Today 16.2

$\boxed{L17}$

Today 16.2 L17

constrained motion

Today 16.2

Thursday 17.1

$\boxed{L17}$

Today 16.2

$\boxed{L17}$

Thursday 17.1

Energy methods for rigid bodies

$\bar{k} \equiv$ Radius of gyration

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<u>Parallel axis theorem</u>

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Before we determine the moment of inertia about points other than G, it will be useful to take another look at the center-of-mass for continuos mass distributions

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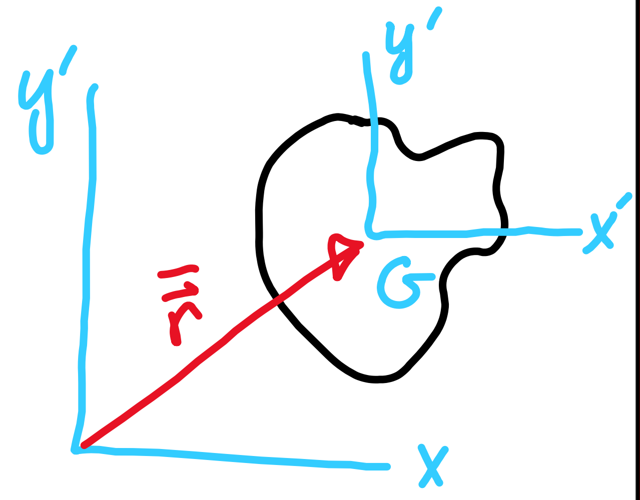
If mass is uniform over area A



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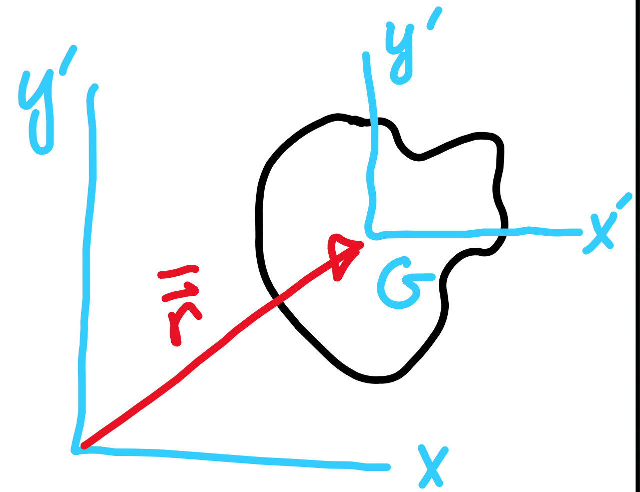

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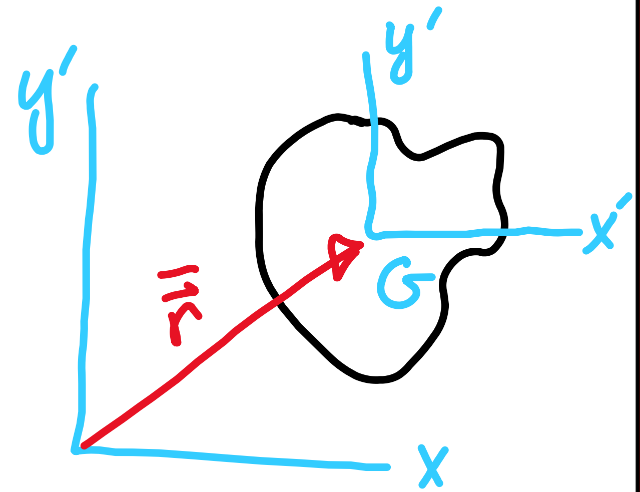

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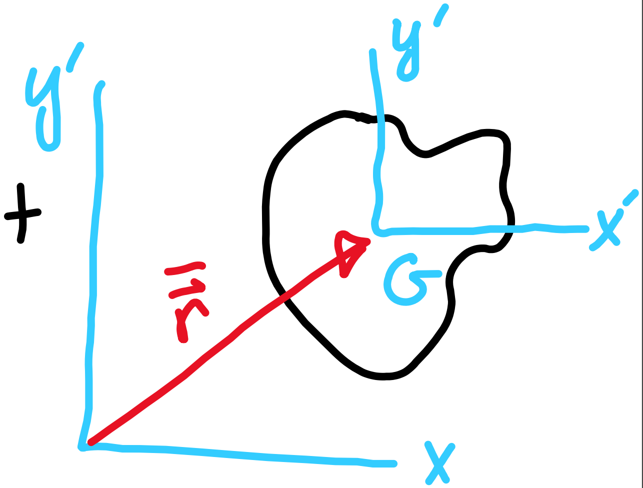

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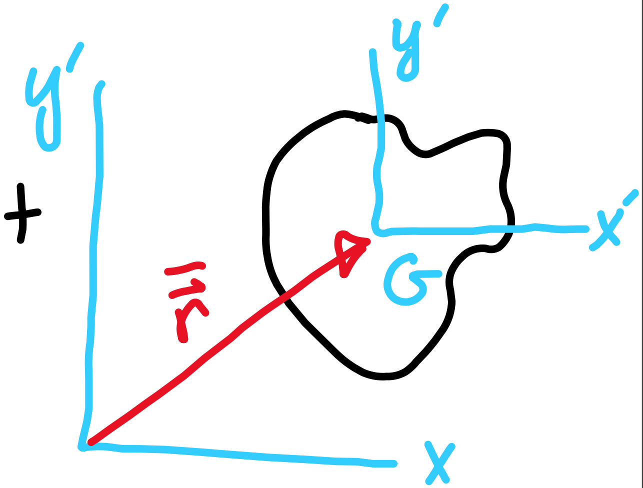

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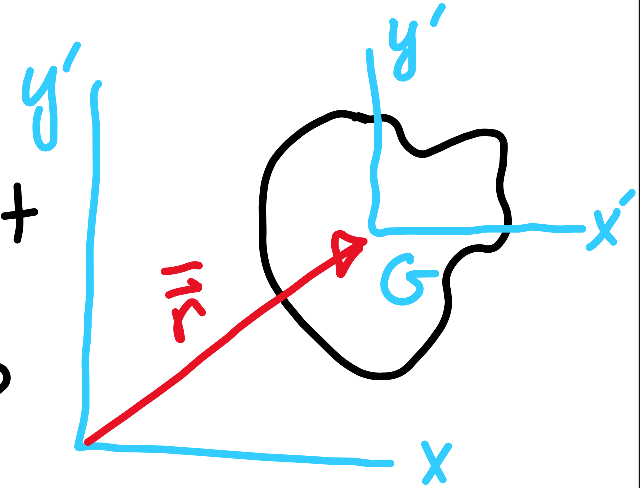

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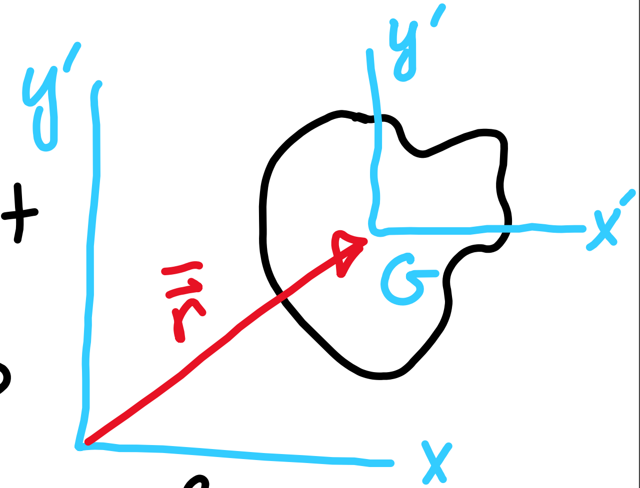

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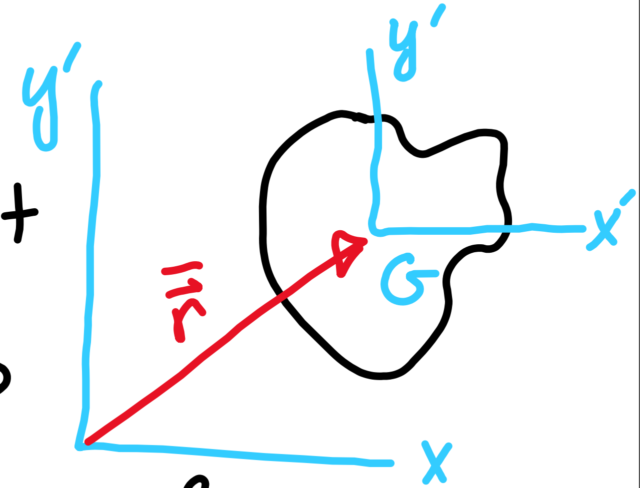

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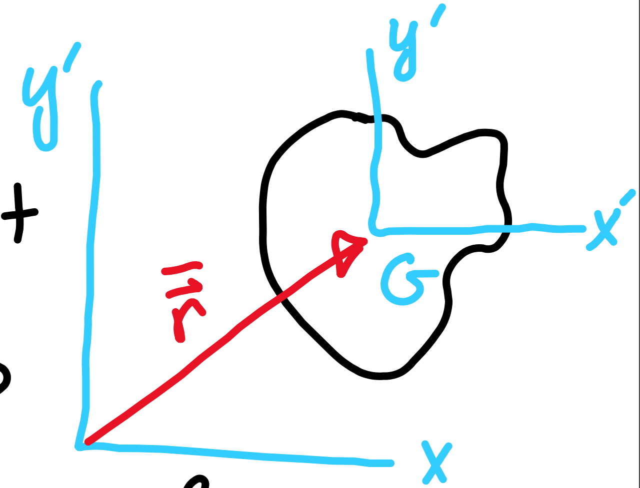

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Parallel axis theorem

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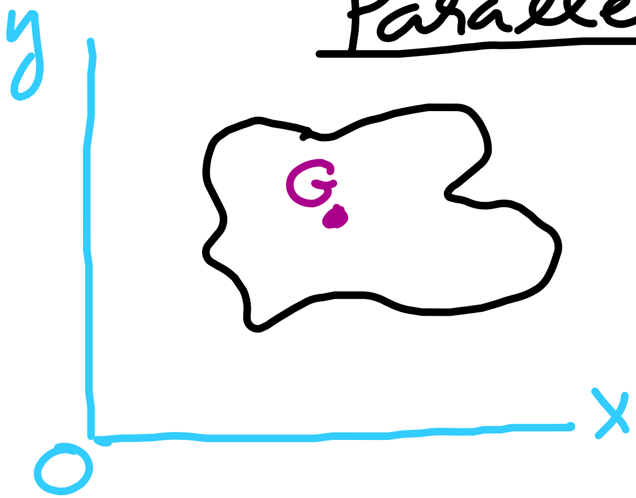

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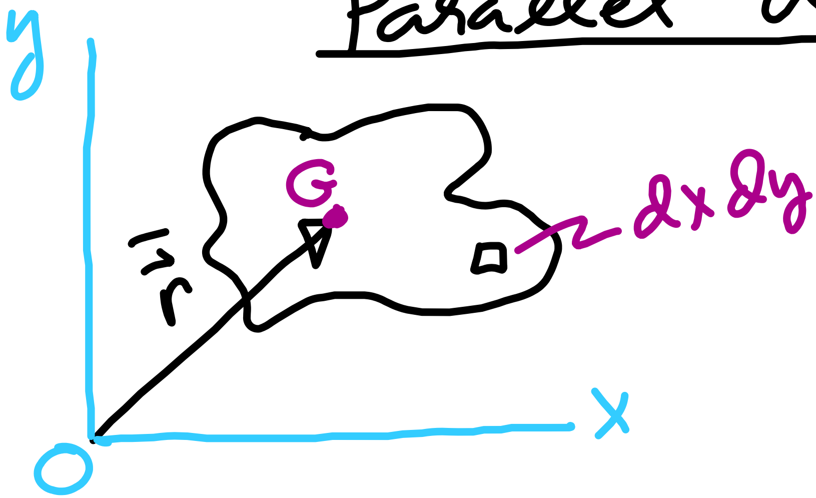

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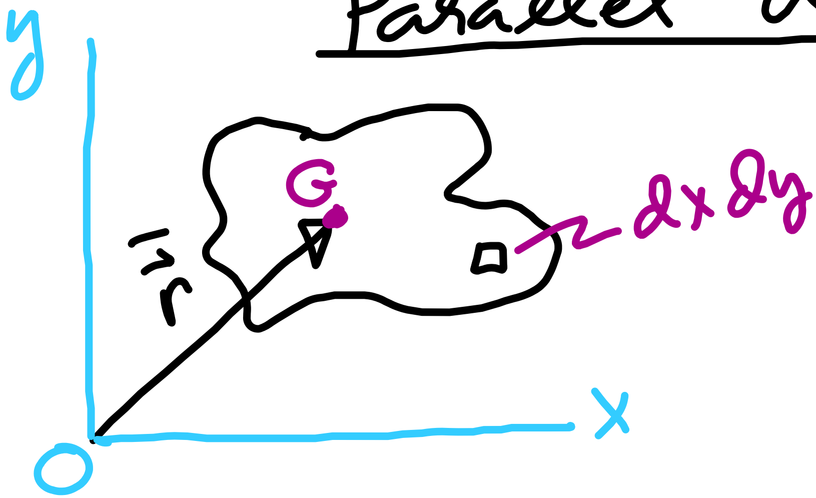

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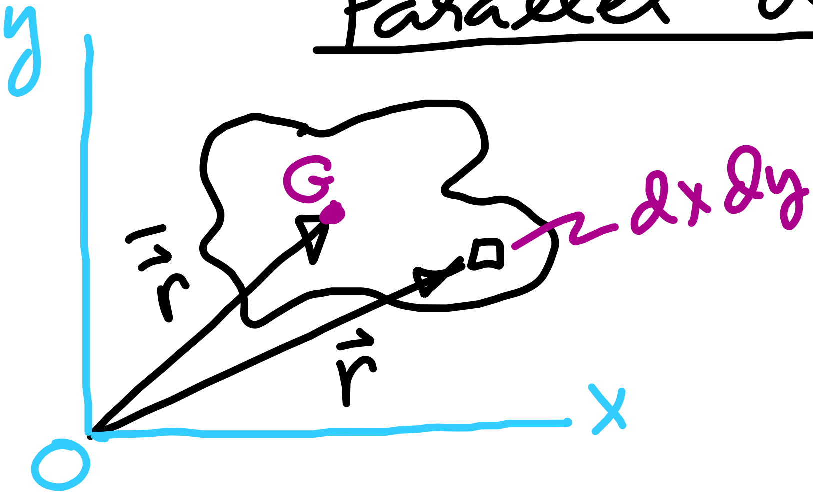

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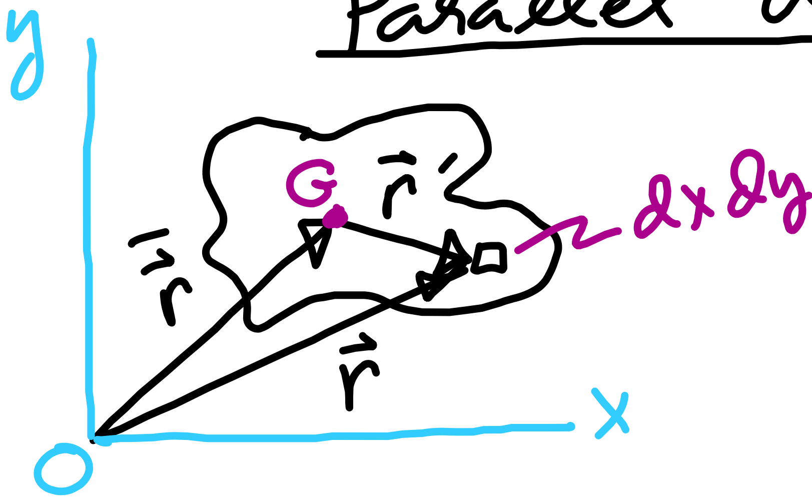

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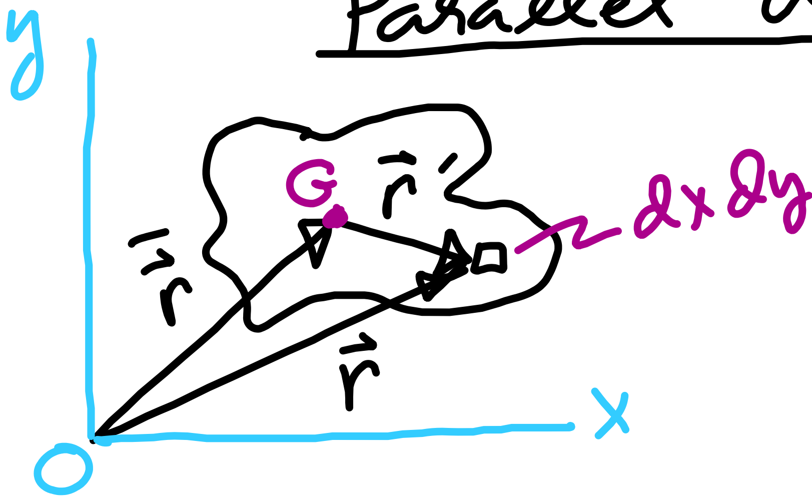

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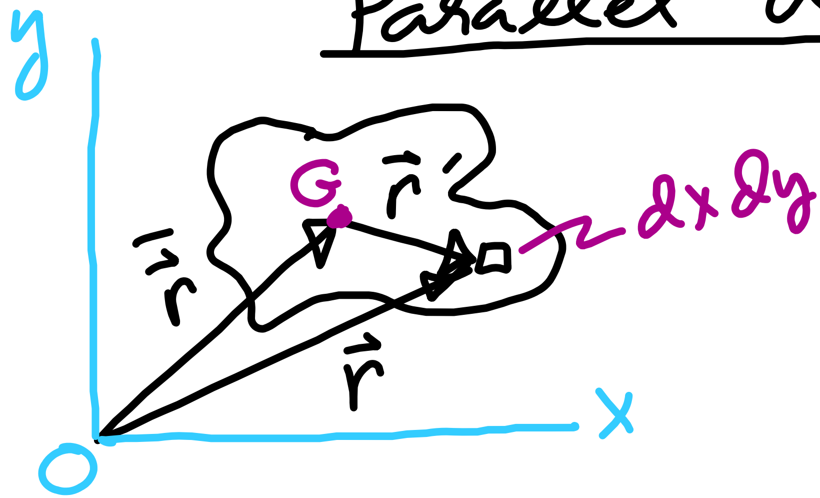

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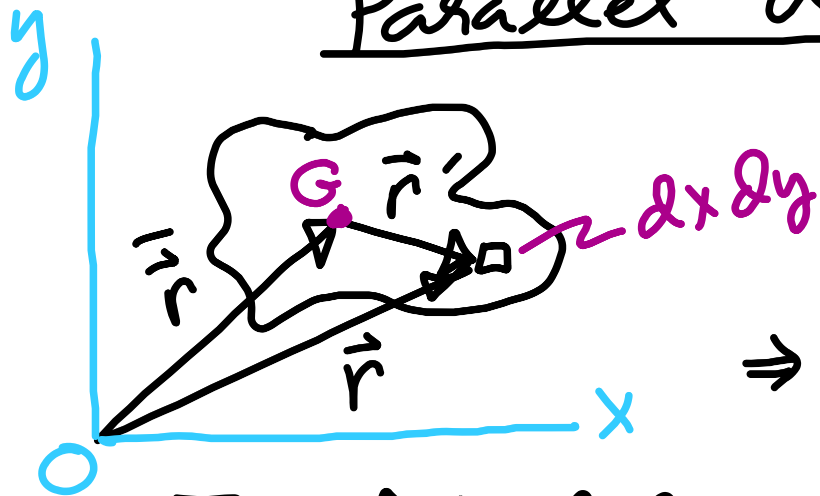

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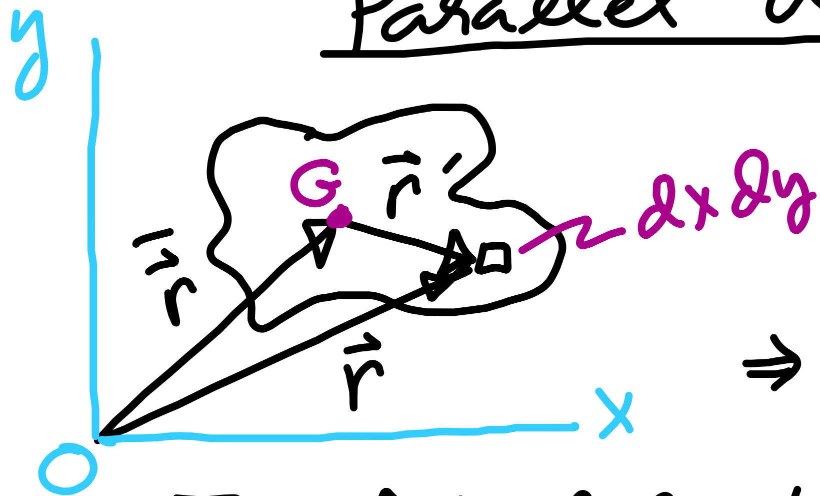

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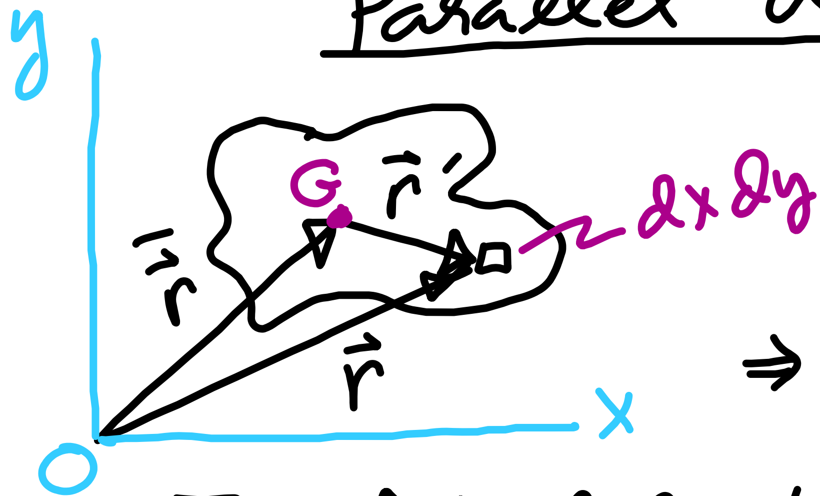

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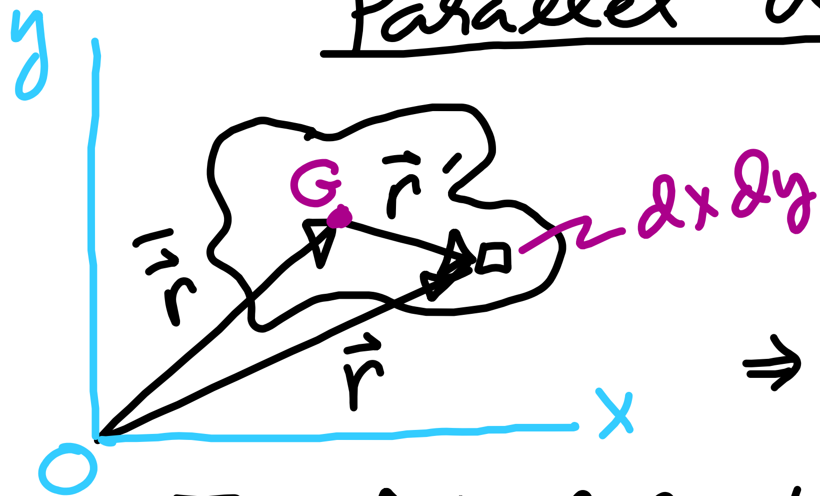

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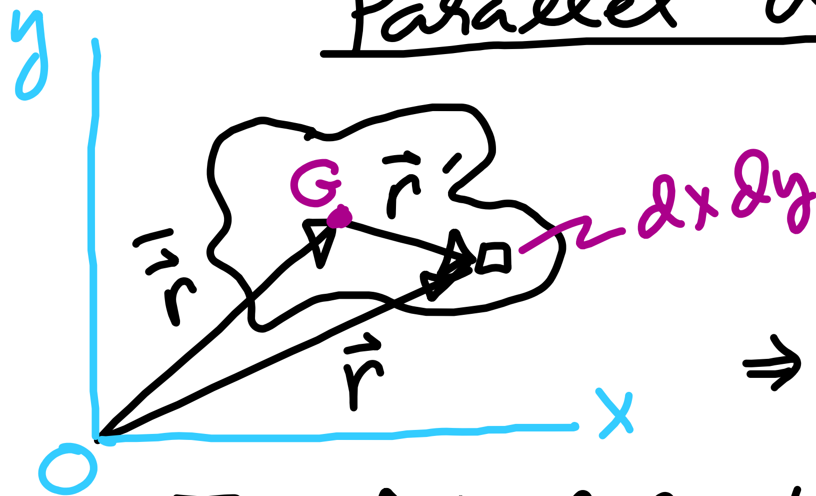

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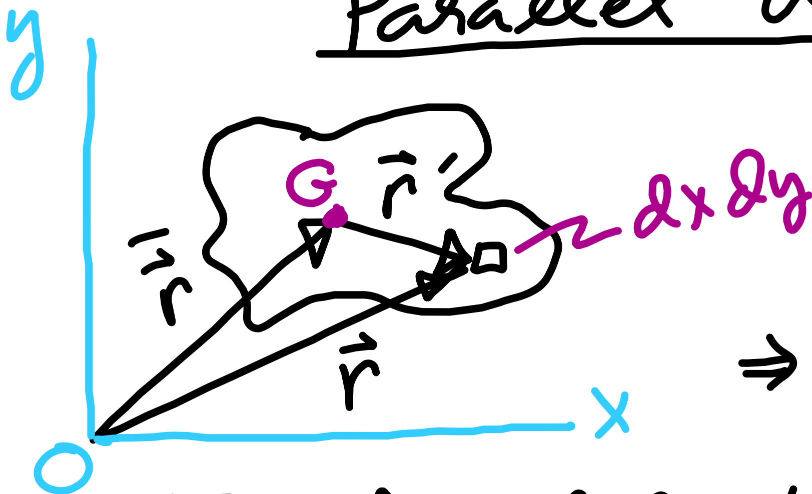

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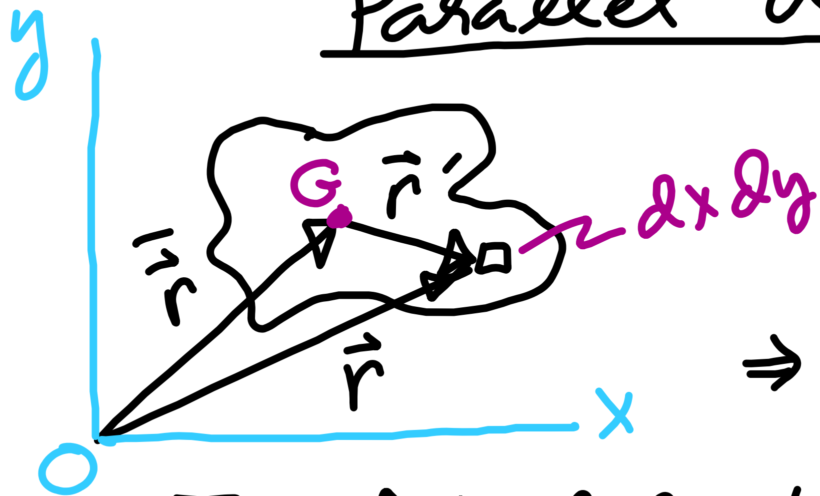

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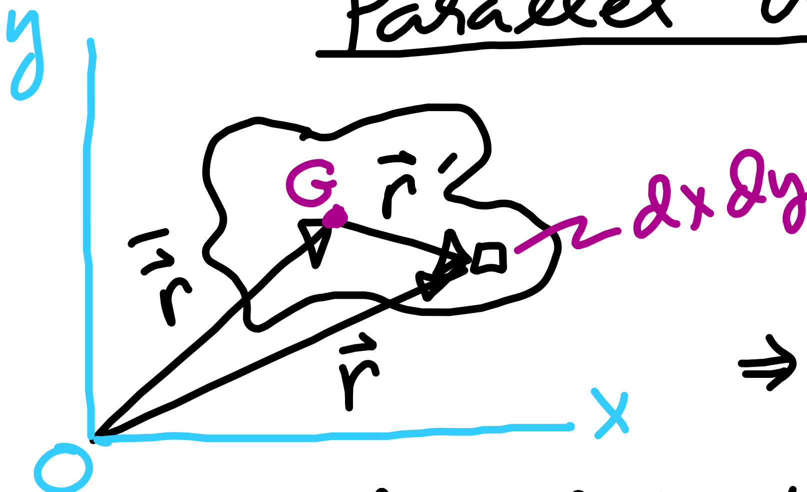
Parallel axis theorem



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& $\frac{M}{A}\int r'^2\,dx\,dy = \bar{I}$ so

$$\boxed{I_o = \bar{I} + M\bar{r}^2}$$

Last time we saw that

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$

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Let an object rotate about some fixed point P

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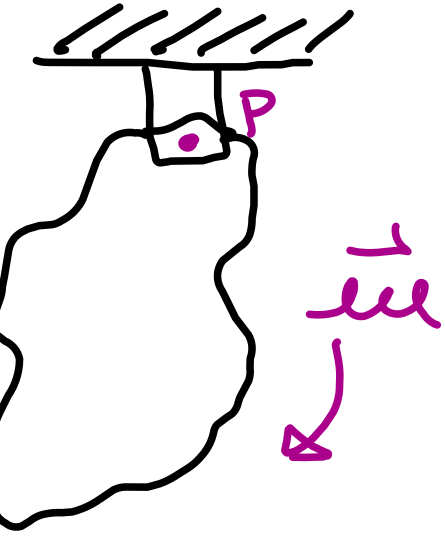

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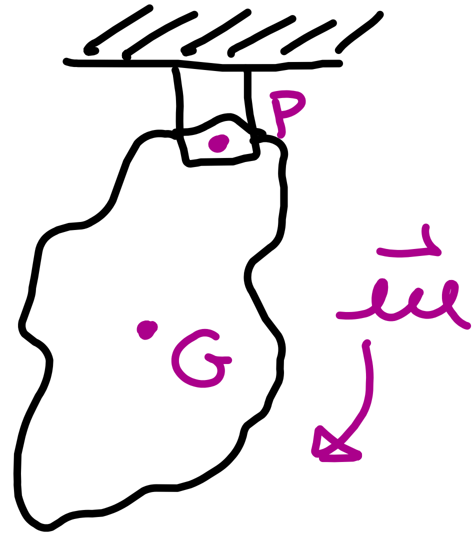

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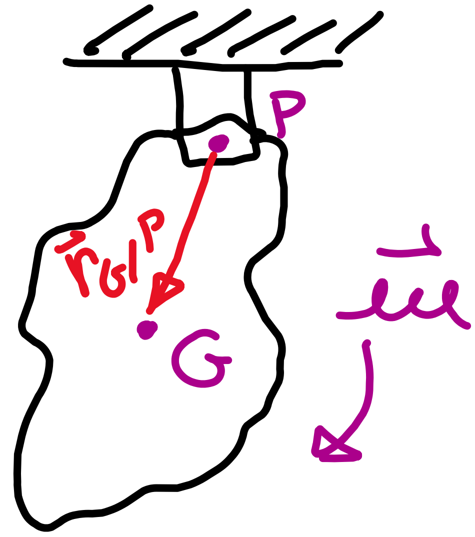

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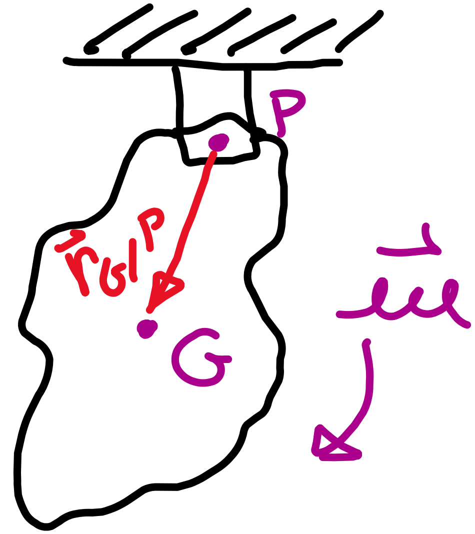

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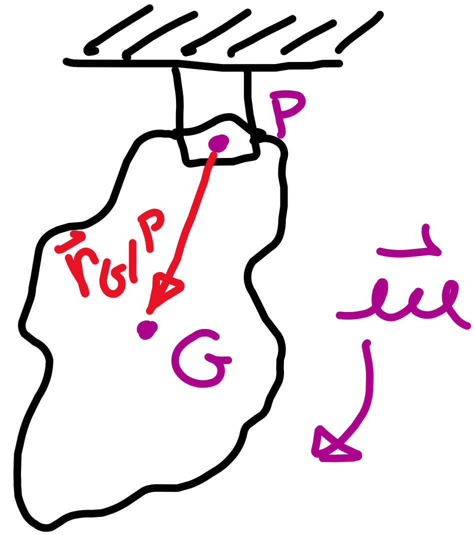

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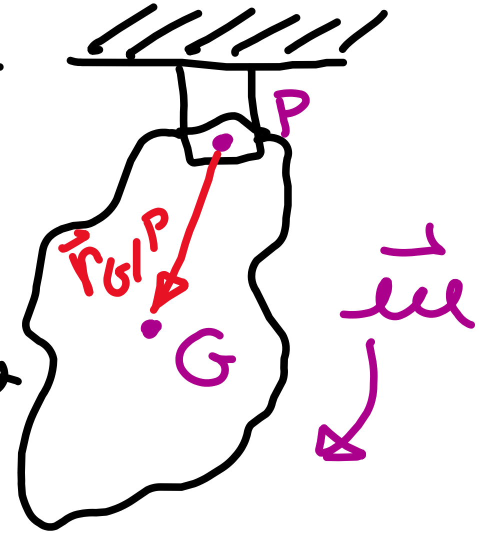
Let an object rotate about some fixed point 

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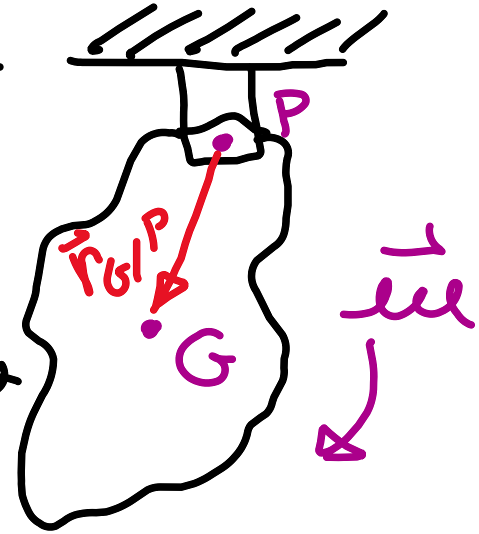

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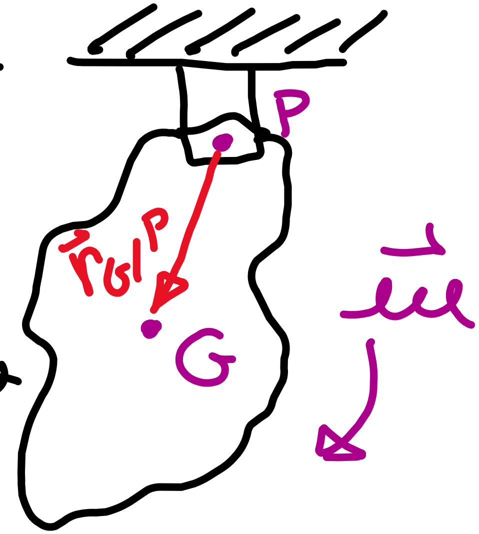

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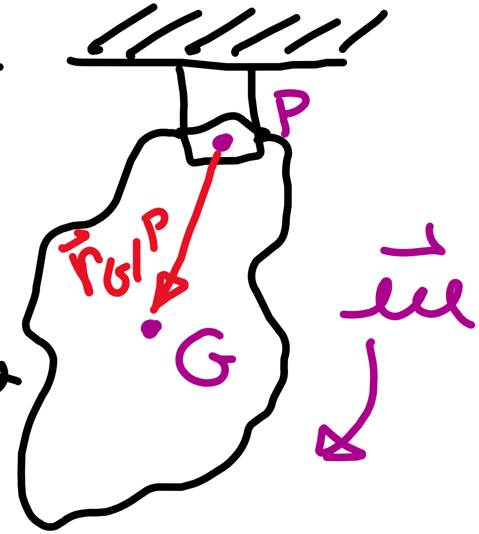


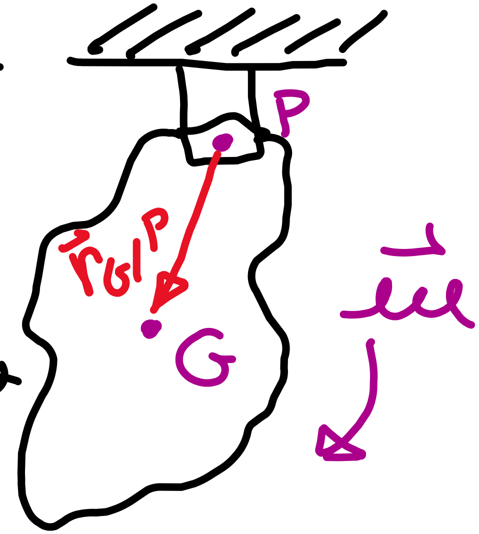
P. $\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$ But

$\vec{\bar{a}} = \bar{a}_n \hat{e}_n + \bar{a}_t \hat{e}_t$ & since $\vec{r}_{G/P} \times \hat{e}_n = 0$

then $\vec{r}_{G/P} \times m\vec{\bar{a}} = \vec{r}_{G/P} \times m\bar{a}_t \hat{e}_t$

& since $\vec{\bar{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$, then $\vec{r}_{G/P} \times m\vec{\bar{a}} = m r_{G/P}^2 \vec{\alpha}$

Let an object rotate about some fixed point P.



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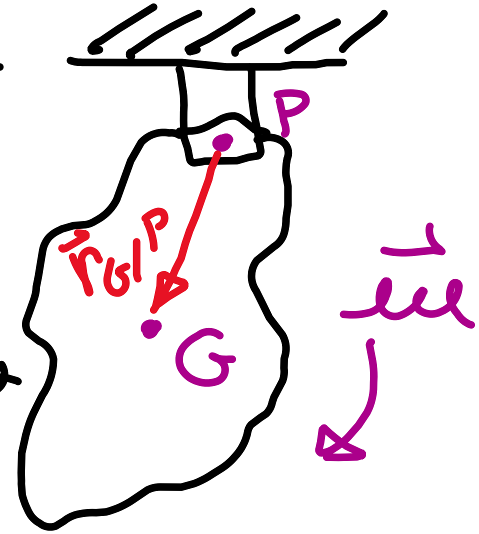
But $r_{G/P} = \bar{r}$

Let an object rotate about some fixed point



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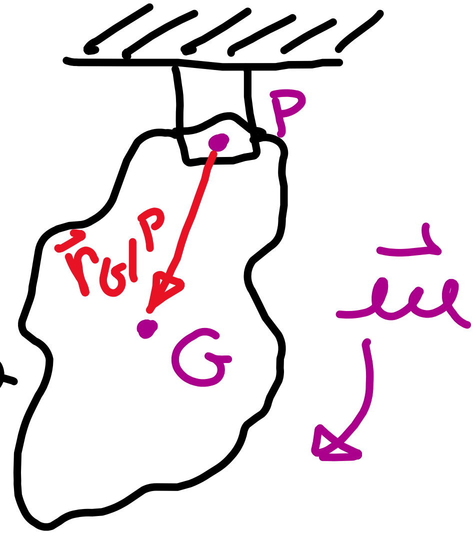
then $\vec{r}_{G/P} \times m\vec{\bar{a}} = \vec{r}_{G/P} \times m\bar{a}_t \hat{e}_t$

& since $\vec{\bar{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$, then $\vec{r}_{G/P} \times m\vec{\bar{a}} = m r_{G/P}^2 \vec{\alpha}$

But $r_{G/P} = \bar{r}$ Now $\vec{r}_{G/P} \times m\vec{\bar{a}} = m\bar{r}^2\vec{\alpha}$

Let an object rotate about some fixed point P.



$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$ But

$\vec{\bar{a}} = \bar{a}_n \hat{e}_n + \bar{a}_t \hat{e}_t$ & since $\vec{r}_{G/P} \times \hat{e}_n = 0$

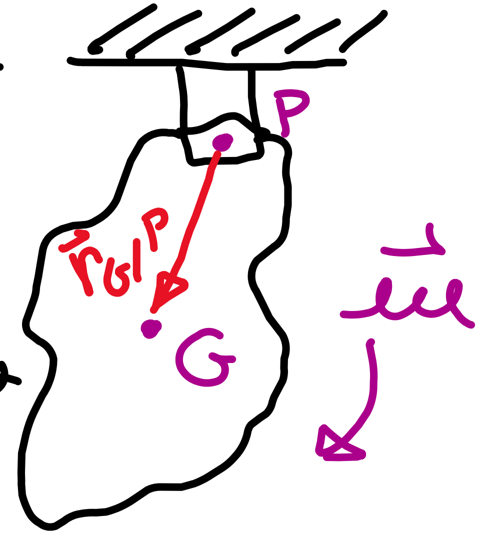
then $\vec{r}_{G/P} \times m\vec{\bar{a}} = \vec{r}_{G/P} \times m \bar{a}_t \hat{e}_t$

& since $\vec{\bar{a}}_t = \vec{\alpha} \times \vec{r}_{G/P}$, then $\vec{r}_{G/P} \times m\vec{\bar{a}} = m r_{G/P}^2 \vec{\alpha}$

But $r_{G/P} = \bar{r}$ Now $\vec{r}_{G/P} \times m\vec{\bar{a}} = m\bar{r}^2\vec{\alpha}$ so

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + m\bar{r}^2\vec{\alpha}$$

Let an object rotate about some fixed point



P. $\boxed{\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}}$ But

$\vec{\bar{a}} = \bar{a}_n \hat{e}_n + \bar{a}_t \hat{e}_t$ & since $\vec{r}_{G/P} \times \hat{e}_n = 0$

then $\vec{r}_{G/P} \times m\vec{\bar{a}} = \vec{r}_{G/P} \times m\bar{a}_t \hat{e}_t$

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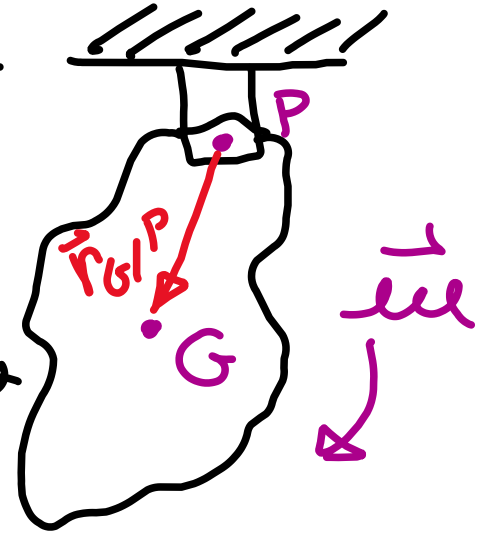
But $r_{G/P} = \bar{r}$ Now $\vec{r}_{G/P} \times m\vec{\bar{a}} = m\bar{r}^2\vec{\alpha}$ so

$\sum \vec{M}_P = \bar{I}\vec{\alpha} + m\bar{r}^2\vec{\alpha}$ <u>or</u> $\boxed{\sum \vec{M}_P = I_P\vec{\alpha}}$

Let an object rotate about some fixed point P.



$\boxed{\Sigma \vec{M}_p = \bar{I}\vec{\alpha} + \vec{r}_{G/p} \times m\bar{\vec{a}}}$ But

$\bar{\vec{a}} = \bar{a}_n \hat{e}_n + \bar{a}_t \hat{e}_t$ & since $\vec{r}_{G/p} \times \hat{e}_n = 0$

then $\vec{r}_{G/p} \times m\bar{\vec{a}} = \vec{r}_{G/p} \times m\bar{a}_t \hat{e}_t$

& since $\bar{\vec{a}}_t = \vec{\alpha} \times \vec{r}_{G/p}$, then $\vec{r}_{G/p} \times m\bar{\vec{a}} = m r_{G/p}^2 \vec{\alpha}$

But $r_{G/p} = \bar{r}$ Now $\vec{r}_{G/p} \times m\bar{\vec{a}} = m\bar{r}^2\vec{\alpha}$ so

$\Sigma \vec{M}_p = \bar{I}\vec{\alpha} + m\bar{r}^2\vec{\alpha}$ or $\boxed{\Sigma \vec{M}_p = I_p \vec{\alpha}}$

so $\underline{\bar{I}\vec{\alpha} + \vec{r}_{G/p} \times m\bar{\vec{a}} = I_p \vec{\alpha}}$

We have now seen that, for a fixed point of rotation

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We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

$$\& \quad \sum \vec{M}_P = I_P \vec{\alpha}, \text{ where } I_P = \bar{I} + m\bar{r}^2$$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

& $\sum \vec{M}_P = I_P \vec{\alpha}$, where $I_P = \bar{I} + m\bar{r}^2$ &

$\bar{r} = r_{G/P}$

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So, do not use **both** types of expressions at the same time!!

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

$$\& \quad \sum \vec{M}_P = I_P \vec{\alpha}, \text{ where } I_P = \bar{I} + m\bar{r}^2 \;\&$$

$\bar{r} = r_{G/P}$

So, do not use <u>both</u> types of expressions at the same time!!

$$\text{Good} \begin{cases} \sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}} \\ \end{cases}$$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

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So, do not use <u>both</u> types of expressions at the same time!!

$$\text{Good} \begin{cases} \sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}} \\ \text{or} \quad \sum \vec{M}_P = I_P \vec{\alpha} \end{cases}$$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$$

& $\sum \vec{M}_P = I_P \vec{\alpha}$, where $I_P = \bar{I} + m\bar{r}^2$ & $\bar{r} = r_{G/P}$

So, do not use <u>both</u> types of expressions at the same time !!

$$\text{Good} \begin{cases} \sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}} \\ \quad \text{or} \quad \sum \vec{M}_P = I_P \vec{\alpha} \end{cases}$$

Bad: $\sum \vec{M}_P = I_P \vec{\alpha} + \vec{r}_{G/P} \times m\bar{\vec{a}}$

We have now seen that, for a fixed point of rotation

$$\sum \vec{M}_p = \bar{I}\vec{\alpha} + \vec{r}_{G/p} \times m\vec{\bar{a}}$$

& $\sum \vec{M}_p = I_p\vec{\alpha}$, where $I_p = \bar{I} + m\bar{r}^2$ &

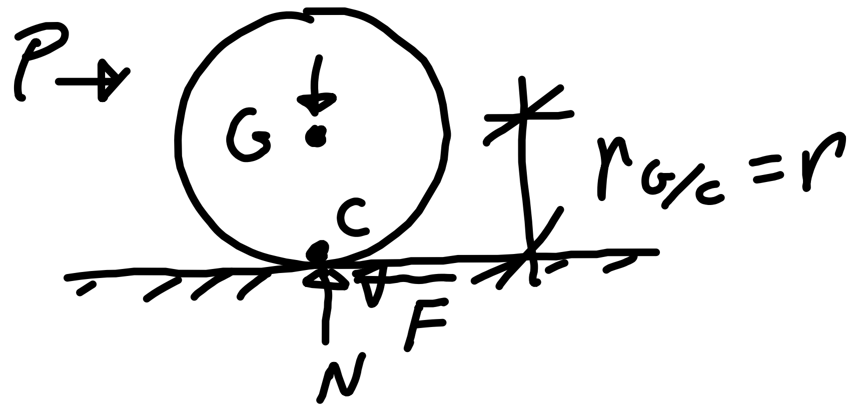
$\bar{r} = r_{G/p}$

So, do not use <u>both</u> types of expressions at the same time !!

Good $\begin{cases} \sum \vec{M}_p = \bar{I}\vec{\alpha} + \vec{r}_{G/p} \times m\vec{\bar{a}} \\ \quad \text{or} \quad \sum \vec{M}_p = I_p\vec{\alpha} \end{cases}$

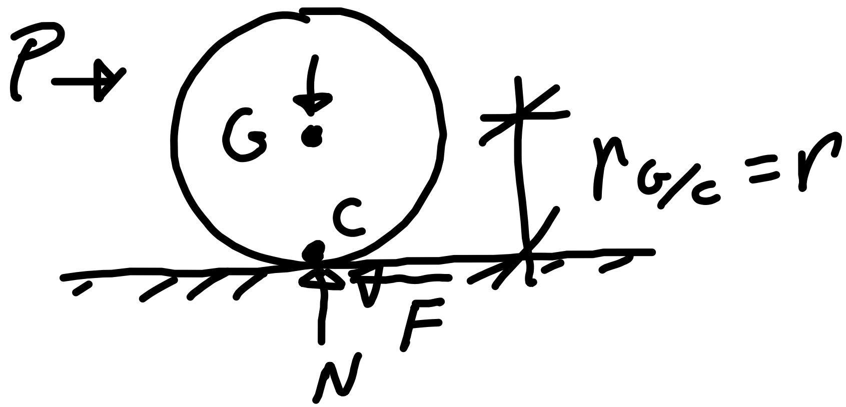
Bad: $\sum \vec{M}_p = \underbrace{I_p\vec{\alpha} + \vec{r}_{G/p} \times m\vec{\bar{a}}}_{\text{¡¡WRONG!!}}$

16.92



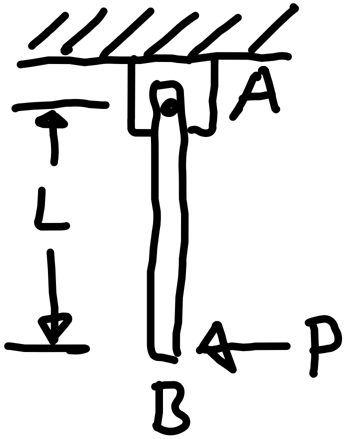
$$\circlearrowleft \sum M_C = \bar{I}\vec{\alpha} + \vec{r}_{G/C} \times m\vec{\bar{a}}$$

Notes on 16.92

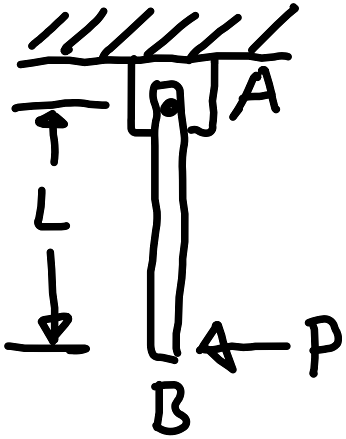


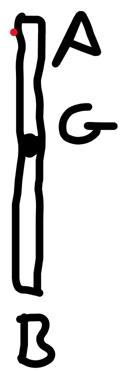
$$\circlearrowleft \sum M_C = \underbrace{\bar{I}\vec{\alpha} + \vec{r}_{G/C} \times m\vec{\bar{a}}}_{\text{Just need to convert this to } (\bar{I} + mr^2)\vec{\alpha} = I_C\vec{\alpha}}$$

Example

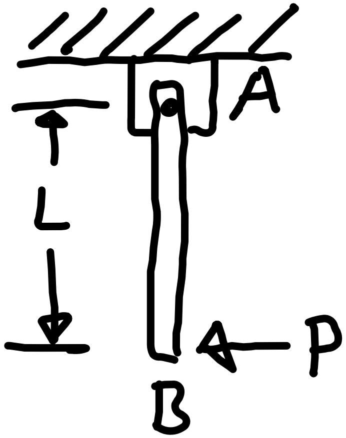


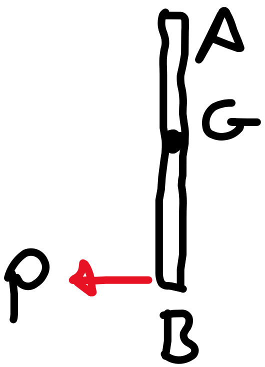
Example



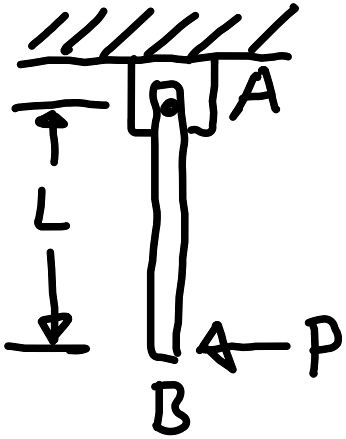


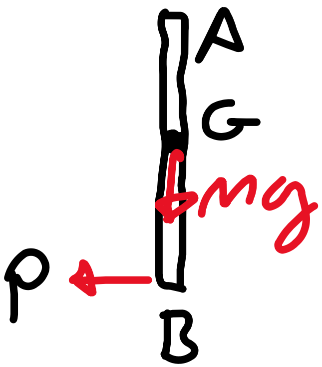
Example



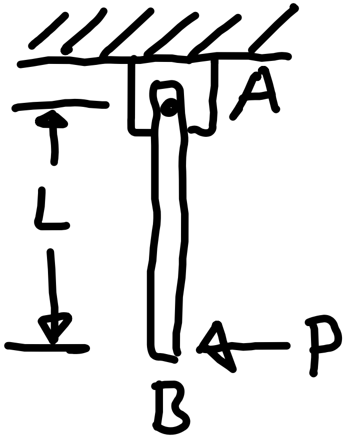


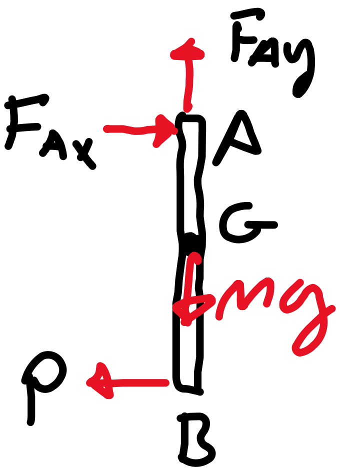
Example



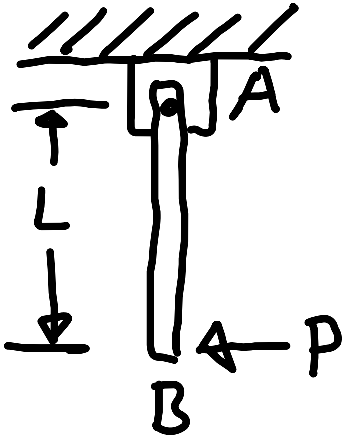


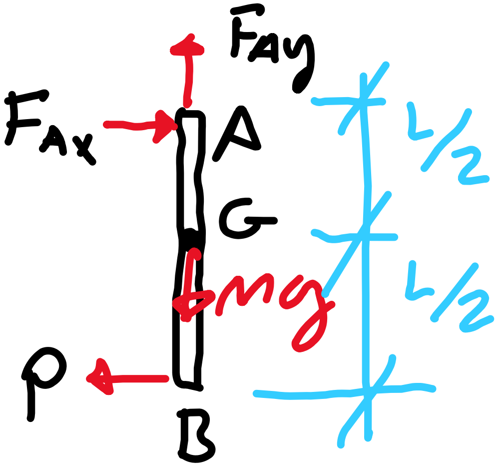
Example





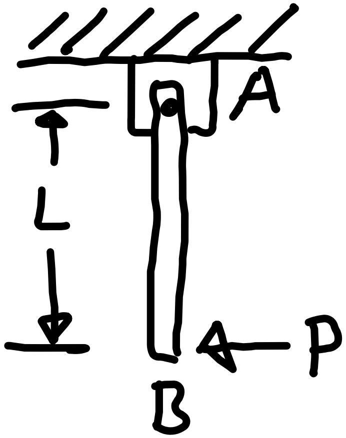
Example

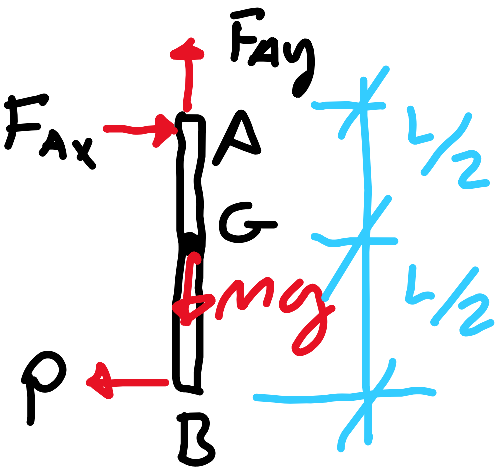




Example

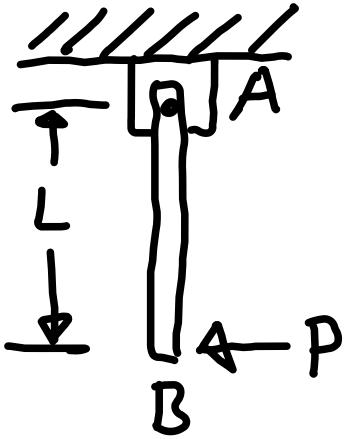
$L = 36\,in$

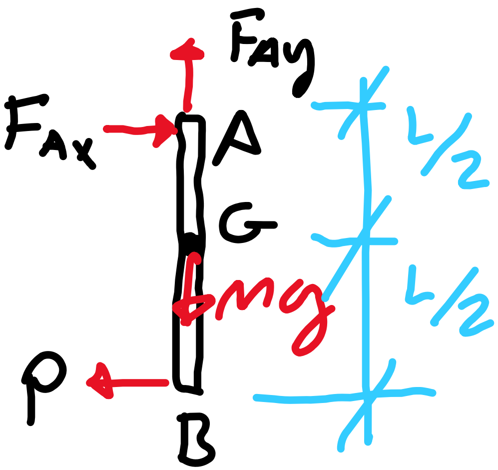




Example

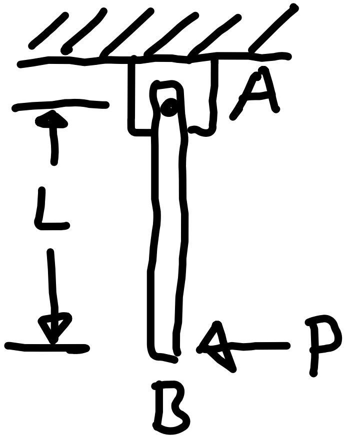
$L = 36\,in, \; w = 4\,lb$

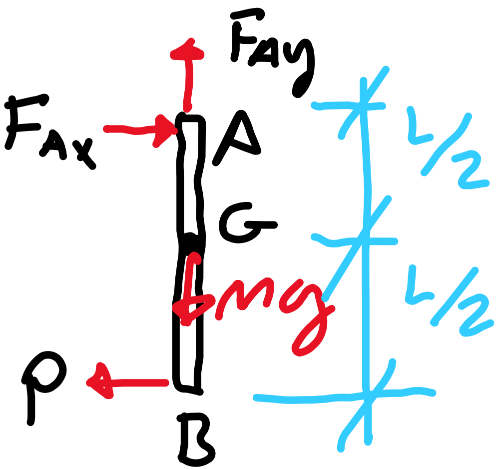




Example

$L = 36\text{ in}, \ W = 4\text{ lb}, \ P = 1.5\text{ lb}$

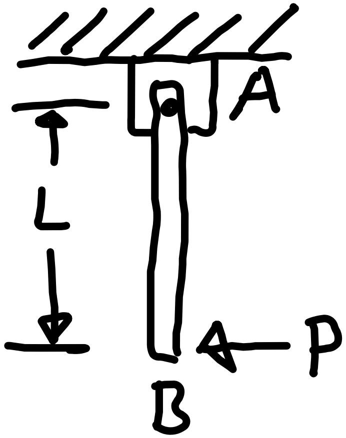


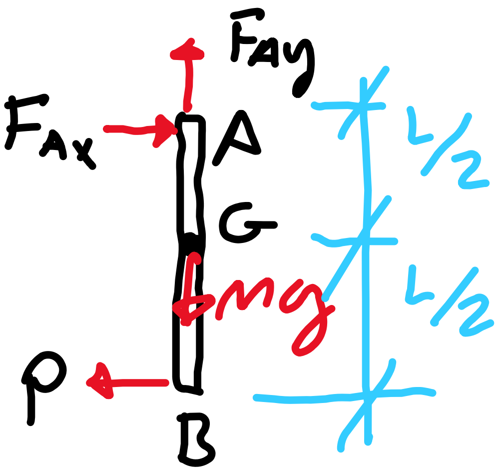


Example

$L = 36\,in$, $w = 4\,lb$, $P = 1.5\,lb$

Find α:

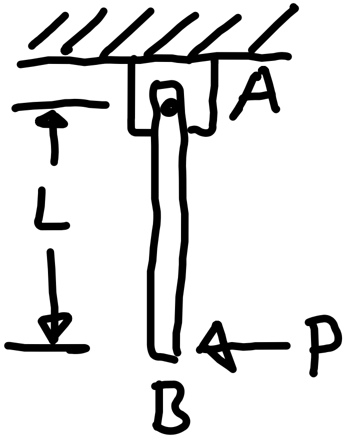


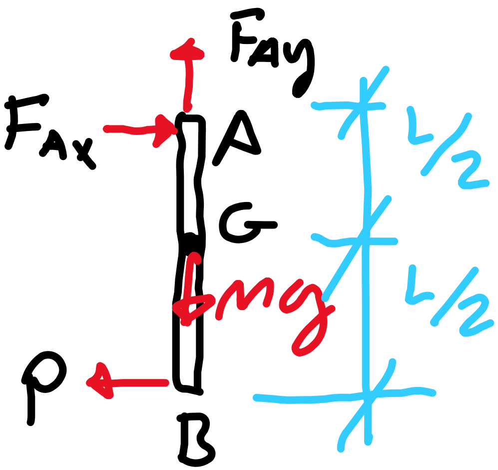


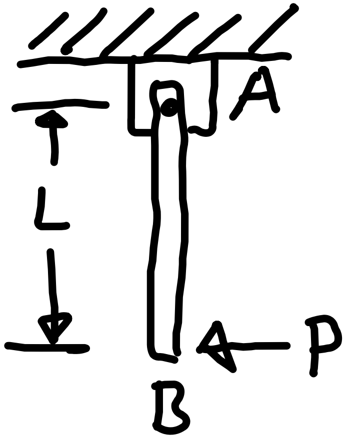
Example

$L = 36\,in,\ W = 4\,lb,\ P = 1.5\,lb$

Find α: $\circlearrowleft \sum M_A = I_A\alpha$





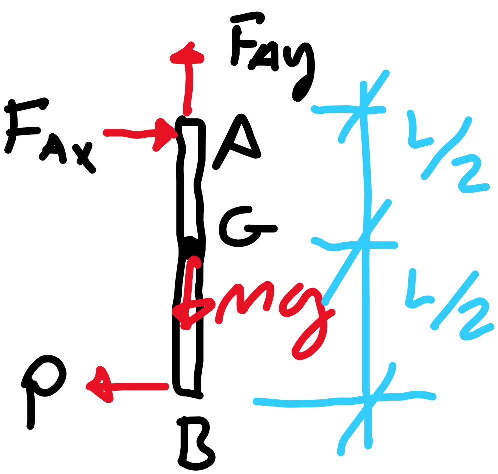


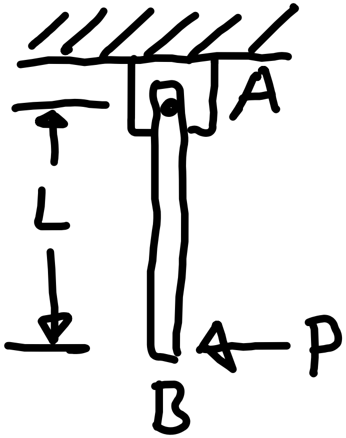
Example

$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$

Find α: $\curvearrowleft \Sigma M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$



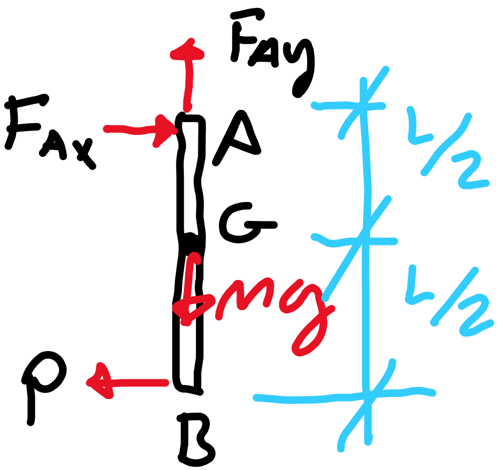


Example

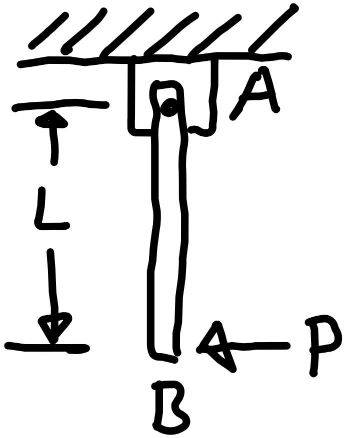
$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$

Find α: $\circlearrowleft \Sigma M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$



Example

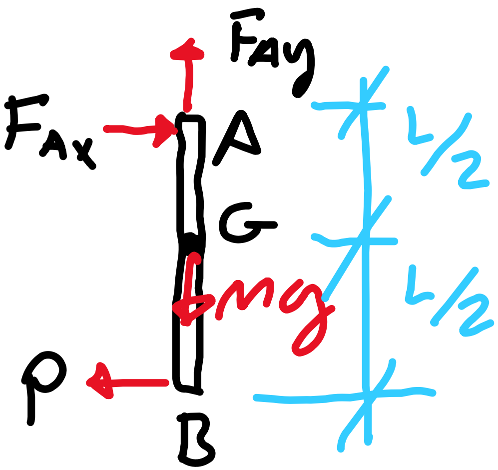


$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$

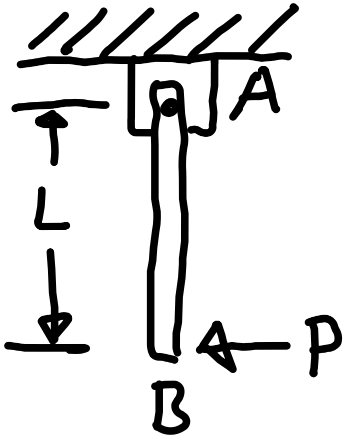
Find α: $\circlearrowleft \Sigma M_A = I_A\alpha$

$\Rightarrow \quad PL = I_A\alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

$\&\ \bar{I} = m\frac{L^2}{12}$

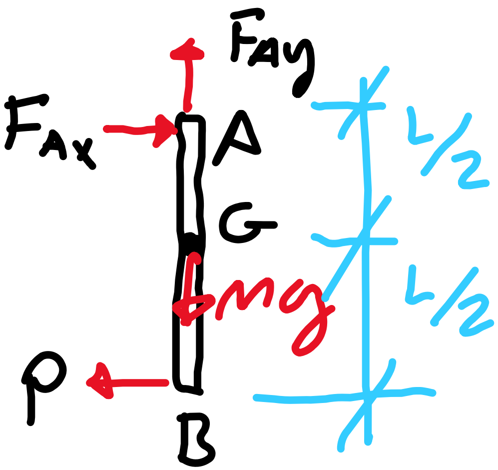


Example



$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$

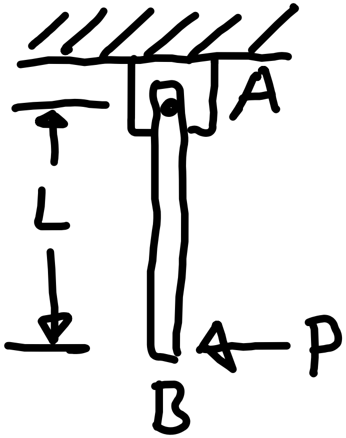
Find α: $\circlearrowleft \sum M_A = I_A\alpha$



$\Rightarrow \quad PL = I_A\alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

& $\bar{I} = m\frac{L^2}{12} \quad \Rightarrow$

$I_A = m\frac{L^2}{12} + m\frac{L^2}{4}$



Example

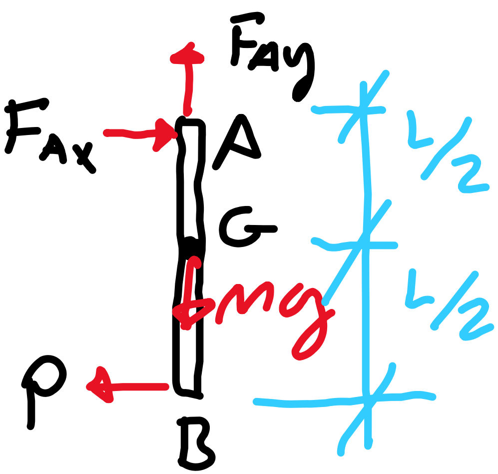
$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$

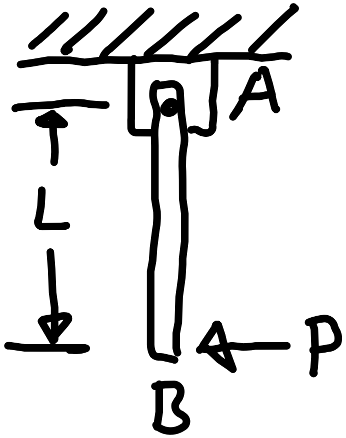
Find α: $\circlearrowleft \Sigma M_A = I_A \alpha$

$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

& $\bar{I} = m\frac{L^2}{12} \quad \Rightarrow$

$I_A = m\frac{L^2}{12} + m\frac{L^2}{4} = \frac{mL^2}{12}(1+3)$





Example

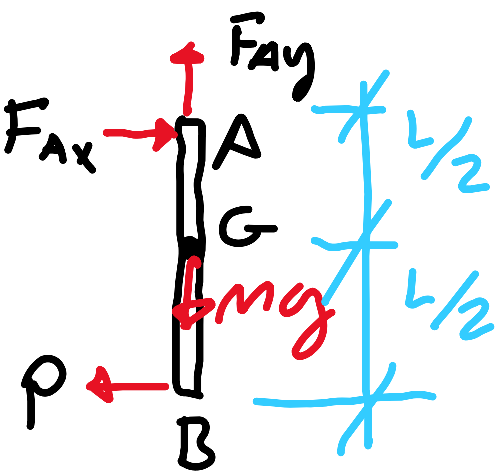
$L = 36\text{ in}, \; w = 4\text{ lb}, \; P = 1.5\text{ lb}$

Find α: $\circlearrowright \Sigma M_A = I_A \alpha$

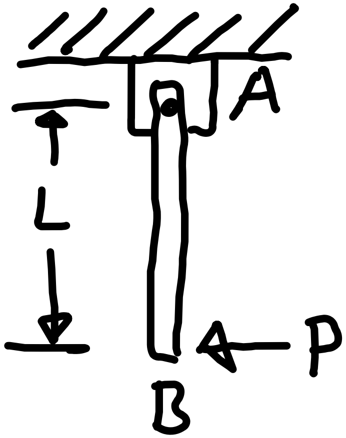
$\Rightarrow \quad PL = I_A \alpha$, where $I_A = \bar{I} + m\frac{L^2}{4}$

& $\bar{I} = m\frac{L^2}{12} \quad \Rightarrow$

$$I_A = m\frac{L^2}{12} + m\frac{L^2}{4} = \frac{mL^2}{12}(1+3) = \frac{mL^2}{3}$$



Example



$$L = 36\,in,\ w = 4\,lb,\ P = 1.5\,lb$$

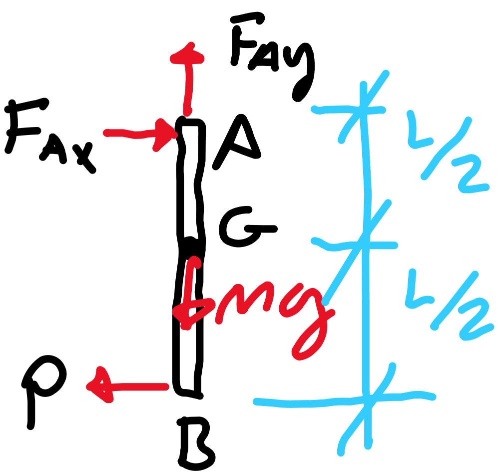
Find α: $\curvearrowright \sum M_A = I_A \alpha$

$$\Rightarrow \quad PL = I_A \alpha, \text{ where } I_A = \bar{I} + m\frac{L^2}{4}$$

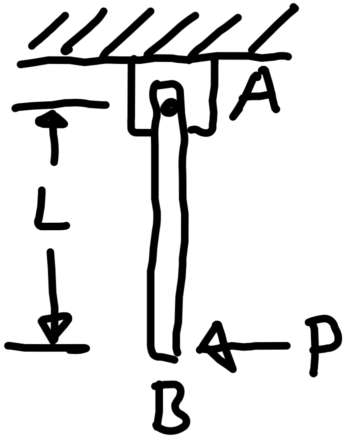
$$\& \quad \bar{I} = m\frac{L^2}{12} \quad \Rightarrow$$

$$I_A = m\frac{L^2}{12} + m\frac{L^2}{4} = \frac{mL^2}{12}(1+3) = \frac{mL^2}{3}$$

$$\text{Now} \quad PL = \left(mL^2/3\right)\alpha$$



Example



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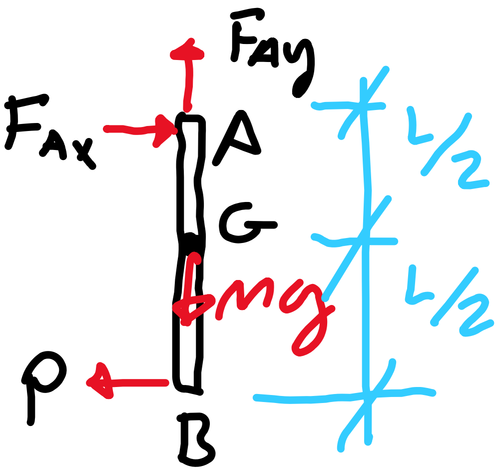
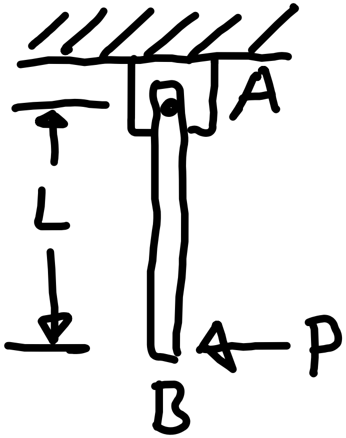
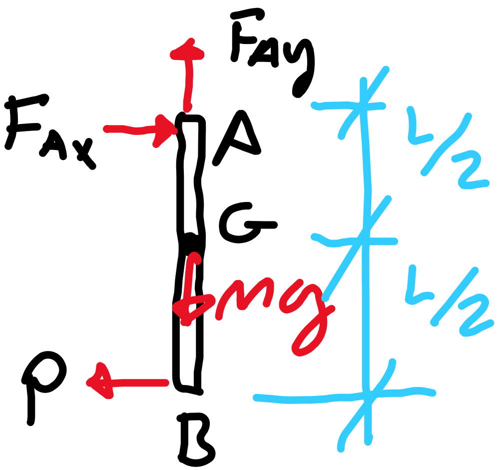
Now $PL = \left(mL^2/3\right)\alpha \Rightarrow$

$$\alpha = \frac{3P}{mL}$$







Example

$3\,ft = 36\,in$

$L = 36\,in$, $W = 4\,lb$, $P = 1.5\,lb$

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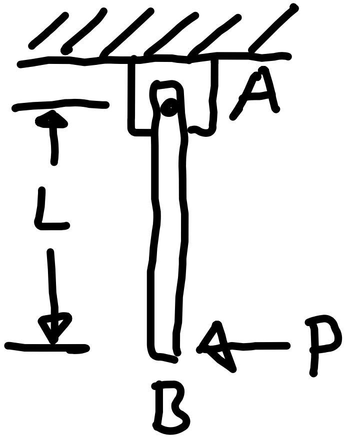
$$I_A = m\frac{L^2}{12} + m\frac{L^2}{4} = \frac{mL^2}{12}(1+3) = \frac{mL^2}{3}$$

Now $PL = (mL^2/3)\alpha \Rightarrow$

$$\alpha = \frac{3P}{mL} = \left(\frac{3 * \frac{3}{2}}{(4/32.2)3}\right)\frac{rad}{s^2}$$



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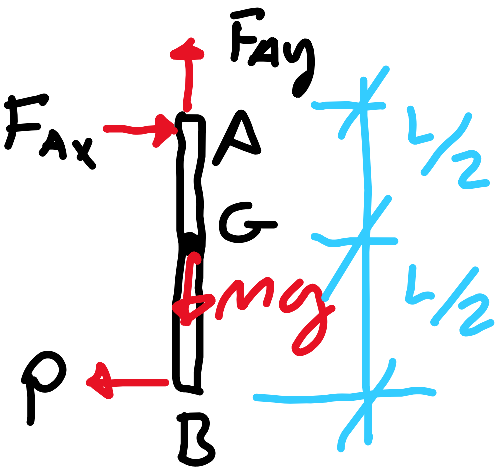
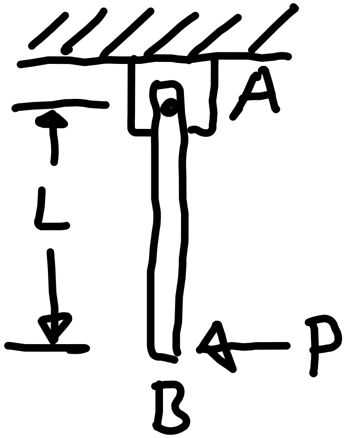


Now $\quad PL = (mL^2/3)\alpha \Rightarrow$

$\alpha = \frac{3P}{mL} = \left(\frac{3 * \frac{3}{2}}{(4/32.2)3}\right)\frac{rad}{s^2}$

$\Rightarrow \quad \alpha = \left(\frac{3}{8}\right)32.2\frac{rad}{s^2}$



Example

$L = 36\,in$, $w = 4\,lb$, $P = 1.5\,lb$ → $3\,ft = 36\,in$

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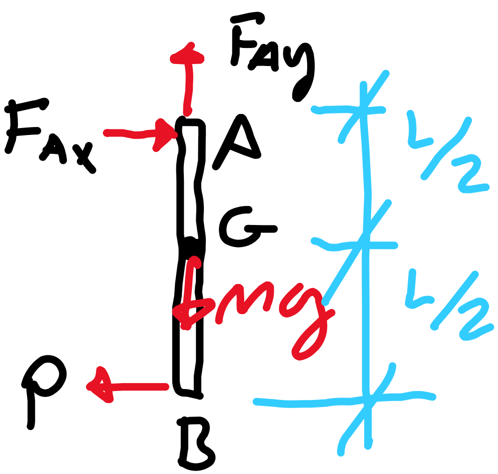
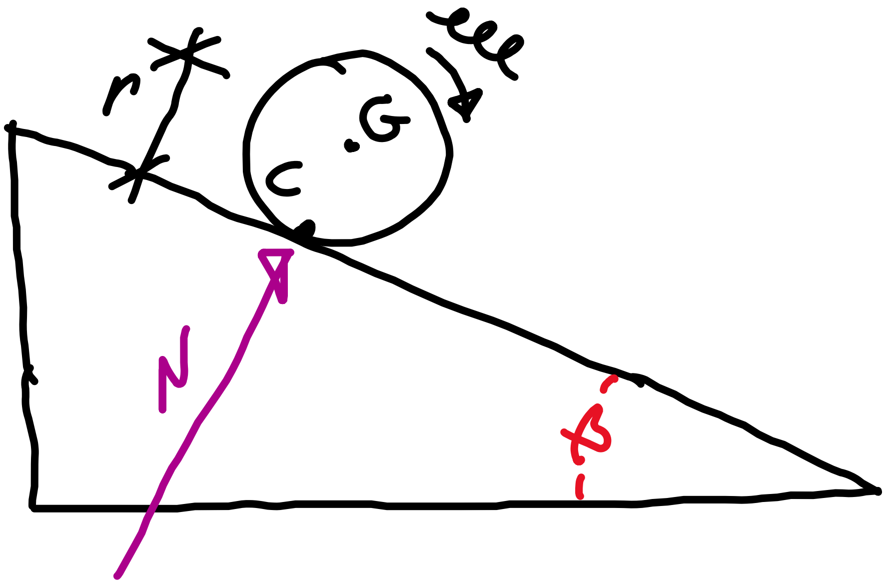
$$\Rightarrow \quad \alpha = \left(\frac{3}{8}\right)32.2\frac{rad}{s^2} = 12.08\,rad/s^2$$



Example: Wheel on incline



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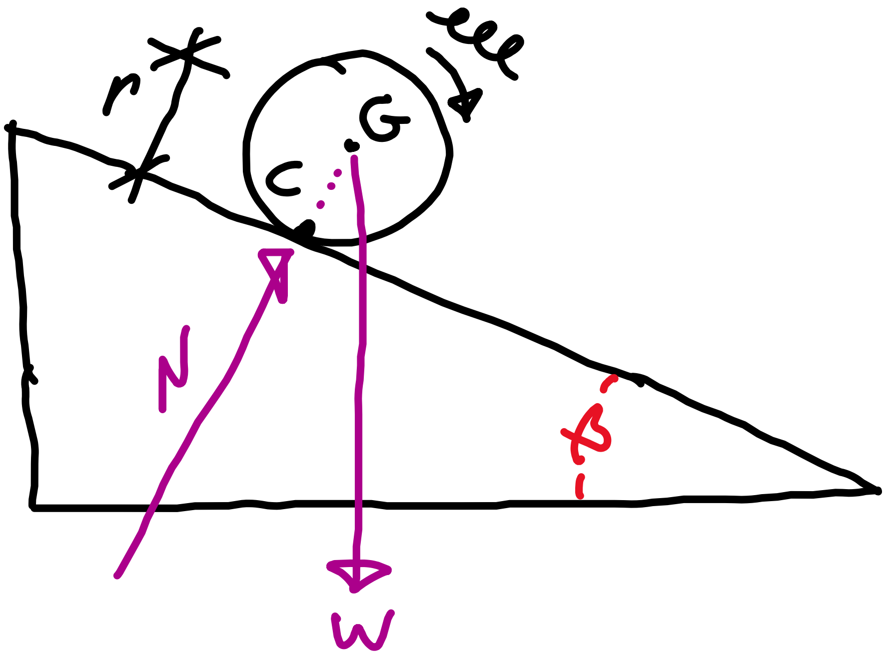

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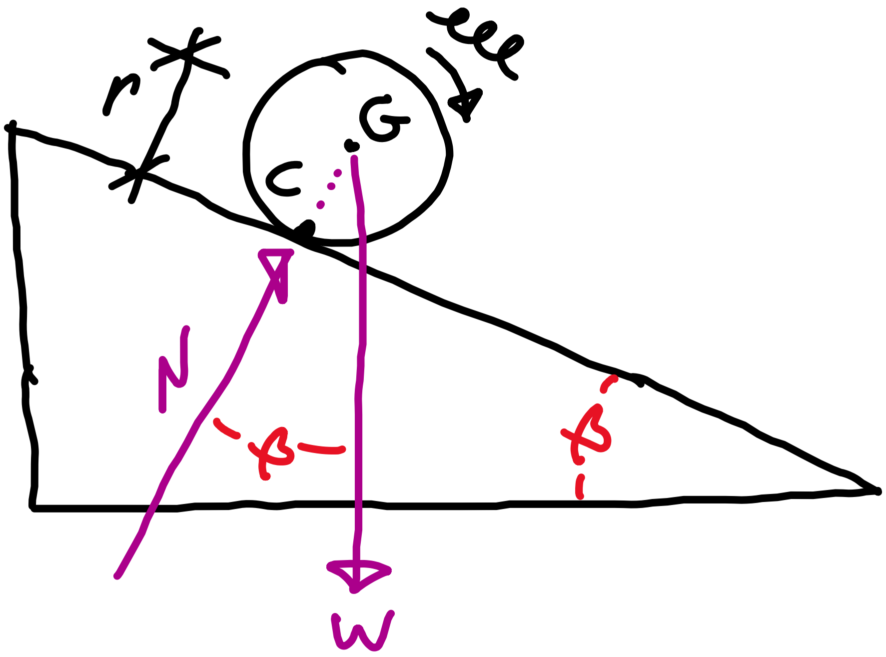
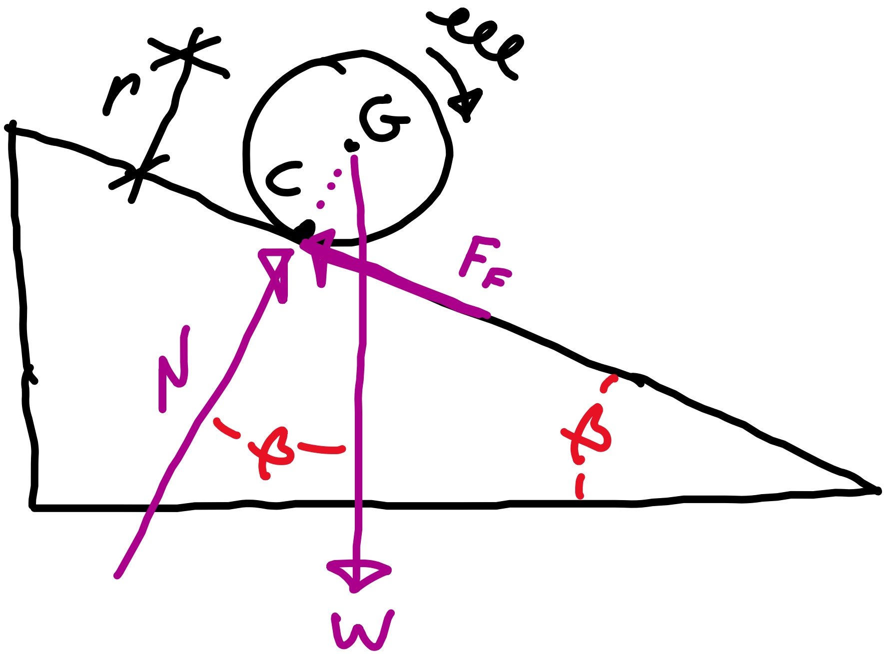


Example: Wheel on incline



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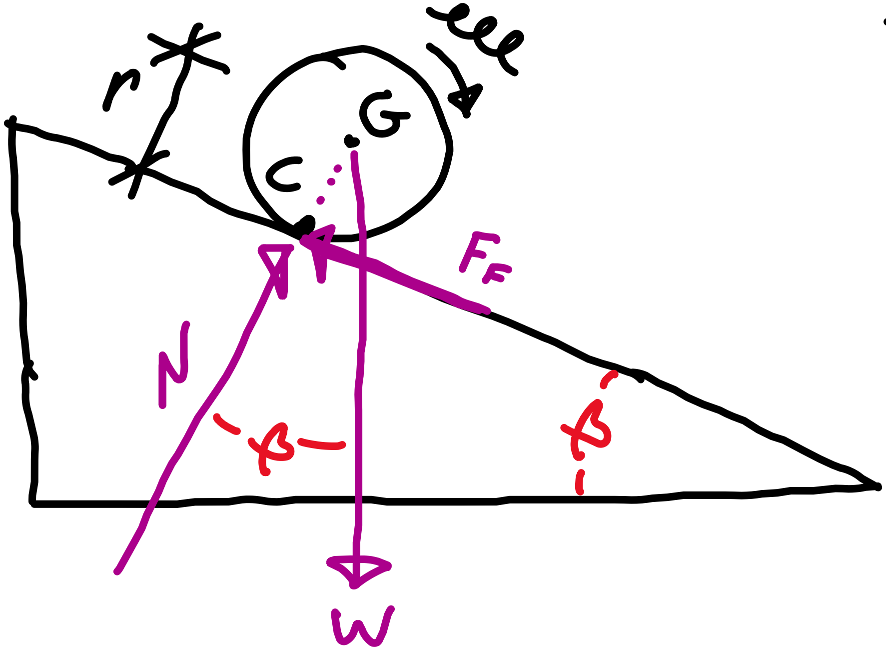

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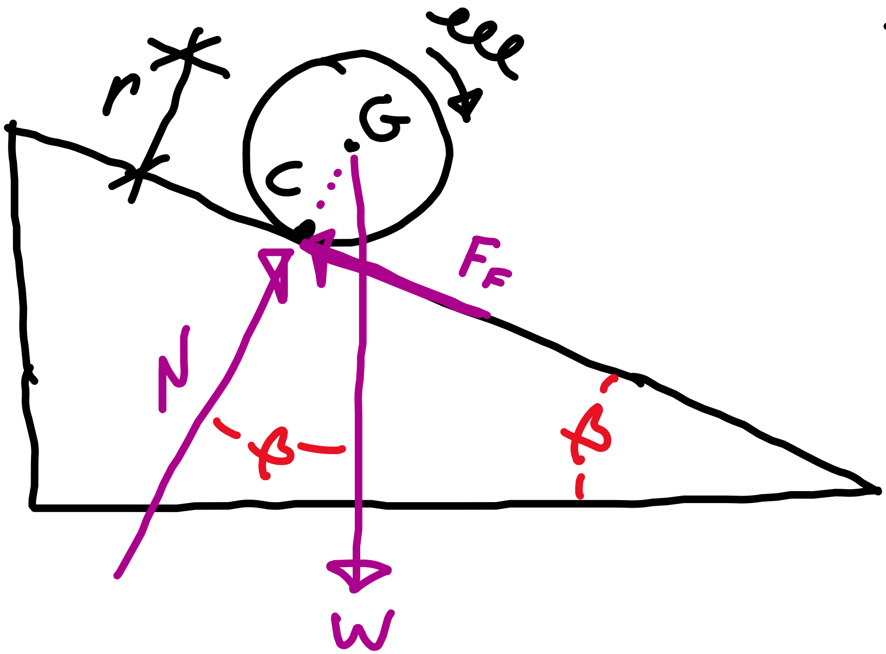
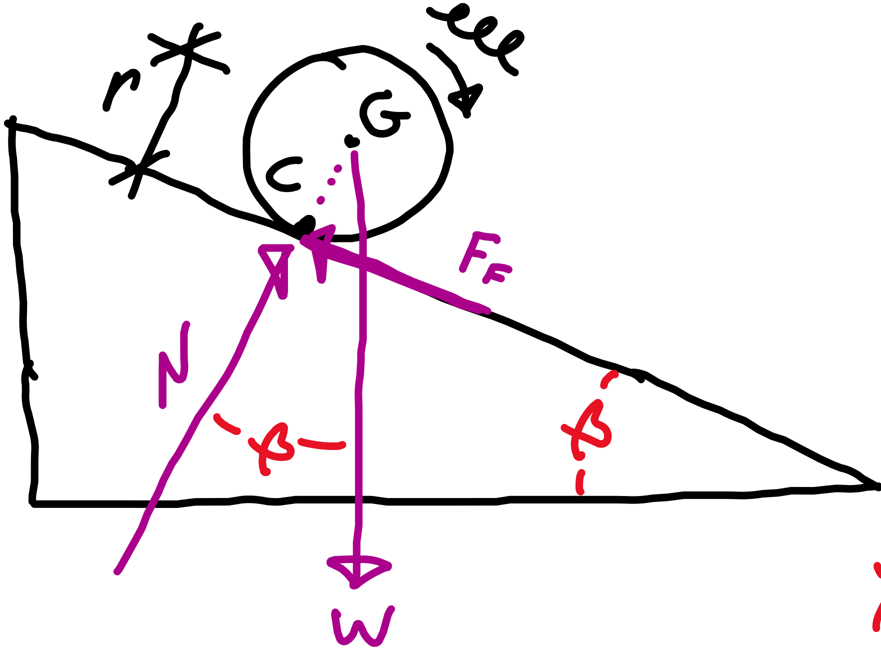


Easiest to sum torques about point C

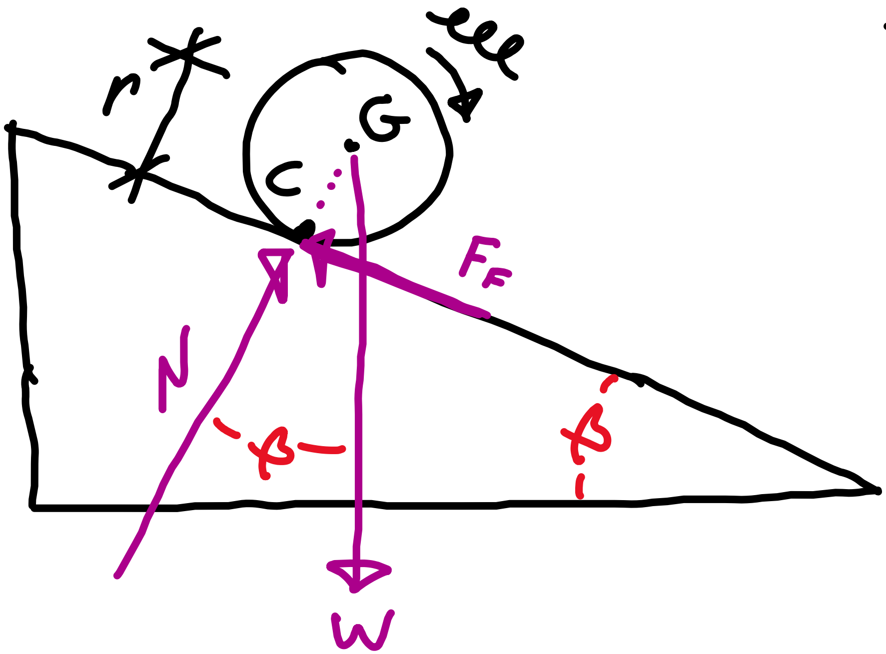
Example: Wheel on incline

Find α:



Easiest to sum torques about point C [can neglect all forces pointing into (or out of) point C]

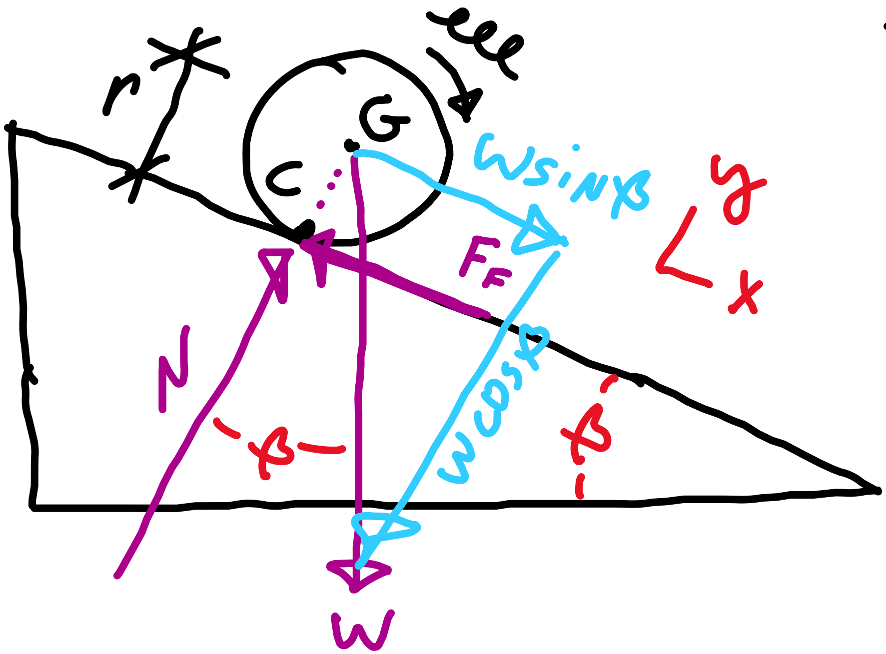
Example: Wheel on incline



Find α:

$$\curvearrowleft \sum M_c = I_c \alpha$$

Example: Wheel on incline



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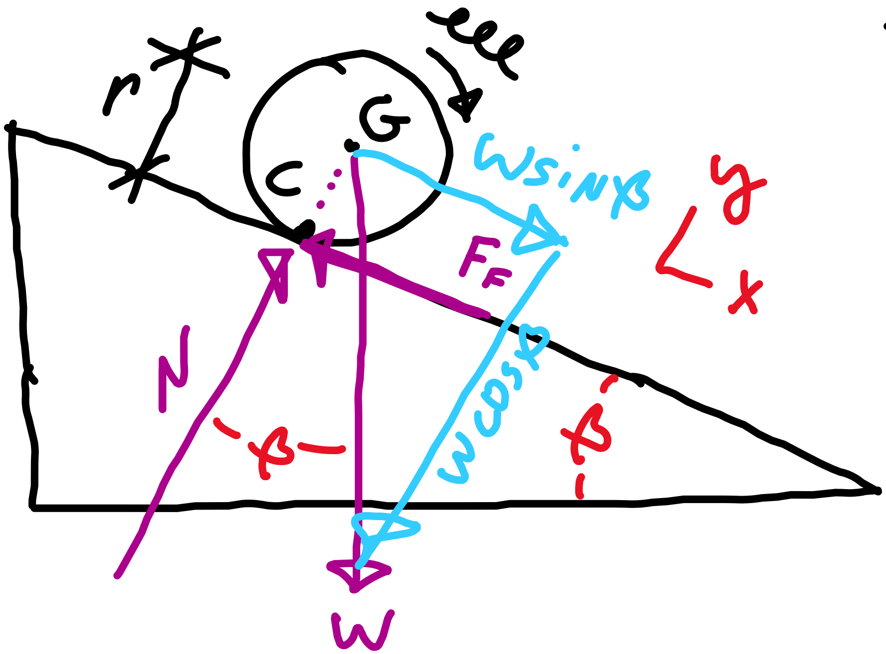

Find α:

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$$\vec{W} = W\sin\beta\,\hat{x} - W\cos\beta\,\hat{y}$$

Example: Wheel on incline

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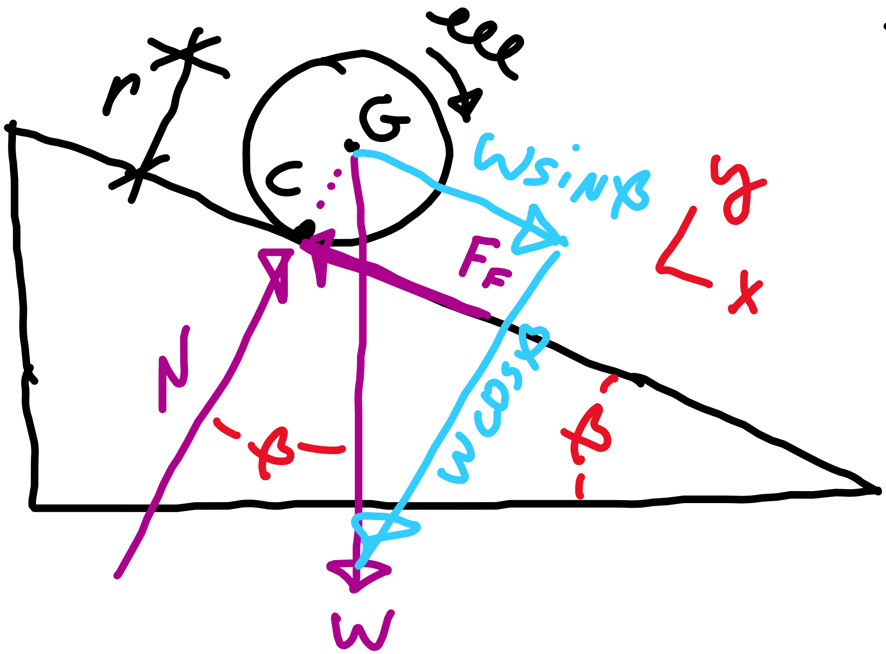


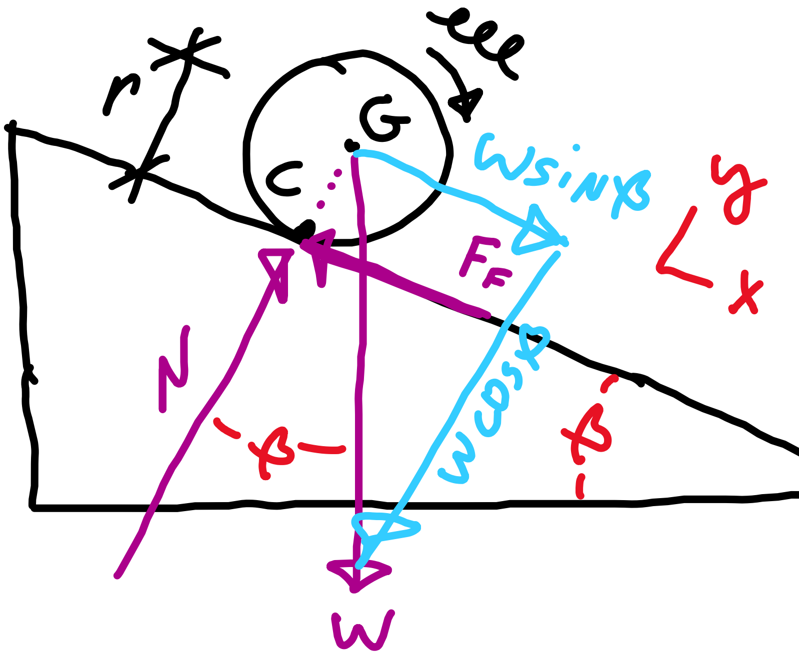
$$\curvearrowright \sum M_c = I_c \alpha$$

$$\vec{W} = W\sin\beta\,\hat{x} - W\cos\beta\,\hat{y}$$

$$\Rightarrow$$

$$W\sin\beta\, r = (\bar{I} + mr^2)\alpha$$

Example: Wheel on incline



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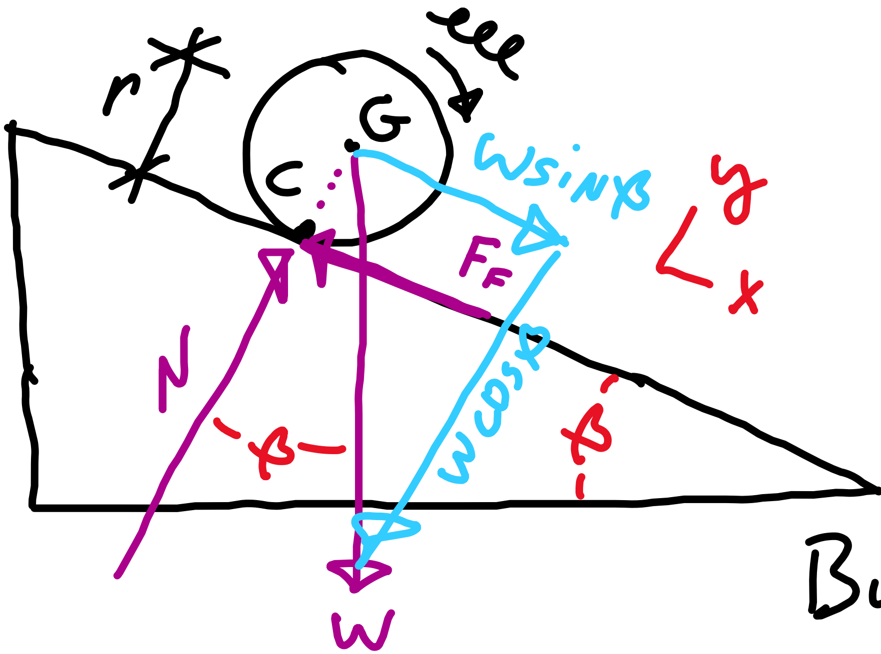
But $\bar{I} = mr^2/2$

Example: Wheel on incline



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$$\vec{W} = W\sin\beta\,\hat{x} - W\cos\beta\,\hat{y}$$

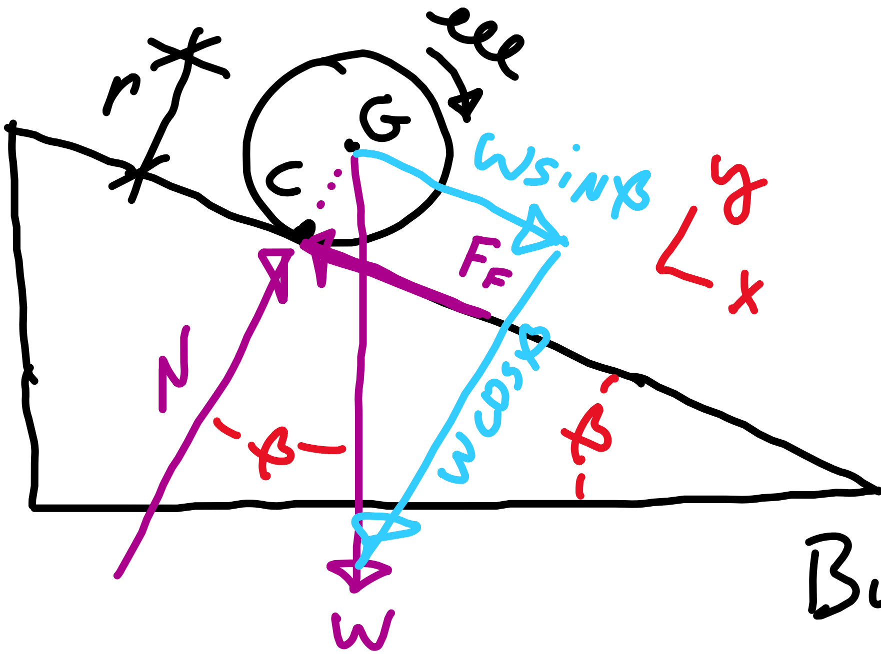
$$\Rightarrow$$

$$W\sin\beta\, r = (\bar{I} + mr^2)\alpha$$

But $\bar{I} = mr^2/2$ so

$$W\sin\beta\, r = \frac{3}{2} m r^2 \alpha$$

Example: Wheel on incline



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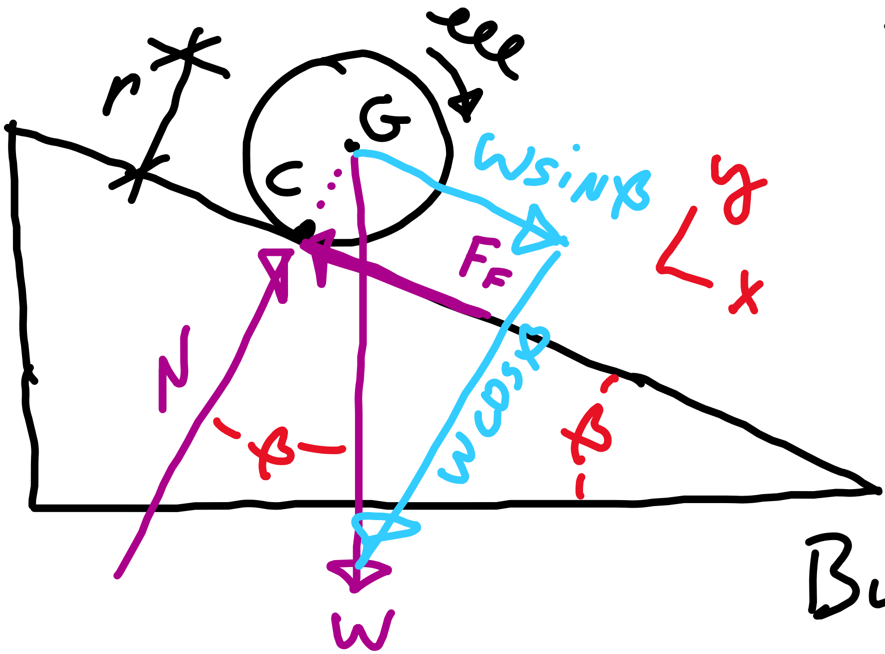
But $\bar{I} = m r^2/2$ so

$$W \sin\beta\, r = \frac{3}{2} m r^2 \alpha \quad \Rightarrow$$

$$\alpha = \frac{2 W \sin\beta}{3 m r}$$

Example: Wheel on incline



Find α:

$$\circlearrowleft \underline{\sum M_c} = \underline{I_c \alpha}$$

$$\vec{W} = w\sin\beta\,\hat{x} - w\cos\beta\,\hat{y}$$

$$\Rightarrow$$

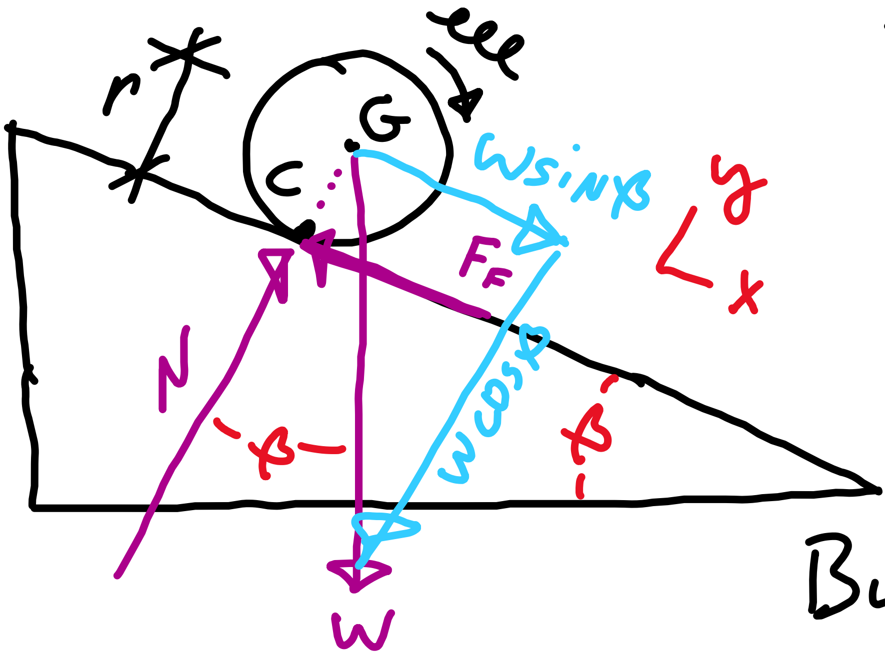
$$w\sin\beta\, r = (\bar{I} + mr^2)\alpha$$

But $\bar{I} = mr^2/2$ so

$$w\sin\beta\, r = \tfrac{3}{2} m r^2 \alpha \Rightarrow$$

$$\alpha = \frac{2w\sin\beta}{3mr} = \frac{2g\sin\beta}{3r}$$

Example: Wheel on incline



Find α:

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$\Rightarrow$

$w\sin\beta\, r = (\bar{I} + mr^2)\alpha$

But $\bar{I} = mr^2/2$ so

$$w\sin\beta\, r = \frac{3}{2} m r^2 \alpha \Rightarrow$$

$$\boxed{\alpha = \frac{2w\sin\beta}{3mr} = \frac{2g\sin\beta}{3r}}$$

we saw that for a

<u>fixed point rotation</u> of a rigid body

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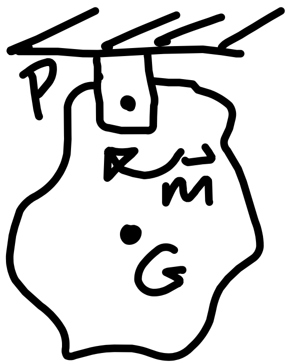
<u>fixed point rotation</u> of a rigid body



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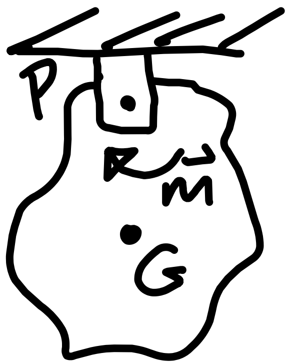
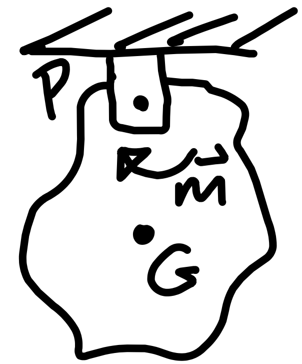


$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$

we saw that for a

<u>Fixed point rotation</u> of a rigid body



$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$ <u>or</u>

$$\sum \vec{M}_P = I_P \vec{\alpha} ,$$

we saw that for a

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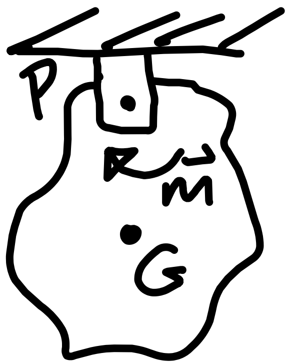

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$ <u>or</u>

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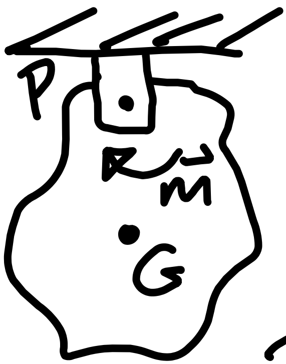
Fixed point rotation of a rigid body



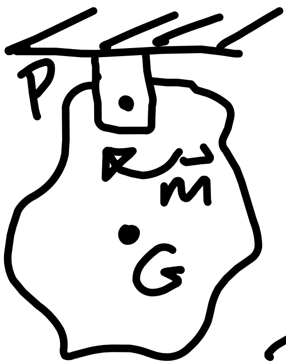
$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$ or

$$\sum \vec{M}_P = I_P \vec{\alpha},$$ where $I_P = \bar{I} + m r_{G/P}^2$

For a fixed point of rotation it is,

we saw that for a

Fixed point rotation of a rigid body



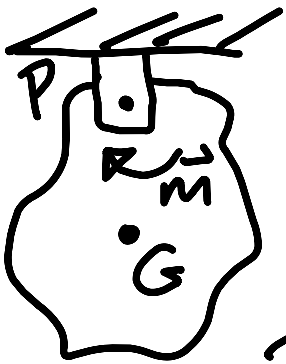
$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$ or

$$\sum \vec{M}_P = I_P\vec{\alpha}, \text{ where } I_P = \bar{I} + m r_{G/P}^2$$

For a fixed point of rotation it is, for me,

we saw that for a

Fixed point rotation of a rigid body



$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}$$ **or**

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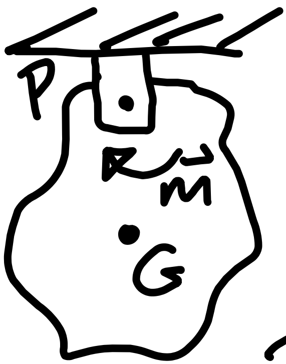
For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$

we saw that for a

<u>fixed point rotation</u> of a rigid body

$$\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \underline{or}$$



$$\Sigma \vec{M}_P = I_P \vec{\alpha}, \text{ where } I_P = \bar{I} + m r_{G/P}^2$$

For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$ and use the form $\Sigma \vec{M}_P = I_P \vec{\alpha}$

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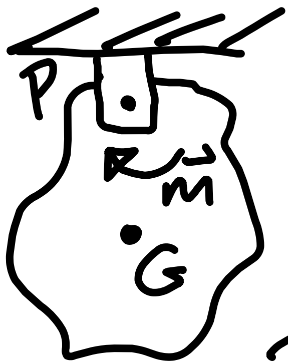

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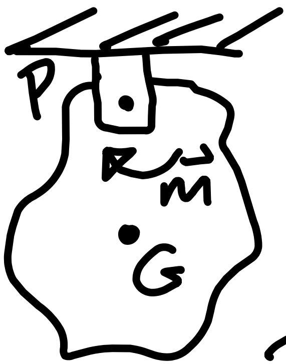
For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$ and use the form $\Sigma \vec{M}_P = I_P\vec{\alpha}$. However,

we saw that for a

<u>fixed point rotation</u> of a rigid body



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For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$ and use the form $\Sigma \vec{M}_P = I_P \vec{\alpha}$. However, for problems that do not have a fixed point of rotation,

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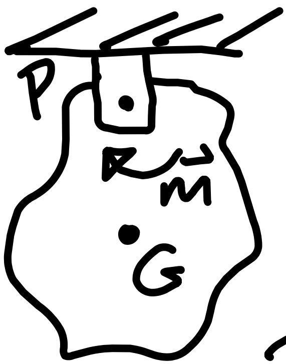
<u>Fixed point rotation</u> of a rigid body



$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \underline{\text{or}}$$

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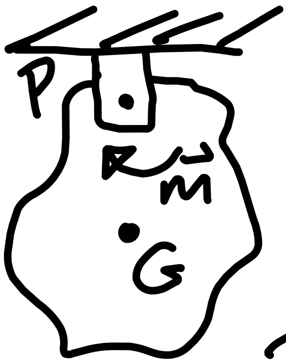
we saw that for a

Fixed point rotation of a rigid body

$$\sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \underline{\text{or}}$$



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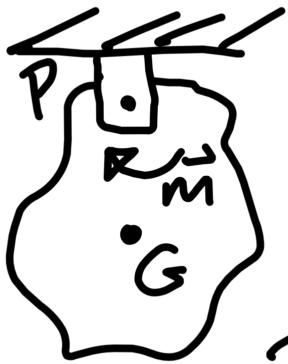
$$\left\{ \sum \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \right\}$$

we saw that for a

<u>Fixed point rotation</u> of a rigid body



$$\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}} \quad \underline{\text{or}}$$

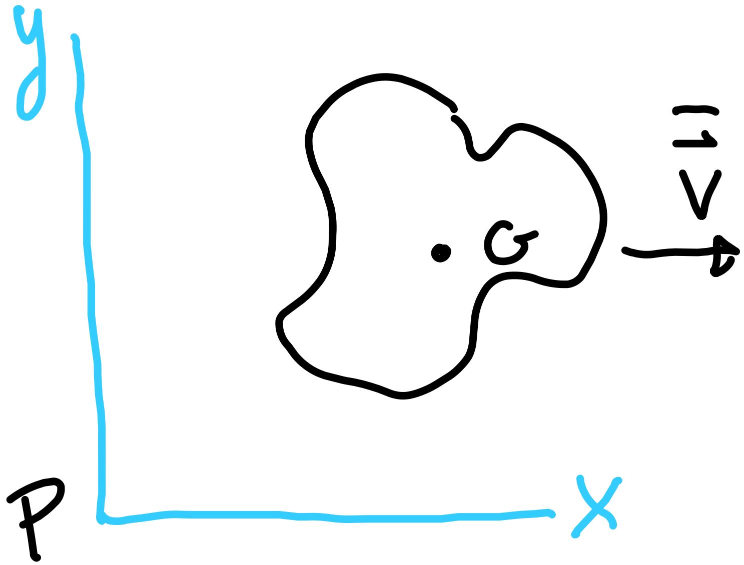
$$\Sigma \vec{M}_P = I_P \vec{\alpha}, \text{ where } I_P = \bar{I} + m r_{G/P}^2$$

For a fixed point of rotation it is, for me, more natural to think about the resulting motion in terms of $I_P\vec{\alpha}$ and use the form $\Sigma \vec{M}_P = I_P \vec{\alpha}$. However, for problems that do not have a fixed point of rotation, the other form

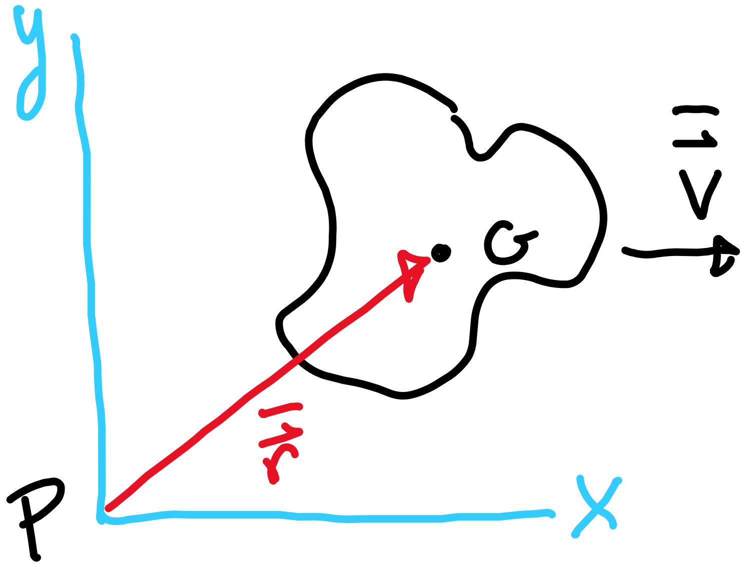
$\{\Sigma \vec{M}_P = \bar{I}\vec{\alpha} + \vec{r}_{G/P} \times m\vec{\bar{a}}\}$ must be used

Example: rigid body in plane motion that is translating & rotating.

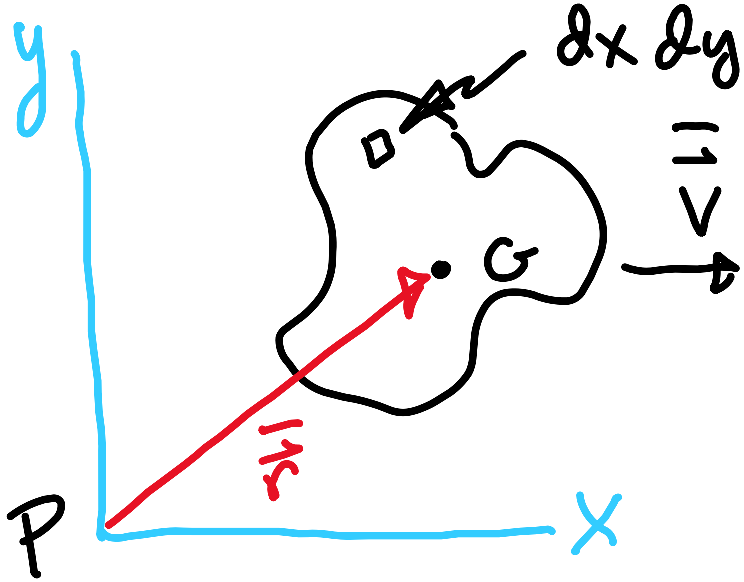
Example: rigid body in plane motion that is translating & rotating.



Example: rigid body in plane motion that is translating & rotating.



Example: rigid body in plane motion that is translating & rotating.



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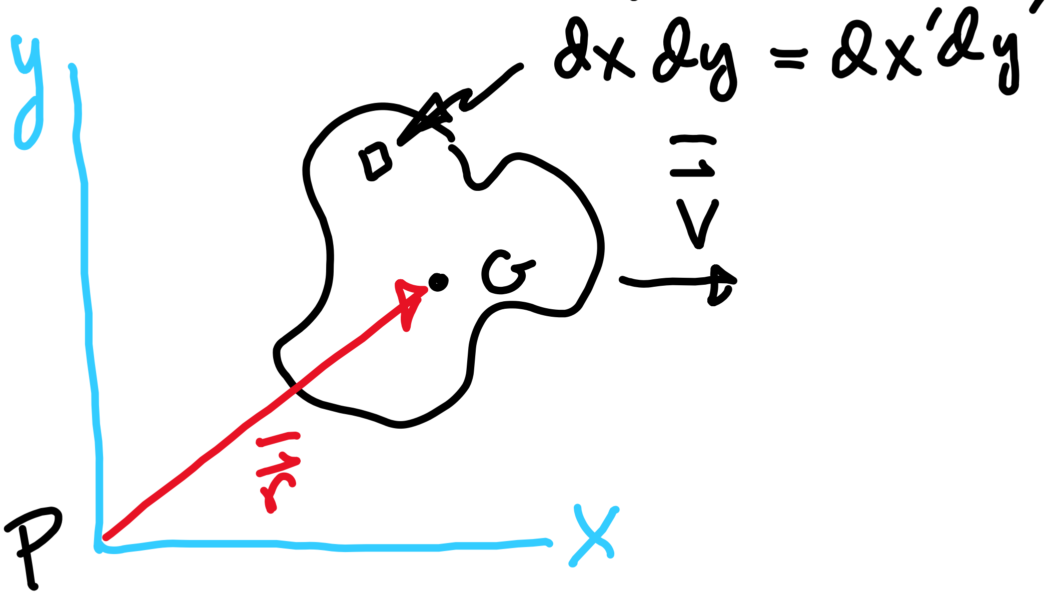
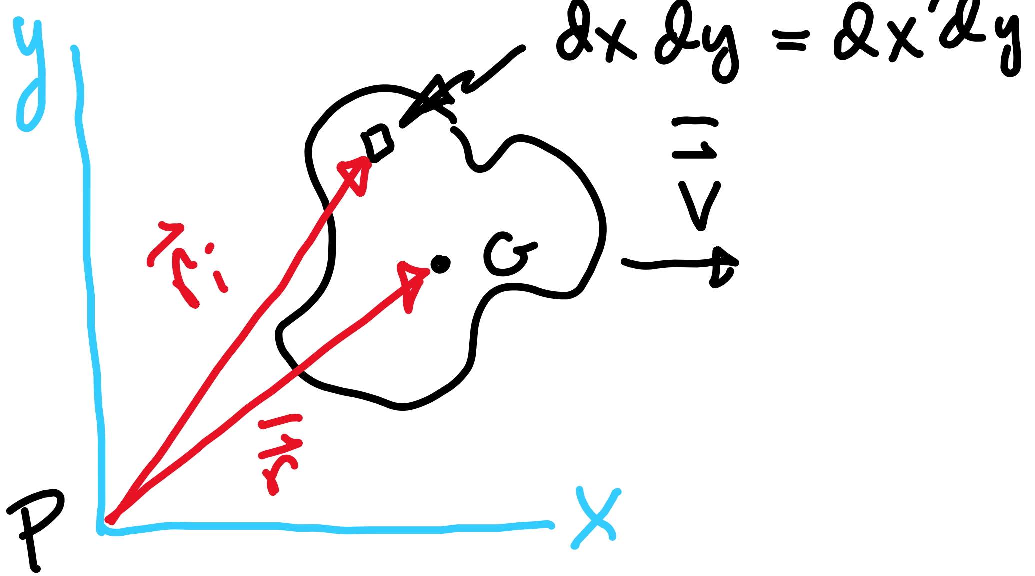


Example: rigid body in plane motion that is translating & rotating.



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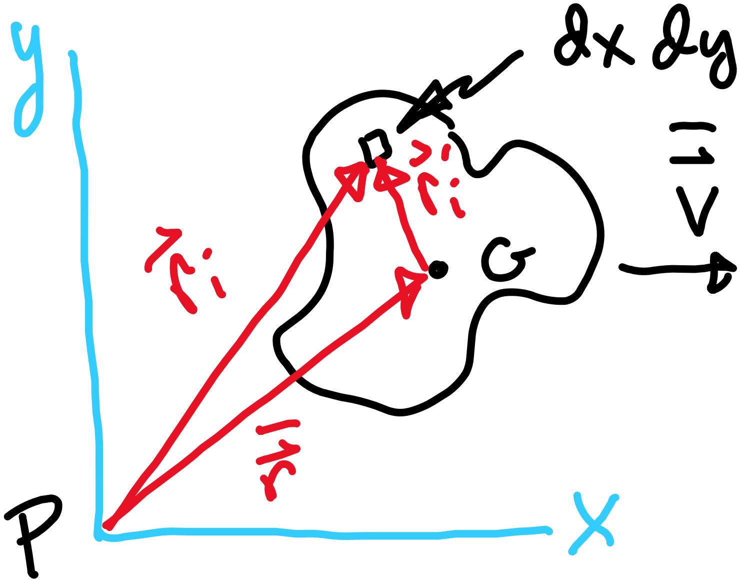

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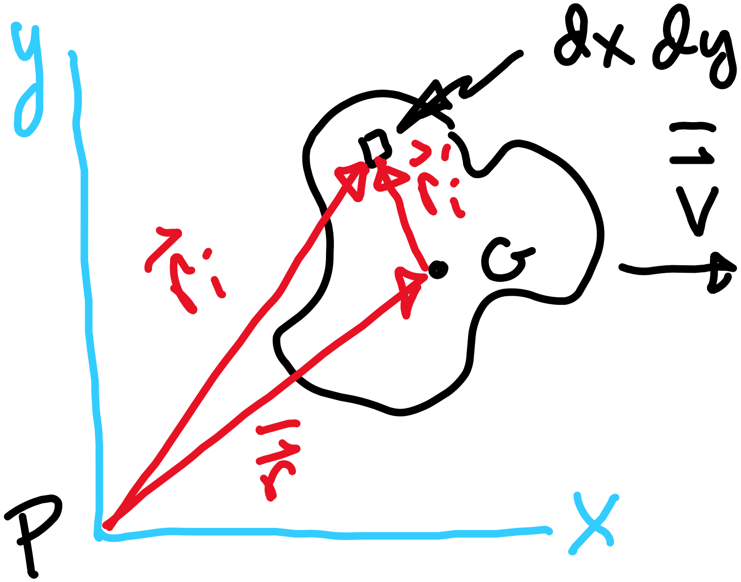


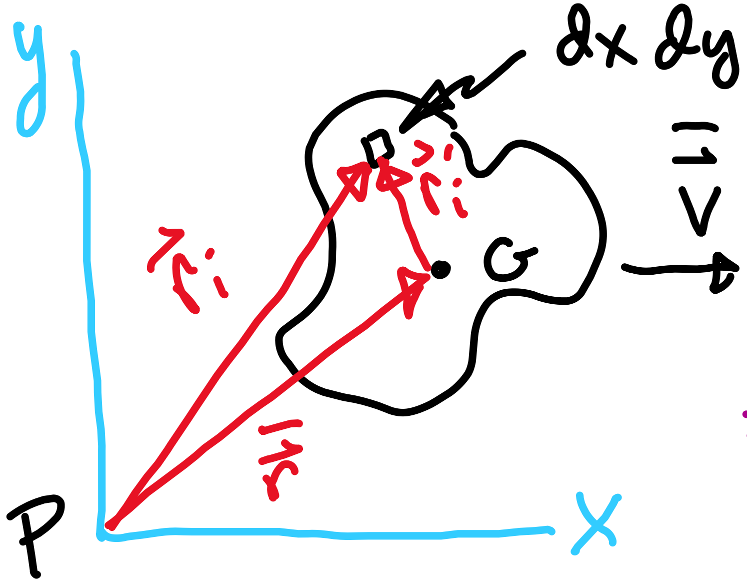
$dx\,dy = dx'dy'$

The angular momentum about p is

$$\vec{H}_p = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum \left(\vec{r}_i \times \frac{m}{A} \vec{V}_i\right) \Delta x_i \Delta y_i$$

Example: rigid body in plane motion that is translating & rotating.

$dx\,dy = dx'dy'$



The angular momentum about P is

$$\vec{H}_P = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum \left(\vec{r}_i \times \frac{m}{A}\vec{V}_i\right) \Delta x_i \Delta y_i$$

$$\Rightarrow \vec{H}_P = \left(\frac{m}{A}\right) \iint \vec{r} \times \vec{V}\, dx'dy'.$$

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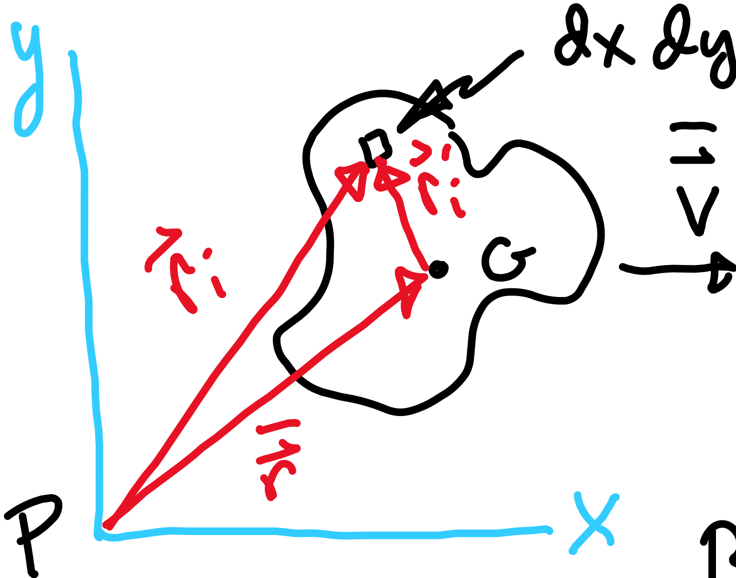

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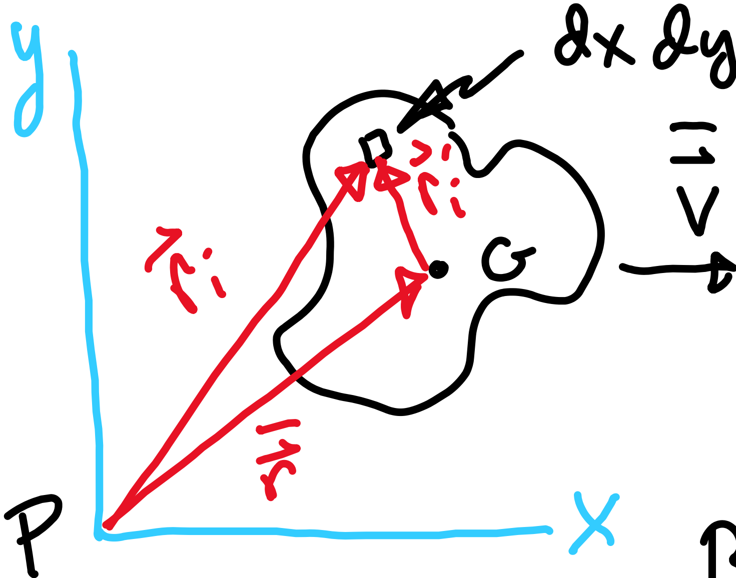
$$\Rightarrow \vec{H}_P = \left(\frac{m}{A}\right) \iint \vec{r} \times \vec{V}\, dx'dy',$$

But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$

Example: rigid body in plane motion that is translating & rotating.



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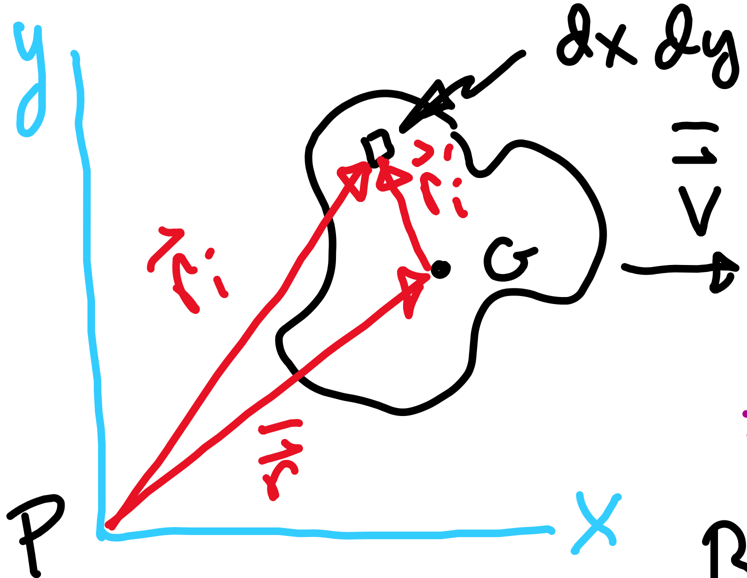
But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

Example: rigid body in plane motion that is translating & rotating.



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But $\vec{r} = \vec{\bar{r}} + \vec{r}\,'$ & $\vec{v} = \vec{\bar{v}} + \vec{v}\,'$

So $\vec{H}_P = \left(\frac{m}{A}\right) \iint \vec{\bar{r}} \times \vec{\bar{v}}\, dx'dy' +$

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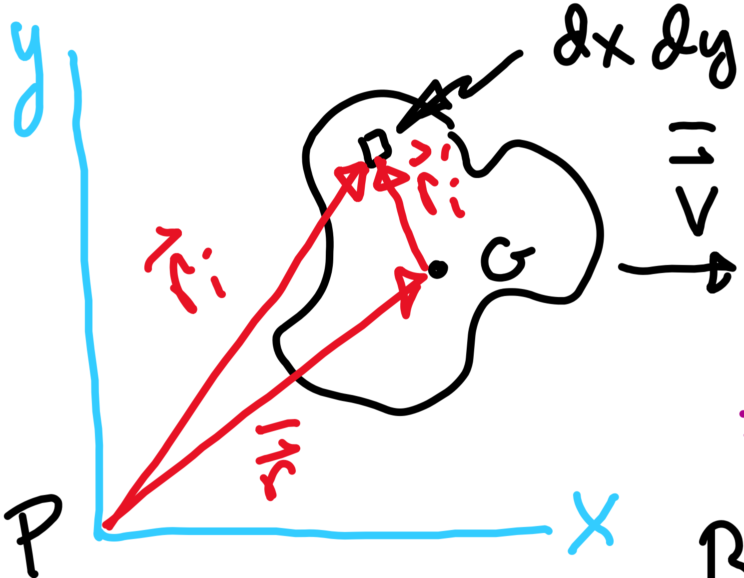

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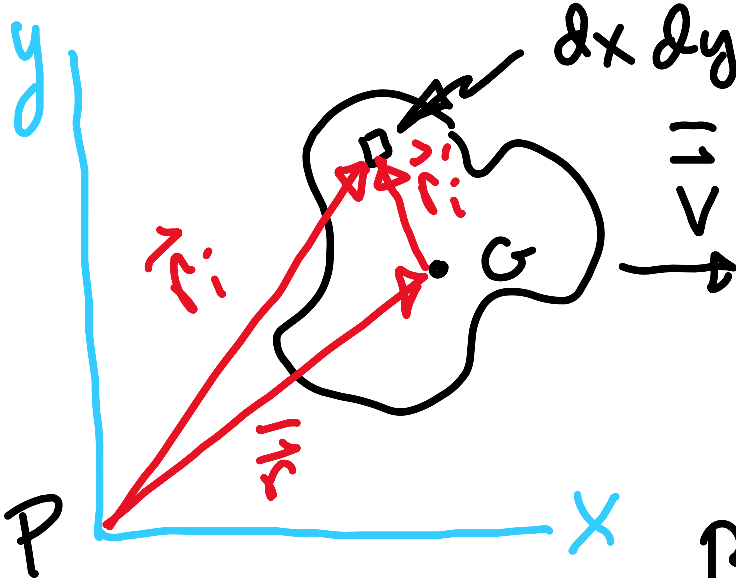
$$\Rightarrow \vec H_p = \left(\frac{m}{A}\right) \iint \vec r \times \vec v \, dx'dy',$$

But $\vec r = \bar{\vec r} + \vec r'$ & $\vec v = \bar{\vec v} + \vec v'$

So $\vec H_p = \left(\frac{m}{A}\right) \iint \bar{\vec r} \times \bar{\vec v} \, dx'dy' + \left(\frac{m}{A}\right) \iint \bar{\vec r} \times \vec v' \, dx'dy' +$

<u>Example</u>: rigid body in plane motion that is translating & rotating.



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But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

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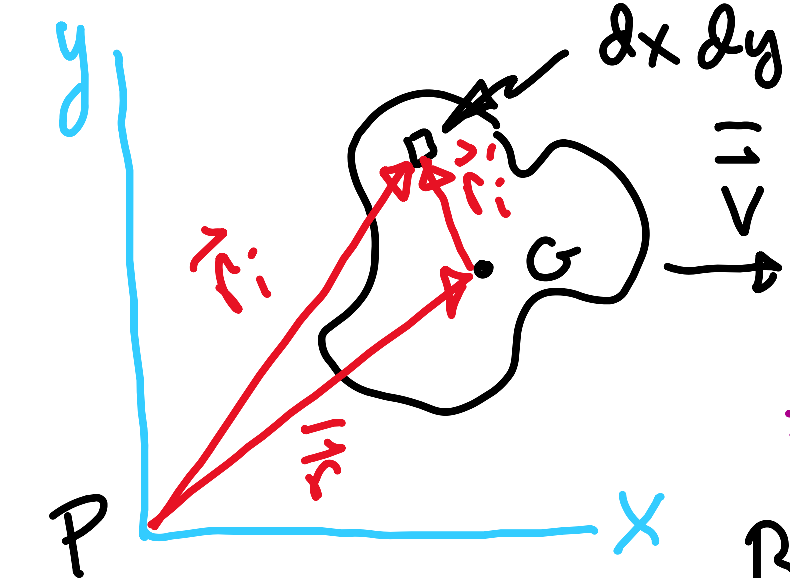
Example: rigid body in plane motion that is translating & rotating.



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So $\vec{H}_P = \left(\frac{m}{A}\right)\iint \bar{\vec{r}}\times\bar{\vec{v}}\,dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec{r}}\times\vec{v}'\,dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}'\times\bar{\vec{v}}\,dx'dy' + \left(\frac{m}{A}\right)\iint \underline{\vec{r}'}\times\underline{\vec{v}'}\,dx'dy'$

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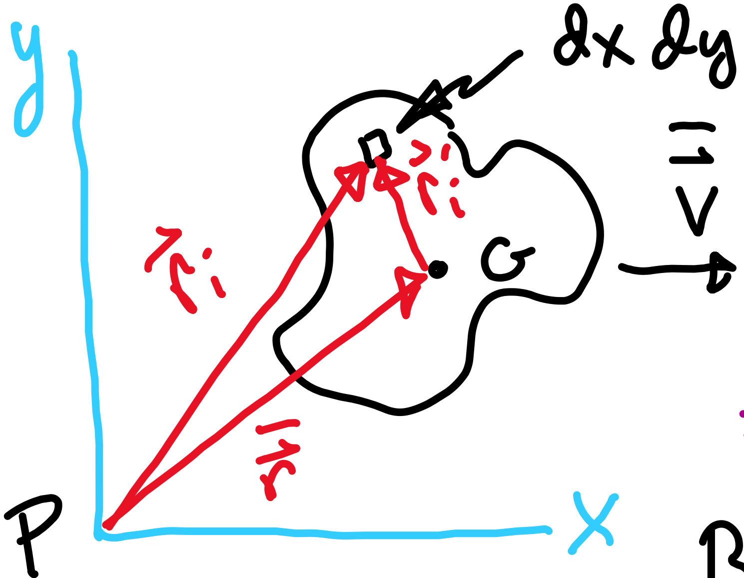

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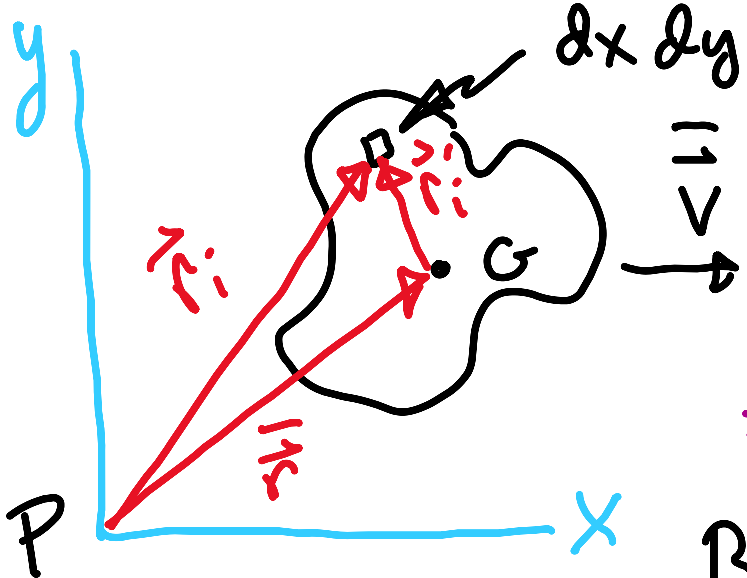
So $\vec{H}_P = \left(\frac{m}{A}\right)\iint \vec{\bar{r}} \times \vec{\bar{v}}\, dx'dy' + \left(\frac{m}{A}\right)\iint \vec{\bar{r}} \times \vec{v}\,'\, dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{\bar{v}}\, dx'dy' +$

$\left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,'\, dx'dy' = \left(\frac{m}{A}\right) \vec{\bar{r}} \times \vec{\bar{v}} \iint dx'dy' +$

Example: rigid body in plane motion that is translating & rotating.



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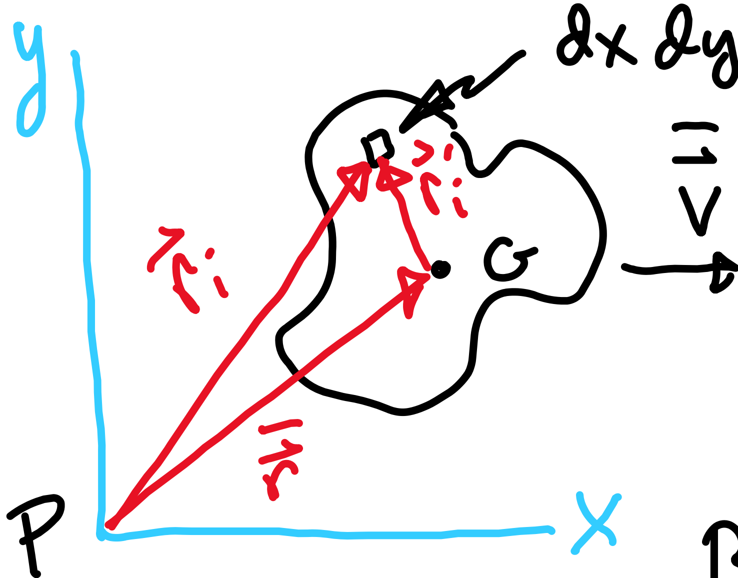
But $\vec r = \bar{\vec r} + \vec r'$ & $\vec v = \bar{\vec v} + \vec v'$

So $\vec H_P = \left(\frac{m}{A}\right)\iint \bar{\vec r} \times \bar{\vec v}\, dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec r} \times \vec v'\, dx'dy' + \left(\frac{m}{A}\right)\iint \vec r' \times \bar{\vec v}\, dx'dy' +$

$\left(\frac{m}{A}\right)\iint \vec r' \times \vec v'\, dx'dy' = \left(\frac{m}{A}\right)\bar{\vec r} \times \bar{\vec v} \iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec r} \times \iint \vec v'\, dx'dy' +$

Example: rigid body in plane motion that is translating & rotating.



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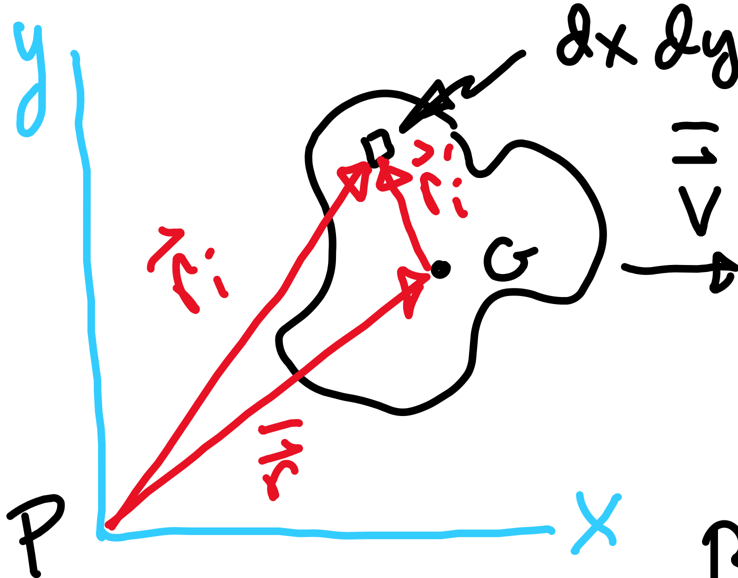
$$\Rightarrow \vec{H}_P = \left(\frac{m}{A}\right) \iint \vec{r} \times \vec{v}\, dx'dy'.$$

But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

So $\vec{H}_P = \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \bar{\vec{v}}\, dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \vec{v}\,'\, dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \bar{\vec{v}}\, dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,'\, dx'dy' = \left(\frac{m}{A}\right)\bar{\vec{r}} \times \bar{\vec{v}} \iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec{r}} \times \iint \vec{v}\,'\, dx'dy' + \left(\frac{m}{A}\right)\left[\iint \vec{r}\,'\, dx'dy'\right] \times \bar{\vec{v}} +$

Example: rigid body in plane motion that is translating & rotating.



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$$\Rightarrow \vec{H}_P = \left(\frac{m}{A}\right) \iint \vec{r} \times \vec{v}\, dx'dy'.$$

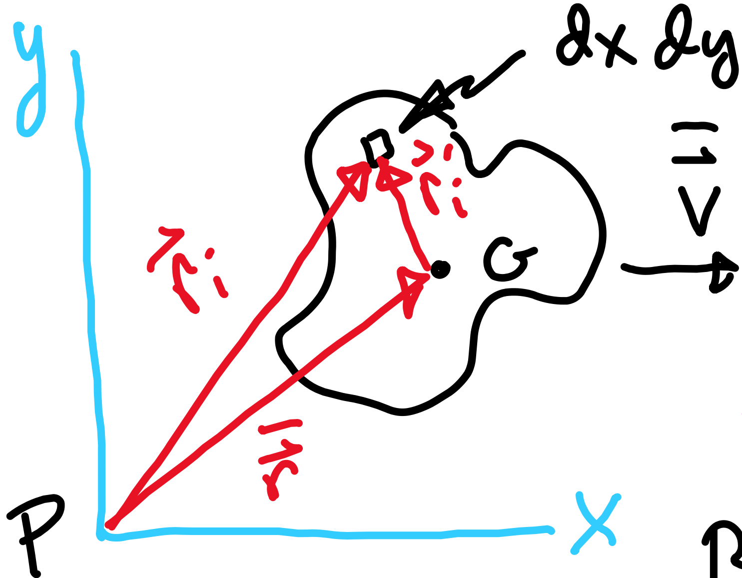
But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

So $\vec{H}_P = \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \bar{\vec{v}}\, dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \vec{v}\,'dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \bar{\vec{v}}\, dx'dy' +$

$\left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,' dx'dy' = \left(\frac{m}{A}\right)\bar{\vec{r}} \times \bar{\vec{v}} \iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec{r}} \times \iint \vec{v}\,' dx'dy' +$

$\left(\frac{m}{A}\right)\left[\iint \vec{r}\,' dx'dy'\right] \times \bar{\vec{v}} + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,' dx'dy' =$

Example: rigid body in plane motion that is translating & rotating.



$dx\,dy = dx'dy'$

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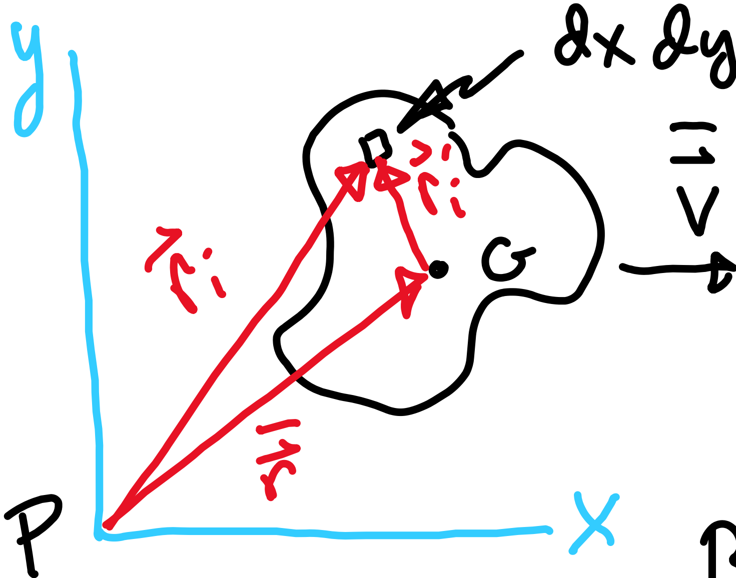
$$\Rightarrow \vec{H}_P = \left(\frac{m}{A}\right)\iint \vec{r}\times\vec{v}\,dx'dy'.$$

But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

So $\vec{H}_P = \left(\frac{m}{A}\right)\iint \bar{\vec{r}}\times\bar{\vec{v}}\,dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec{r}}\times\vec{v}\,'dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,'\times\bar{\vec{v}}\,dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,'\times\vec{v}\,'dx'dy' = \left(\frac{m}{A}\right)\bar{\vec{r}}\times\bar{\vec{v}}\iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec{r}}\times\iint \vec{v}\,'dx'dy' + \left(\frac{m}{A}\right)\left[\iint \vec{r}\,'dx'dy'\right]\times\bar{\vec{v}} + \left(\frac{m}{A}\right)\iint \vec{r}\,'\times\vec{v}\,'dx'dy' = m\bar{\vec{r}}\times\bar{\vec{v}} +$

Example: rigid body in plane motion that is translating & rotating.



The angular momentum about p is

$$\vec{H}_p = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum \left(\vec{r}_i \times \tfrac{m}{A}\vec{v}_i\right)\Delta x_i \Delta y_i$$

$$\Rightarrow \vec{H}_p = \left(\tfrac{m}{A}\right)\iint \vec{r} \times \vec{v}\, dx'dy'.$$

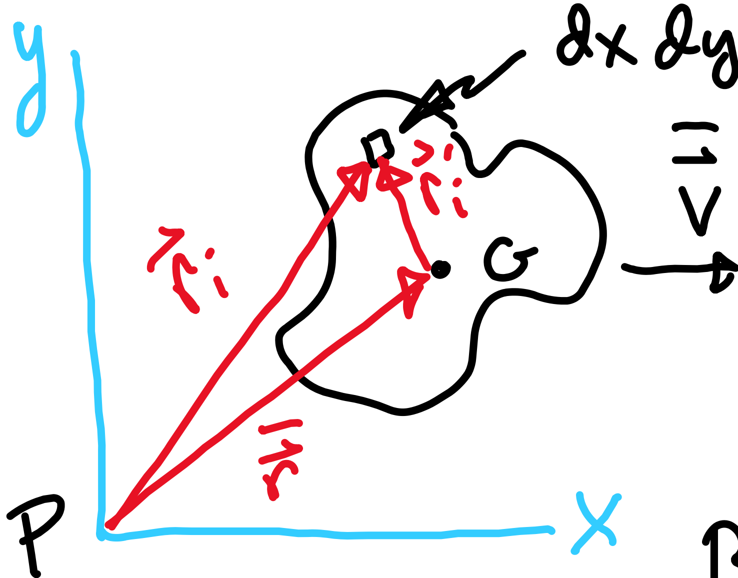
But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

So
$$\vec{H}_p = \left(\tfrac{m}{A}\right)\iint \bar{\vec{r}} \times \bar{\vec{v}}\, dx'dy' + \left(\tfrac{m}{A}\right)\iint \bar{\vec{r}} \times \vec{v}\,'dx'dy' + \left(\tfrac{m}{A}\right)\iint \vec{r}\,' \times \bar{\vec{v}}\, dx'dy' +$$
$$\left(\tfrac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,' dx'dy' = \left(\tfrac{m}{A}\right)\bar{\vec{r}} \times \bar{\vec{v}} \iint dx'dy' + \left(\tfrac{m}{A}\right)\bar{\vec{r}} \times \iint \vec{v}\,'dx'dy' +$$
$$\left(\tfrac{m}{A}\right)\left[\iint \vec{r}\,'dx'dy'\right] \times \bar{\vec{V}} + \left(\tfrac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,'dx'dy' = m\,\bar{\vec{r}} \times \bar{\vec{v}} + 0$$

ASU

Example: rigid body in plane motion that is translating & rotating.

$dx\,dy = dx'dy'$



The angular momentum about P is

$$\vec{H}_P = \lim_{\substack{\Delta x_i \to 0 \\ \Delta y_i \to 0}} \sum \left(\vec{r}_i \times \frac{m}{A}\vec{v}_i\right) \Delta x_i \Delta y_i$$

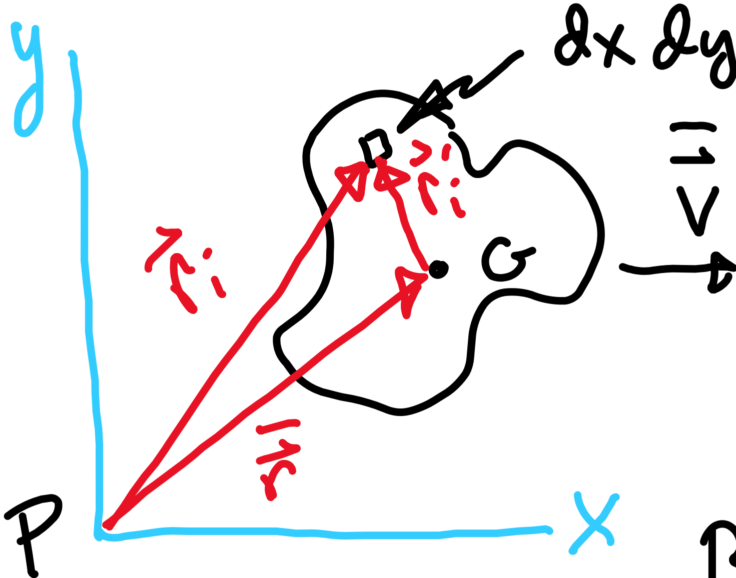
$$\Rightarrow \vec{H}_P = \left(\frac{m}{A}\right) \iint \vec{r} \times \vec{v}\, dx'dy'.$$

But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

So
$$\vec{H}_P = \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \bar{\vec{v}}\, dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \vec{v}\,'\, dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \bar{\vec{v}}\, dx'dy' +$$
$$\left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,'\, dx'dy' = \left(\frac{m}{A}\right)\bar{\vec{r}} \times \bar{\vec{v}} \iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec{r}} \times \iint \vec{v}\,'\, dx'dy' +$$
$$\left(\frac{m}{A}\right)\left[\iint \vec{r}\,'\, dx'dy'\right] \times \bar{\vec{v}} + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,'\, dx'dy' = m\,\bar{\vec{r}} \times \bar{\vec{v}} + 0 + 0 +$$

ASU

Example: rigid body in plane motion that is translating & rotating.



$dx\,dy = dx'dy'$

The angular momentum about P is

$$\vec H_P = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \sum \left(\vec r_i \times \frac{m}{A} \vec v_i\right) \Delta x_i \Delta y_i$$

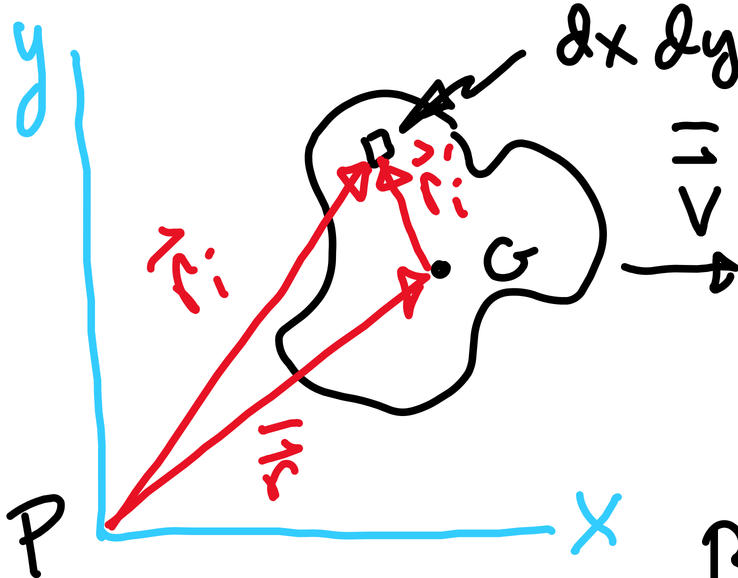
$$\Rightarrow \vec H_P = \left(\frac{m}{A}\right) \iint \vec r \times \vec v \, dx'dy'.$$

But $\vec r = \bar{\vec r} + \vec r'$ & $\vec v = \bar{\vec v} + \vec v'$

So

$$\vec H_P = \left(\frac{m}{A}\right)\iint \bar{\vec r} \times \bar{\vec v}\, dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec r} \times \vec v'\, dx'dy' + \left(\frac{m}{A}\right)\iint \vec r' \times \bar{\vec v}\, dx'dy' + \left(\frac{m}{A}\right)\iint \vec r' \times \vec v'\, dx'dy' = \left(\frac{m}{A}\right)\bar{\vec r} \times \bar{\vec v} \iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec r} \times \iint \vec v'\, dx'dy' + \left(\frac{m}{A}\right)\left[\iint \vec r'\, dx'dy'\right] \times \bar{\vec V} + \left(\frac{m}{A}\right)\iint \vec r' \times \vec v'\, dx'dy' = m\,\bar{\vec r} \times \bar{\vec v} + 0 + 0 + \vec H_G$$

Example: rigid body in plane motion that is translating & rotating.



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The angular momentum about P is

$$\vec{H}_P = \lim_{\substack{\Delta x_i \to 0 \\ \Delta y_i \to 0}} \sum \left(\vec{r}_i \times \frac{m}{A}\vec{v}_i\right) \Delta x_i \Delta y_i$$

$$\Rightarrow \vec{H}_P = \left(\frac{m}{A}\right) \iint \vec{r} \times \vec{v}\, dx'dy'.$$

But $\vec{r} = \bar{\vec{r}} + \vec{r}\,'$ & $\vec{v} = \bar{\vec{v}} + \vec{v}\,'$

So
$$\vec{H}_P = \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \bar{\vec{v}}\, dx'dy' + \left(\frac{m}{A}\right)\iint \bar{\vec{r}} \times \vec{v}\,' dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \bar{\vec{v}}\, dx'dy' + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,' dx'dy' = \left(\frac{m}{A}\right)\bar{\vec{r}} \times \bar{\vec{v}} \iint dx'dy' + \left(\frac{m}{A}\right)\bar{\vec{r}} \times \iint \vec{v}\,' dx'dy' + \left(\frac{m}{A}\right)\left[\iint \vec{r}\,' dx'dy'\right] \times \bar{\vec{v}} + \left(\frac{m}{A}\right)\iint \vec{r}\,' \times \vec{v}\,' dx'dy' = m\bar{\vec{r}} \times \bar{\vec{v}} + 0 + 0 + \vec{H}_G$$

$$\Rightarrow \boxed{\vec{H}_P = m\bar{\vec{r}} \times \bar{\vec{v}} + \vec{H}_G}$$

From previous slide

From previous slide

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$\vec{H}_P = m\bar{\vec{r}} \times \bar{\vec{v}} + \vec{H}_G$ so $\dot{\vec{H}}_P = m\frac{d}{dt}\left[\bar{\vec{r}} \times \bar{\vec{v}}\right] + \dot{\vec{H}}_G$

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$= m\vec{v}\times\vec{v} + m\,\vec{r}\times\vec{a} +$

From previous slide

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ASU

<u>From previous slide</u>

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From previous slide

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From previous slide

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From previous slide

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For this case our moments [torques]

From previous slide

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then $\Sigma\vec{M}_P = \dot{\vec{H}}_P$ becomes

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For this case our moments [torques] can cause rotational motion about the center-of-mass G

From previous slide

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then $\sum\vec{M}_P = \dot{\vec{H}}_P$ becomes

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For this case our moments [torques] can cause rotational motion about the center-of-mass G & linear acceleration of the body

Remember,

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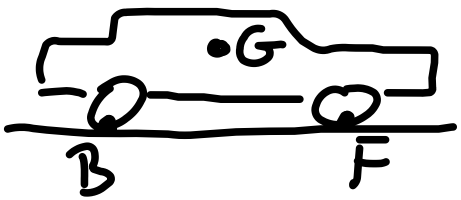

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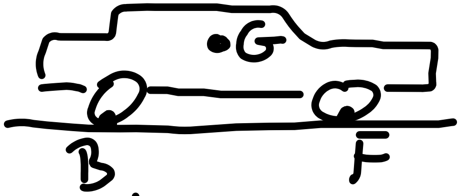

we could determine the acceleration of the car using sum of torques about points B or F while taking $\alpha_{car} = 0$.

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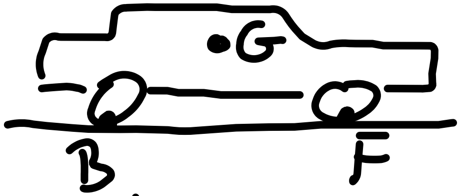

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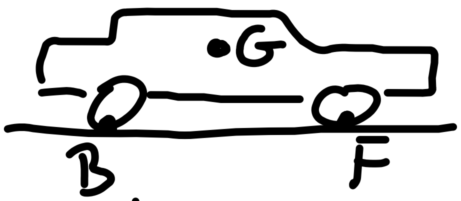

we could determine the acceleration of the car using sum of torques about points B or F while taking $\alpha_{car} = 0$. For a sum about B, we end up with $\Sigma \vec{M}_B = m\vec{r}_{G/B} \times \vec{a}$

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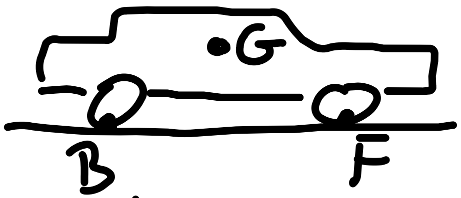

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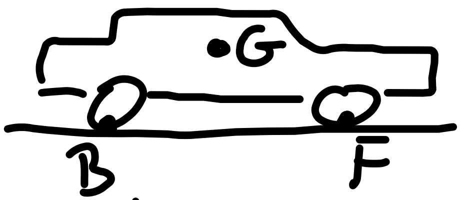

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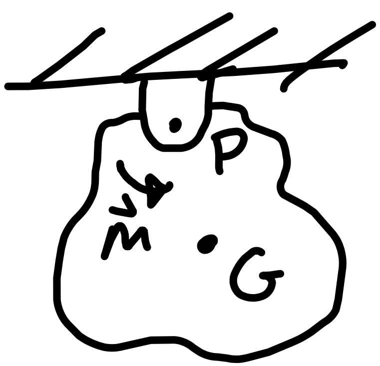

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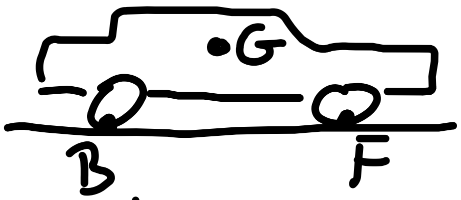

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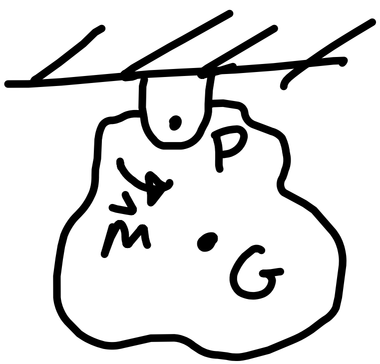

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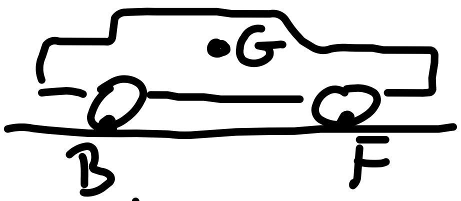

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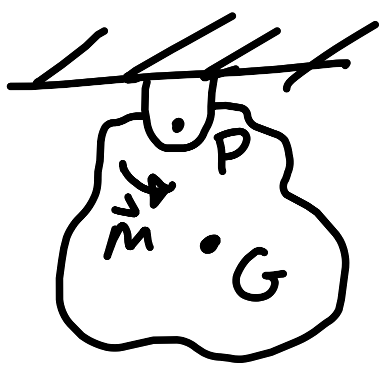

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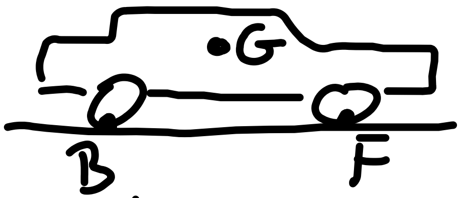

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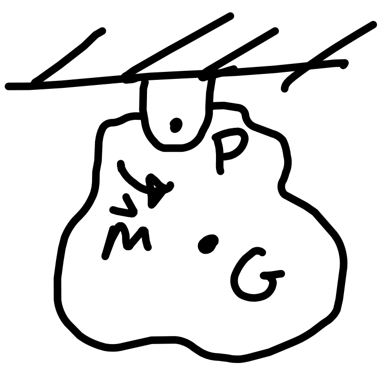

$$\Sigma \vec{M}_P = I_P \vec{\alpha}, \text{ where}$$

$$I_P = \bar{I} + m r_{G/P}^2$$