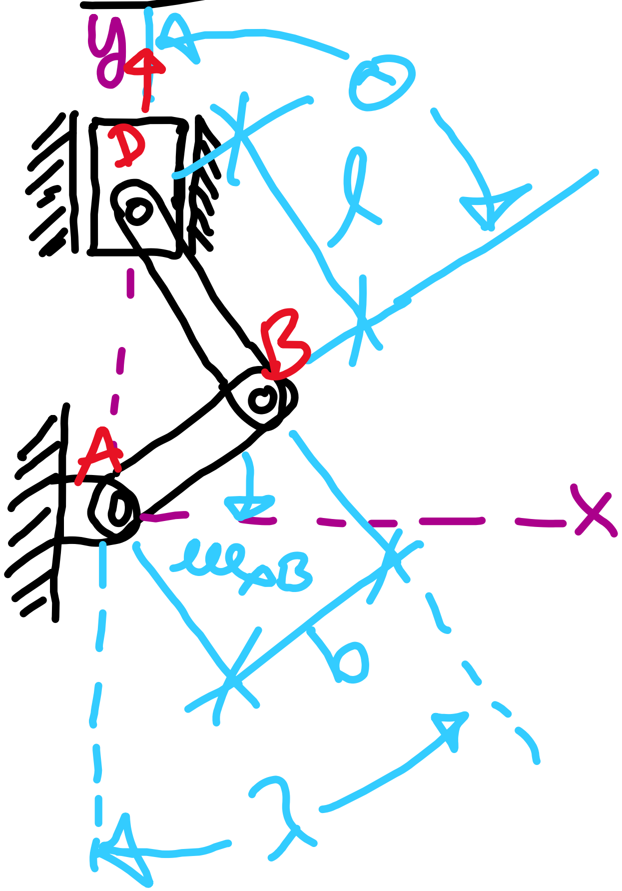

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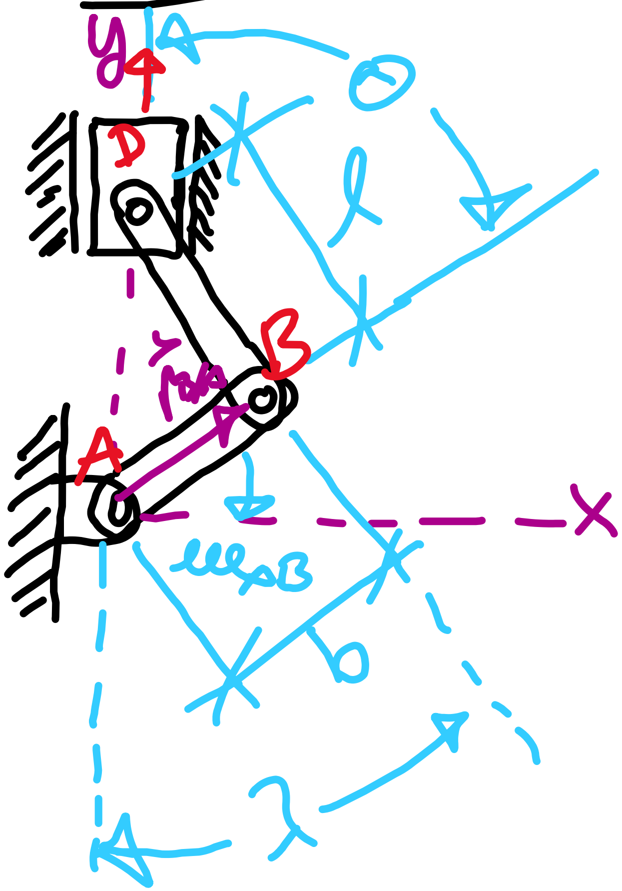

Given $\theta = 60°, b = 60mm$
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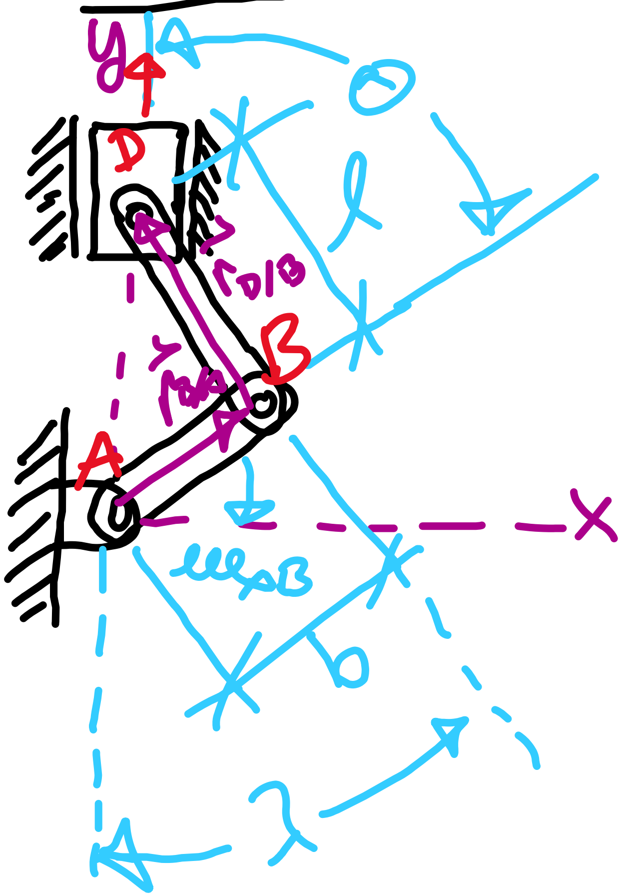
So $\vec{r}_{B/A} = \hat{x} b \sin\theta + \hat{y} b \cos\theta$

Notes on 15.62:



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So $\vec{r}_{B/A} = \hat{x}\, b \sin\Theta + \hat{y}\, b \cos\Theta$

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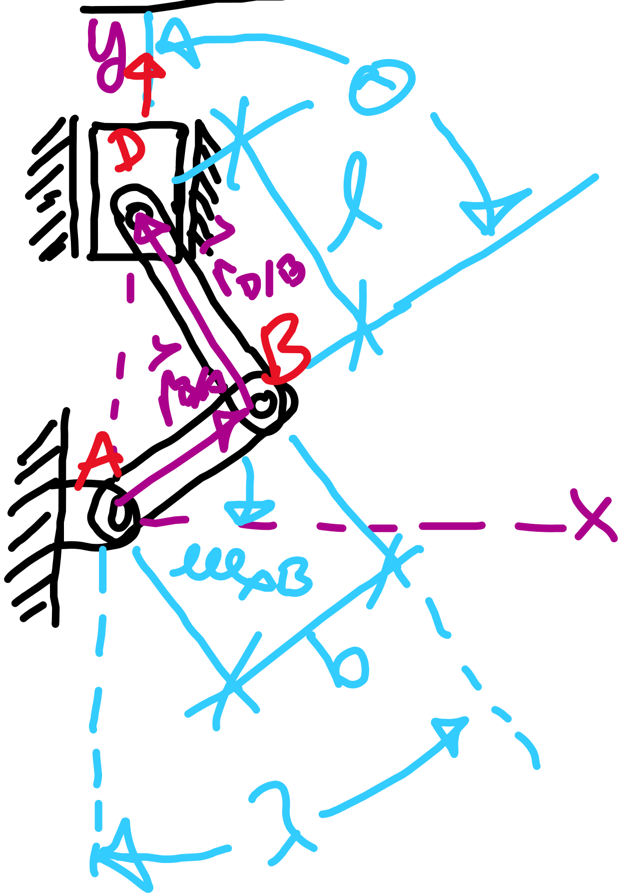

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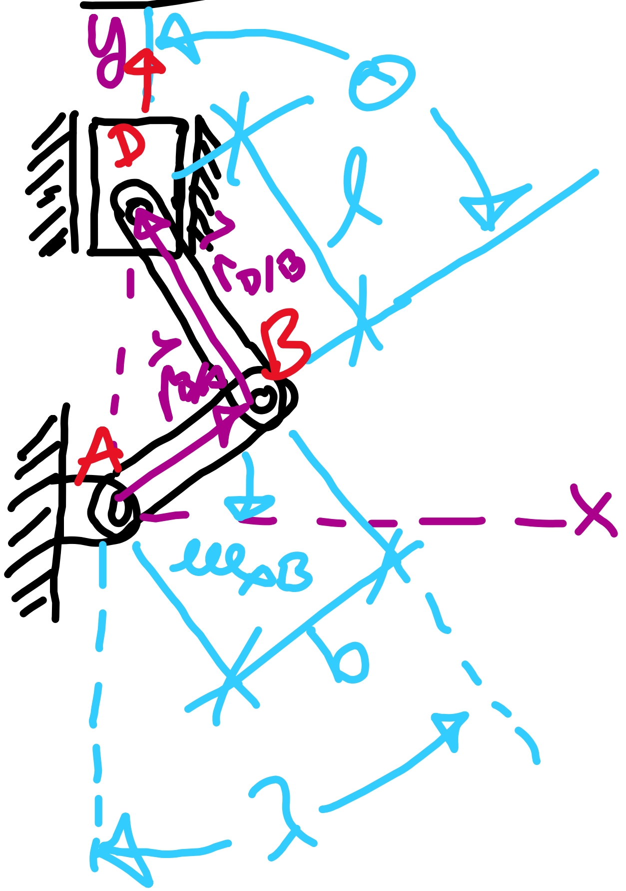

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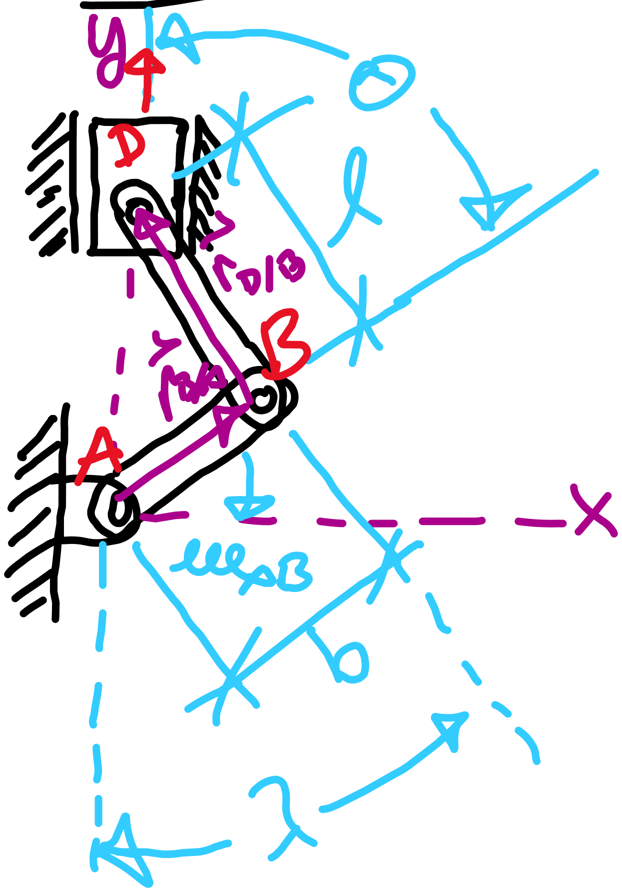

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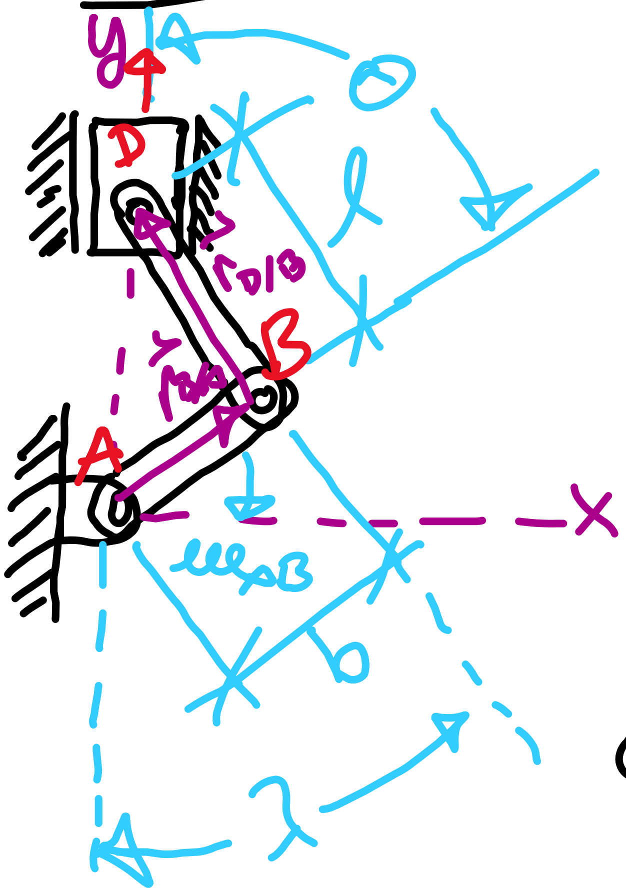

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One way to solve for V_D:

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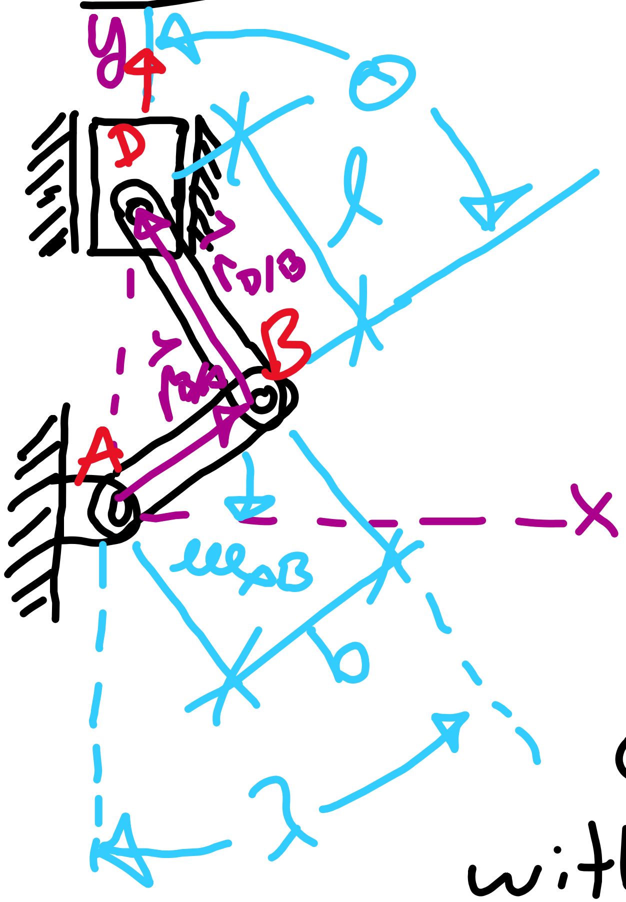

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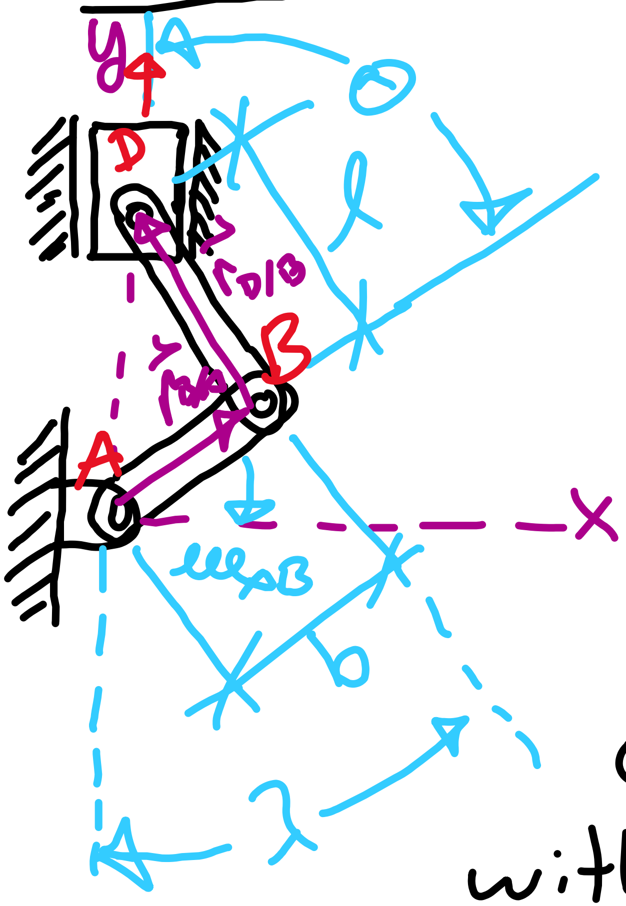
One way to solve for V_D: $\vec{V}_D = \vec{V}_{D/B} + \vec{V}_B$

with $\vec{V}_{D/B} = \vec{\omega}_{DB} \times \vec{r}_{D/B}$ & $\vec{V}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A}$

Notes on 15.62:



Given $\theta = 60°$, $b = 60\text{mm}$

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Your eqn for $\vec{V}_D$ will have 3 unknowns ($\lambda, \vec{V}_D, \omega_{DB}$) so you need two more equations.

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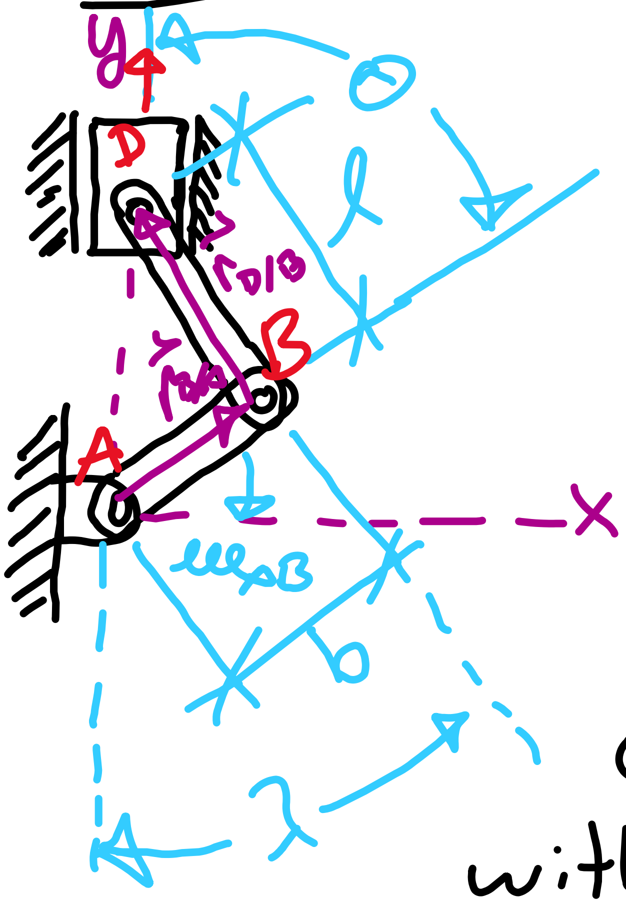

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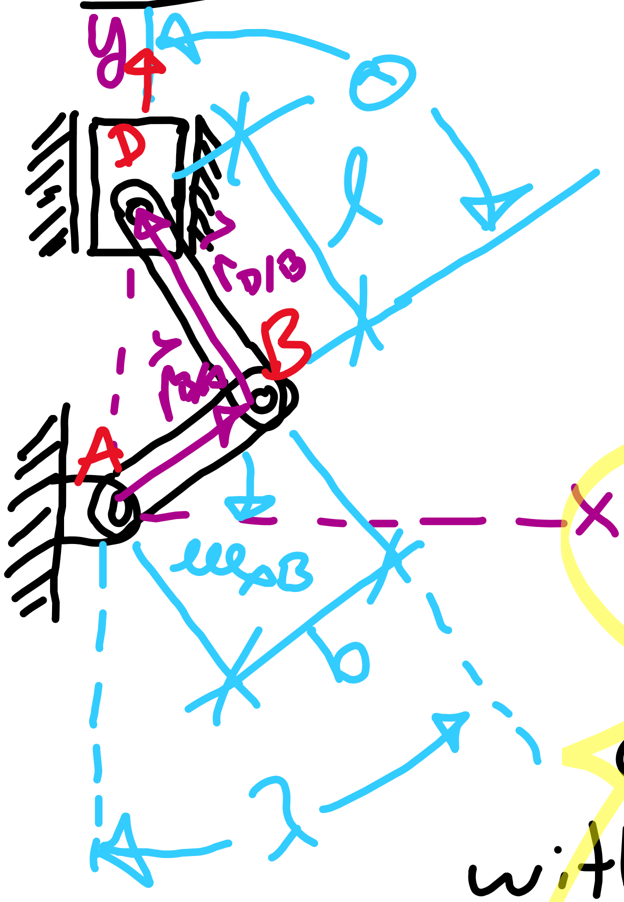

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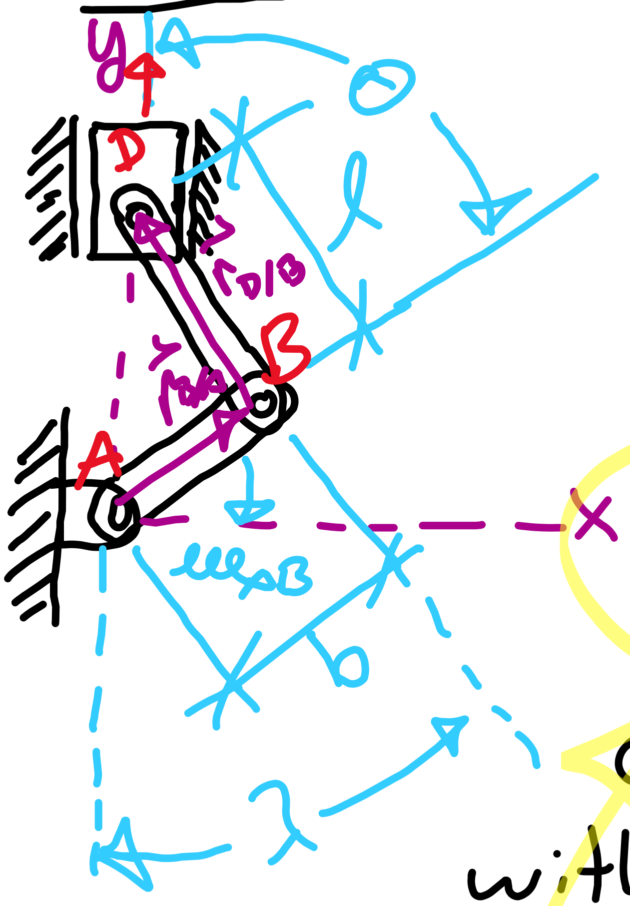

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Your eqn for $\vec{V}_D$ will have 3 unknowns ($\lambda, \vec{V}_D, \omega_{DB}$) so you need two more equations. Here are the other two when you set $V_{Dx} = 0$ you can get ω_{DB} and then use that for $\vec{V}_D = V_{Dy} \hat{y}$.