Today 15.3, 15.4

L15

Today 15.3, 15.4

instantaneous center of rotation

L15

Today 15.3, 15.4

L15

Rigid

body acceleration

Today 15.3, 15.4

L15

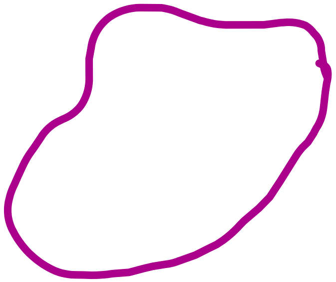
Thursday 16.1

Kinetics of rigid bodies

Instantaneous center of rotation

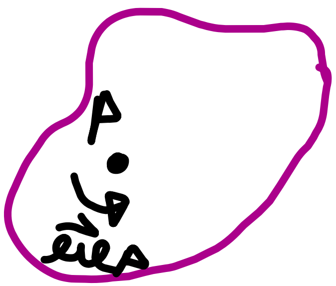
Instantaneous center of rotation

Take a rigid body



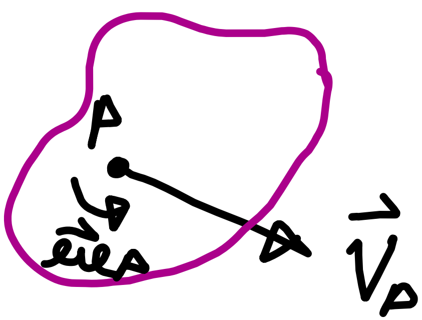
Instantaneous center of rotation

Take a rigid body that is in plane motion with a rotation ω_A about point A



Instantaneous center of rotation

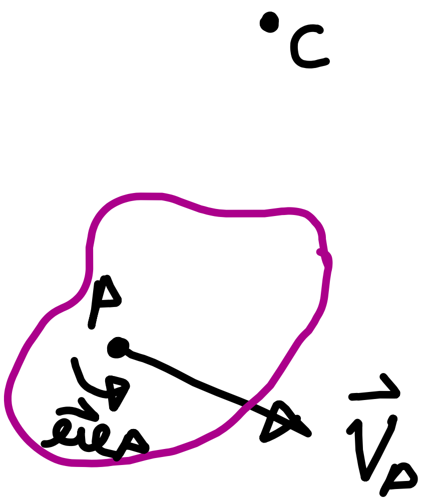
Take a rigid body that is in plane motion with a rotation ω_A about point A that has a velocity $\vec{V}_A$



Instantaneous center of rotation

Take a rigid body that is in plane motion with a rotation ω_A about point A that has a velocity $\vec{v}_A$.

For that point A there exists another point C



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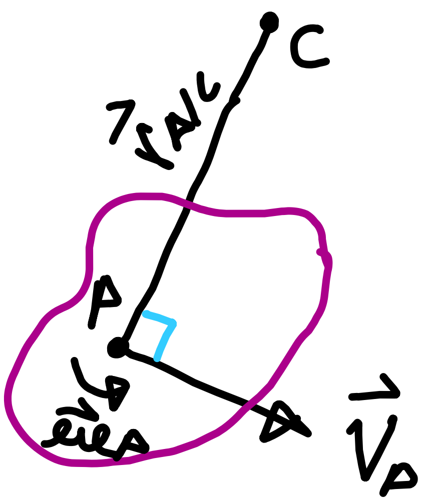
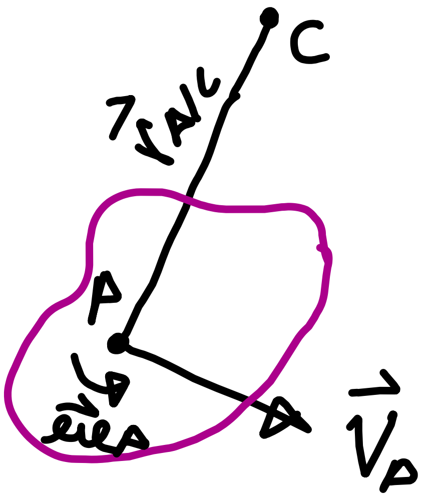


For that point A there exists another point C such that

$\vec{V}_A = \vec{\omega}_A \times \vec{r}_{A/C}$

Instantaneous center of rotation

Take a rigid body that is in plane motion with a rotation ω_A about point A that has a velocity $\vec{V}_A$.



For that point A there exists another point C such that

$$\vec{V}_A = \vec{\omega}_A \times \vec{r}_{A/C}$$

Note:

since $\vec{r}_{A/C} \cdot \vec{V}_A = \vec{r}_{A/C} \cdot (\vec{\omega}_A \times \vec{r}_{A/C})$

$\vec{\omega}_A \times \vec{r}_{A/C}$: perpendicular to both $\vec{\omega}_A$ & $\vec{r}_{A/C}$

Instantaneous center of rotation

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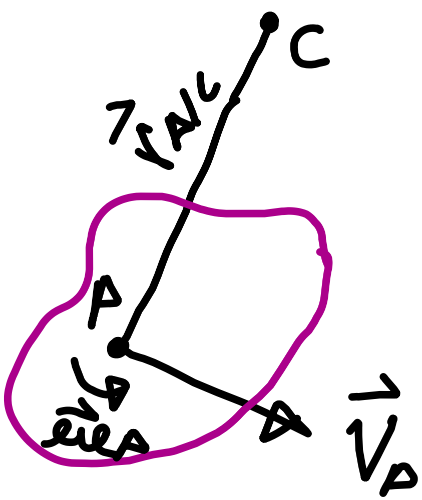

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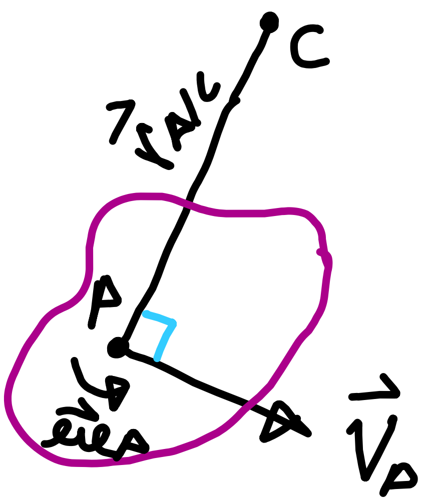

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Note:

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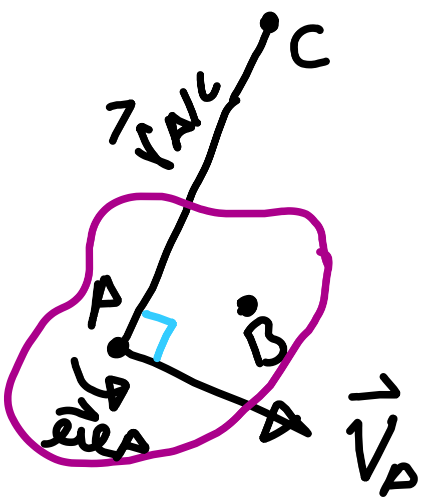
$\vec{V}_A$ & $\vec{r}_{A/C}$ are perpendicular. What about some other point on that rigid body?

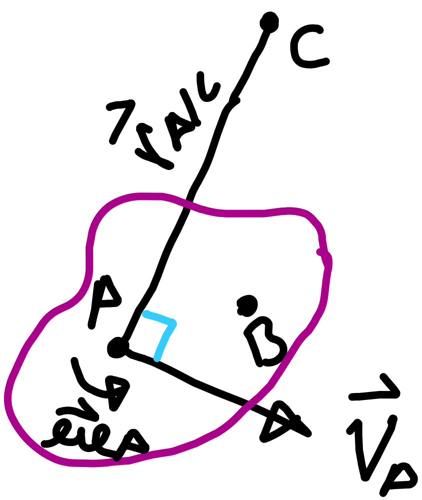
Instantaneous center of rotation

Take some point B on the rigid body



Instantaneous center of rotation

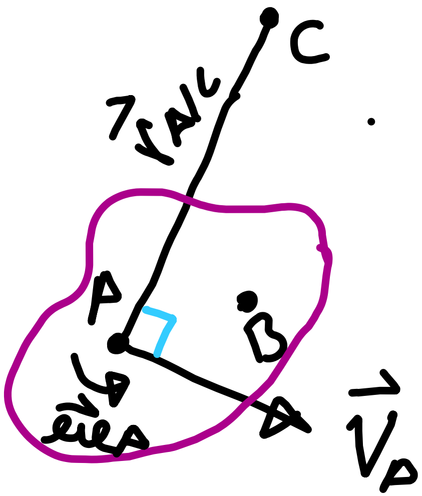
Take some point B on the rigid body. We have $\vec{v}_A = \vec{\omega}_A \times \vec{r}_{A/C}$ & want to construct $\vec{v}_B$:



Instantaneous center of rotation

Take some point B on the rigid body. We have $\vec{v}_A = \vec{\omega}_A \times \vec{r}_{A/C}$ & want to construct $\vec{v}_B$:

$$\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$



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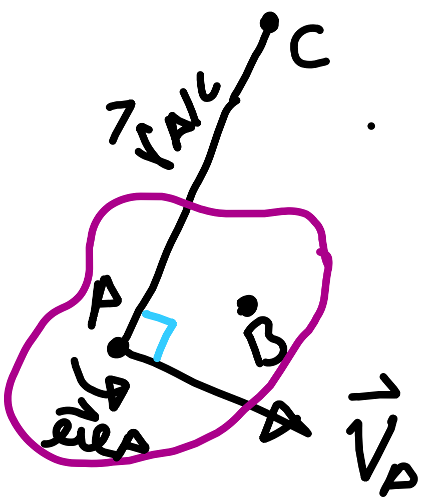


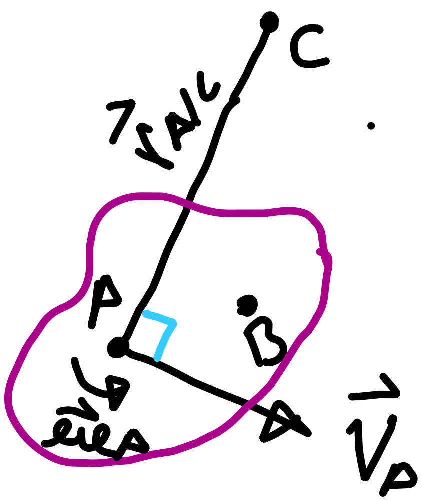
$$\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$

relative velocity of B about point A

Instantaneous center of rotation

Take some point B on the rigid body. We have $\vec{V}_A = \vec{\omega}_A \times \vec{r}_{A/C}$ & want to construct $\vec{V}_B$:

$$\vec{V}_B = \vec{V}_{B/A} + \vec{V}_A = \vec{\omega}_A \times \vec{r}_{B/A} + \vec{\omega}_A \times \vec{r}_{A/C}$$



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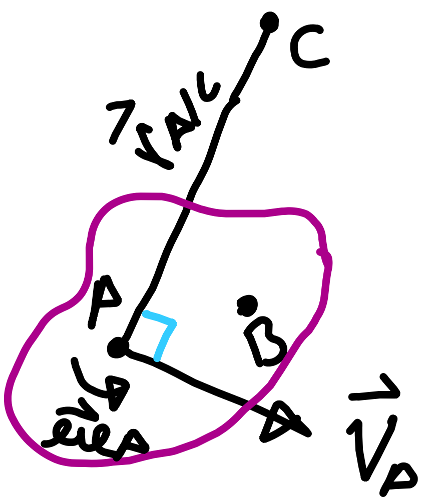

$$\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A = \vec{\omega}_A \times \vec{r}_{B/A} + \vec{\omega}_A \times \vec{r}_{A/C}$$

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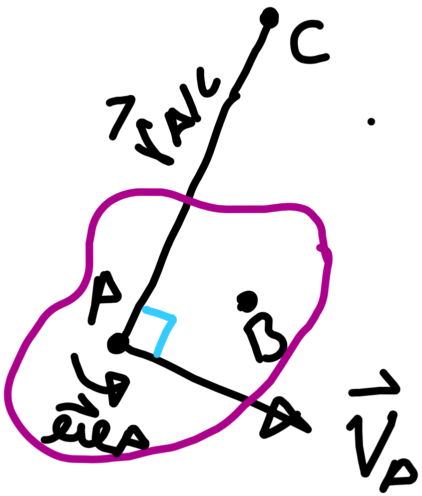

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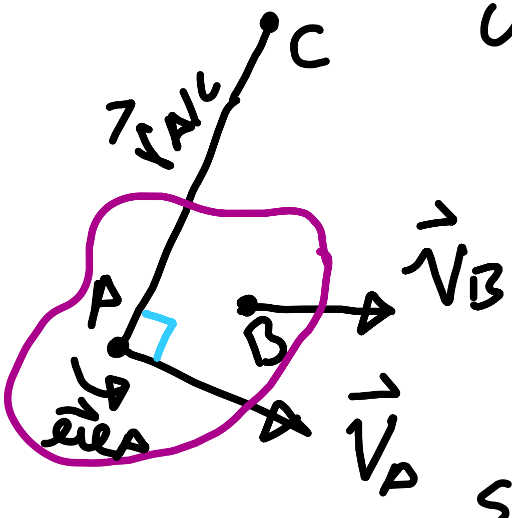

$$\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A = \vec{\omega}_A \times \vec{r}_{B/A} + \vec{\omega}_A \times \vec{r}_{A/C}$$

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So $\vec{v}_B =$

Instantaneous center of rotation

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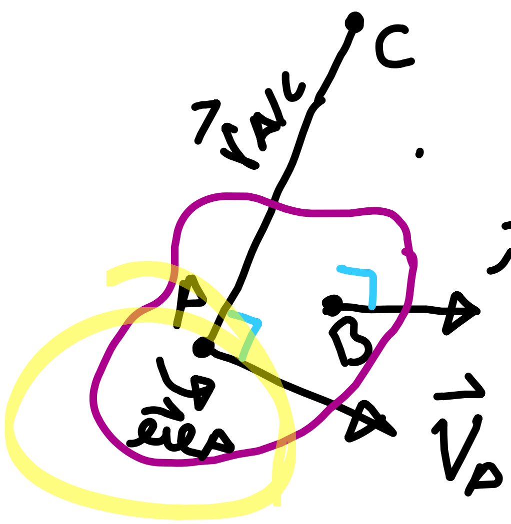

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so $\vec{V}_B = \vec{\omega}_A \times$

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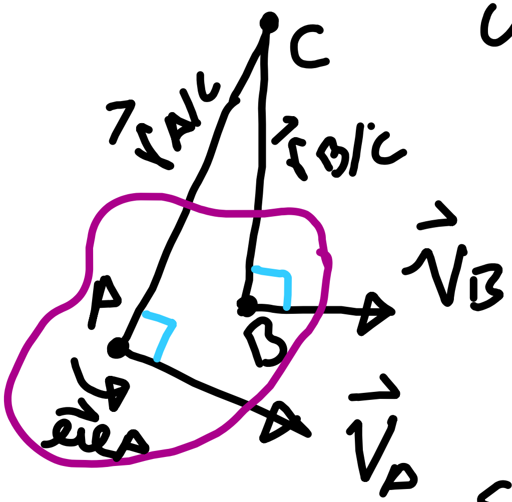

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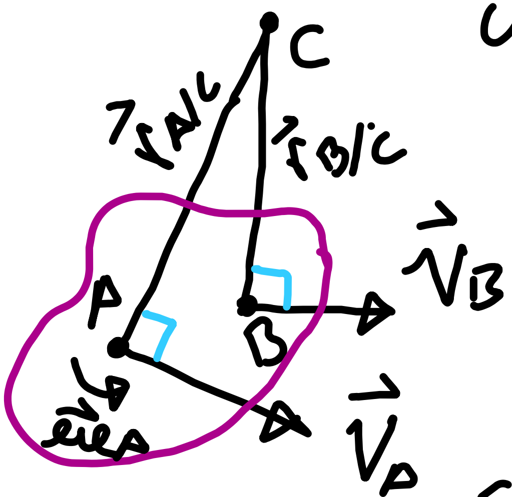

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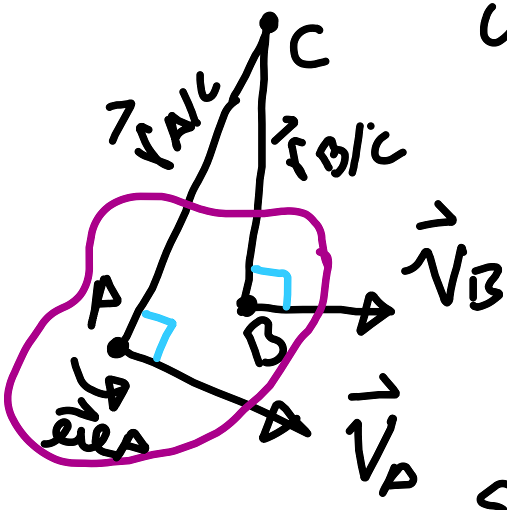

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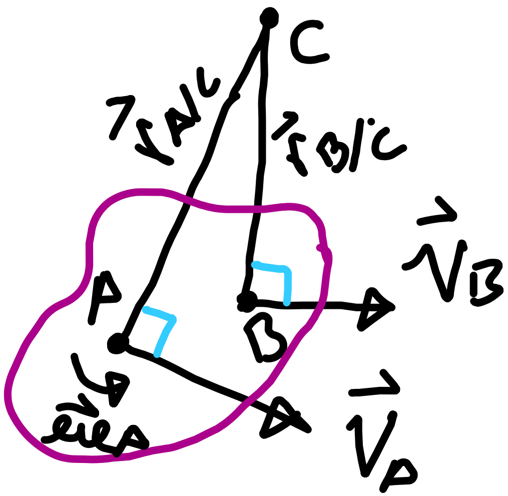
So $\vec{v}_B = \vec{\omega}_A \times \vec{r}_{B/C}$ We now have $\vec{v}_B$ constructed as a rotation about point C. Since B was arbitrary, this will work for <u>any</u> point on the rigid body!*

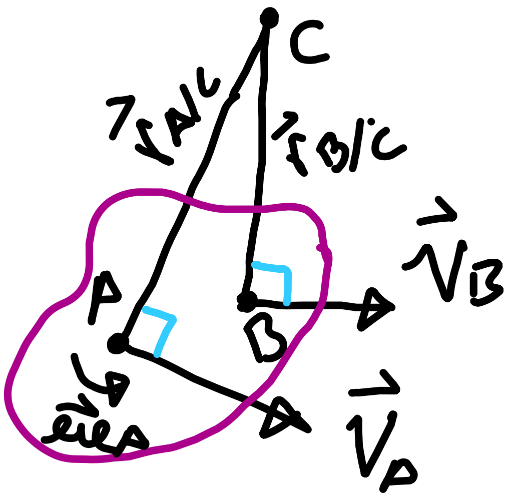
Instantaneous center of rotation

∃ for this one instance in time



Instantaneous center of rotation

For this one instance in time we have a single center of rotation for all points on the rigid body.



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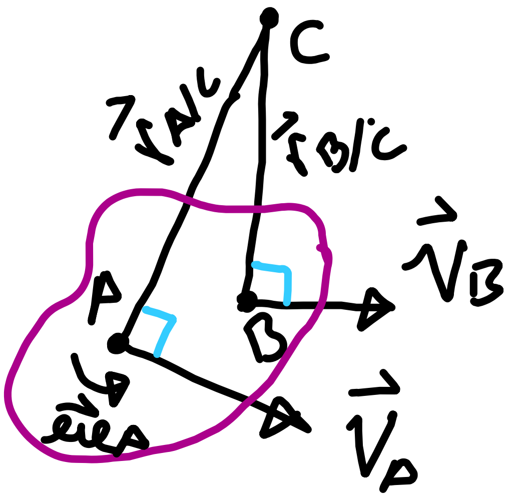
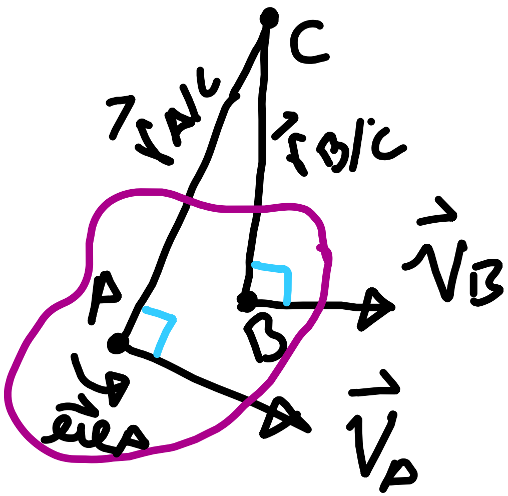
For this one instance in time we have a single center of rotation for all points on the rigid body. As with point A,



Instantaneous center of rotation

For this one instance in time we have a single center of rotation for all points on the rigid body. As with point A, the velocity of point B is perpendicular to the line connecting it to point C



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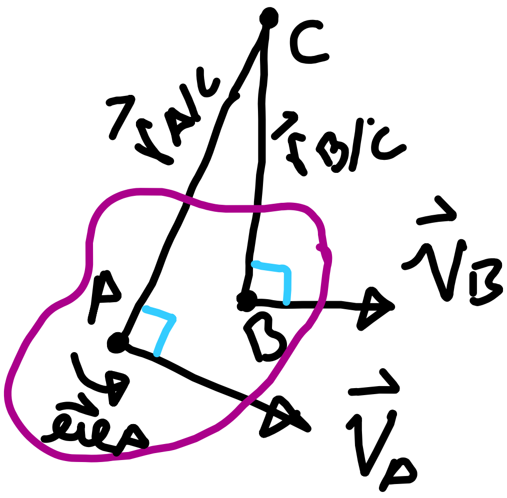

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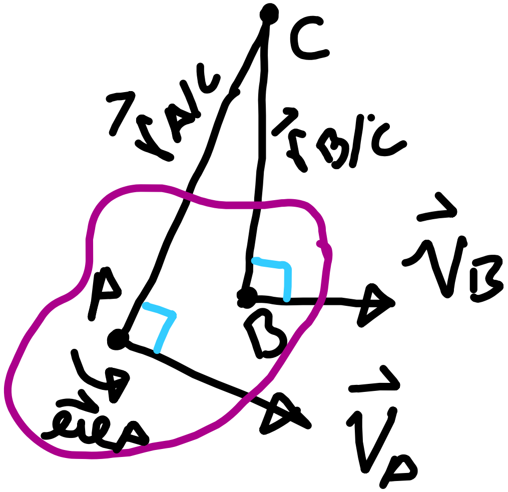
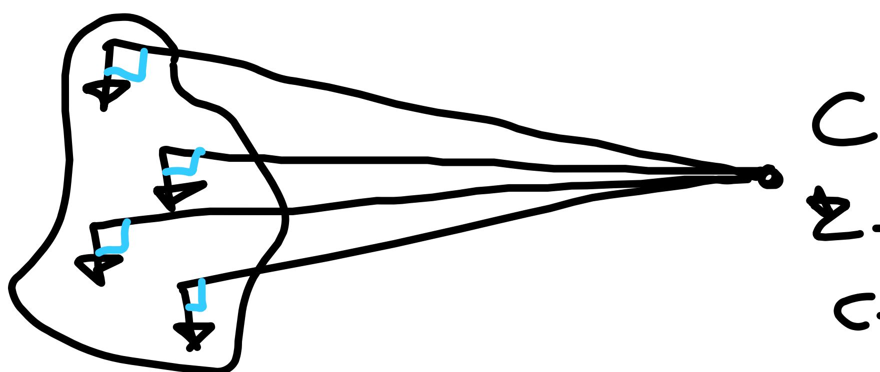




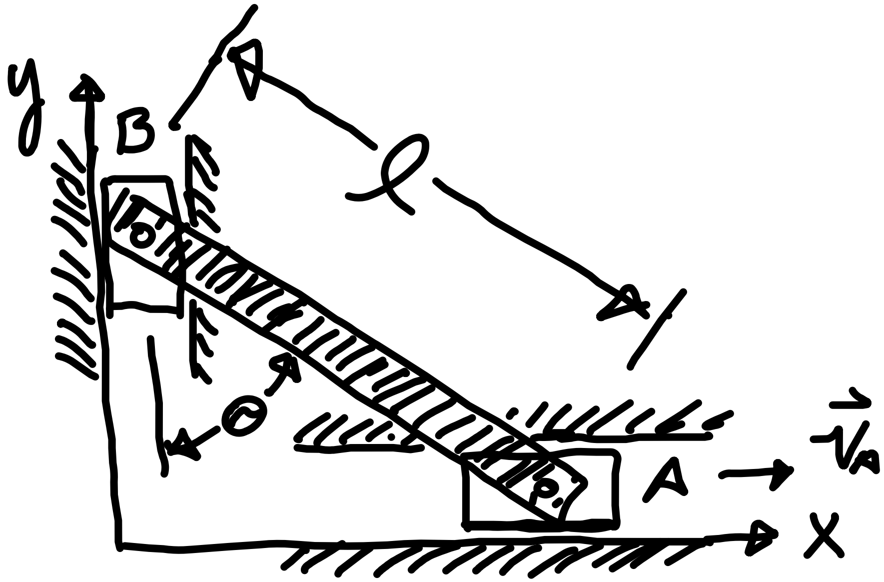
← Instantaneous center of rotation

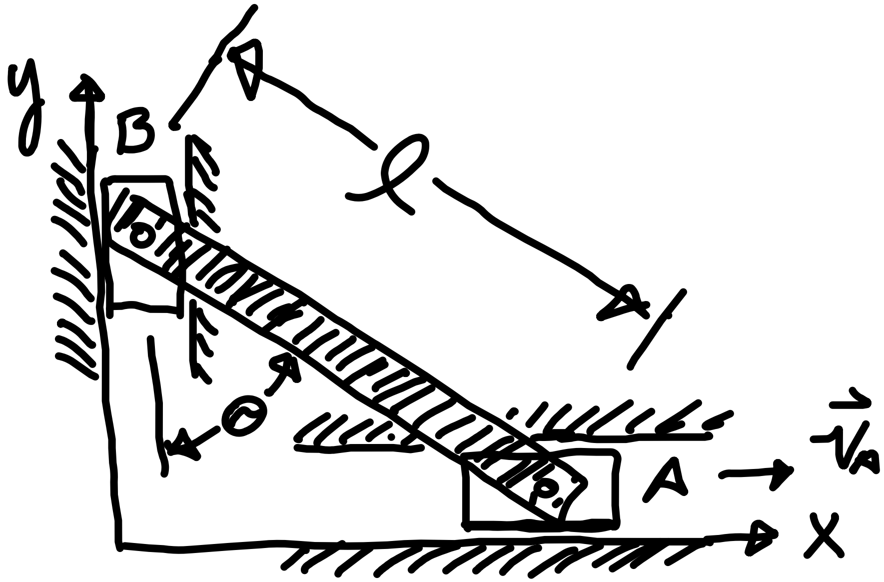
Example

Given l, $\vec{V}_A = V_A \hat{x}$ &

θ

Find $\vec{V}_B$ & ω_{ARM}:





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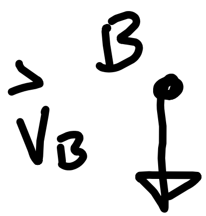

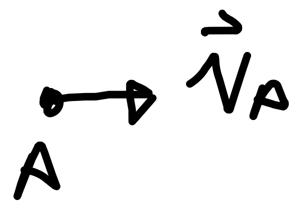

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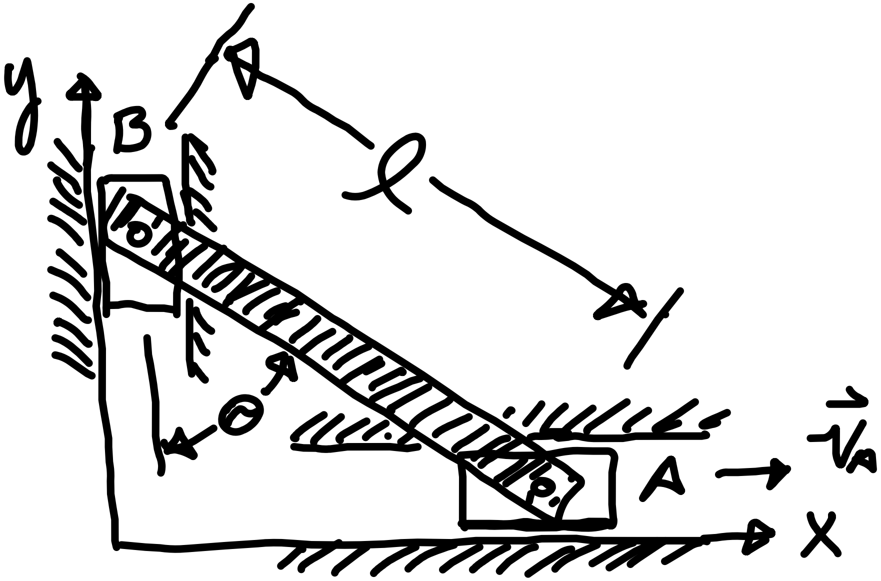

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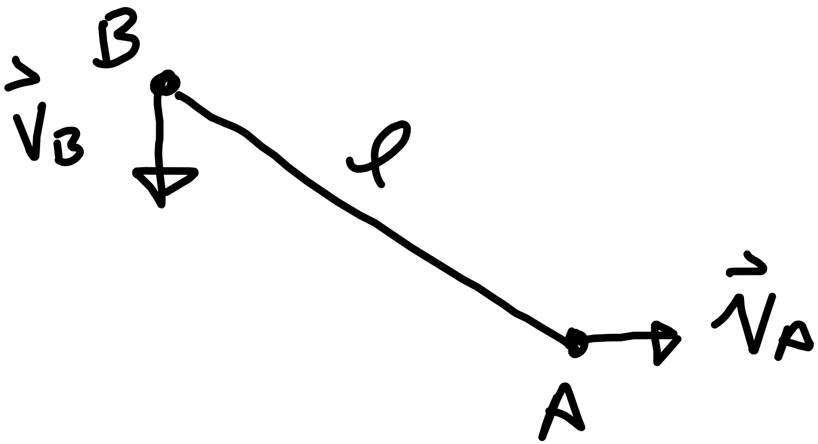

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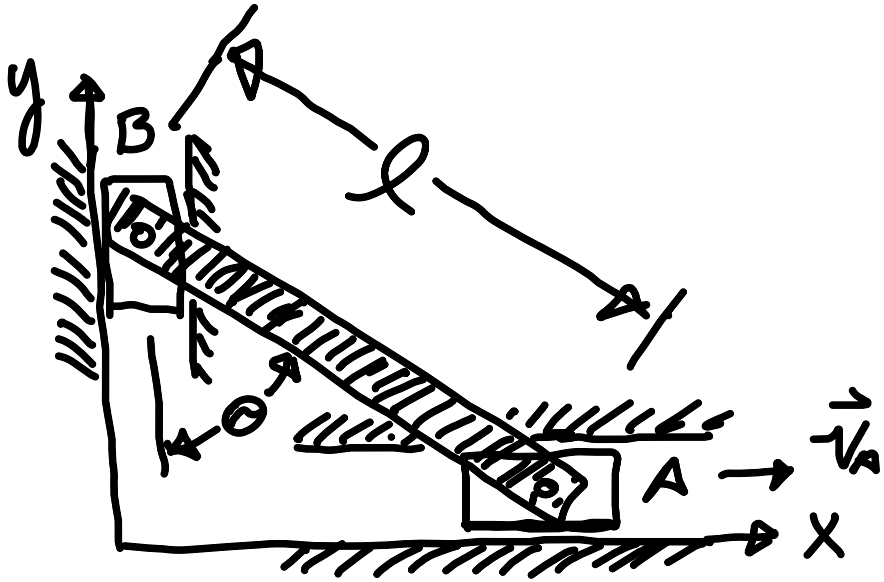

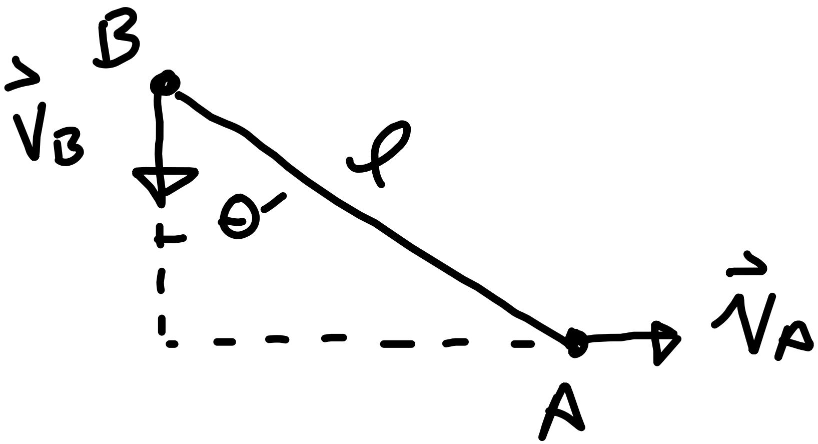

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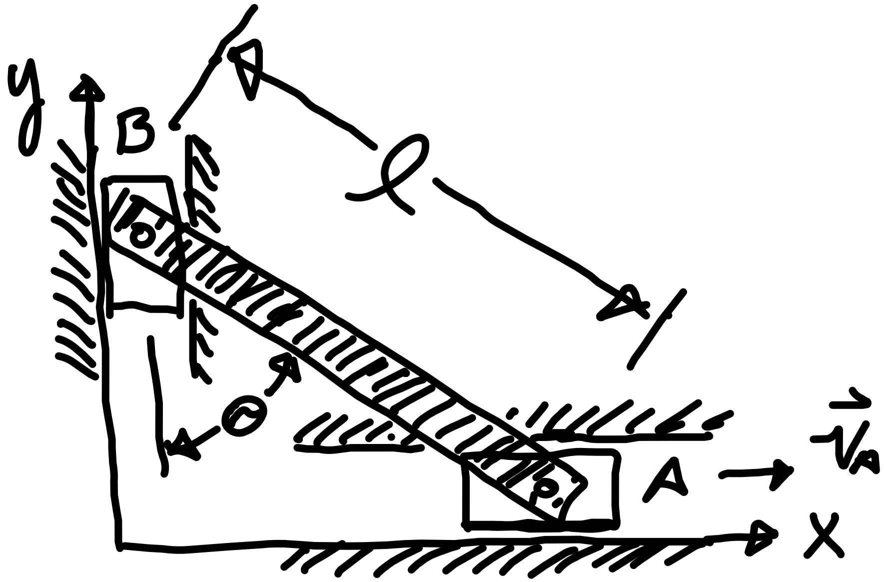

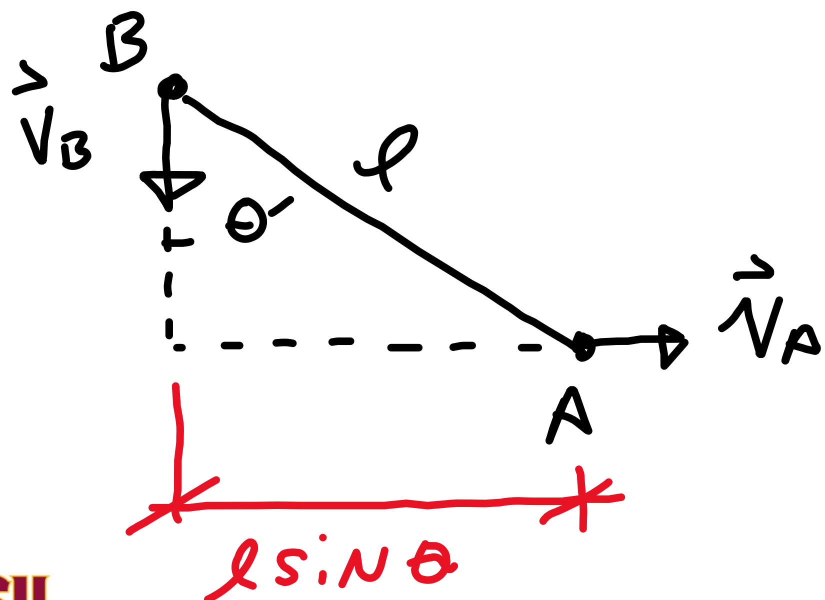

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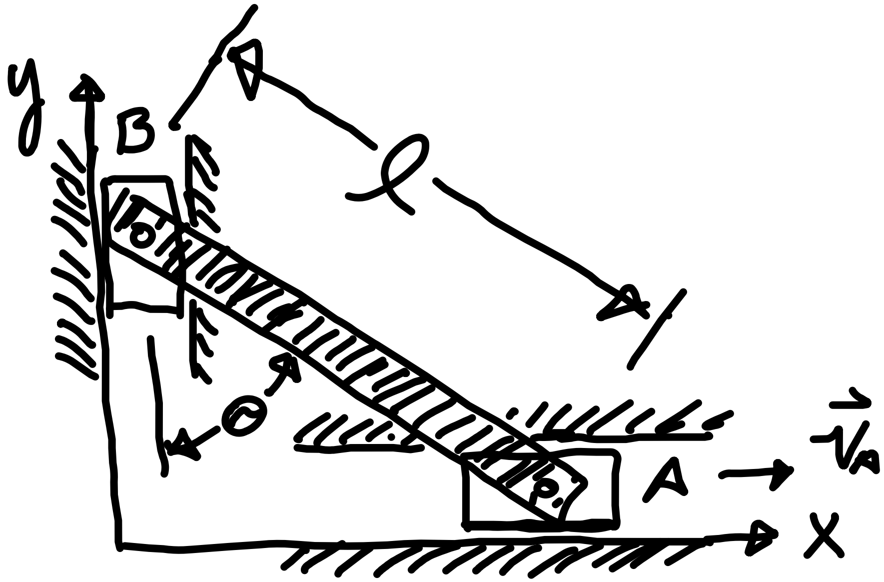

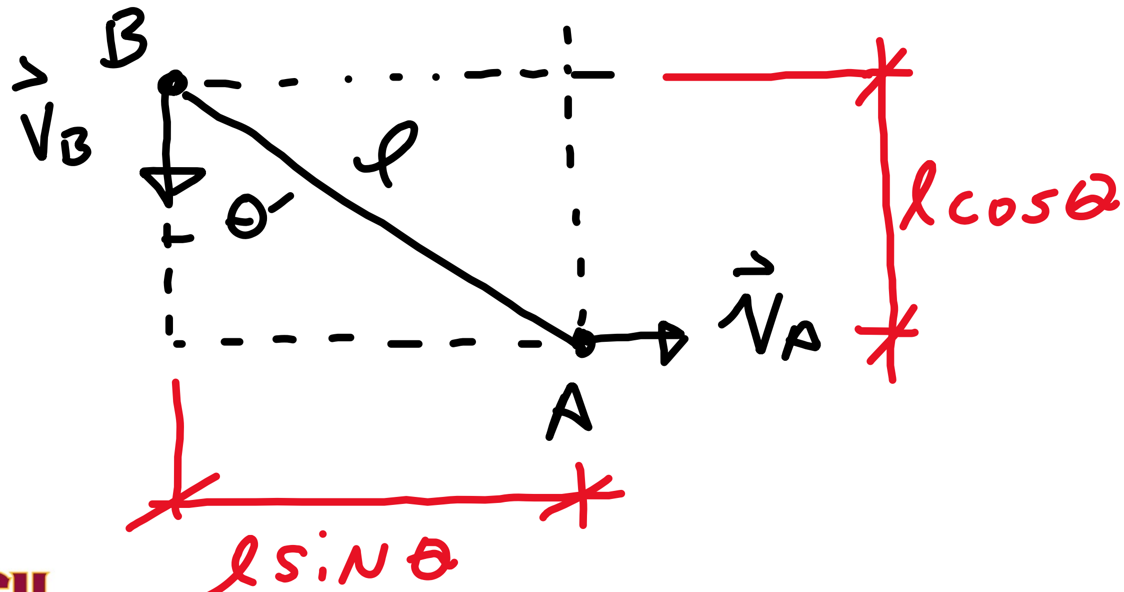

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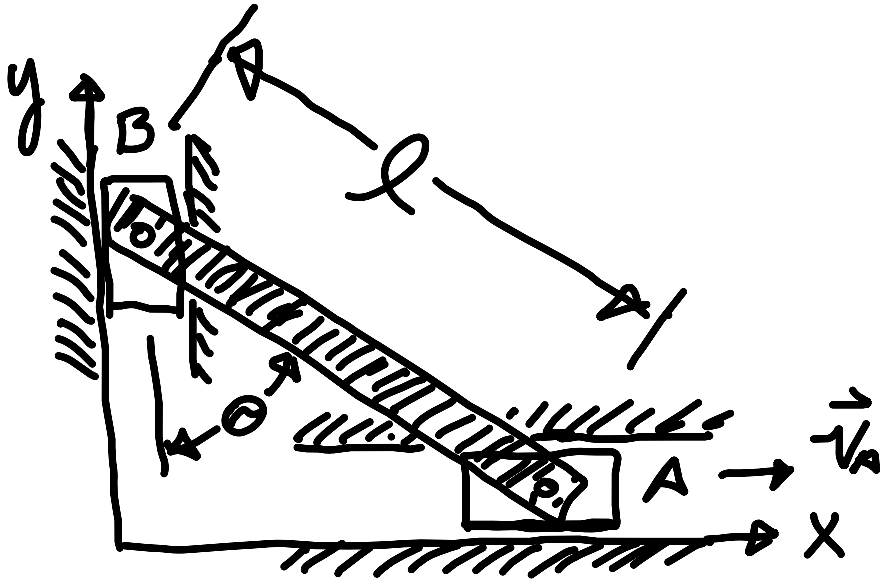
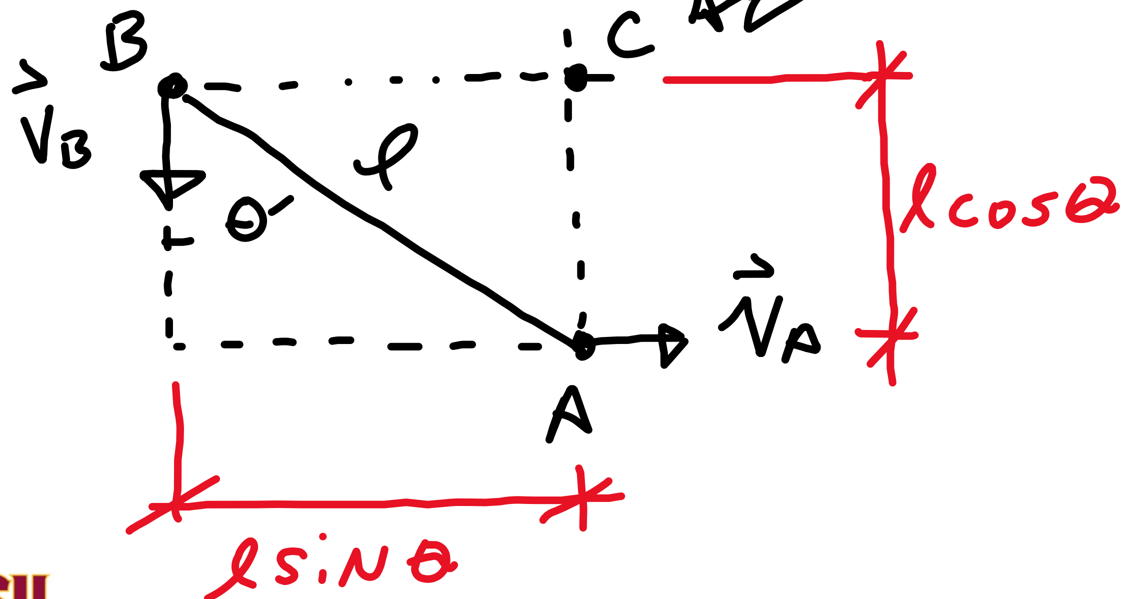


C AZ instantaneous center of rotation



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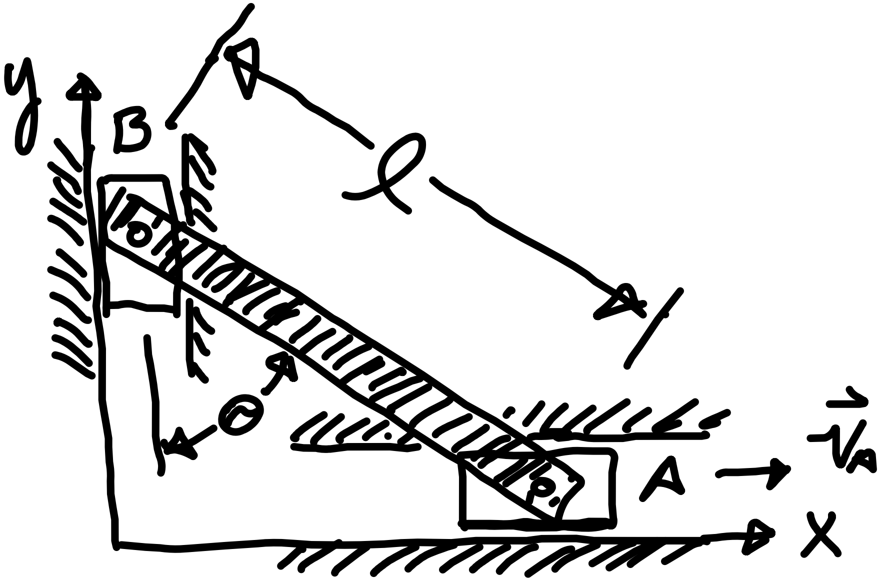

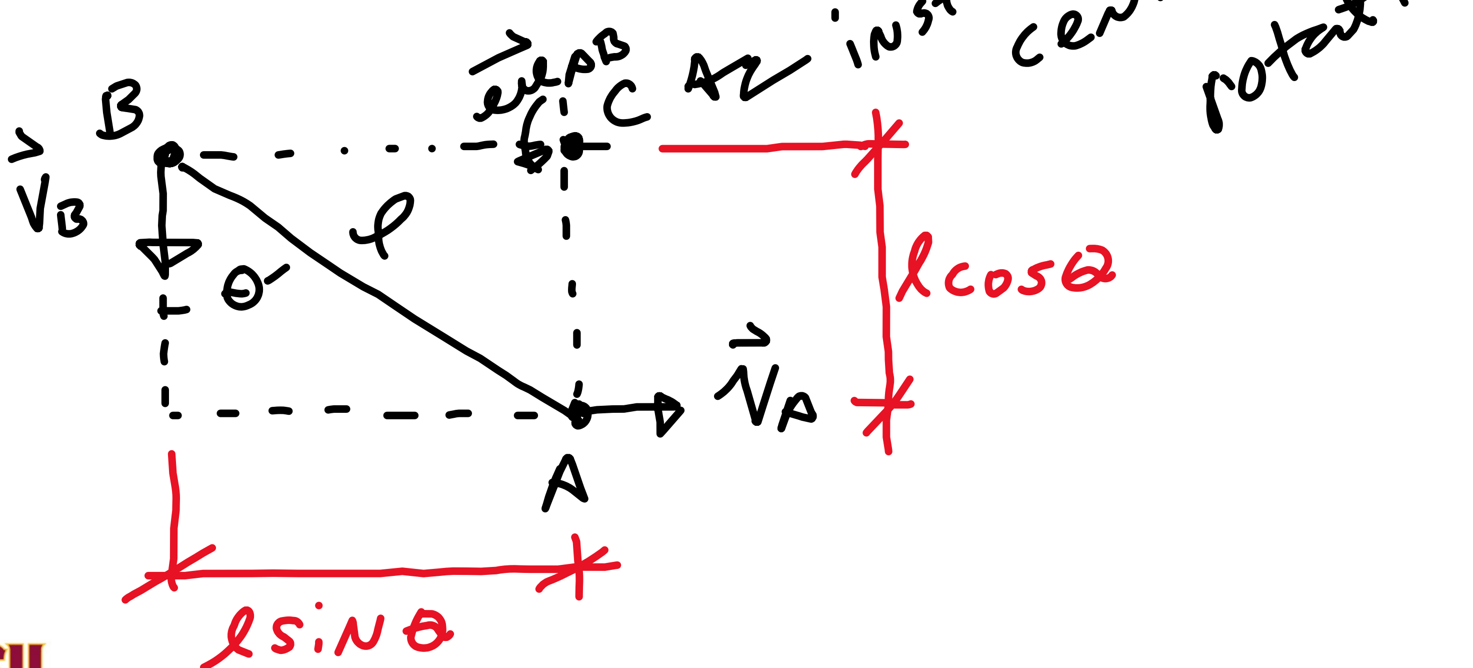

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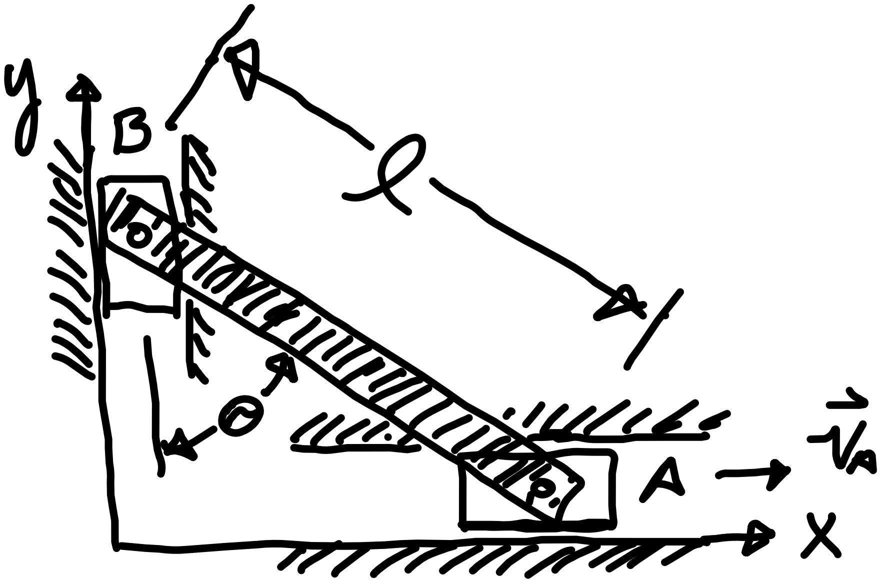
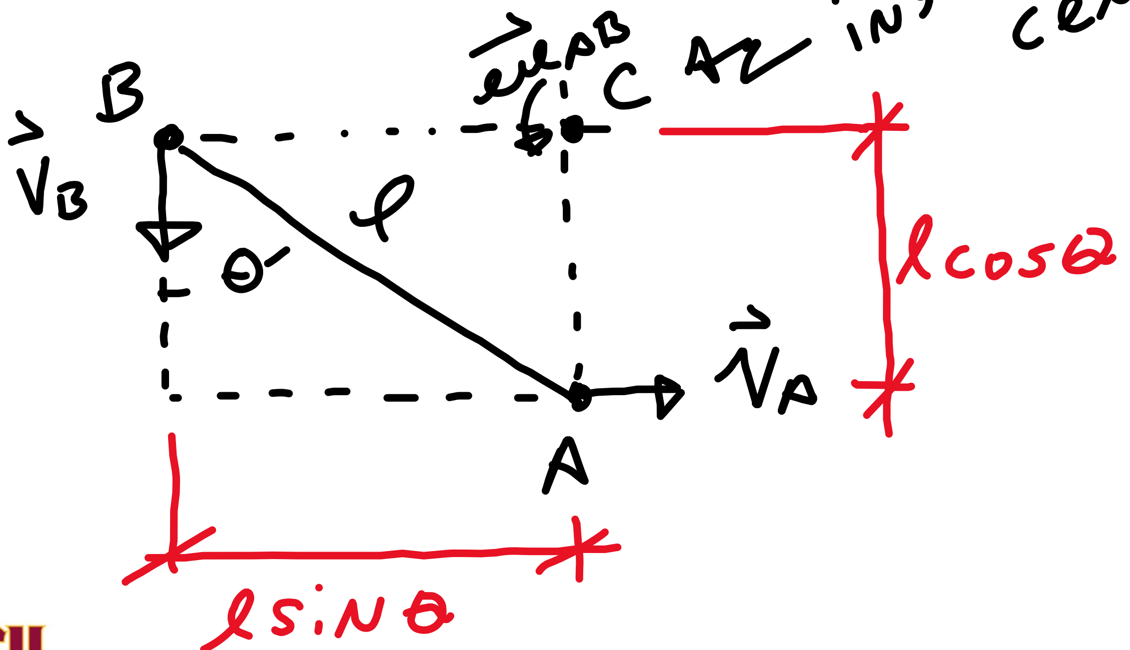
Find $\vec{V}_B$ & ω_{ARM}:





C AZ instantaneous center of rotation

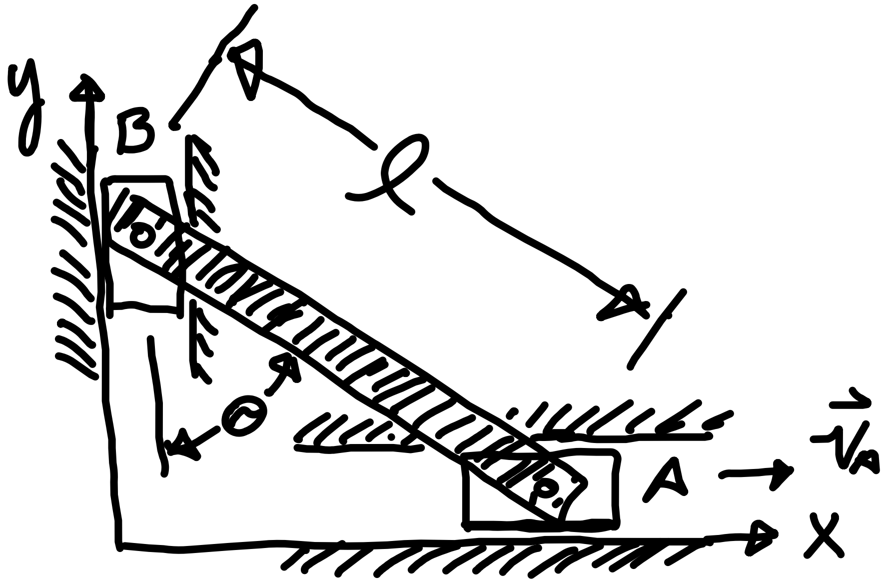
so

$V_A = \omega_{AB}\, l \cos\theta$

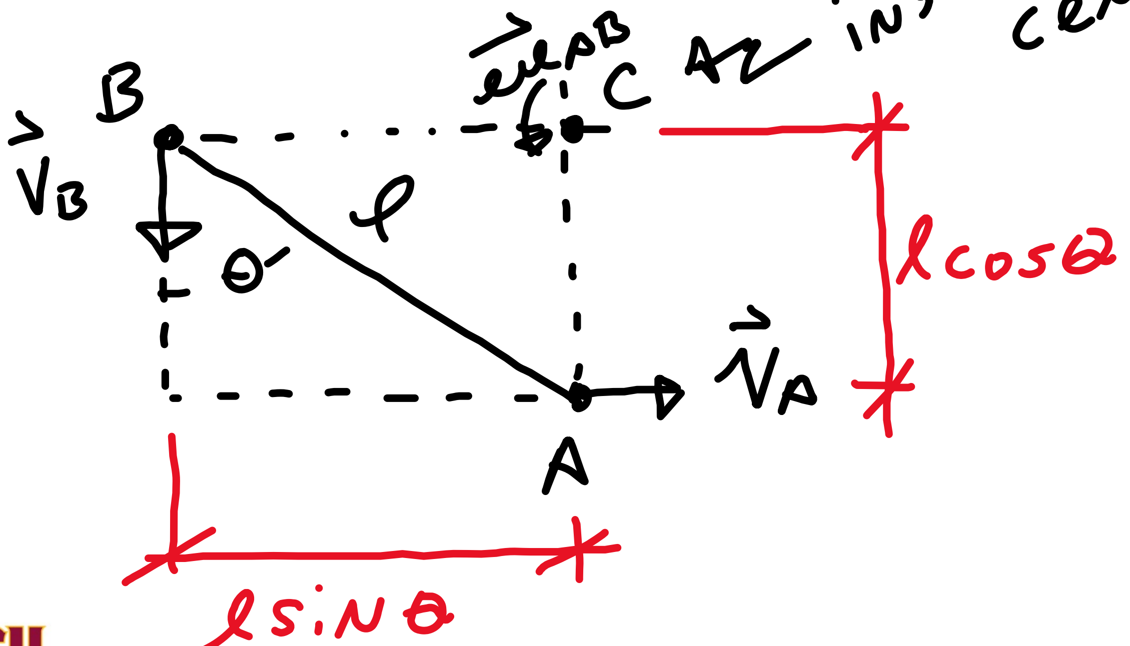
Example

Given l, $\vec{V}_A = V_A\hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:



C A2 instantaneous center of rotation



so

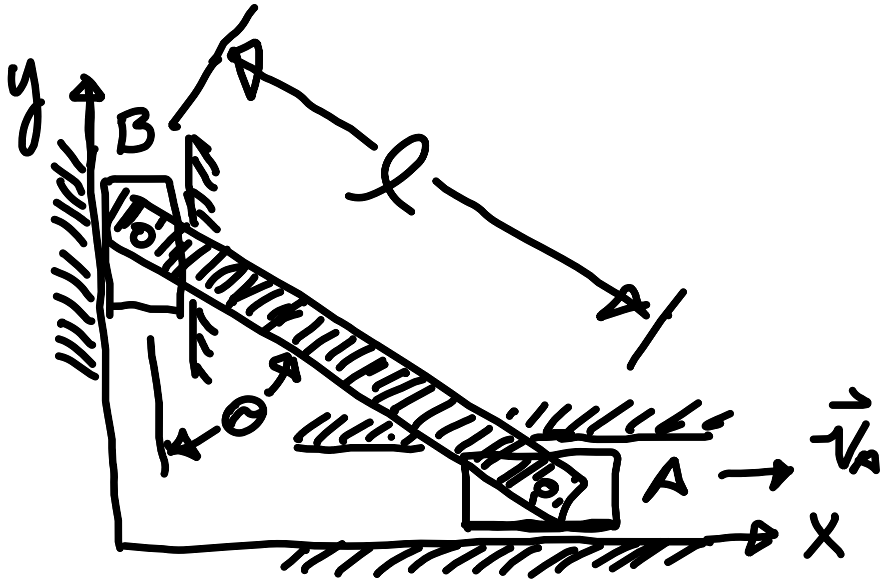
$$V_A = \omega_{AB}\, l\cos\theta$$

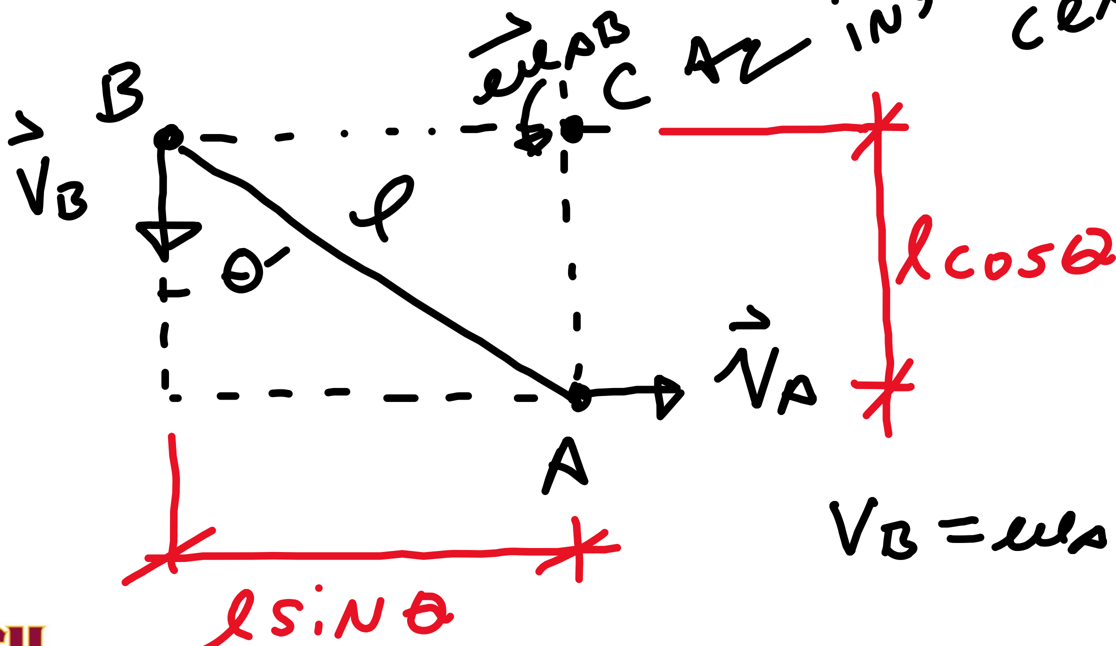
$$\Rightarrow \omega_{AB} = \frac{V_A \circlearrowright}{l\cos\theta}$$

Example

Given l, $\vec{V}_A = V_A \hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:





C Az instantaneous center of rotation

so

$$V_A = \omega_{AB}\, l \cos\theta$$

$$\Rightarrow \omega_{AB} = \frac{V_A}{l\cos\theta}$$

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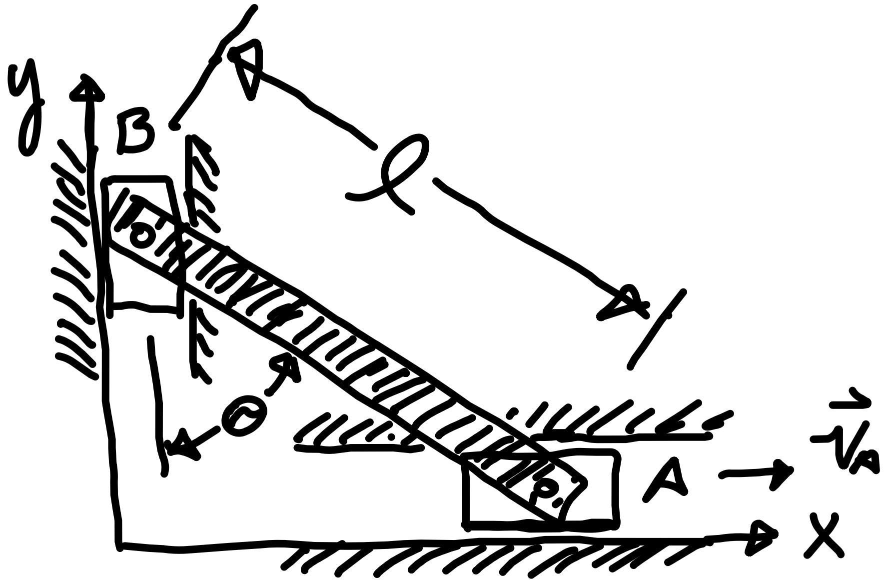

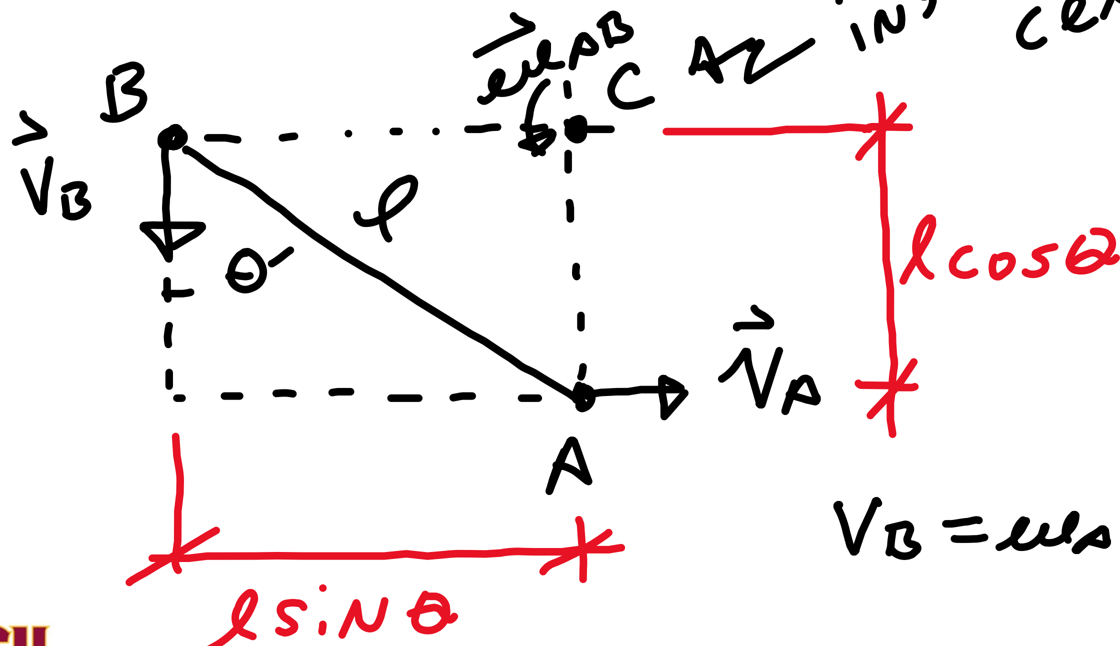

C ← instantaneous center of rotation

so

$V_A = \omega_{AB}\, l \cos\theta$

$$\Rightarrow \omega_{AB} = \frac{V_A}{l\cos\theta}$$

$$V_B = \omega_{AB}\, l \sin\theta = \frac{V_A}{l\cos\theta}\, l \sin\theta$$

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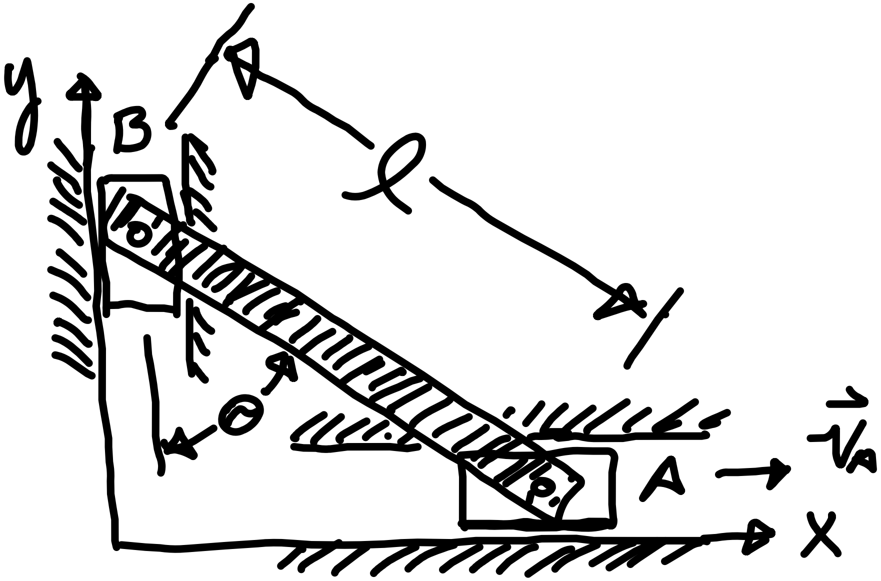
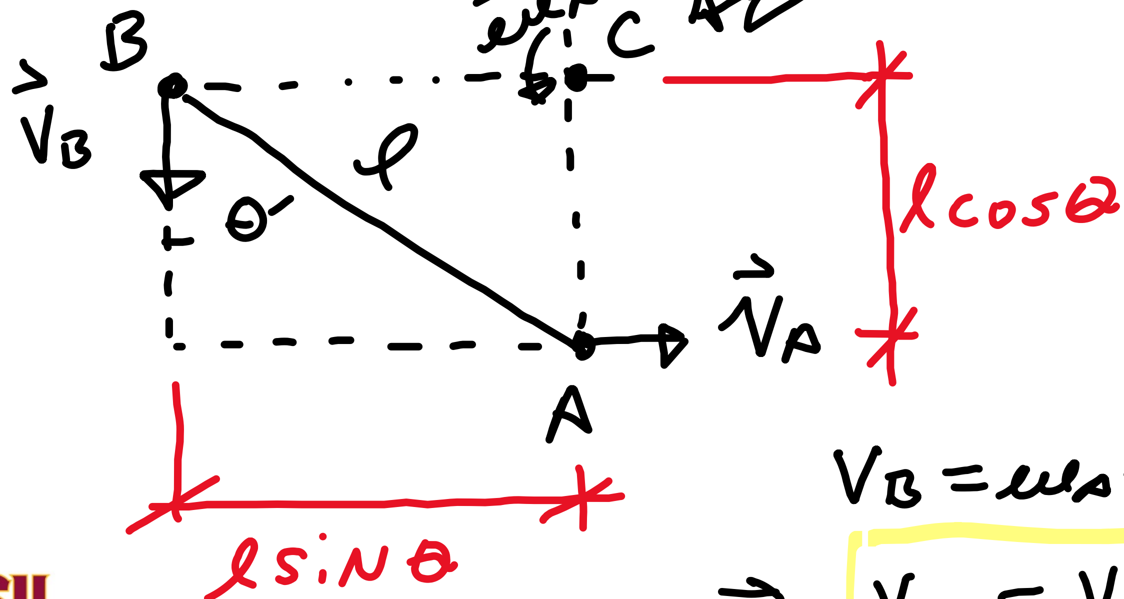


C ≡ instantaneous center of rotation



so

$$V_A = \omega_{AB}\, l \cos\theta$$

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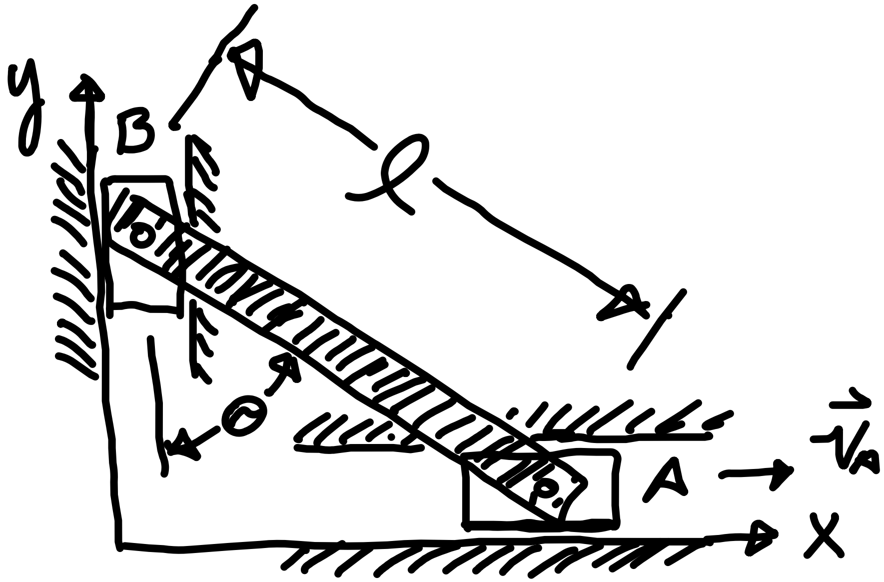
$$V_B = \omega_{AB}\, l \sin\theta = \frac{V_A}{l\cos\theta}\, l \sin\theta$$

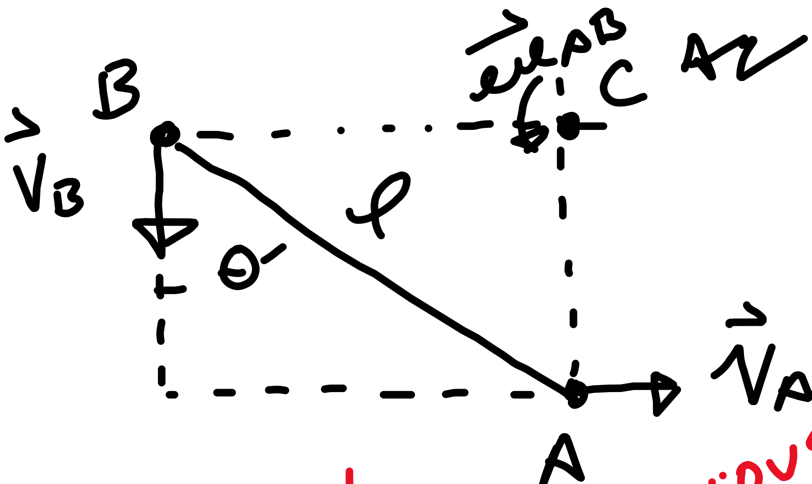
$$\Rightarrow V_B = V_A \tan\theta$$

Example

Given ℓ, $\vec{V}_A = V_A \hat{x}$ & θ

Find $\vec{V}_B$ & ω_{ARM}:





C as instantaneous center of rotation

so

$V_A = \omega_{AB} \ell \cos\theta$

$\Rightarrow \omega_{AB} = \dfrac{V_A}{\ell \cos\theta}$ ↻

$V_B = \omega_{AB} \ell \sin\theta = \dfrac{V_A}{\ell \cos\theta} \ell \sin\theta$

Easier to solve than previous method ⇒ $V_B = V_A \tan\theta$

For every point on the rigid body (in plane motion that is rotating) $V_p = \omega r_{p/c}$

For every point on the rigid body (in plane motion that is rotating) $v_P = \omega r_{P/C}$

Here ω is fixed $\Rightarrow v_P$ scales with distance $r_{P/C}$

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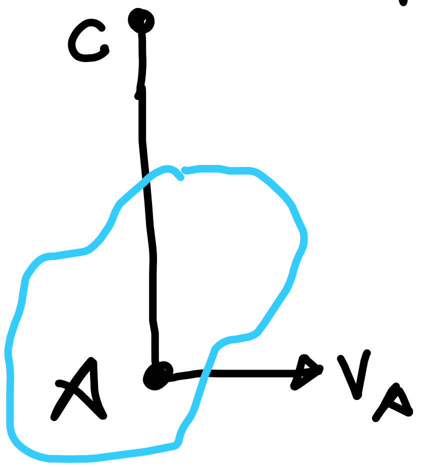

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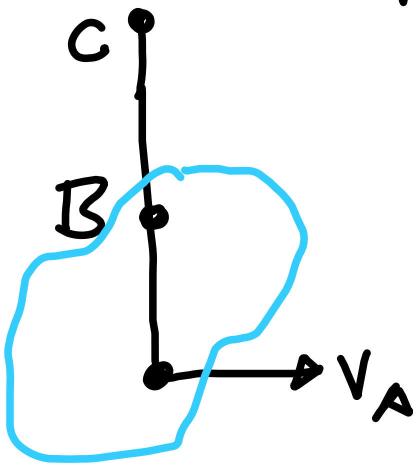

If point B is $\frac{1}{2}$ distance from C as is A

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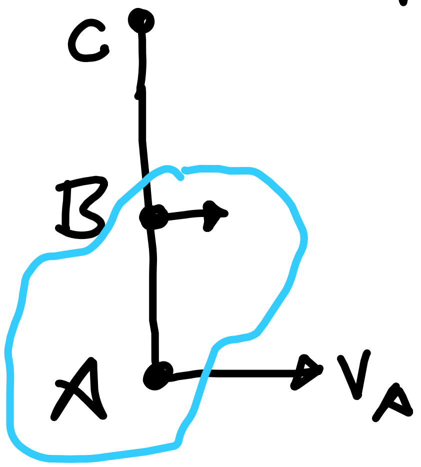

If point B is $\frac{1}{2}$ distance from C as is A, then V_B is $\frac{1}{2} V_A$.

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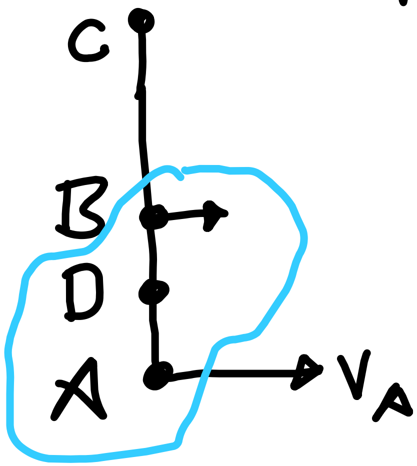

If point B is $\frac{1}{2}$ distance from C as is A, then V_B is $\frac{1}{2} V_A$.

If point D is 3/4 distance from C as A is

For every point on the rigid body (in plane motion that is rotating) $V_P = \omega r_{P/C}$

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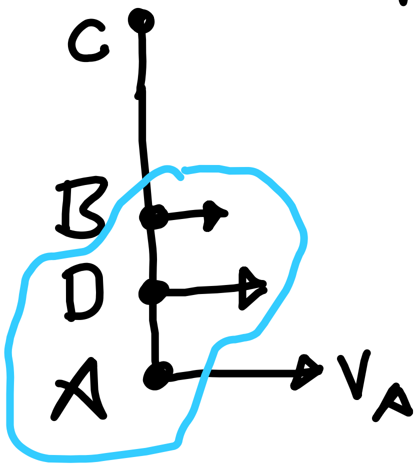

If point B is $\frac{1}{2}$ distance from C as is A, then V_B is $\frac{1}{2} V_A$.

If point D is $3/4$ distance from C as A is, then

$$V_D = (3/4) V_A$$

For every point on the rigid body (in plane motion that is rotating) $V_P = \omega r_{P/C}$

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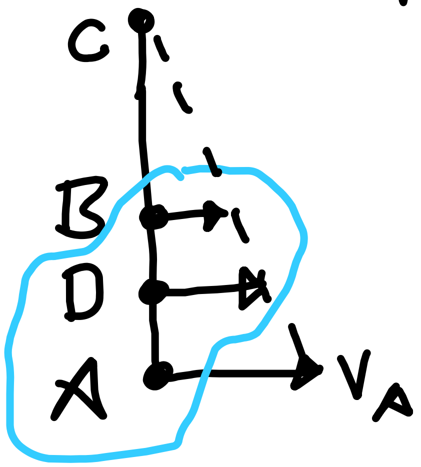

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If point D is $\frac{3}{4}$ distance from C as A is, then $V_D = (\frac{3}{4}) V_A$. And on & on

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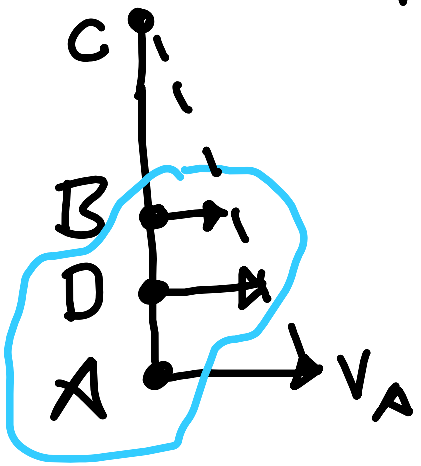

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This form similar triangles

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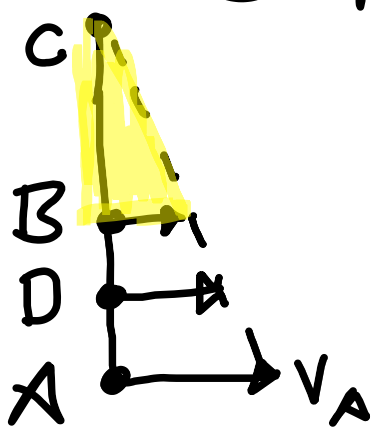

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$$\frac{V_B}{r_{B/C}} = \frac{V_D}{r_{D/C}} = \frac{V_A}{r_{A/C}}$$

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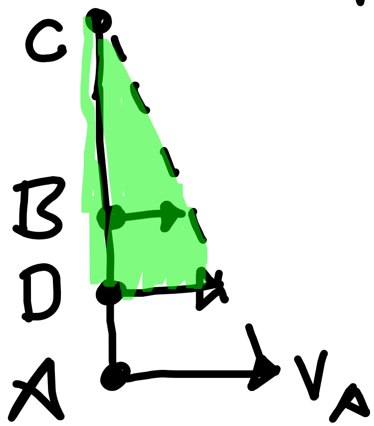

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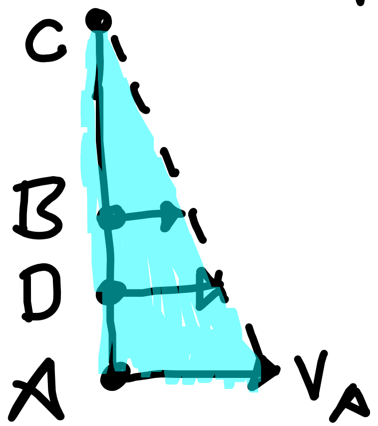

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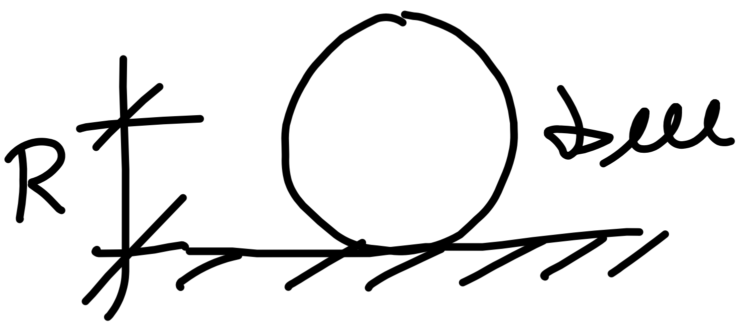
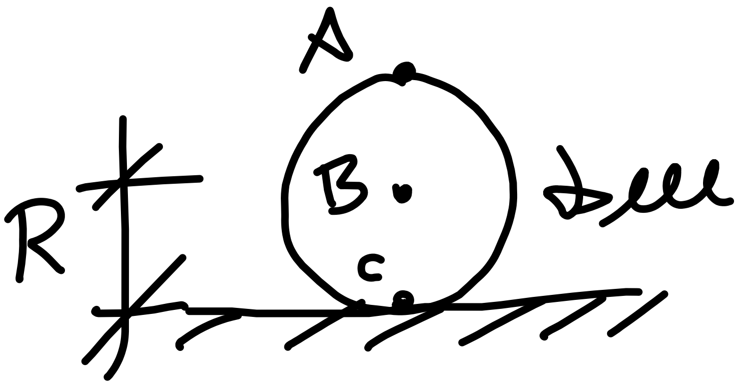
Wheel rolling w/no slipping



Wheel rolling w/no slipping

We know that

$V_A = 2V_B$ & $V_c = 0$.



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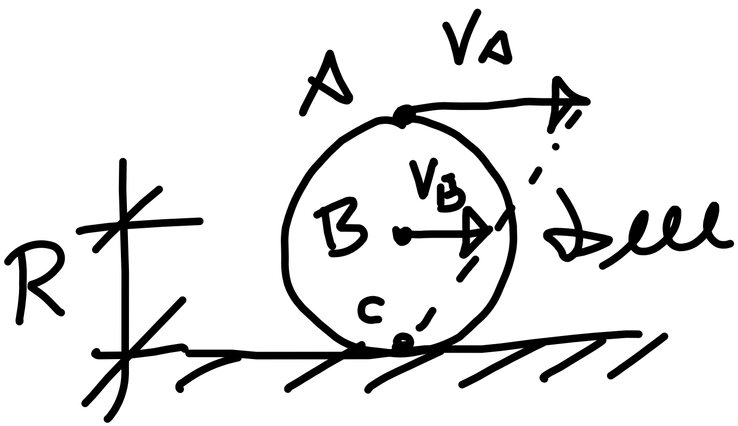

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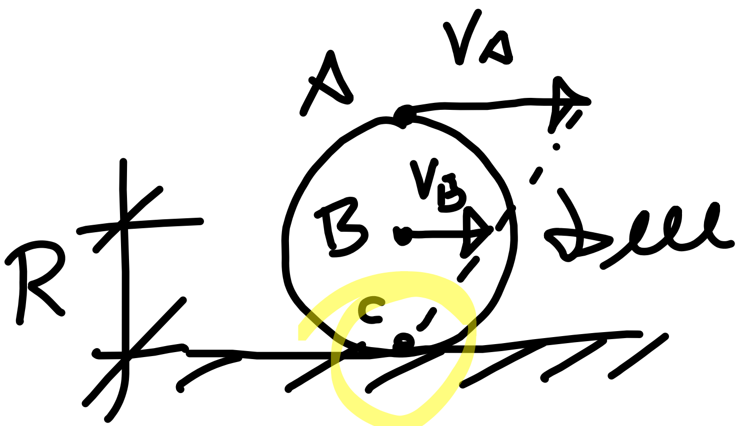

we know that

$V_A = 2V_B$ & $V_C = 0$.

In fact, we now know that point c is the instantaneous center of rotation

Relative motion for two points:

<u>Relative motion for two points</u>:

$$\vec{r}_{A/B} \equiv \vec{r}_A - \vec{r}_B$$

Relative motion for two points:

$$\vec{r}_{A/B} \equiv \vec{r}_A - \vec{r}_B \Rightarrow \vec{r}_A = \vec{r}_{A/B} + \vec{r}_B$$

ASU

Relative motion for two points:

$\vec{r}_{A/B} \equiv \vec{r}_A - \vec{r}_B \Rightarrow \vec{r}_A = \vec{r}_{A/B} + \vec{r}_B$ for a rigid body, the distance between A & B is fixed

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Acceleration

$\vec{a} = \frac{d\vec{v}}{dt}$

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Acceleration

$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow$

$\vec{a}_A = \vec{a}_{A/B} + \vec{a}_B$

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so $\vec{v}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}$

Acceleration

$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow$

$\vec{a}_A = \vec{a}_{A/B} + \vec{a}_B$, where

$$\vec{a}_{A/B} = \vec{\alpha} \times \vec{r}_{A/B} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{A/B}]$$

ASU

In the past we used $\vec{a} = a_n \hat{e}_n + a_t \hat{e}_t$ for circular motion

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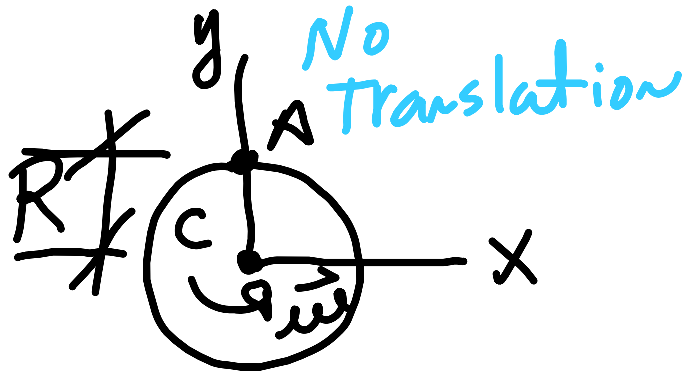

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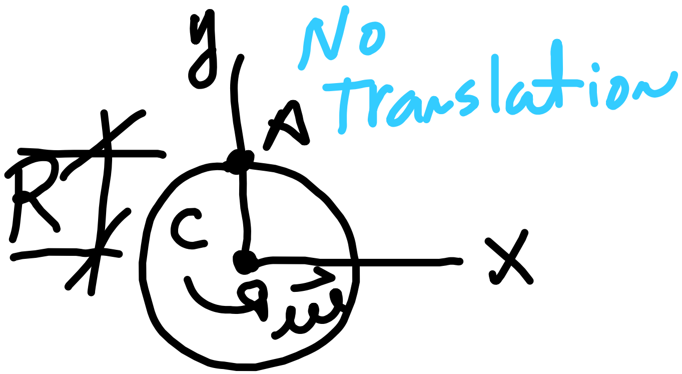
$\vec{\omega} = \omega \hat{z}$



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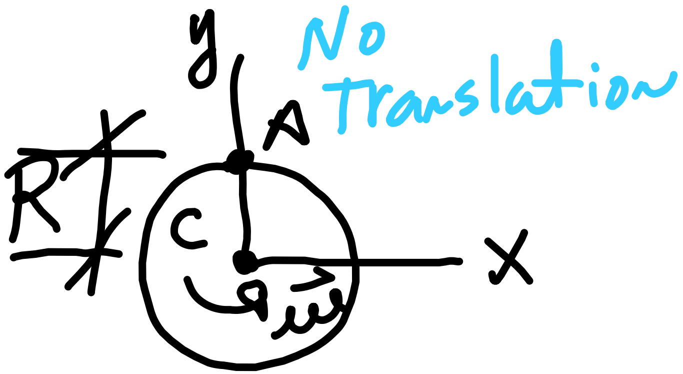
$\vec{\omega} = \omega \hat{z}$, with $\omega = const.$



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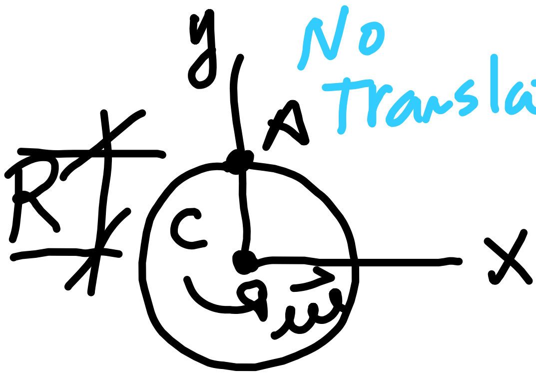
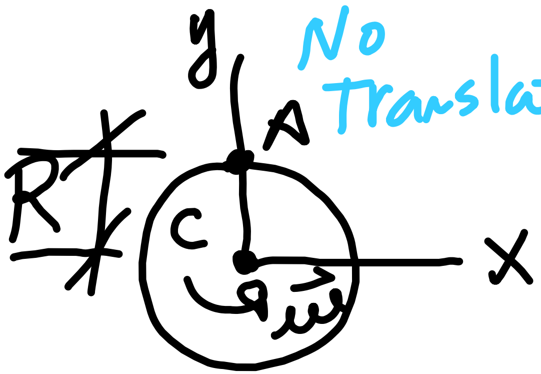


$\vec{\omega} = \omega \hat{z}$, with $\omega = const.$

$\vec{a}_A = a_n \hat{e}_n + a_t \hat{e}_t$, where

$a_n = \frac{V_A^2}{R}$

In the past we used $\vec{a} = a_n \hat{e}_n + a_t \hat{e}_t$ for circular motion. How does this relate to our new expression?



$\vec{\omega} = \omega \hat{z}$, with $\omega = \text{const.}$

$\vec{a}_A = a_n \hat{e}_n + a_t \hat{e}_t$, where

$a_n = \frac{V_A^2}{R}$ & $a_t = \frac{dV_A}{dt}$

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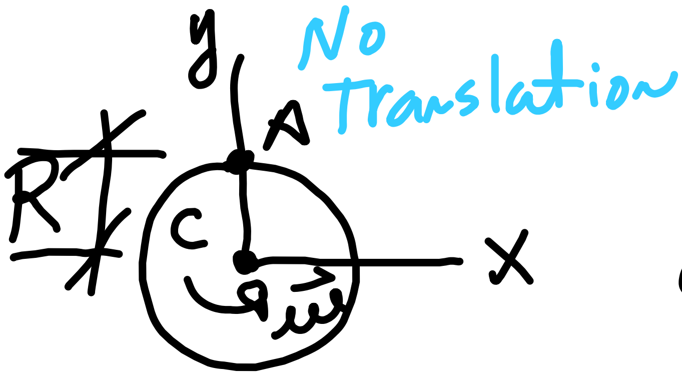


$\vec{\omega} = \omega\hat{z}$, with $\omega = \text{const.}$

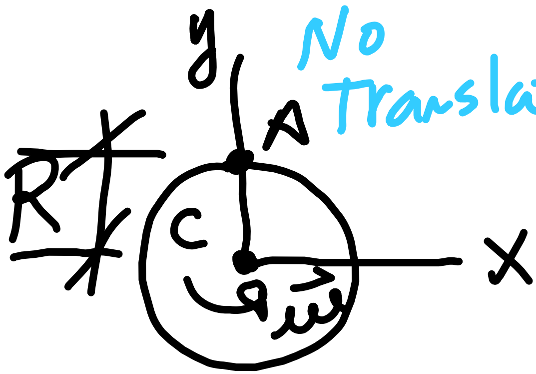
$\vec{a}_A = a_n\hat{e}_n + a_t\hat{e}_t$, where

$$a_n = \frac{v_A^2}{R} \quad \& \quad a_t = \frac{dv_A}{dt}$$

$\hat{e}_n$ points from A to C

In the past we used $\vec{a} = a_n \hat{e}_n + a_t \hat{e}_t$ for circular motion. How does this relate to our new expression?

No Translation



$\vec{\omega} = \omega \hat{z}$, with $\omega = \text{const.}$

$\vec{a}_A = a_n \hat{e}_n + a_t \hat{e}_t$, where

$$a_n = \frac{V_A^2}{R} \quad \& \quad a_t = \frac{dV_A}{dt}$$

$\hat{e}_n$ points from A to C and $\hat{e}_t$ is tangential to the path that A takes.

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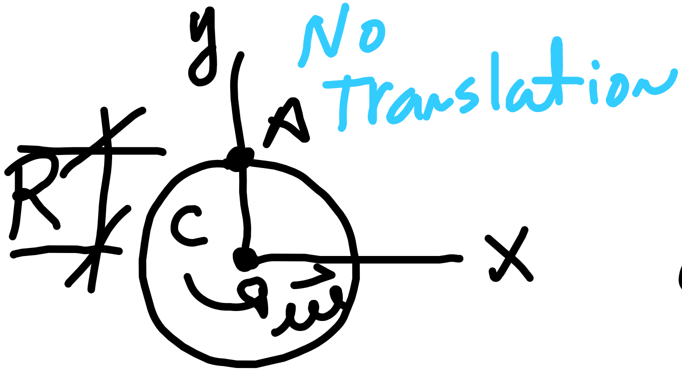

$\vec{\omega} = \omega\hat{z}$, with $\omega = \text{const.}$

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$a_n = \frac{v_A^2}{R}$ & $a_t = \frac{dv_A}{dt}$

$\hat{e}_n$ points from A to C and $\hat{e}_t$ is tangential to the path that A takes. For uniform circular motion $v_A = R\omega$

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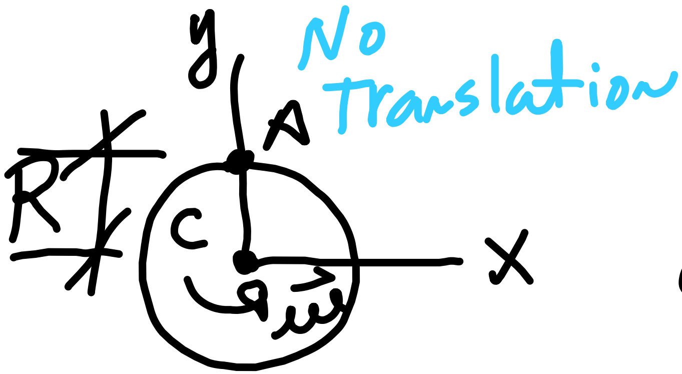

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$\hat{e}_N$ points from A to C and $\hat{e}_t$ is tangential to the path that A takes. For uniform circular motion $V_A = R\omega$ & $\frac{dV_A}{dt} = 0$

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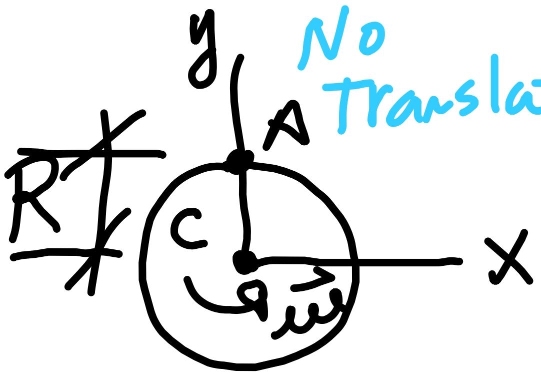


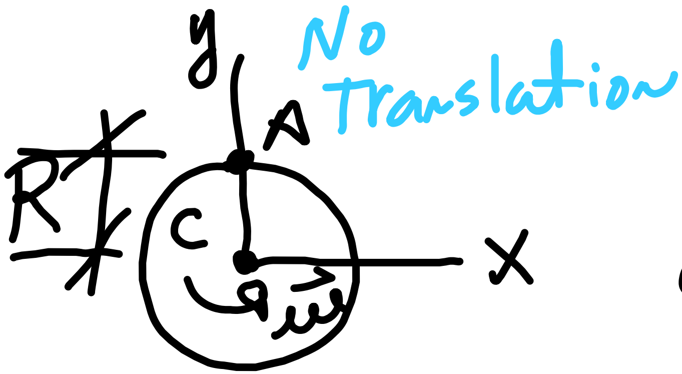
$\vec{\omega} = \omega \hat{z}$, with $\omega = \text{const.}$

$\vec{a}_N = a_N \hat{e}_N + a_t \hat{e}_t$, where

$$a_N = \frac{V_A^2}{R} \quad \& \quad a_t = \frac{dV_A}{dt}$$

$\hat{e}_N$ points from A to C and $\hat{e}_t$ is tangential to the path that A takes. For uniform circular motion $V_A = R\omega$ & $\frac{dV_A}{dt} = 0$ so $a_N = R\omega^2$

In the past we used $\vec{a} = a_n \hat{e}_n + a_t \hat{e}_t$ for circular motion. How does this relate to our new expression?



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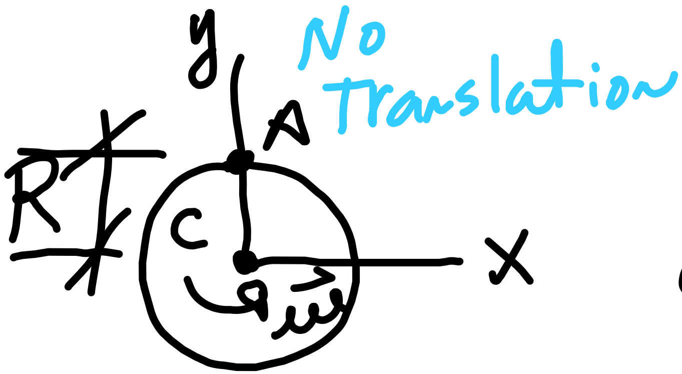
$\vec{a}_A = a_n \hat{e}_n + a_t \hat{e}_t$, where

$$a_n = \frac{V_A^2}{R} \;\; \& \;\; a_t = \frac{dV_A}{dt}$$

$\hat{e}_n$ points from A to C and $\hat{e}_t$ is tangential to the path that A takes. For uniform circular motion $V_A = R\omega$ & $\frac{dV_A}{dt} = 0$ so

$a_n = R\omega^2$ & $a_t = 0$

In the past we used $\vec{a} = a_n \hat{e}_n + a_t \hat{e}_t$ for circular motion. How does this relate to our new expression?



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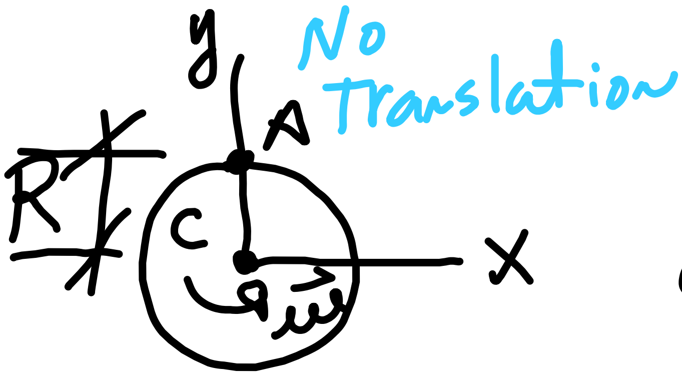


$\vec{\omega} = \omega \hat{z}$, with $\omega = \text{const.}$

$\vec{a}_N = a_N \hat{e}_N + a_t \hat{e}_t$, where

$$a_N = \frac{V_A^2}{R} \;\&\; a_t = \frac{dV_A}{dt}$$

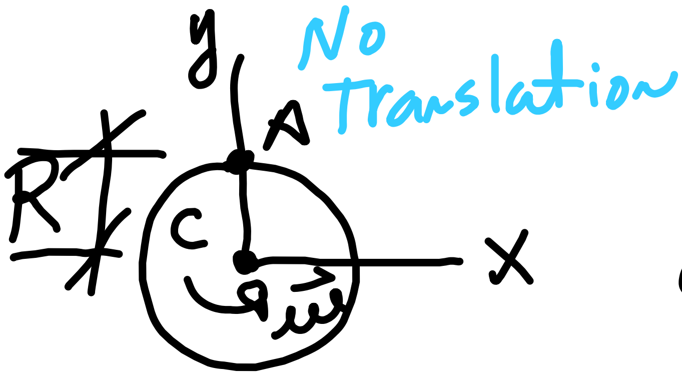
$\hat{e}_N$ points from A to C and $\hat{e}_t$ is tangential to the path that A takes. For uniform circular motion $V_A = R\omega$ & $\frac{dV_A}{dt} = 0$ so

$a_N = R\omega^2$ & $a_t = 0$ & $\hat{e}_N = -\hat{y}$ $\Rightarrow$

$\vec{a}_{A/C} = \vec{a}_A$

ASU

In the past we used $\vec{a} = a_n \hat{e}_n + a_t \hat{e}_t$ for circular motion. How does this relate to our new expression?



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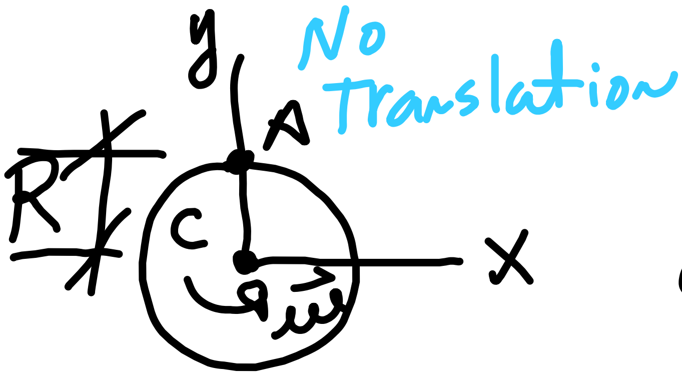
$a_n = R\omega^2$ & $a_t = 0$ & $\hat{e}_n = -\hat{y}$ ⇒

$$\vec{a}_{A/C} = \vec{a}_A = R\omega^2(-\hat{y})$$

In the past we used $\vec{a} = a_n\hat{e}_n + a_t\hat{e}_t$ for circular motion. How does this relate to our new expression?



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$\vec{a}_{A/C} = \vec{a}_A = R\omega^2(-\hat{y})$ If $\vec{\omega} \neq \text{const.}$

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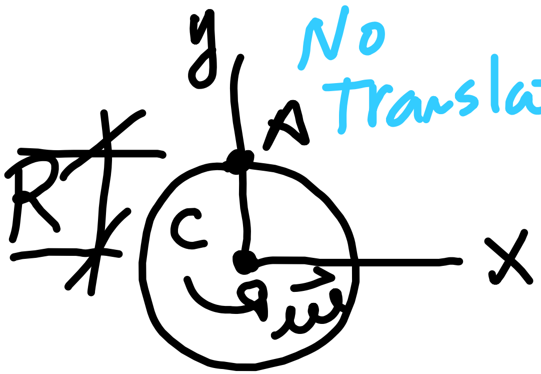

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$\vec{a}_{A/C} = \vec{a}_A = R\omega^2(-\hat{y})$ If $\vec{\omega} \neq \text{const.}$ we get

$$\frac{dV_A}{dt} = R\frac{d\omega}{dt}$$

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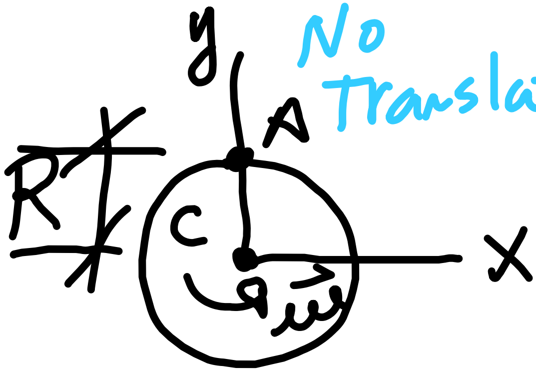

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$$\frac{dV_A}{dt} = R\frac{d\omega}{dt} = R\alpha$$

ASU

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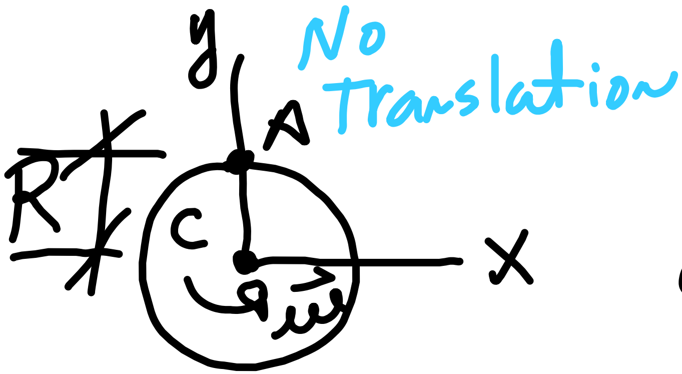

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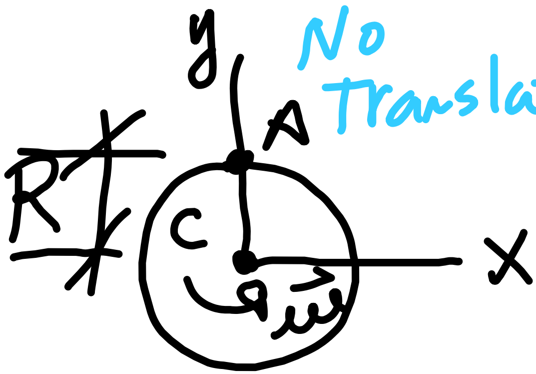

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$\frac{dV_A}{dt} = R\frac{d\omega}{dt} = R\alpha$ and $\hat{e}_t = -\hat{x}$ ⇒ $\vec{a}_{A/C} = \alpha R(-\hat{x}) +$

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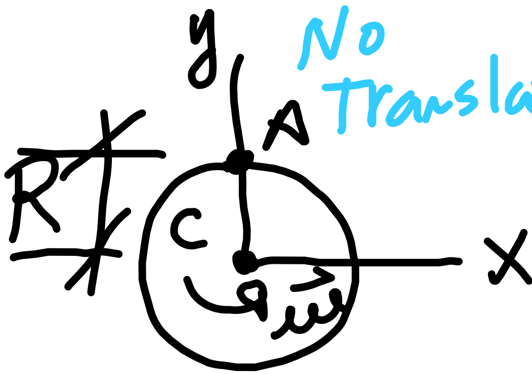

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$\vec{a}_{A/C} = \vec{a}_A = R\omega^2(-\hat{y})$ If $\underline{\underline{\vec{\omega} \neq \text{const.}}}$ we get

$\frac{dV_A}{dt} = R\frac{d\omega}{dt} = R\alpha$ and $\hat{e}_t = -\hat{x} \Rightarrow \vec{a}_{A/C} = \alpha R(-\hat{x}) + R\omega^2(-\hat{y})$

Our expression $\underline{\vec{a}_{A/C} = \alpha R(-\hat{x}) + R\omega^2(-\hat{y})}$

Our expression $\vec{a}_{A/C} = \alpha R(-\hat{x}) + R\omega^2(-\hat{y})$ has to match what we would get using

$$\vec{a}_{A/C} = \vec{\alpha} \times \vec{r}_{A/C} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{A/C}]$$

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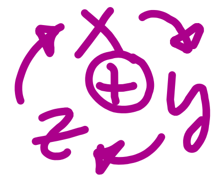

$$\vec{a}_{A/C} = \alpha R\,\hat{z}\times\hat{y} + \omega^2 R\,\hat{z}\times[\hat{z}\times\hat{y}]$$

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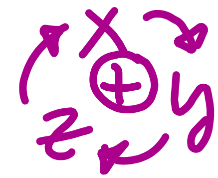

$$\vec{a}_{A/C} = \alpha R \, \hat{z} \times \hat{y} + \omega^2 R \, \hat{z} \times [\hat{z} \times \hat{y}]$$

But $\hat{z} \times \hat{y} = -\hat{x}$

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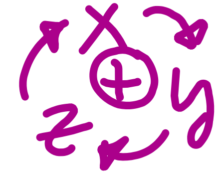

$$\vec{a}_{A/C} = \alpha R\, \hat{z} \times \hat{y} + \omega^2 R\, \hat{z} \times [\hat{z} \times \hat{y}]$$

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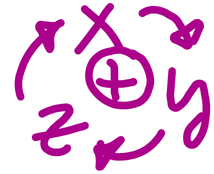

$$\vec{a}_{A/C} = \alpha R \, \hat{z} \times \hat{y} + \omega^2 R \, \hat{z} \times [\hat{z} \times \hat{y}]$$

But $\hat{z} \times \hat{y} = -\hat{x}$ & $\hat{z} \times (-\hat{x}) = -\hat{y}$ so

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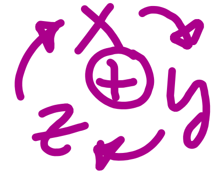

$$\vec{a}_{A/C} = \alpha R \, \hat{z} \times \hat{y} + \omega^2 R \, \hat{z} \times [\hat{z} \times \hat{y}]$$

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$$\vec{a}_{A/C} = \alpha R(-\hat{x}) + \omega^2 R \, \hat{z} \times (-\hat{x}) \Rightarrow$$

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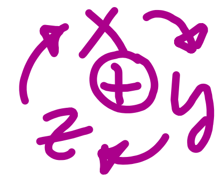


$$\vec{a}_{A/C} = \alpha R\,\hat{z}\times\hat{y} + \omega^2 R\,\hat{z}\times[\hat{z}\times\hat{y}]$$

But $\hat{z}\times\hat{y} = -\hat{x}$ & $\hat{z}\times(-\hat{x}) = -\hat{y}$ so

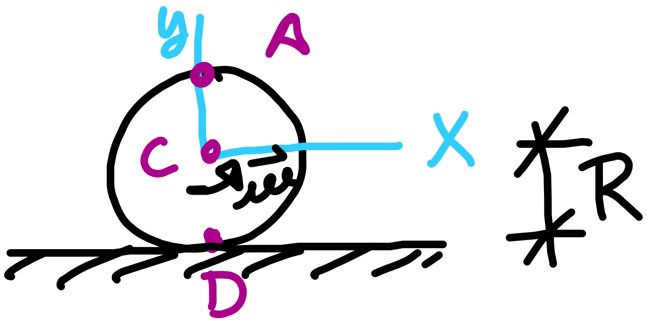
$$\vec{a}_{A/C} = \alpha R(-\hat{x}) + \omega^2 R\,\hat{z}\times(-\hat{x}) \Rightarrow$$

$$\vec{a}_{A/C} = \alpha R(-\hat{x}) + \omega^2 R(-\hat{y})$$

matches previous expression

Example: Wheel rolling w/no slip

Example: Wheel rolling w/no slip



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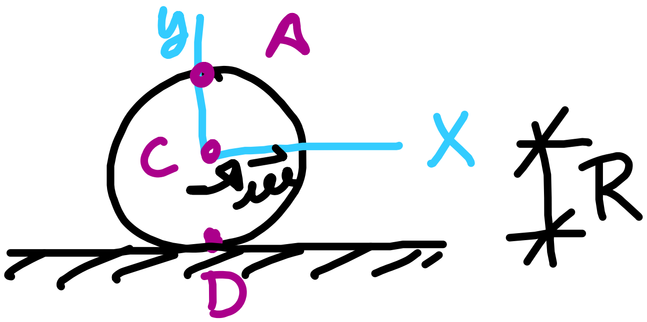
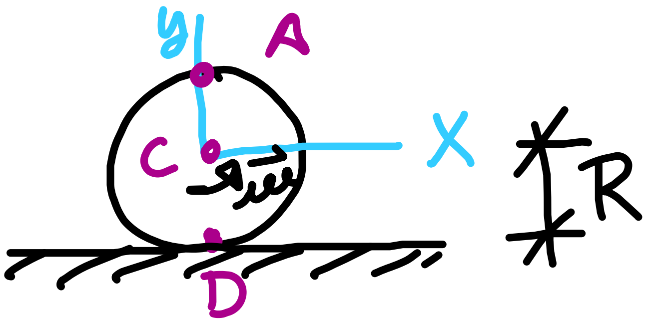


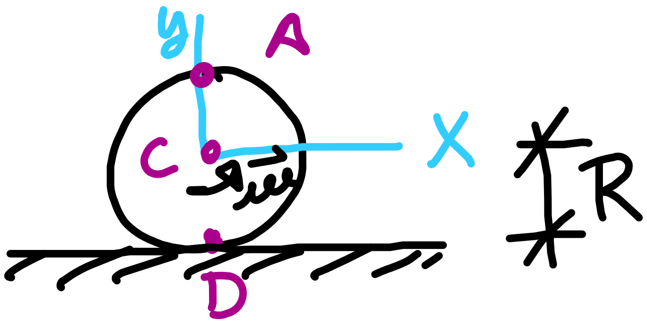
$$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$$

Example: Wheel rolling w/no slip



$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$ **If** $\vec{v}_C = \text{const.}$

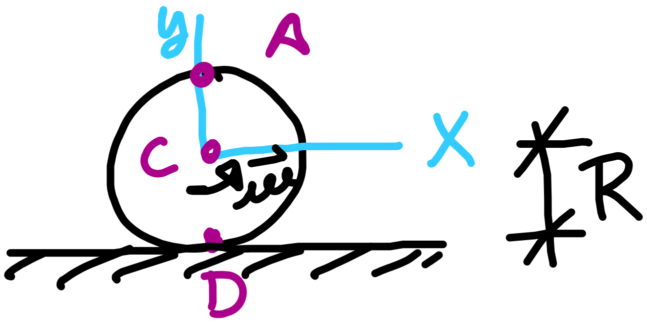
Example: Wheel rolling w/no slip



$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$ If $\vec{v}_c = \text{const.}$

then $\vec{a}_c = 0$

Example: Wheel rolling w/no slip



$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$ **If** $\vec{v}_C = \text{const.}$

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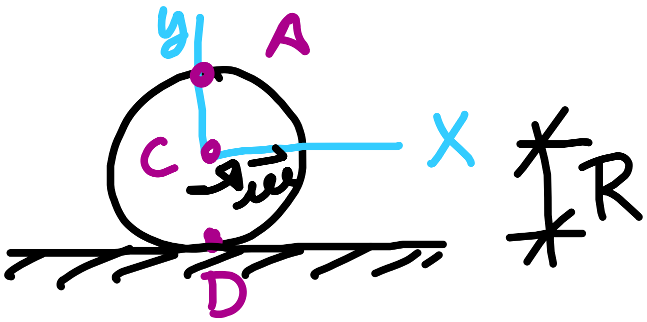
Example: Wheel rolling w/no slip

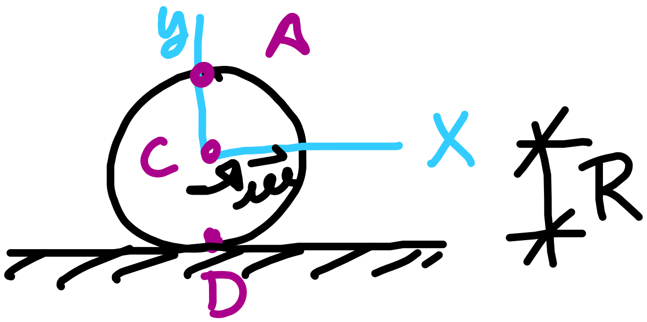


$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$ **If** $\vec{v}_C = const.$

then $\vec{a}_C = 0$ & $\vec{\alpha} = 0$

& $v_C = R\omega$

Example: Wheel rolling w/no slip

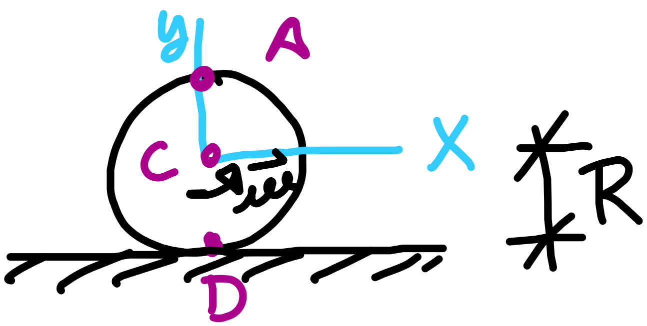


$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$ **If** $\vec{v}_C = \text{const.}$

then $\vec{a}_C = 0$ & $\vec{\alpha} = 0$

& $v_C = R\omega$ **If** $v_C \neq \text{const.}$

Example: Wheel rolling w/no slip

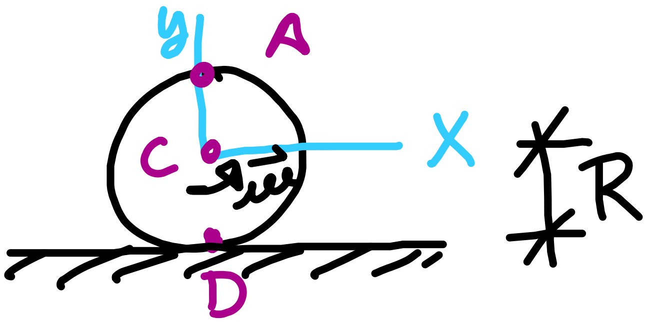


$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$

If $\vec{v}_C = \text{const.}$ then $\vec{a}_C = \emptyset$ & $\vec{\alpha} = \emptyset$ & $v_C = R\omega$

If $v_C \neq \text{const.}$ then $\vec{\alpha} \neq \emptyset$

Example: Wheel rolling w/no slip



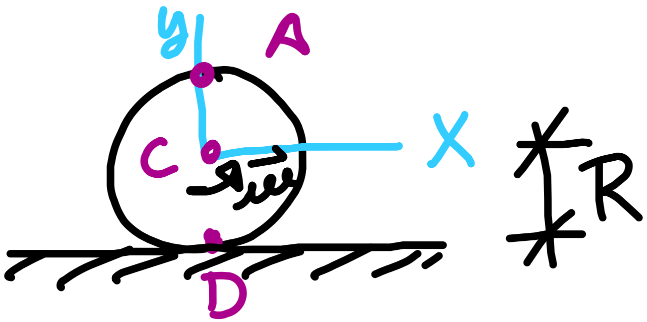
$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$ **If** $\vec{v}_C = \text{const.}$

then $\vec{a}_C = 0$ & $\vec{\alpha} = 0$

& $v_C = R\omega$ **If** $v_C \neq \text{const.}$

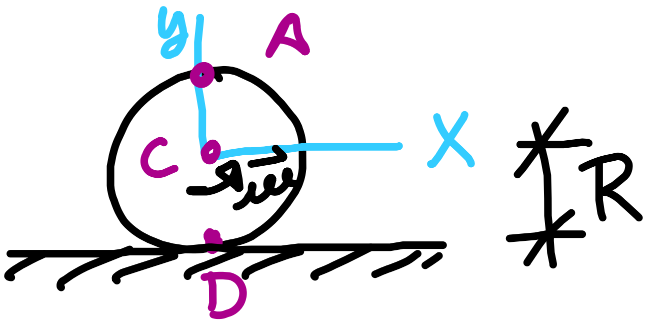
then $\vec{\alpha} \neq 0$ & $a_C \neq 0$

Example: Wheel rolling w/no slip



$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$ **If** $\vec{v}_C = \text{const.}$ then $\vec{a}_C = 0$ & $\vec{\alpha} = 0$ & $v_C = R\omega$ **If** $v_C \neq \text{const.}$ then $\vec{\alpha} \neq 0$ & $a_C \neq 0 \Rightarrow \vec{a}_A = \alpha R(-\hat{x}) + \omega^2 R(-\hat{y}) + \vec{a}_C$

Example: Wheel rolling w/no slip



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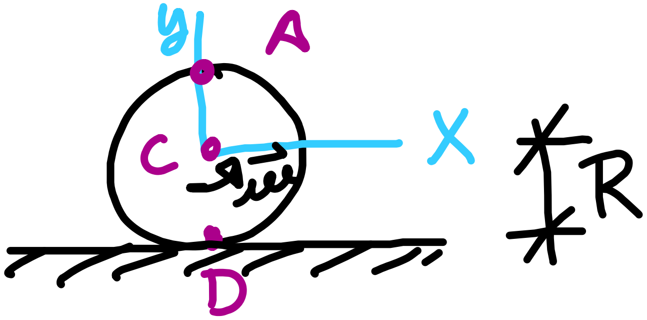
Example: Flywheel on shaft rolling on rail

Example: Wheel rolling w/no slip



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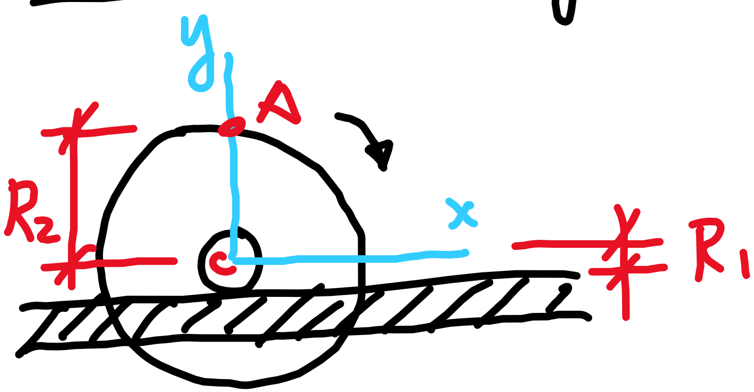
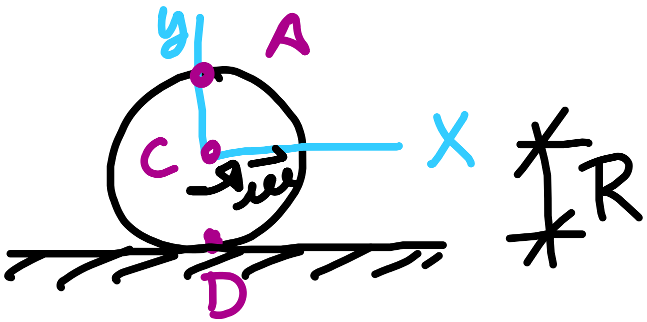


Example: Wheel rolling w/no slip



$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$ **If** $\vec{v}_C = const.$

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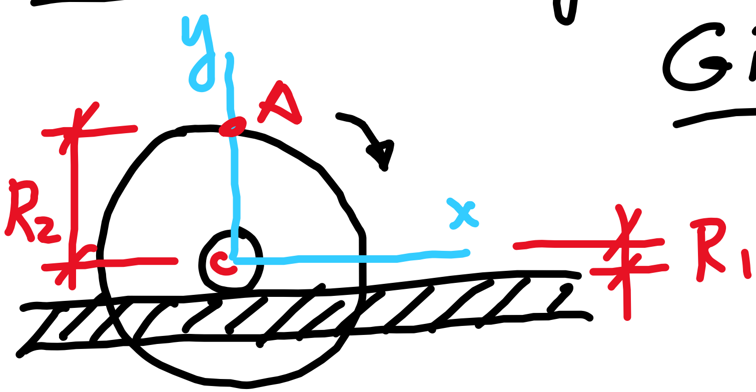
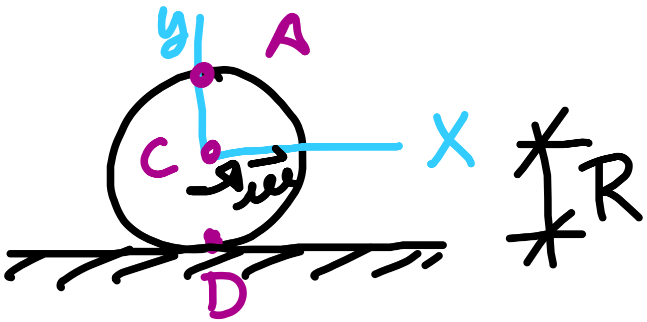


Given v_C, a_C, R_1 & R_2

Example: Wheel rolling w/no slip



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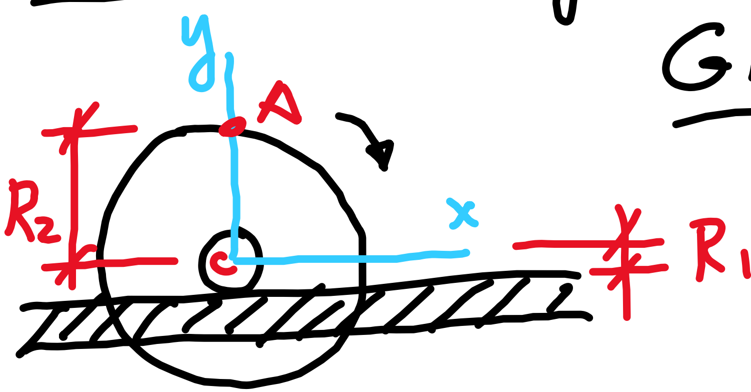
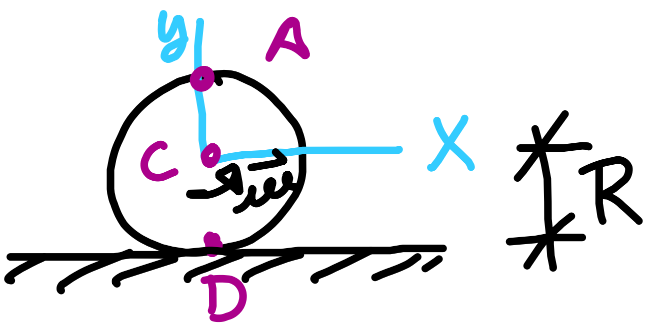
Example: Flywheel on shaft rolling on rail



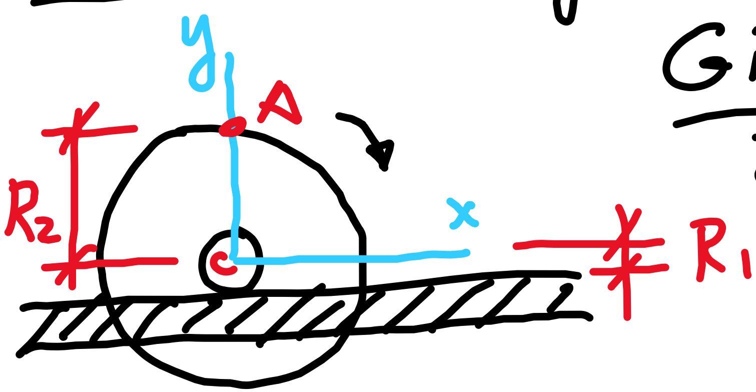
Given v_C, a_C, R_1 & R_2 Find $\vec{a}_A$:

Example: Wheel rolling w/no slip



$\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C$ If $\vec{v}_C = const.$ then $\vec{a}_C = 0$ & $\vec{\alpha} = 0$ & $v_C = R\omega$ If $v_C \neq const.$ then $\vec{\alpha} \neq 0$ & $a_C \neq 0 \Rightarrow \vec{a}_A = \alpha R(-\hat{x}) + \omega^2 R(-\hat{y}) + \vec{a}_C$

Example: Flywheel on shaft rolling on rail

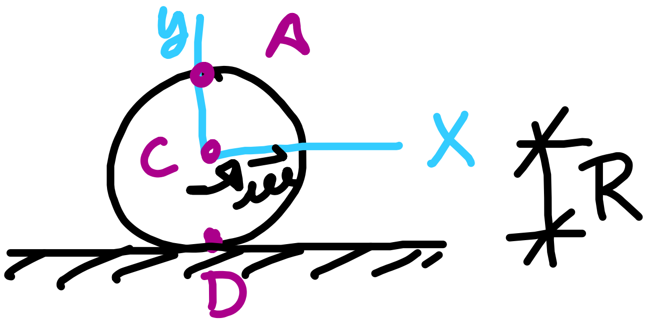


Given v_C, a_C, R_1 & R_2 Find $\vec{a}_A$:

$\vec{a}_{A/C} = \vec{\alpha} \times \vec{r}_{A/C} + \vec{\omega} \times [\vec{\omega} \times \vec{r}_{A/C}]$

Need to obtain $\vec{\alpha}, \vec{r}_{A/C}$ & $\vec{\omega}$ from v_C, a_C, R_1 & R_2

Example: Wheel rolling w/no slip



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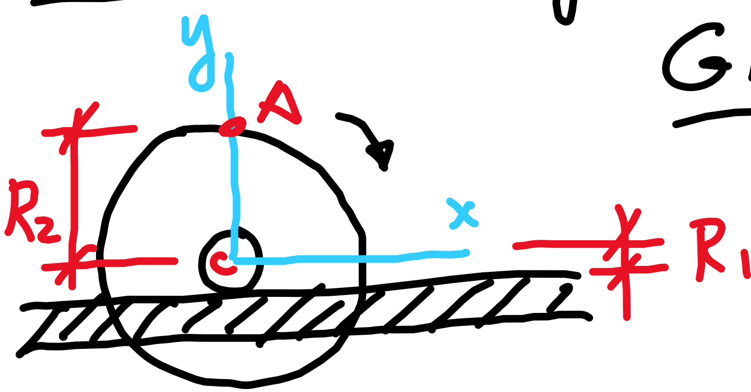

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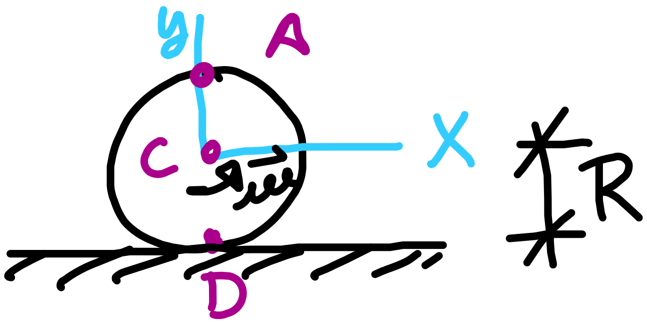

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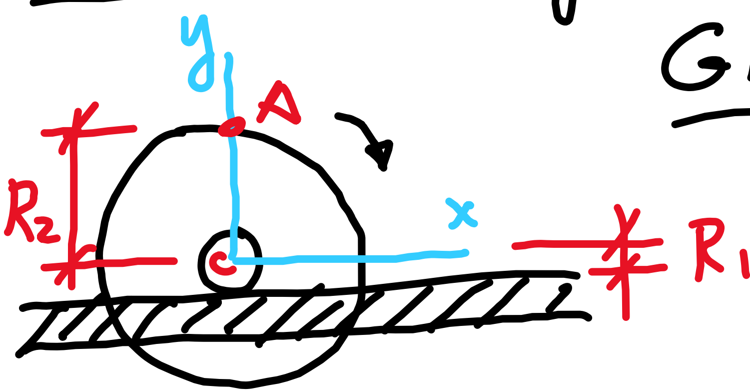

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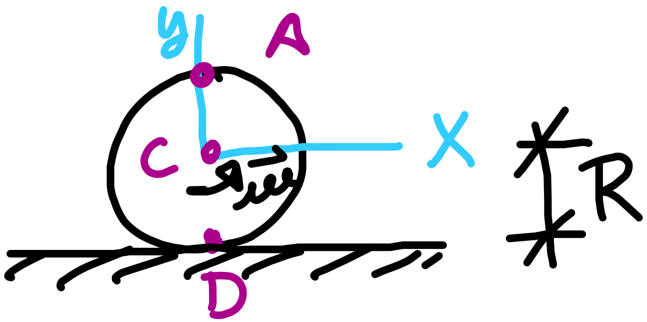

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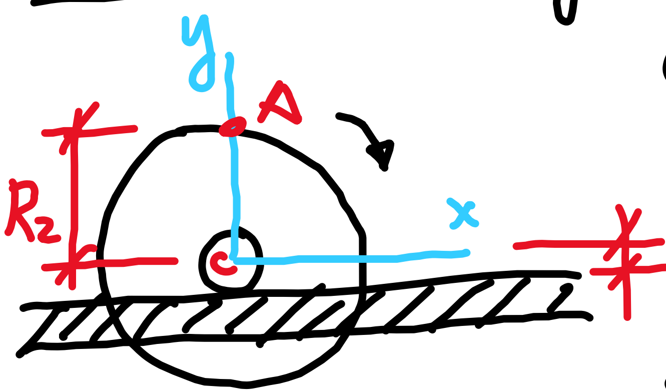

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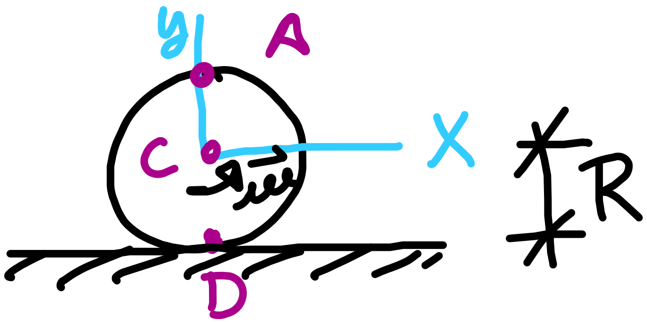

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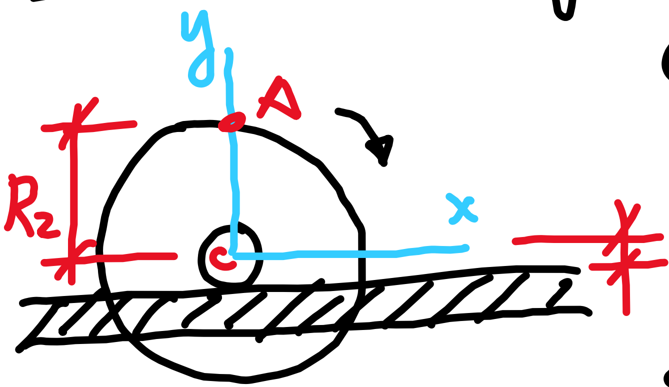

Given v_C, a_C, R_1 & R_2 Find $\vec{a}_A$:

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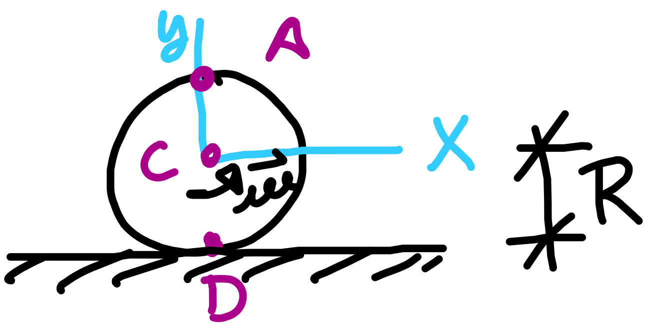

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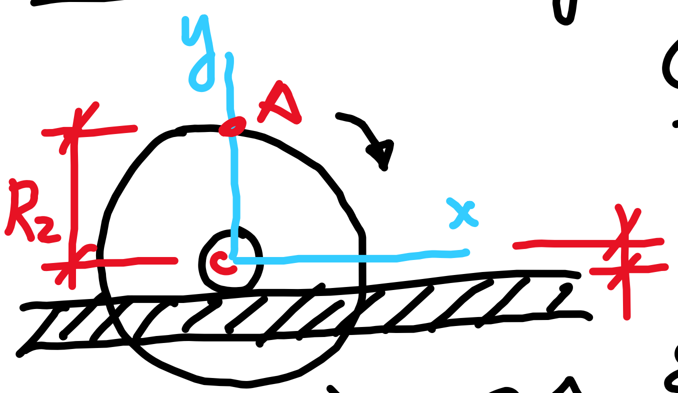

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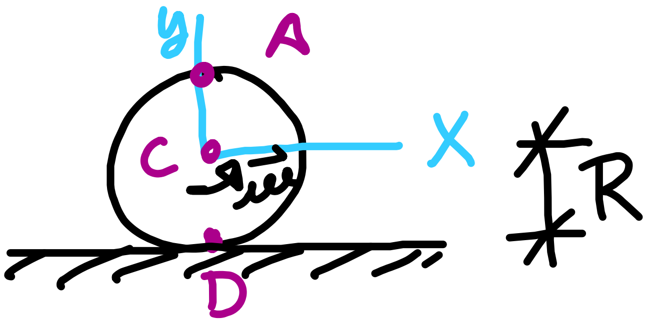
$\vec{v}_C = \omega R_1 \hat{x}$ so $\vec{\omega} = (v_C/R)(-\hat{z})$

& $\vec{a}_C = \alpha R_1 \hat{x}$ so $\vec{\alpha} = (a_C/R)(-\hat{z})$

Also $\vec{r}_{A/C} = R_1 \hat{y}$

Example: Wheel rolling w/no slip



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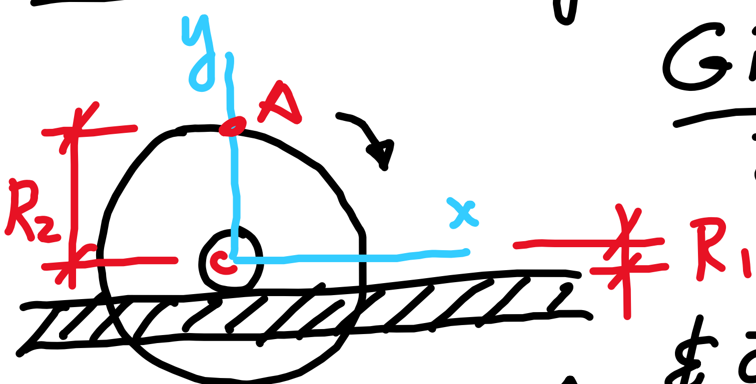
Example: Flywheel on shaft rolling on rail



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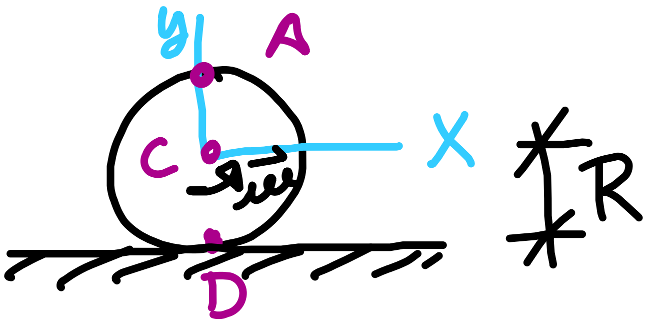
ASU

Example: Wheel rolling w/no slip



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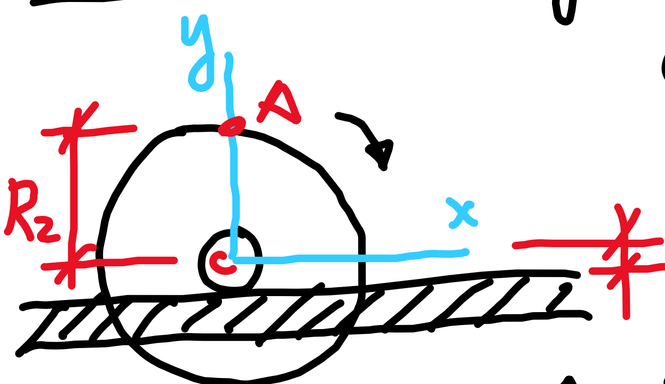

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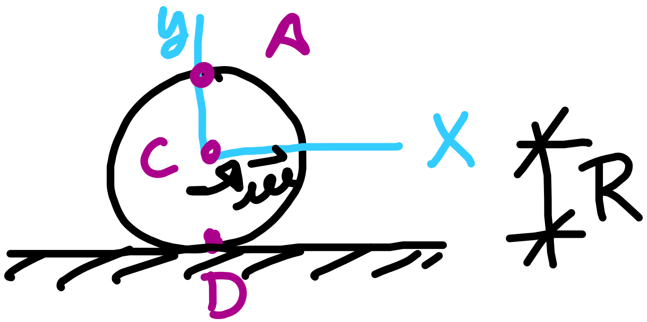
& $\vec{a}_C = \alpha R_1 \hat{x}$ so $\vec{\alpha} = (a_C/R)(-\hat{z})$

Also $\vec{r}_{A/C} = R_1 \hat{y}$ so $\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C = \frac{a_C}{R_1} R_2 (-\hat{z}) \times \hat{y} +$

$\frac{v_C^2}{R_1^2} R_2 \hat{z} \times [\hat{z} \times \hat{y}] + \vec{a}_C$

Example: Wheel rolling w/no slip



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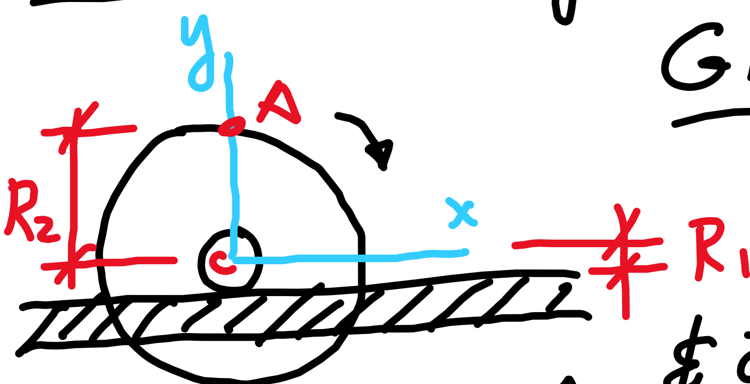
Example: Flywheel on shaft rolling on rail



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& $\vec{a}_C = \alpha R_1 \hat{x}$ so $\vec{\alpha} = (a_C/R)(-\hat{z})$

Also $\vec{r}_{A/C} = R_1 \hat{y}$ so $\vec{a}_A = \vec{a}_{A/C} + \vec{a}_C = \frac{a_C}{R_1} R_2 (-\hat{z}) \times \hat{y} +$

$\frac{v_C^2}{R_1^2} R_2 \hat{z} \times [\hat{z} \times \hat{y}] + \vec{a}_C = \left(\frac{a_C R_2}{R_1}\right)\hat{x} + a_C \hat{x} + \left(\frac{v_C^2}{R_1^2}\right)(R_2)(-\hat{y})$

Note: In this case $\vec{a}_c = a_c \hat{x}$

<u>Note</u>: In this case $\vec{a}_c = a_c \hat{x}$ If rail was tilted some angle θ with respect to horizontal

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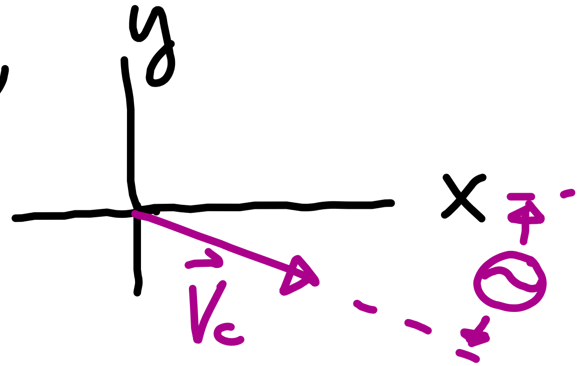
Then



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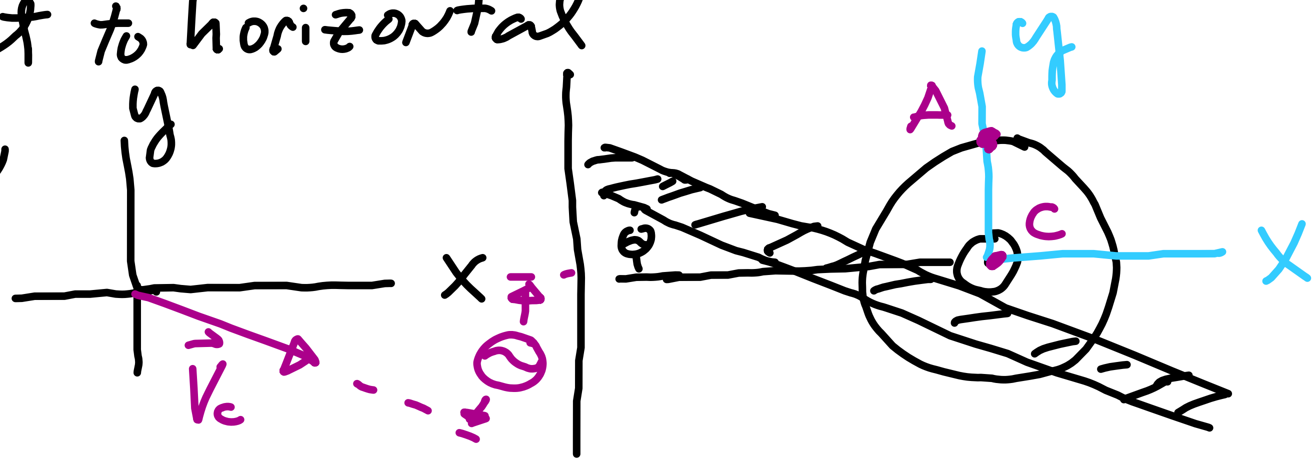

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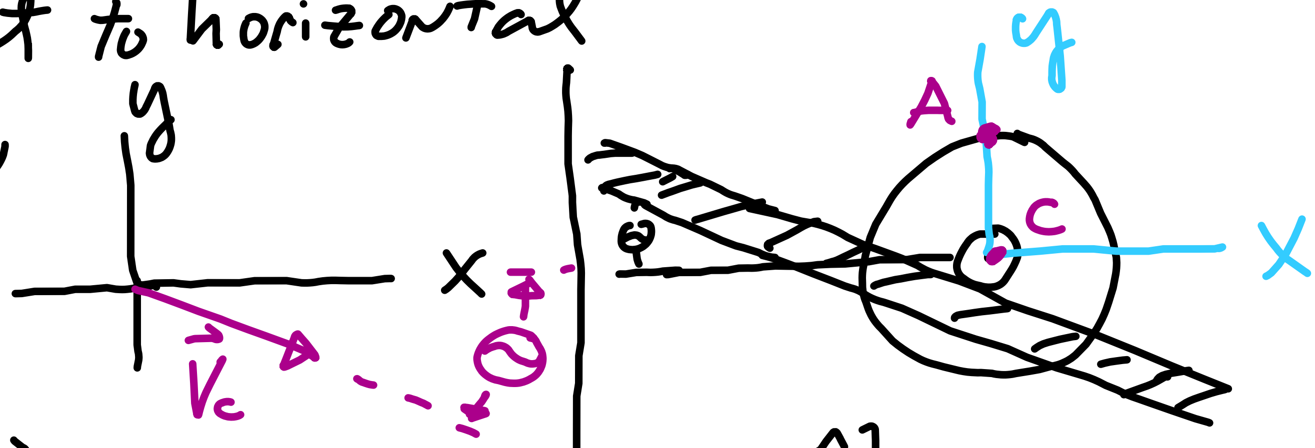

& $\vec{V}_c = V_c[\cos\theta \hat{x} - \sin\theta \hat{y}]$

Note: In this case $\vec{a}_c = a_c \hat{x}$ If rail was tilted some angle θ with respect to horizontal

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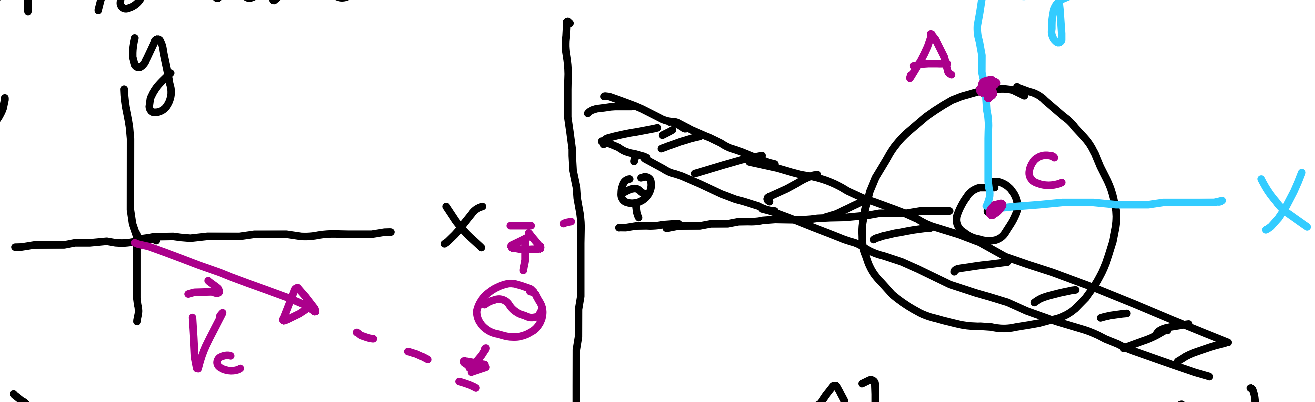

& $\vec{V}_c = V_c[\cos\theta\,\hat{x} - \sin\theta\,\hat{y}]$ Just have to be consistent with the sign convention

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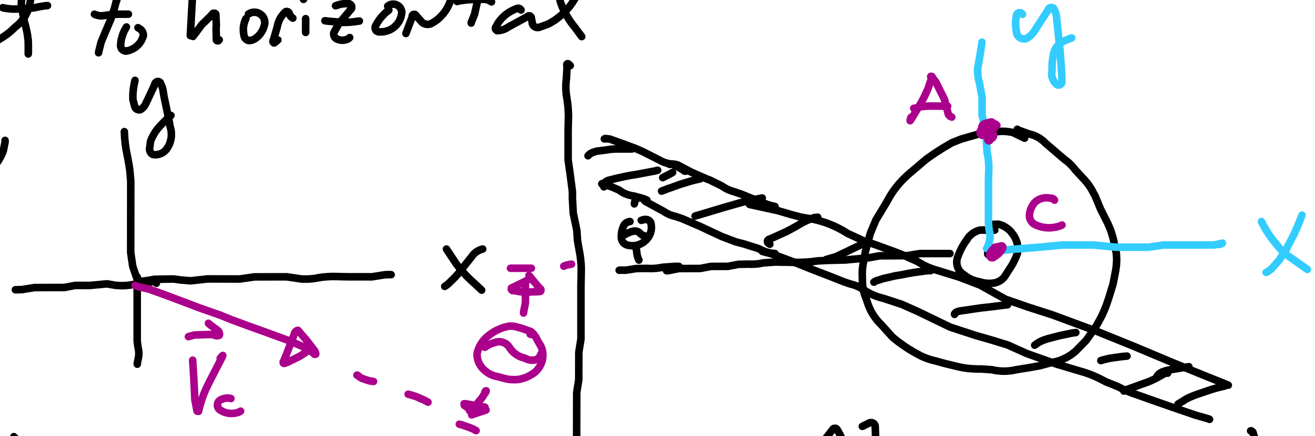
Then

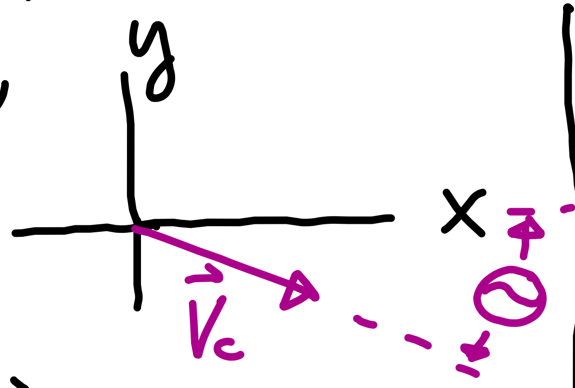


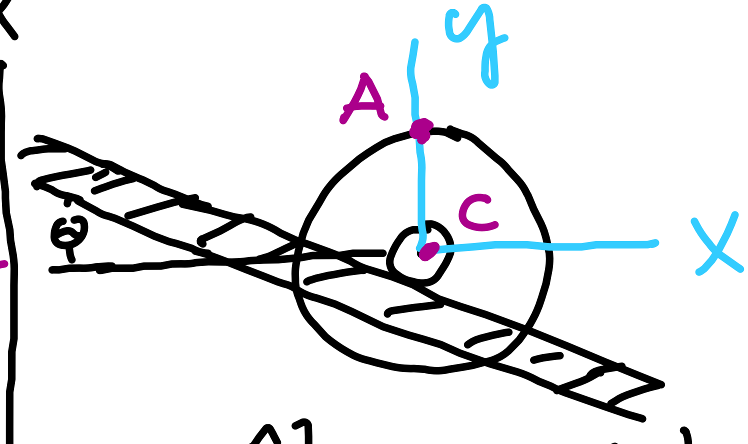
& $\vec{V}_c = V_c[\cos\theta\,\hat{x} - \sin\theta\,\hat{y}]$ Just have to be consistent with the sign convention

Also [Keeping $\theta > 0$] $\vec{a}_c = a_c\cos\theta\,\hat{x} - a_c\sin\theta\,\hat{y}$

Note: In this case $\vec{a}_c = a_c \hat{x}$ If rail was tilted some angle θ with respect to horizontal

Then





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Also [Keeping $\theta > 0$] $\vec{a}_c = a_c \cos\theta \hat{x} - a_c \sin\theta \hat{y}$

Now

$$\vec{a}_A = \left\{ \left[\frac{a_c R_2}{R_1} \right] \hat{x} + a_c \cos\theta \hat{x} \right\} + \left\{ \left[\frac{V_c^2}{R_1^2} R_2 \right] (-\hat{y}) + a_c \sin\theta \hat{y} \right\}$$