Today : 17.1

$\boxed{L18}$

$\boxed{L18}$

<u>Today</u> : 17.1

<u>Tuesday</u> : 17.2

Today : 17.1

L18

Tuesday : 17.2

Thursday 3-25 : Review

L18

Today : 17.1

Tuesday : 17.2

Thursday 3-25 : Review

Tuesday 3-30 : Exam #3

Work and energy

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$$U_{1\to 2} = T_2 - T_1 ,$$

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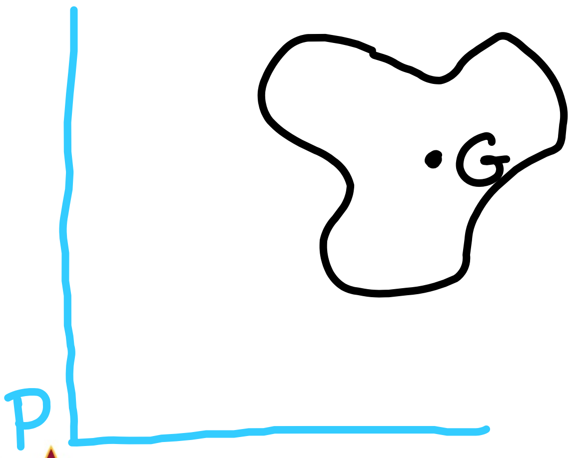

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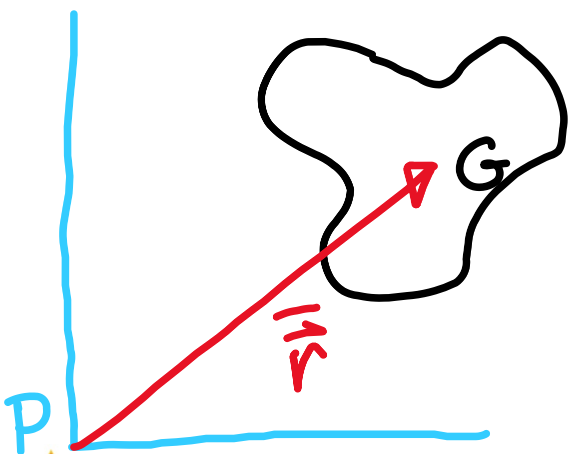

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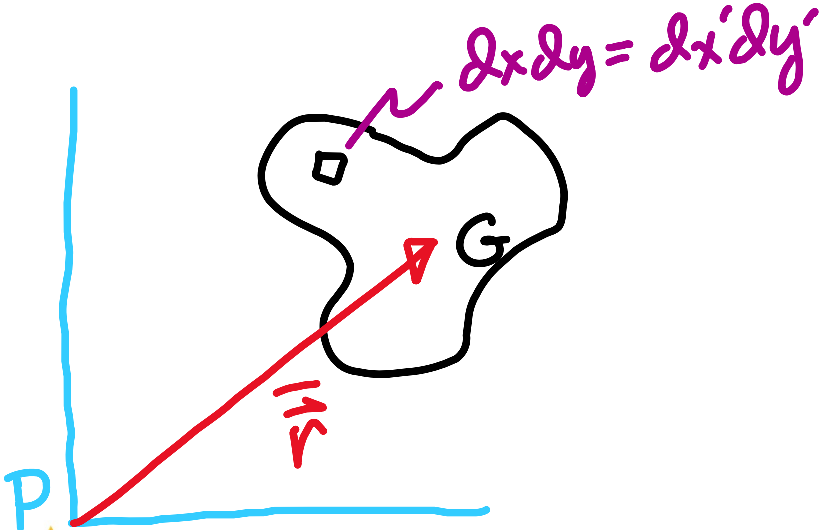

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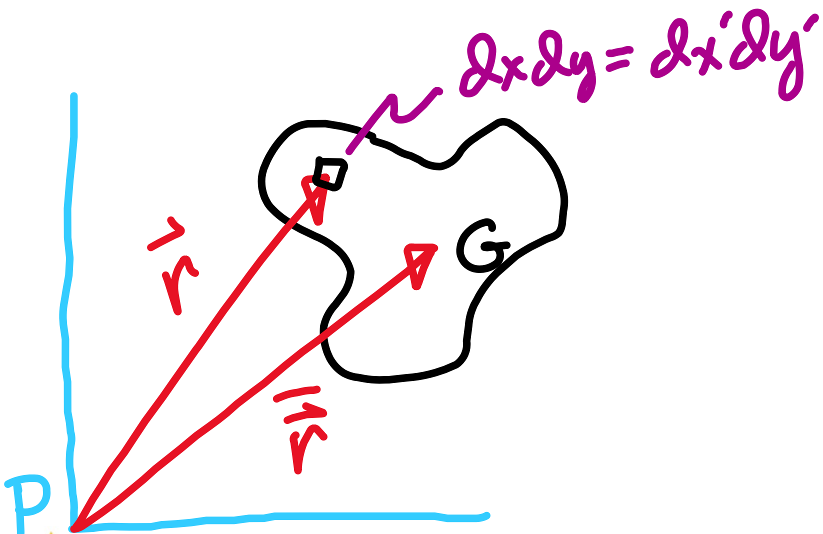

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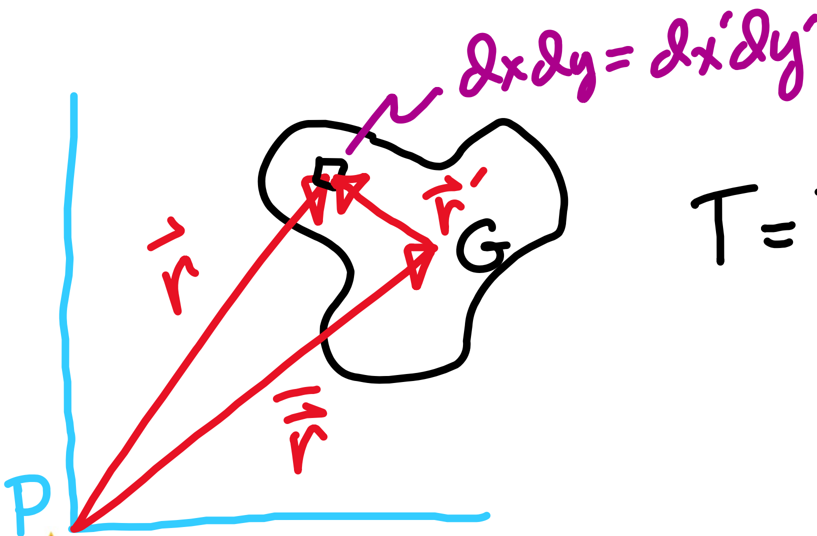

$$T = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{1}{2}\left(\frac{m}{A}\right) \sum v_i^2 \, \Delta x_i \, \Delta y_i$$

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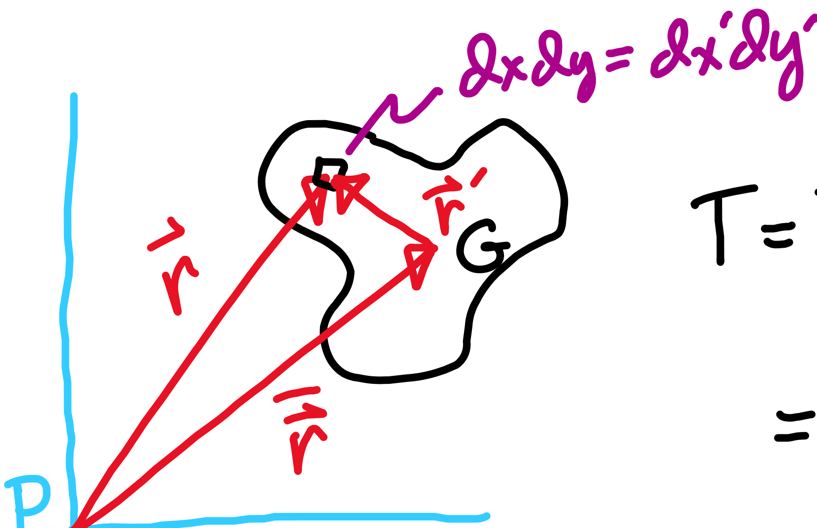

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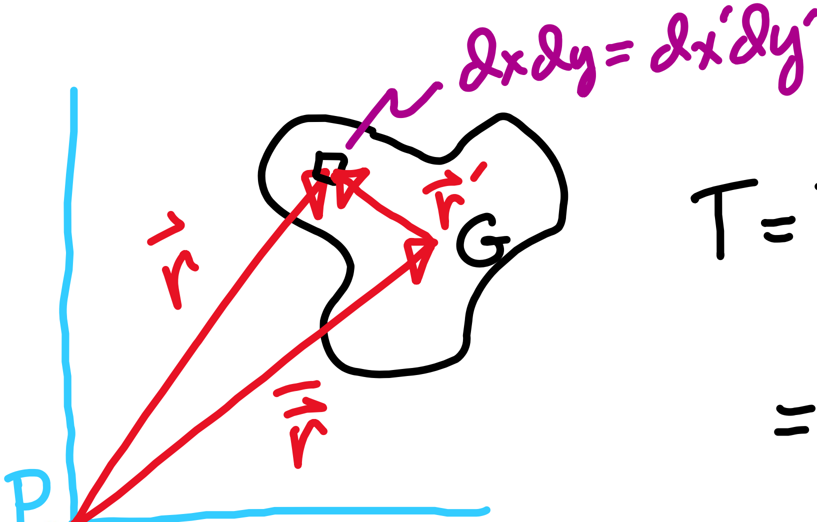

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Note: $\vec{r} = \vec{\bar{r}} + \vec{r}\,'$

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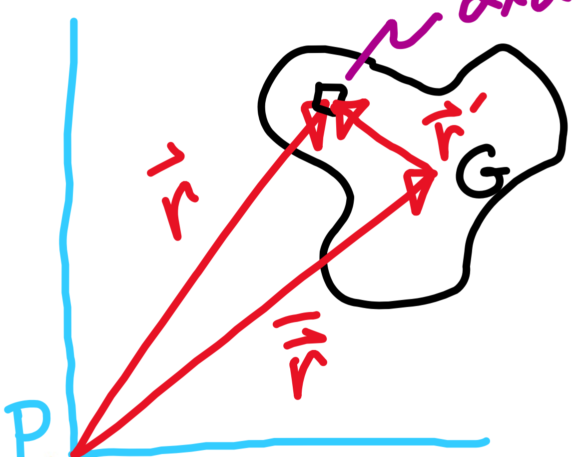
$dx\,dy = dx'\,dy'$



$$T = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{1}{2}\left(\frac{m}{A}\right) \sum v_i^2 \,\Delta x_i \,\Delta y_i$$

$$= \frac{1}{2}\left(\frac{m}{A}\right) \iint v^2 \,dx'\,dy'$$

Note: $\vec{r} = \vec{\bar{r}} + \vec{r}\,' \Rightarrow \vec{v} = \vec{\bar{v}} + \vec{v}\,'$

From previous slide $T = \frac{M}{A} \iint v^2 \, dx' \, dy'$

From previous slide $T = \frac{M}{A}\iint V^2\,dx'\,dy'$ &

$\vec{V} = \bar{\vec{V}} + \vec{V}'$

From previous slide $T = \frac{M}{A} \int\int \frac{1}{2} V^2 \, dx' dy'$ &

$$\vec{V} = \bar{\vec{V}} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\bar{\vec{V}} + \vec{V}') \cdot (\bar{\vec{V}} + \vec{V}')$$

From previous slide $T = \frac{m}{A} \int\int \frac{1}{2} V^2 \, dx' dy'$ &

$$\vec{V} = \vec{\bar{V}} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\vec{\bar{V}} + \vec{V}') \cdot (\vec{\bar{V}} + \vec{V}') \Rightarrow$$

$$V^2 = \bar{V}^2 + V'^2 + 2\vec{\bar{V}} \cdot \vec{V}'$$

From previous slide $T = \rho \iint \frac{1}{2} V^2 \, dx' dy'$ &

$\vec{V} = \bar{\vec{V}} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\bar{\vec{V}} + \vec{V}') \cdot (\bar{\vec{V}} + \vec{V}') \Rightarrow$

$V^2 = \bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}'$ so

$T = \left(\frac{1}{2}\rho\right) \iint \left(\bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}'\right) dx' dy'$

From previous slide $T = \frac{\rho}{A} \iint \frac{1}{2} V^2 \, dx' dy'$ &

$\vec{V} = \bar{\vec{V}} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\bar{\vec{V}} + \vec{V}') \cdot (\bar{\vec{V}} + \vec{V}') \Rightarrow$

$V^2 = \bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}'$ so

$$T = \left(\frac{1}{2}\frac{\rho}{A}\right) \iint \left(\bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}'\right) dx' dy'$$

Constant over $dx'dy'$ integration

From previous slide $T = \frac{m}{A}\iint \frac{1}{2} V^2 \, dx' \, dy'$ &

$$\vec{V} = \vec{\bar{V}} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\vec{\bar{V}} + \vec{V}')\cdot(\vec{\bar{V}} + \vec{V}') \Rightarrow$$

$$V^2 = \bar{V}^2 + V'^2 + 2\vec{\bar{V}}\cdot\vec{V}' \quad \text{so}$$

$$T = \left(\frac{1}{2}\frac{m}{A}\right)\iint \left(\bar{V}^2 + V'^2 + 2\vec{\bar{V}}\cdot\vec{V}'\right) dx' \, dy' \Rightarrow$$

$$T = \left(\frac{1}{2}\frac{m}{A}\right)\bar{V}^2 \iint dx' \, dy' +$$

From previous slide $T = \frac{m}{A}\iint \frac{1}{2} V^2 \, dx' \, dy'$ &

$$\vec{V} = \bar{\vec{V}} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\bar{\vec{V}} + \vec{V}') \cdot (\bar{\vec{V}} + \vec{V}') \Rightarrow$$

$$V^2 = \bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}' \rightsquigarrow 0$$

$$T = \left(\frac{1}{2}\frac{m}{A}\right) \iint \left(\bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}'\right) dx' \, dy' \Rightarrow$$

$$T = \left(\frac{1}{2}\frac{m}{A}\right) \bar{V}^2 \iint dx' \, dy' + \left(\frac{m}{A}\right) \iint V'^2 \, dx' \, dy'$$

From previous slide $T = \frac{m}{A}\iint \frac{1}{2} V^2 \, dx' \, dy'$ &

$$\vec{V} = \bar{\vec{V}} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\bar{\vec{V}} + \vec{V}') \cdot (\bar{\vec{V}} + \vec{V}') \Rightarrow$$

$$V^2 = \bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}'$$

so

$$T = \left(\frac{1}{2}\frac{m}{A}\right)\iint \left(\bar{V}^2 + V'^2 + 2\bar{\vec{V}} \cdot \vec{V}'\right) dx' \, dy' \Rightarrow$$

$$T = \left(\frac{1}{2}\frac{m}{A}\right)\bar{V}^2 \iint dx' \, dy' + \left(\frac{m}{A}\right)\iint V'^2 \, dx' \, dy' + \left(\frac{m}{A}\right)\bar{\vec{V}} \cdot \int \vec{V}' \, dx' \, dy'$$

From previous slide $T = \frac{m}{A}\iint \frac{1}{2} V^2 \, dx' dy'$ &

$$\vec{V} = \bar{\vec{V}} + \vec{V}' \Rightarrow |\vec{V}|^2 = (\bar{\vec{V}} + \vec{V}')\cdot(\bar{\vec{V}} + \vec{V}') \Rightarrow$$

$$V^2 = \bar{V}^2 + V'^2 + 2\bar{\vec{V}}\cdot\vec{V}'$$ so 0

$$T = \left(\frac{1}{2}\frac{m}{A}\right)\iint\left(\bar{V}^2 + V'^2 + 2\bar{\vec{V}}\cdot\vec{V}'\right)dx'dy' \Rightarrow$$

$$T = \left(\frac{1}{2}\frac{m}{A}\right)\bar{V}^2 \underbrace{\iint dx'dy'}_{A} + \left(\frac{m}{A}\right)\iint V'^2 dx'dy' + \left(\frac{m}{A}\right)\bar{\vec{V}}\cdot\overbrace{\int \vec{V}' dx'dy'}^{0}$$

ASU

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$$\Rightarrow T = \frac{1}{2} M \bar{V}^2 + \left(\frac{m}{A}\right)\iint V'^2 dx' dy'$$

From previous slide $T = \frac{M}{A}\int\int \frac{1}{2} v^2 dx'dy'$ &

$\vec{v} = \vec{\bar{v}} + \vec{v}' \Rightarrow |\vec{v}|^2 = (\vec{\bar{v}} + \vec{v}')\cdot(\vec{\bar{v}} + \vec{v}') \Rightarrow$

$v^2 = \bar{v}^2 + v'^2 + 2\vec{\bar{v}}\cdot\vec{v}'$ so

$T = \left(\frac{1}{2}\frac{M}{A}\right)\int\int(\bar{v}^2 + v'^2 + 2\vec{\bar{v}}\cdot\vec{v}')dx'dy' \Rightarrow$

$T = \left(\frac{1}{2}\frac{M}{A}\right)\bar{v}^2\underbrace{\int\int dx'dy'}_{A} + \left(\frac{M}{A}\right)\int\int v'^2 dx'dy' + \left(\frac{M}{A}\right)\vec{\bar{v}}\cdot\overbrace{\int\vec{v}'dx'dy'}^{0}$

$\Rightarrow T = \frac{1}{2}M\bar{v}^2 + \left(\frac{M}{A}\right)\int\int v'^2 dx'dy'$

we can rewrite $v'^2 = r'^2 \dot{\theta}^2$

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we can rewrite $v'^2 = r'^2\omega^2$ & since ω has no position dependence over surface

From previous slide $T=\frac{M}{A}\iint \frac{1}{2}v^2\,dx'dy'$ &

$\vec{v} = \bar{\vec{v}} + \vec{v}\,'$ $\Rightarrow$ $|\vec{v}|^2 = (\bar{\vec{v}} + \vec{v}\,')\cdot(\bar{\vec{v}} + \vec{v}\,')$ $\Rightarrow$

$v^2 = \bar{v}^2 + v'^2 + 2\bar{\vec{v}}\cdot\vec{v}\,'$ so

$$T = \left(\frac{1}{2}\frac{M}{A}\right)\iint\left(\bar{v}^2 + v'^2 + 2\bar{\vec{v}}\cdot\vec{v}\,'\right)dx'dy' \Rightarrow$$

$$T = \left(\frac{1}{2}\frac{M}{A}\right)\bar{v}^2\underbrace{\iint dx'dy'}_{A} + \left(\frac{M}{A}\right)\iint v'^2\,dx'dy' + \left(\frac{M}{A}\right)\bar{\vec{v}}\cdot\overbrace{\int \vec{v}\,'\,dx'dy'}^{0}$$

$\Rightarrow T = \frac{1}{2}M\bar{v}^2 + \left(\frac{M}{A}\right)\iint v'^2\,dx'dy'$ we can rewrite $v'^2 = r'^2\omega^2$ &

since ω has no position dependence over surface, then $T = \frac{1}{2}M\bar{v}^2 + \frac{1}{2}\omega^2\frac{M}{A}\iint r'^2\,dx'dy'$

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we can rewrite $v'^2 = r'^2\omega^2$ & since ω has no position dependence over surface, then

$$T = \frac{1}{2}M\bar{V}^2 + \frac{1}{2}\omega^2\frac{M}{A}\iint r'^2\,dx'dy'$$

But $I \equiv \left(\frac{M}{A}\right)\int r'^2\,dx'dy'$

From previous slide $T=\frac{M}{A}\iint \frac{1}{2}v^2\,dx'dy'$ &

$\vec{v} = \vec{\bar{v}} + \vec{v}\,' \Rightarrow |\vec{v}|^2 = (\vec{\bar{v}} + \vec{v}\,')\cdot(\vec{\bar{v}} + \vec{v}\,') \Rightarrow$

$v^2 = \bar{v}^2 + v'^2 + 2\vec{\bar{v}}\cdot\vec{v}\,'$ so

$$T=\left(\frac{1}{2}\frac{M}{A}\right)\iint\left(\bar{v}^2 + v'^2 + 2\vec{\bar{v}}\cdot\vec{v}\,'\right)dx'dy' \Rightarrow$$

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we can rewrite $v'^2 = r'^2\omega^2$ &

since ω has no position dependence over surface, then $T=\frac{1}{2}M\bar{v}^2 + \frac{1}{2}\omega^2\frac{M}{A}\iint r'^2\,dx'dy'$

But $I \equiv \left(\frac{M}{A}\right)\int r'^2\,dx'dy'$ so

$$T=\frac{1}{2}M\bar{v}^2 + \frac{1}{2}I\omega^2$$

From previous slide $T = \frac{m}{A}\iint \frac{1}{2} v^2 \, dx' dy'$ &

$\vec{v} = \bar{\vec{v}} + \vec{v}' \Rightarrow |\vec{v}|^2 = (\bar{\vec{v}} + \vec{v}') \cdot (\bar{\vec{v}} + \vec{v}') \Rightarrow$

$v^2 = \bar{v}^2 + v'^2 + 2\bar{\vec{v}} \cdot \vec{v}'$ so

$T = \left(\frac{1}{2}\frac{m}{A}\right)\iint \left(\bar{v}^2 + v'^2 + 2\bar{\vec{v}} \cdot \vec{v}'\right) dx' dy' \Rightarrow$

$$T = \left(\frac{1}{2}\frac{m}{A}\right)\bar{v}^2 \underbrace{\iint dx' dy'}_{A} + \left(\frac{m}{A}\right)\iint v'^2 \, dx' dy' + \left(\frac{m}{A}\right)\bar{\vec{v}} \cdot \overbrace{\int \vec{v}' \, dx' dy'}^{0}$$

$\Rightarrow T = \frac{1}{2} m \bar{v}^2 + \left(\frac{m}{A}\right)\iint v'^2 \, dx' dy'$ we can rewrite $v'^2 = r'^2 \omega^2$ &

since ω has no position dependence over surface, then $T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2}\omega^2 \frac{m}{A}\iint r'^2 \, dx' dy'$

But $I \equiv \left(\frac{m}{A}\right)\int r'^2 \, dx' dy'$ so

$$T = \underbrace{\frac{1}{2} m \bar{v}^2}_{\text{Translational K.E.}} + \frac{1}{2} I \omega^2$$

From previous slide $T = \frac{m}{A}\iint \frac{1}{2} v^2 \, dx' dy'$ &

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$v^2 = \bar{v}^2 + v'^2 + 2\vec{\bar{v}} \cdot \vec{v}'$ so

$T = \left(\frac{1}{2}\frac{m}{A}\right)\iint (\bar{v}^2 + v'^2 + 2\vec{\bar{v}} \cdot \vec{v}')\, dx' dy' \Rightarrow$

$$T = \left(\frac{1}{2}\frac{m}{A}\right)\bar{v}^2 \underbrace{\iint dx' dy'}_{A} + \left(\frac{m}{A}\right)\iint v'^2 \, dx' dy' + \left(\frac{m}{A}\right)\vec{\bar{v}} \cdot \overbrace{\int \vec{v}' \, dx' dy'}^{0}$$

$\Rightarrow T = \frac{1}{2} m \bar{v}^2 + \left(\frac{m}{A}\right)\iint v'^2 \, dx' dy'$ we can rewrite $v'^2 = r'^2 \omega^2$ & since ω has no position dependence over surface, then $T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2}\omega^2 \frac{m}{A}\iint r'^2 \, dx' dy'$

But $I \equiv \left(\frac{m}{A}\right)\int r'^2 \, dx' dy'$ so

$$T = \underbrace{\frac{1}{2} m \bar{v}^2}_{\text{Translational K.E.}} + \underbrace{\frac{1}{2} I \omega^2}_{\text{Rotational K.E.}}$$

ASU

If rotating about fixed point P:

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$\bar{v} = r_{G/P}$ eee

If rotating about fixed point P:

$$\bar{v} = r_{G/P}\,\omega \Rightarrow \frac{1}{2}m\bar{v}^2 = \frac{1}{2}m r_{G/P}^2\,\omega^2$$

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so $T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$ becomes

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so $T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$ becomes

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ASU

If rotating about fixed point P:

$$\bar{v} = r_{G/P}\,\omega \Rightarrow \frac{1}{2}m\bar{v}^2 = \frac{1}{2}m r_{G/P}^2\,\omega^2$$

so $T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$ becomes

$$T = \frac{1}{2}m r_{G/P}^2\,\omega^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}\left[m r_{G/P}^2 + \bar{I}\right]\omega^2$$

If rotating about fixed point P:

$\bar{v} = r_{G/P}\,\omega \Rightarrow \frac{1}{2}m\bar{v}^2 = \frac{1}{2}m r_{G/P}^2\,\omega^2$

so $T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$ becomes

$$T = \frac{1}{2}m r_{G/P}^2\,\omega^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}\left[m r_{G/P}^2 + \bar{I}\right]\omega^2$$ But

$$\bar{I} + m r_{G/P}^2 = I_P$$

If rotating about fixed point P:

$$\bar{v} = r_{G/P}\,\omega \Rightarrow \frac{1}{2}m\bar{v}^2 = \frac{1}{2}m r_{G/P}^2\,\omega^2$$

so $T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$ becomes

$$T = \frac{1}{2}m r_{G/P}^2\,\omega^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}\left[m r_{G/P}^2 + \bar{I}\right]\omega^2$$ But

$$\bar{I} + m r_{G/P}^2 = I_P$$

So $\boxed{T = \frac{1}{2}\bar{I}_P\,\omega^2}$

So far we have

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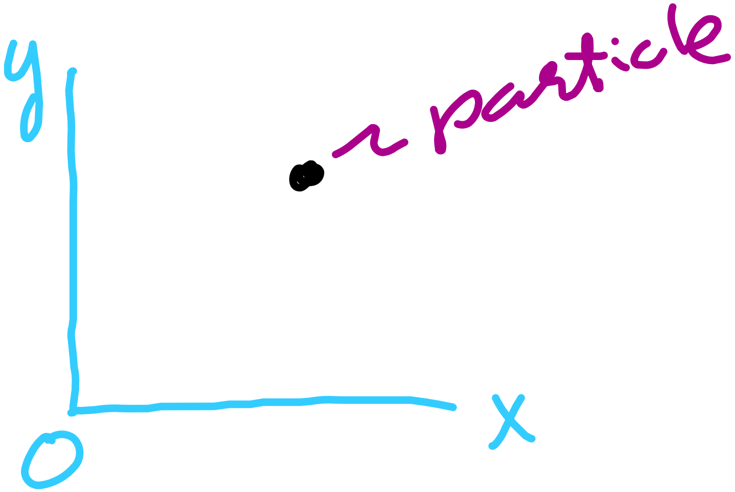
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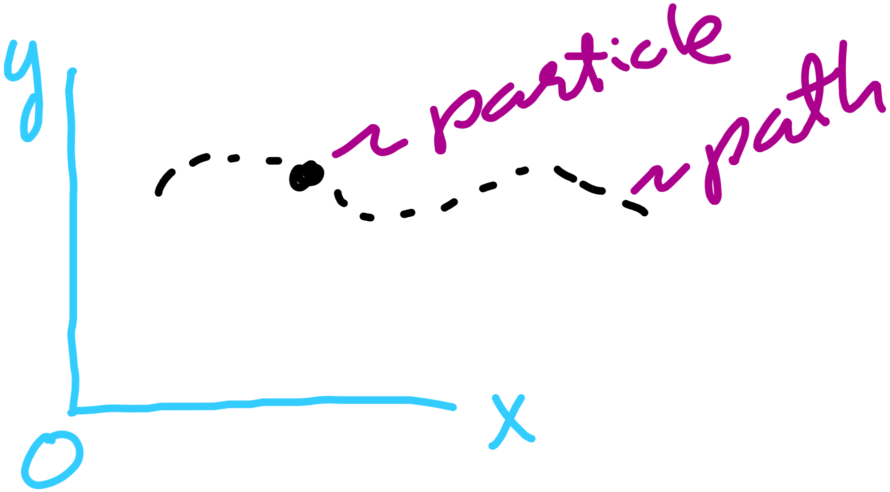
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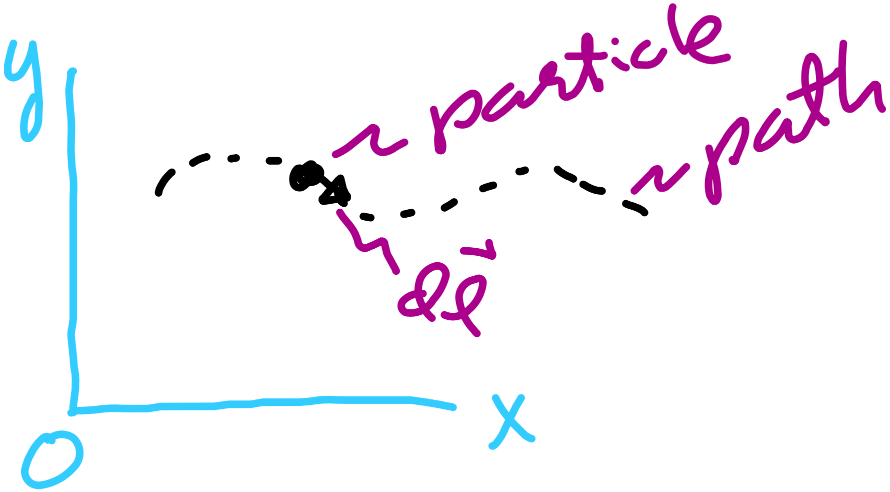
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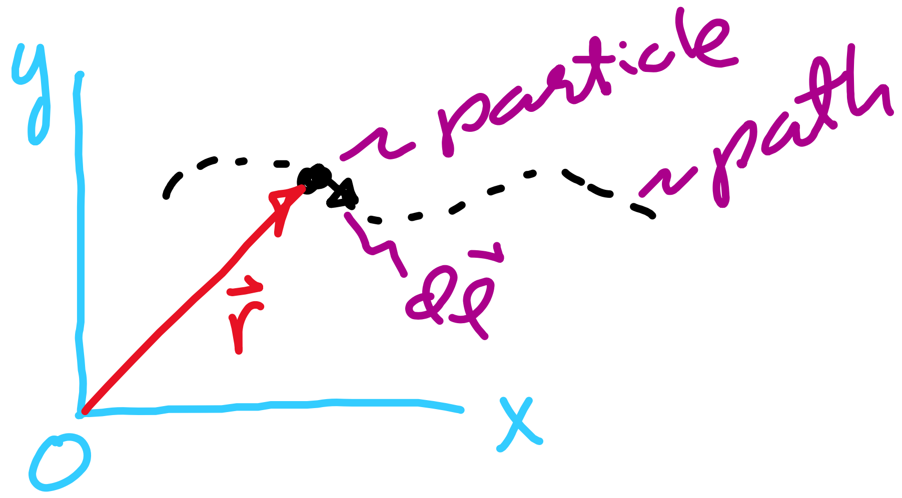

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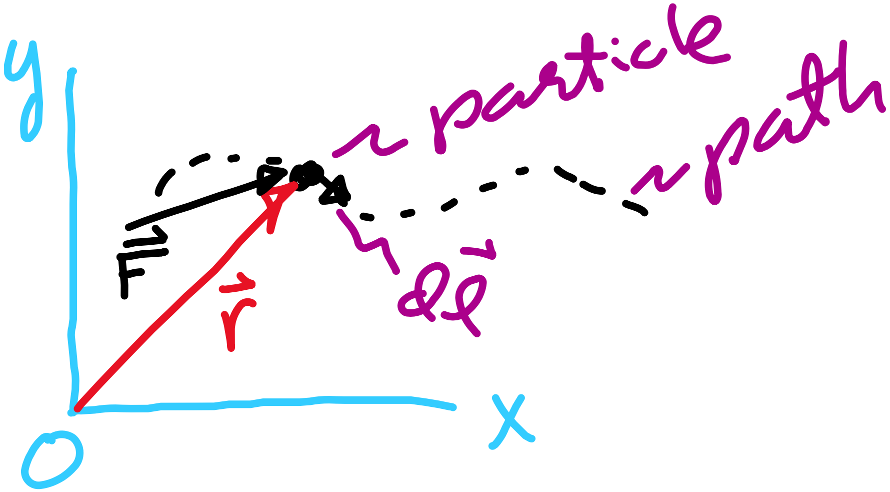
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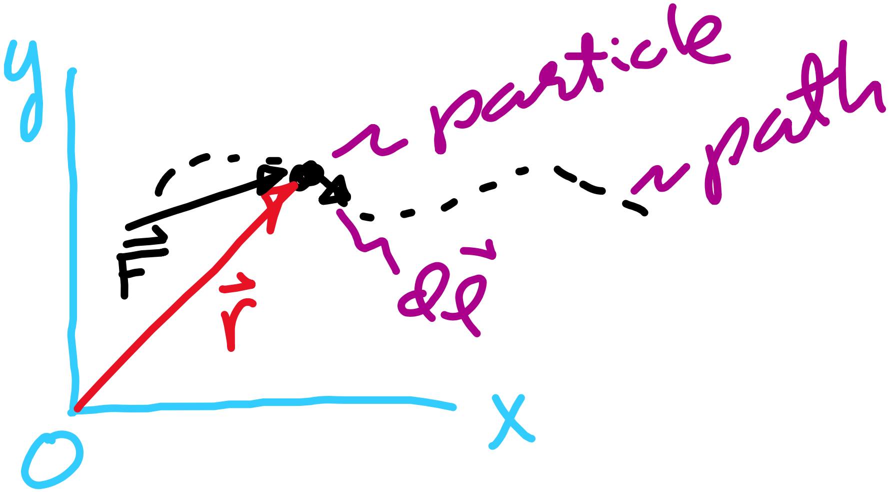

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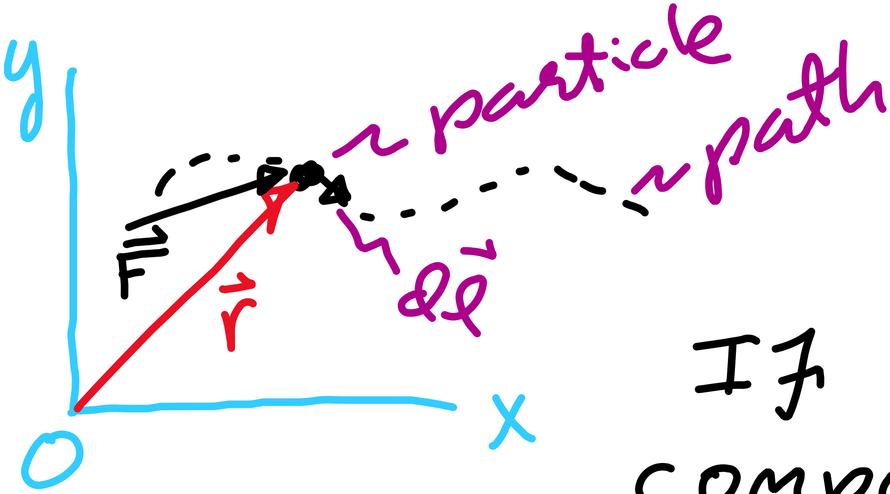

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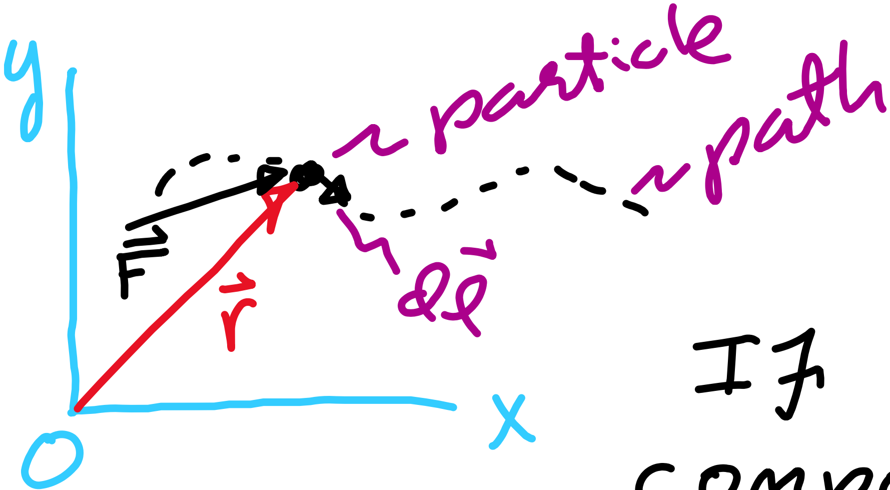

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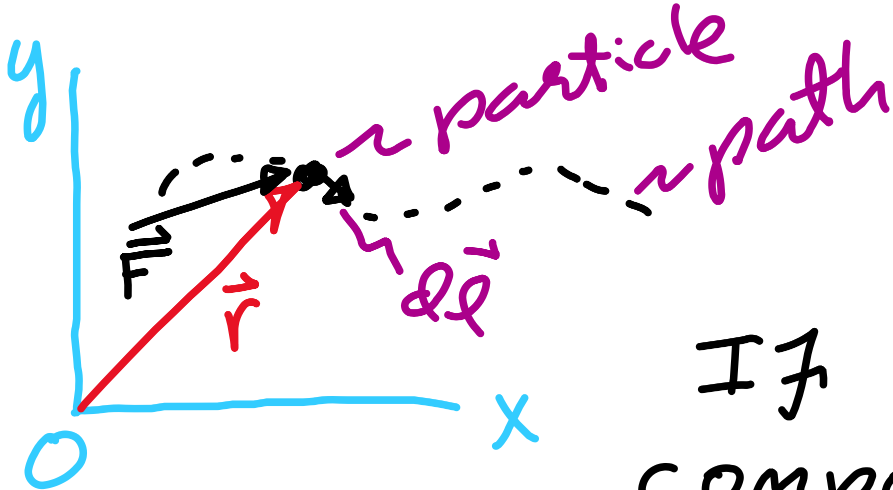

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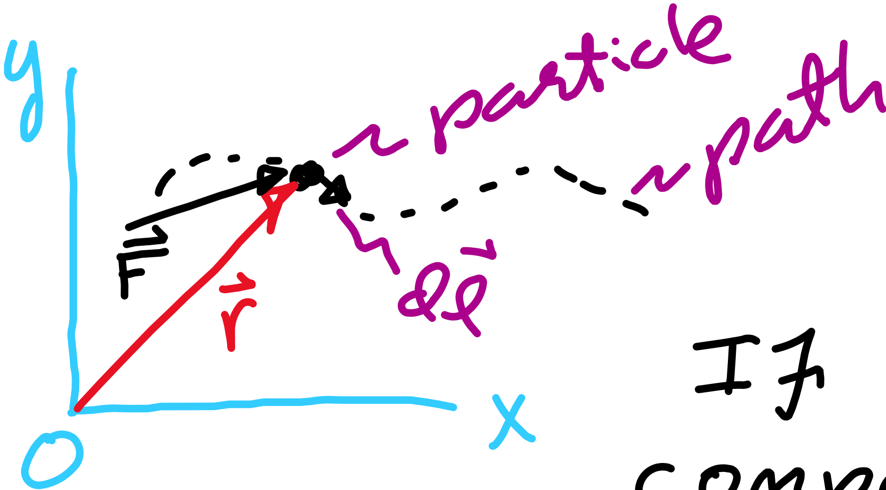

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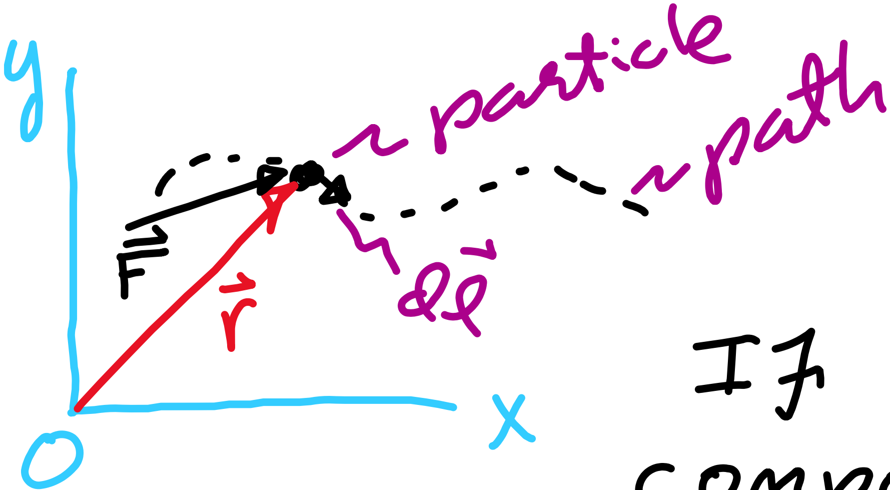

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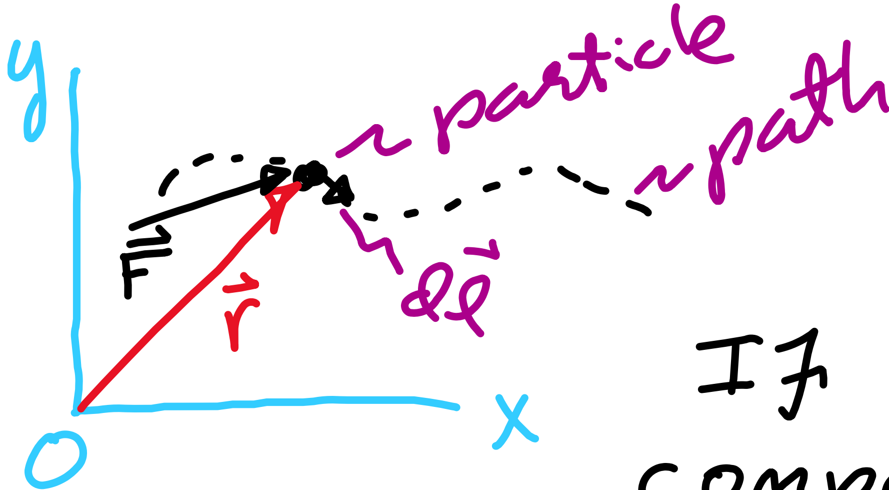

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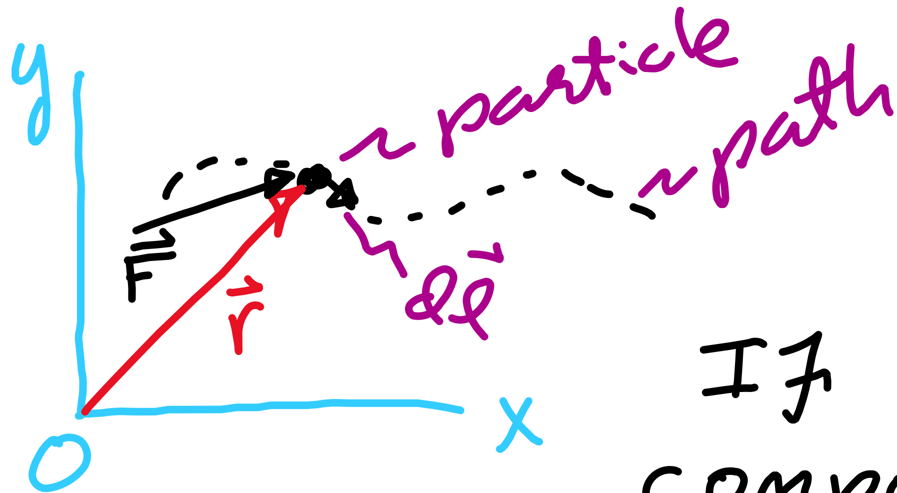

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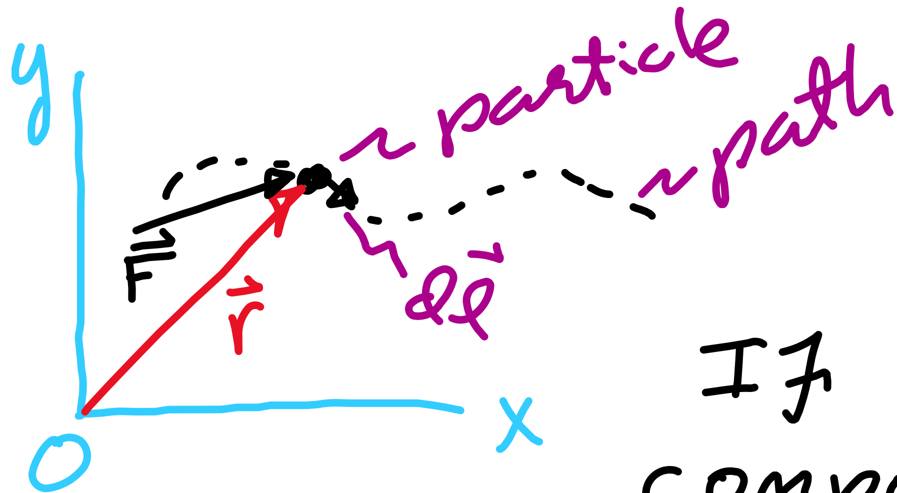

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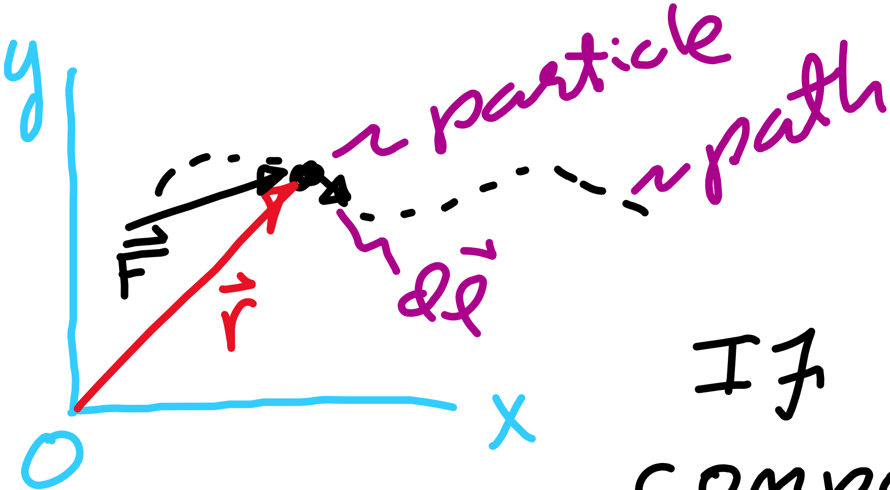

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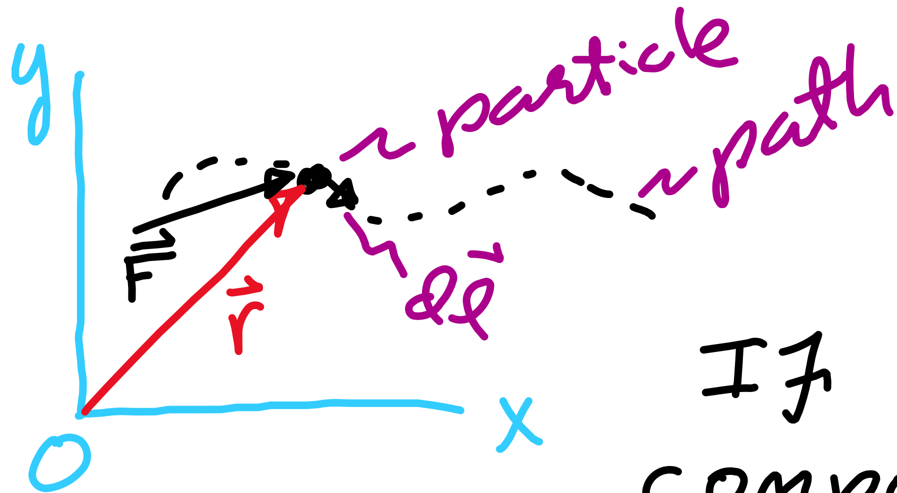

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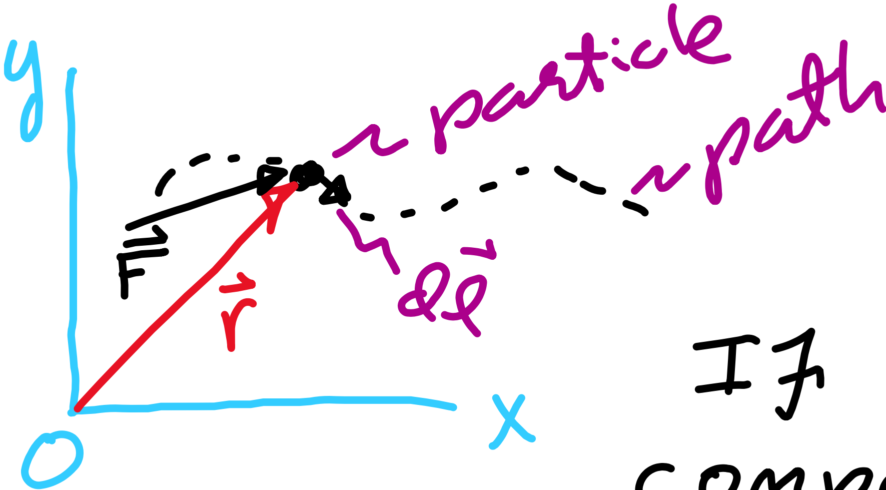

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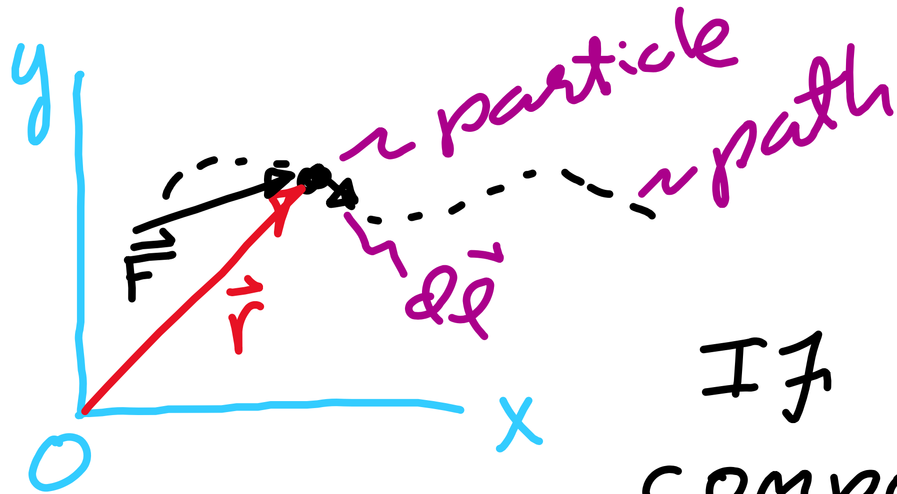
ASU



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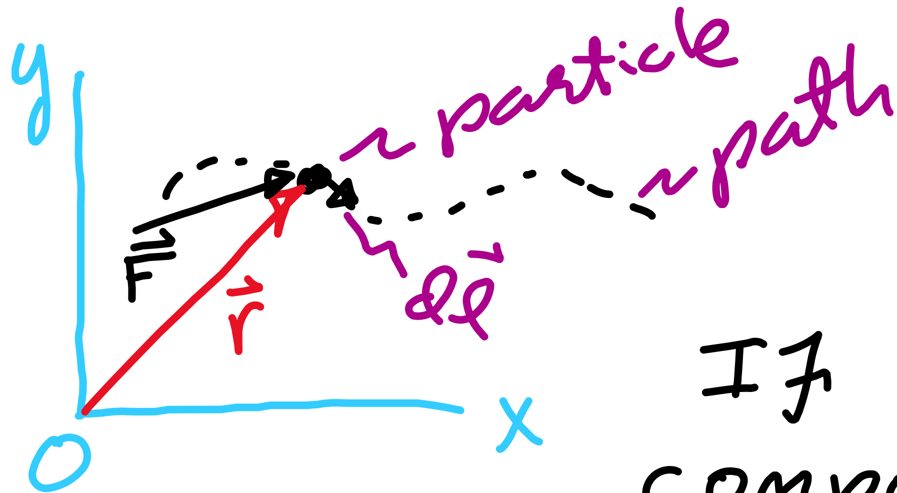
Part of work <u>NOT</u> associated with torque about point O



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For torques (moments) about point O

$$U_{1\to 2} = \int_{\theta_1}^{\theta_2} M_O \, d\theta$$

ASU

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Now I want to show that there is no energy lost due to friction for a wheel that is rolling without slipping.

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We will analyze the motion of a disk rolling down an incline 2 ways

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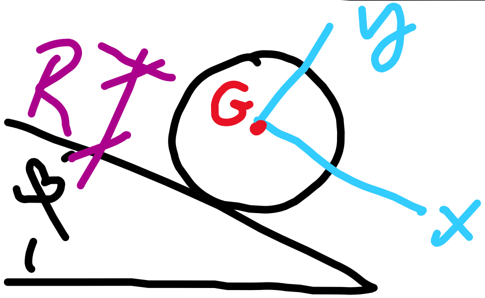

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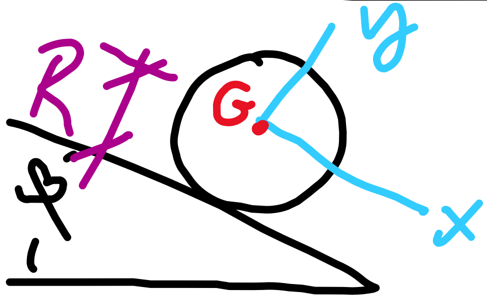

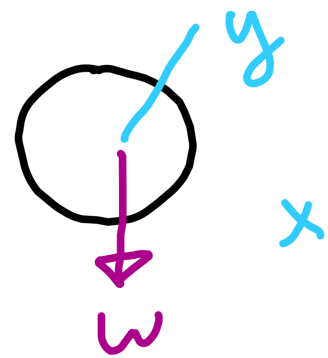

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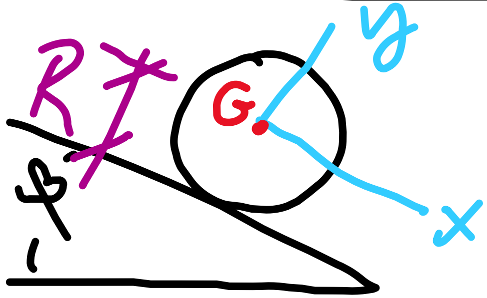

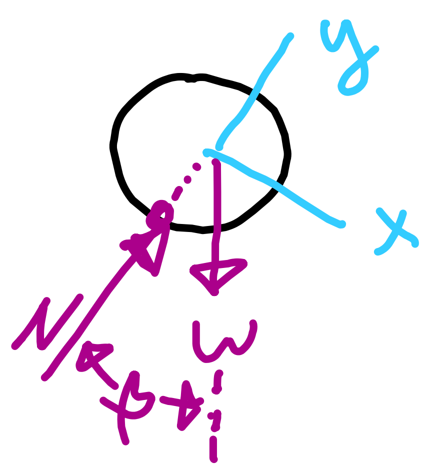

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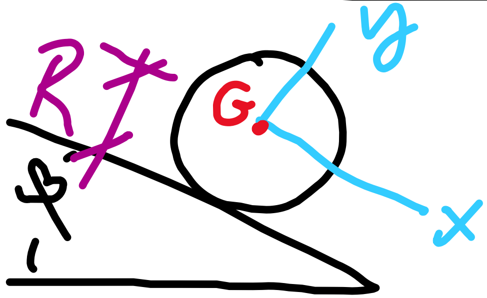

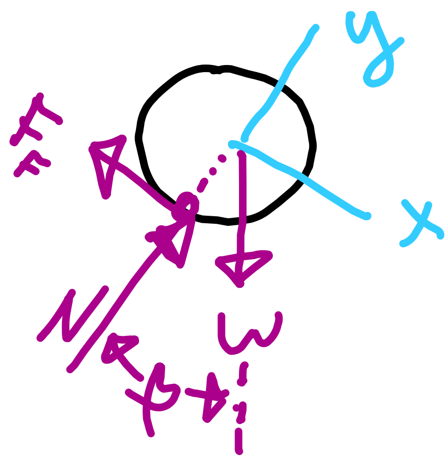

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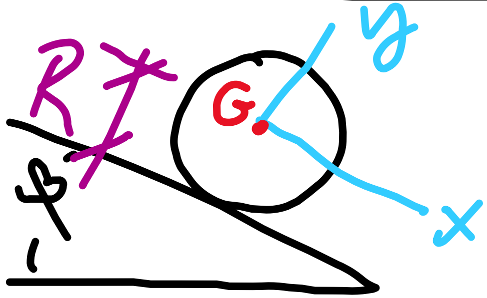

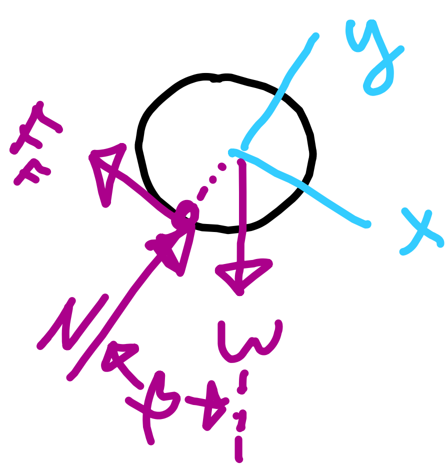

$\Rightarrow W_x = W\sin\beta$

Now I want to show that there is no energy lost due to friction for a wheel that is rolling without slipping.

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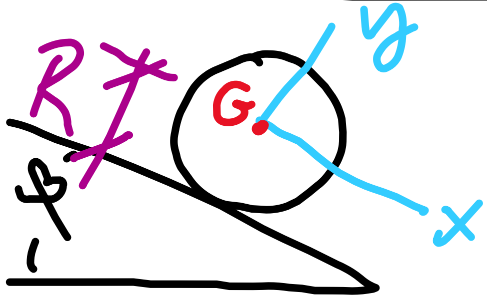

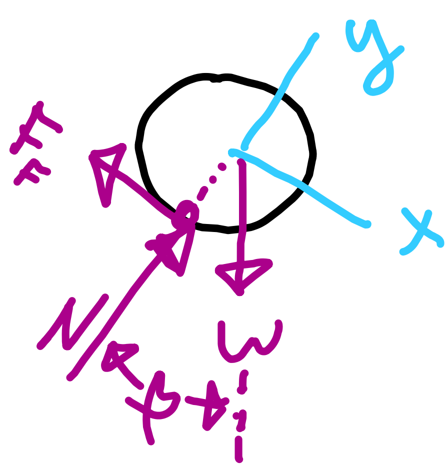

$$\Rightarrow W_x = W\sin\beta$$
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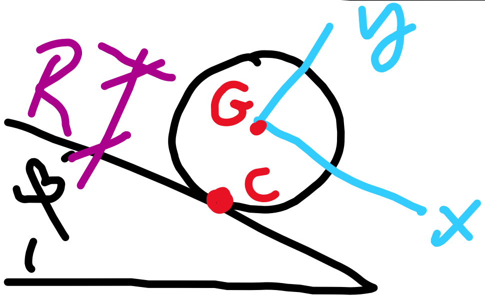

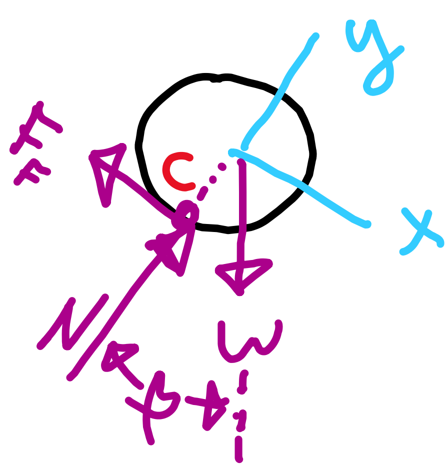

$\Rightarrow W_x = W\sin\beta$

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$\sum M_c = I_c \alpha$

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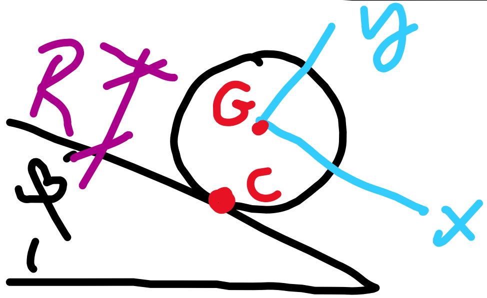
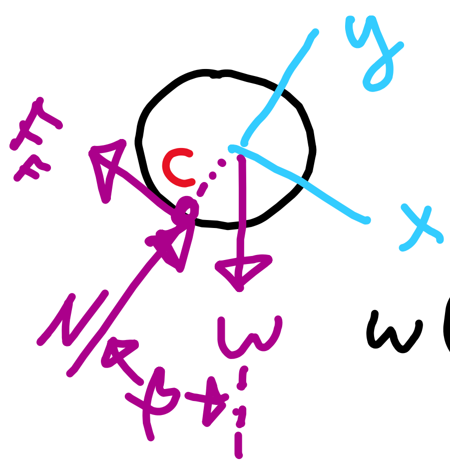


$\Rightarrow W_x r = I_c \alpha$ $\qquad \Rightarrow$



$\Rightarrow W_x = W \sin\beta$

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$W (\sin\beta) R = I_c \alpha$

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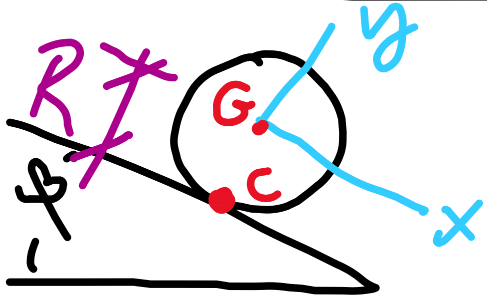

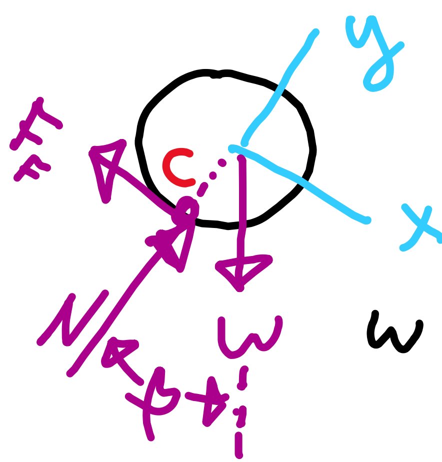

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Take $\alpha = \omega \frac{d\omega}{d\theta}$

Now I want to show that there is no energy lost due to friction for a wheel that is rolling without slipping.

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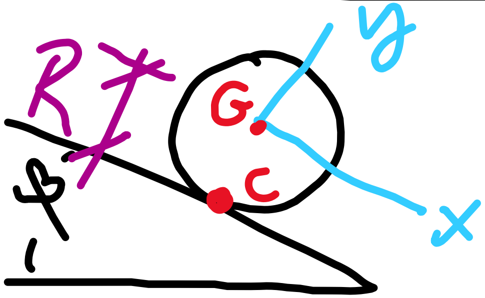

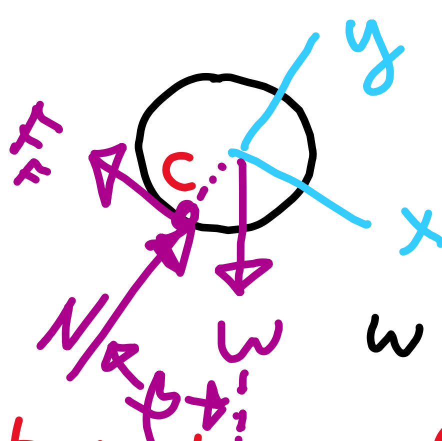

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Take $\alpha = \omega\frac{d\omega}{d\theta}$ & integrate over θ

Now I want to show that there is no energy lost due to friction for a wheel that is rolling without slipping.

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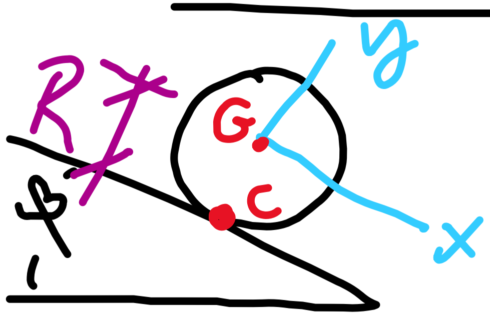
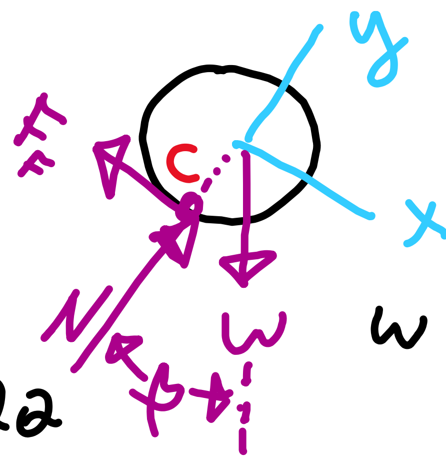
<u>1st way</u>: Without use of energy conservation





$\Rightarrow W_x = W\sin\beta$

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$w(\sin\beta)R\int d\theta = I_c\int \omega\, d\theta$

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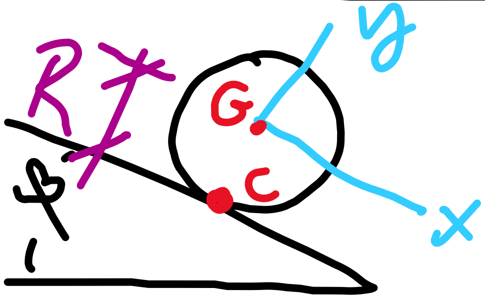

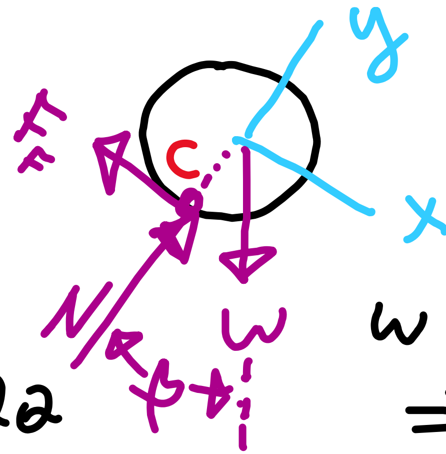

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$w(\sin\beta)R\int d\theta = I_c\int \omega\, d\omega \Rightarrow$

$W(\sin\beta)R\,\Delta\theta = \frac{1}{2}I_c(\omega_F^2 - \omega_I^2)$

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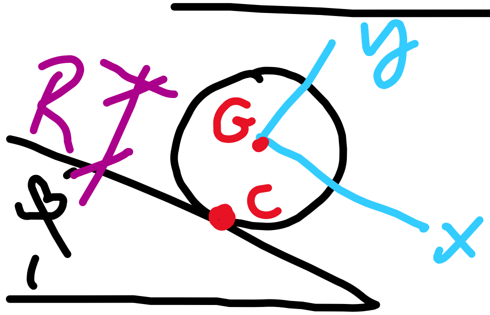

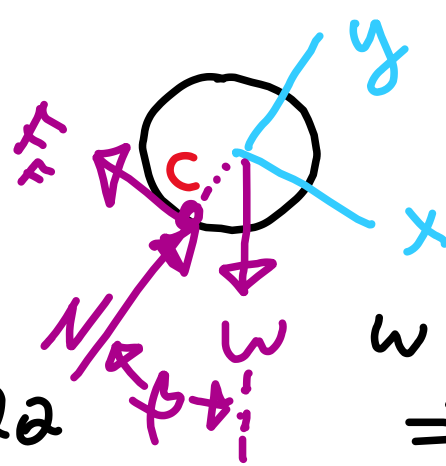

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$W(\sin\beta)R\int d\theta = I_c\int \omega\, d\omega$

$\Rightarrow$

$$W(\sin\beta)R\,\Delta\theta = \underbrace{\tfrac{1}{2}I_c(\omega_F^2 - \omega_I^2)}_{\text{Looks like kinetic energy}}$$

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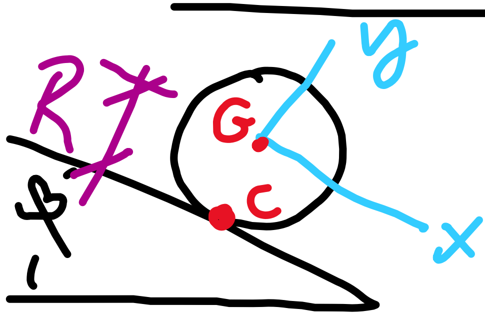

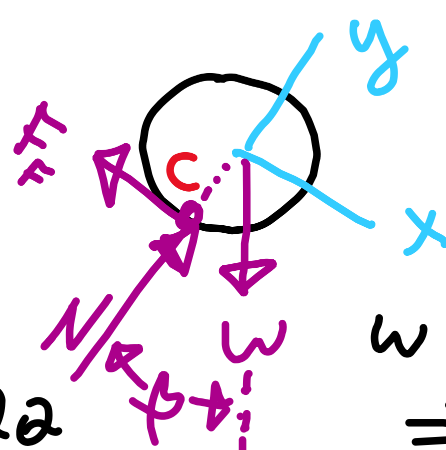

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$W(\sin\beta)R\int d\theta = I_c\int \omega\, d\omega$

$\left[W(\sin\beta)R\,\Delta\theta\right] = \underbrace{\tfrac{1}{2}I_c\left(\omega_F^2 - \omega_I^2\right)}_{\text{Looks like kinetic energy}}$

want to see this → in terms of potential energy

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c \left(\omega_F^2 - \omega_I^2\right)$

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The disk has rolled down the incline a distance $l = R\Delta\theta$

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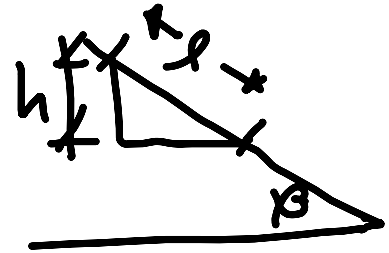

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c \left(\omega_F^2 - \omega_I^2\right)$

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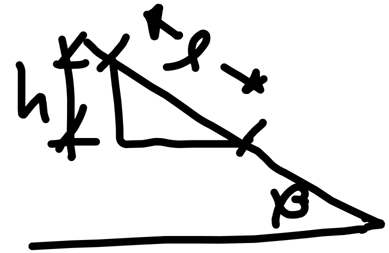

since $\sin\beta = \frac{l}{h}$

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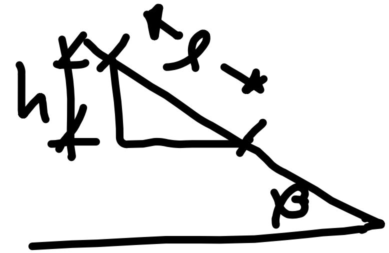

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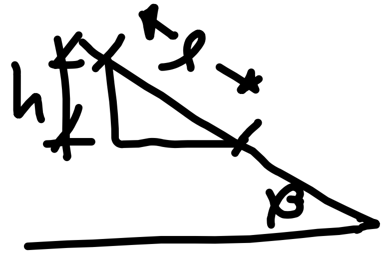

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$\Rightarrow \quad h = R\Delta\theta\sin\beta$

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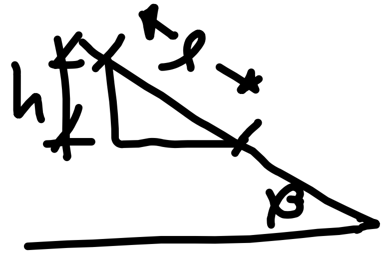

since $\sin\beta = \frac{l}{h}$
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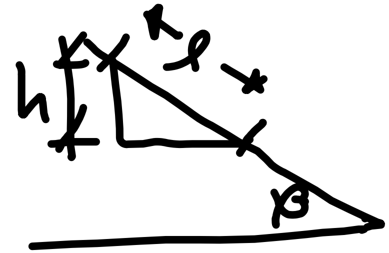
Now $wR\Delta\theta\sin\beta = wh$

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c \left(\omega_F^2 - \omega_I^2\right)$

The disk has rolled down the incline a distance $\ell = R\Delta\theta$



since $\sin\beta = \frac{\ell}{h}$

then $h = \ell \sin\beta$

$\Rightarrow \quad h = R\Delta\theta\sin\beta$

Now $wR\Delta\theta\sin\beta = wh$ & since $w = mg$

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c (\omega_F^2 - \omega_I^2)$

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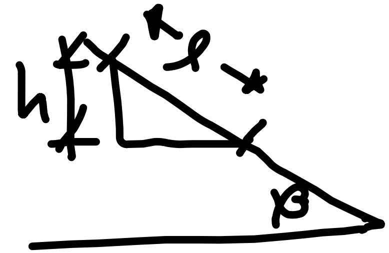

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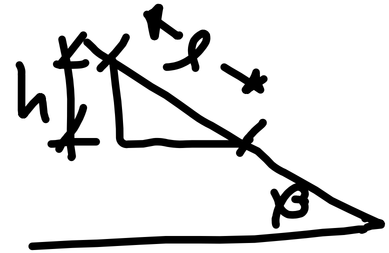

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Now $wR\Delta\theta\sin\beta = wh$ & since $w = mg$ we have $mgh = \frac{1}{2} I_c (\omega_F^2 - \omega_I^2)$, But

$I_c = \bar{I} + MR^2$

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c (\omega_F^2 - \omega_I^2)$

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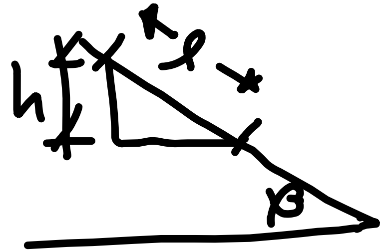

since $\sin\beta = \frac{h}{l}$ then $h = l\sin\beta$

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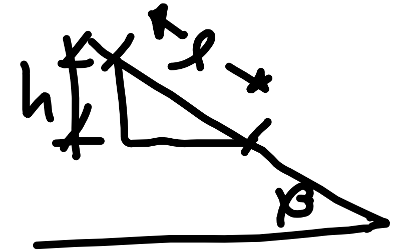

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But $\bar{v} = R\omega$ so $mgh = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{1}{2}M(\bar{v}_F^2 - \bar{v}_I^2)$

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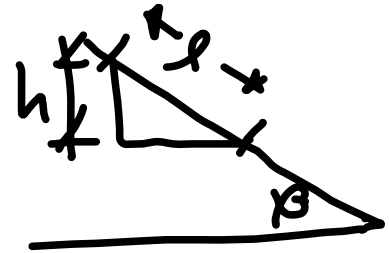

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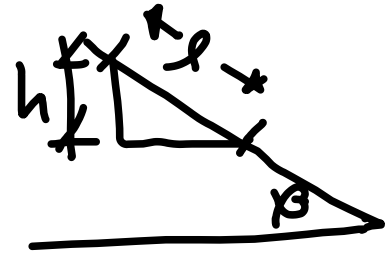
Now use conservation of energy:

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c (\omega_F^2 - \omega_I^2)$

The disk has rolled down the incline a distance $l = R\Delta\theta$



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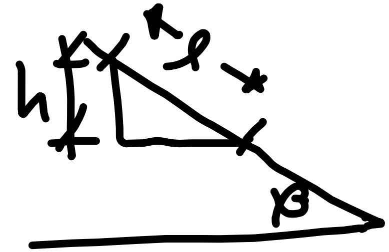
Now use conservation of energy:

$T_I = \frac{1}{2}\bar{I}\omega_I^2 + \frac{1}{2}M\bar{v}_I^2$

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c (\omega_F^2 - \omega_I^2)$

The disk has rolled down the incline a distance $l = R\Delta\theta$



since $\sin\beta = \frac{l}{h}$ then $h = l\sin\beta$

$\Rightarrow \; h = R\Delta\theta\sin\beta$

Now $wR\Delta\theta\sin\beta = wh$ & since $w = mg$ we have $mgh = \frac{1}{2} I_c(\omega_F^2 - \omega_I^2)$, But $I_c = \bar{I} + MR^2$ so $mgh = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{MR^2}{2}(\omega_F^2 - \omega_I^2)$

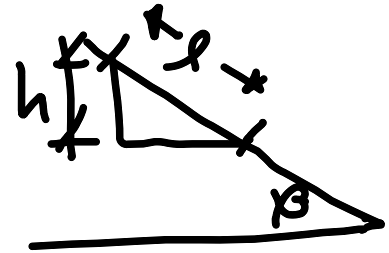
But $\bar{v} = R\omega$ so $mgh = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{1}{2}M(\bar{v}_F^2 - \bar{v}_I^2)$

Now use conservation of energy:

$T_I = \frac{1}{2}\bar{I}\omega_I^2 + \frac{1}{2}M\bar{v}_I^2$, $T_F = \frac{1}{2}\bar{I}\omega_F^2 + \frac{1}{2}M\bar{v}_F^2$

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c (\omega_F^2 - \omega_I^2)$

The disk has rolled down the incline a distance $l = R\Delta\theta$



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then $h = l\sin\beta$

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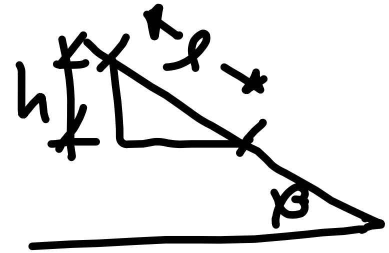
Now use conservation of energy:

$T_I = \frac{1}{2}\bar{I}\omega_I^2 + \frac{1}{2}m\bar{v}_I^2$, $T_F = \frac{1}{2}\bar{I}\omega_F^2 + \frac{1}{2}m\bar{v}_F^2$, $V_I = mgy_I$

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c (\omega_F^2 - \omega_I^2)$

The disk has rolled down the incline a distance $\ell = R\Delta\theta$



since $\sin\beta = \frac{\ell}{h}$ then $h = \ell\sin\beta$

$\Rightarrow$ $h = R\Delta\theta\sin\beta$

Now $wR\Delta\theta\sin\beta = wh$ & since $w = mg$ we have $mgh = \frac{1}{2} I_c(\omega_F^2 - \omega_I^2)$, But $I_c = \bar{I} + MR^2$ so $mgh = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{MR^2}{2}(\omega_F^2 - \omega_I^2)$

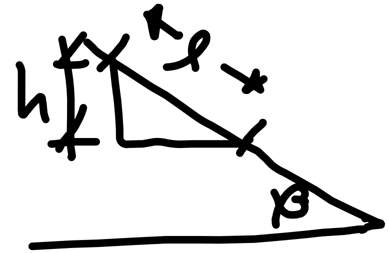
But $\bar{v} = R\omega$ so $mgh = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{1}{2}M(\bar{v}_F^2 - \bar{v}_I^2)$

Now use conservation of energy:

$T_I = \frac{1}{2}\bar{I}\omega_I^2 + \frac{1}{2}m\bar{v}_I^2$, $T_F = \frac{1}{2}\bar{I}\omega_F^2 + \frac{1}{2}m\bar{v}_F^2$, $V_I = mgy_I$ & $V_F = mgy_F$

We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c (\omega_F^2 - \omega_I^2)$

The disk has rolled down the incline a distance $l = R\Delta\theta$



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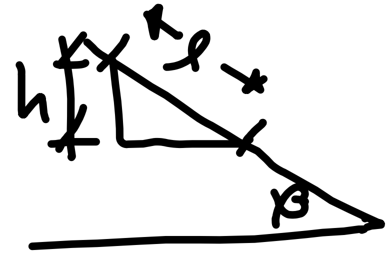
We have $wR(\sin\beta)\Delta\theta = \frac{1}{2} I_c(\omega_F^2 - \omega_I^2)$

The disk has rolled down the incline a distance $l = R\Delta\theta$



since $\sin\beta = \frac{h}{l}$ then $h = l\sin\beta$

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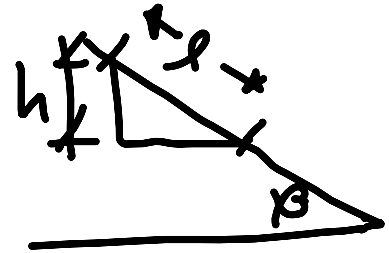
Now use conservation of energy:

$T_I = \frac{1}{2}\bar{I}\omega_I^2 + \frac{1}{2}M\bar{v}_I^2$, $T_F = \frac{1}{2}\bar{I}\omega_F^2 + \frac{1}{2}M\bar{v}_F^2$, $V_I = mgy_I$ & $V_F = mgy_F$ & $T_I + V_I + U^{NC}_{I\to F} = T_F + V_F \Rightarrow$

$$Mgh + U^{NC}_{I\to F} = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{1}{2}M(\bar{v}_F^2 - \bar{v}_I^2)$$

We have $\omega R(\sin\beta)\Delta\theta = \frac{1}{2} I_c (\omega_F^2 - \omega_I^2)$

The disk has rolled down the incline a distance $l = R\Delta\theta$



since $\sin\beta = \frac{l}{h}$ then $h = l\sin\beta$

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$$\boxed{mgh = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{mR^2}{2}(\omega_F^2 - \omega_I^2)}$$

Comparing these

$$\boxed{mgh + U^{NC}_{I\to F} = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{1}{2}m(\bar{v}_F^2 - \bar{v}_I^2)}$$

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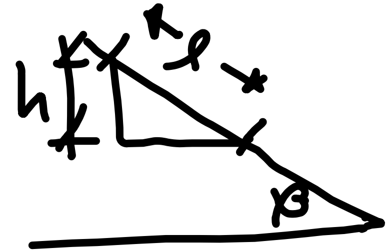
The disk has rolled down the incline a distance $l = R\Delta\theta$



since $\sin\beta = \frac{l}{h}$ then $h = l\sin\beta$

$\Rightarrow$ $h = R\Delta\theta\sin\beta$

$$\boxed{mgh = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{mR^2}{2}(\omega_F^2 - \omega_I^2)}$$

Comparing these

MUST have $U^{NC}_{I\to F} = 0$

$$\boxed{Mgh + U^{NC}_{I\to F} = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{1}{2}M(\bar{v}_F^2 - \bar{v}_I^2)}$$

So wheel rolling without slipping conserves energy such that $U^{NC}_{I\to F} = 0$.

$$\boxed{mgh = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{mR^2}{2}(\omega_F^2 - \omega_I^2)}$$

MUST have $U^{NC}_{I\to F} = 0$

$$\boxed{mgh + \boxed{U^{NC}_{I\to F}} = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{1}{2}m(\bar{v}_F^2 - \bar{v}_I^2)}$$

ASU

So wheel rolling without slipping conserves energy such that $U^{NC}_{I\to F} = 0$.

No energy lost due to frictional force.

$$\boxed{mgh = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{mR^2}{2}(\omega_F^2 - \omega_I^2)}$$

Must have $U^{NC}_{I\to F} = 0$

$$\boxed{mgh + \boxed{U^{NC}_{I\to F}} = \frac{1}{2}\bar{I}(\omega_F^2 - \omega_I^2) + \frac{1}{2}m(\bar{v}_F^2 - \bar{v}_I^2)}$$

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To figure out why this is true, we can look at the path of the contact point for a disk rolling without slipping

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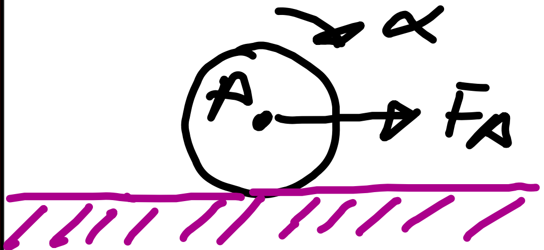
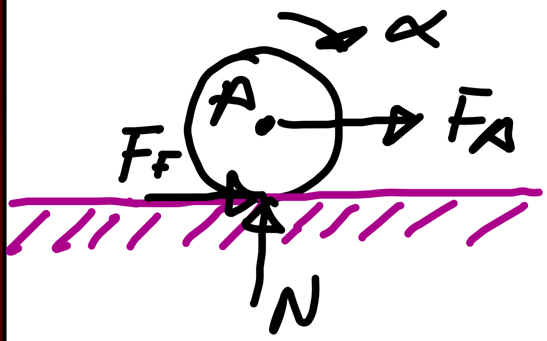
To figure out why this is true, we can look at the path of the contact point for a disk rolling without slipping that has force applied to the axel.

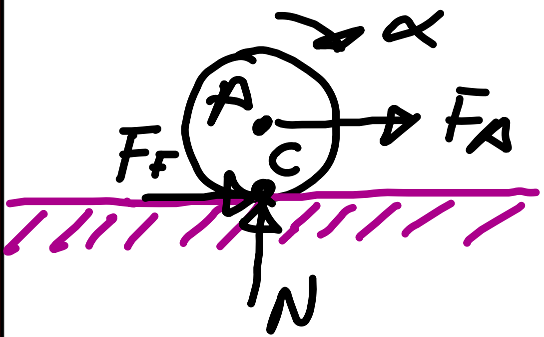

A



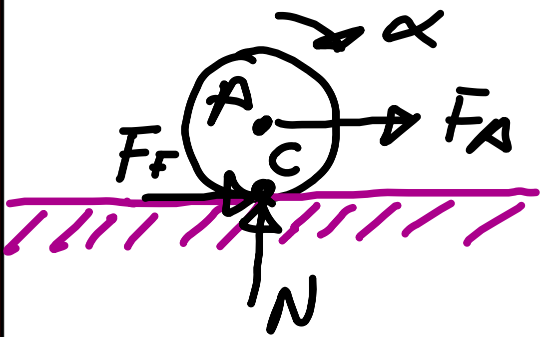
α
A
F_A



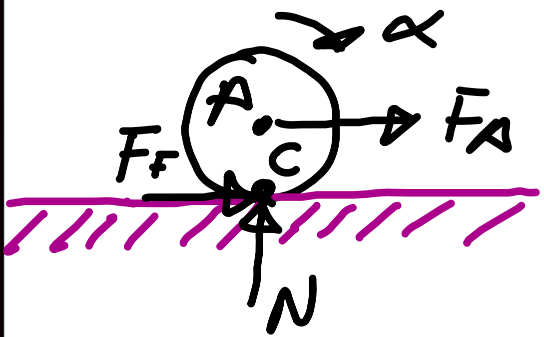
α
A
F_A
F_F
N



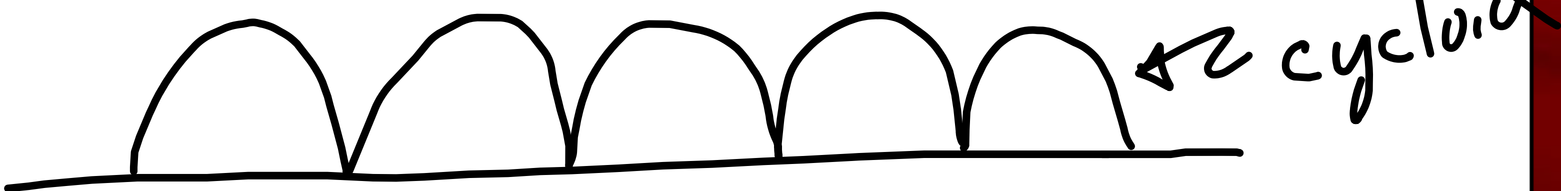
Frictional force applied at point C

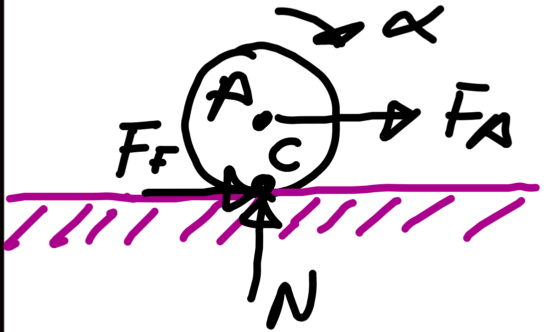


Frictional force applied at point c, but the path of point c is a cycloid

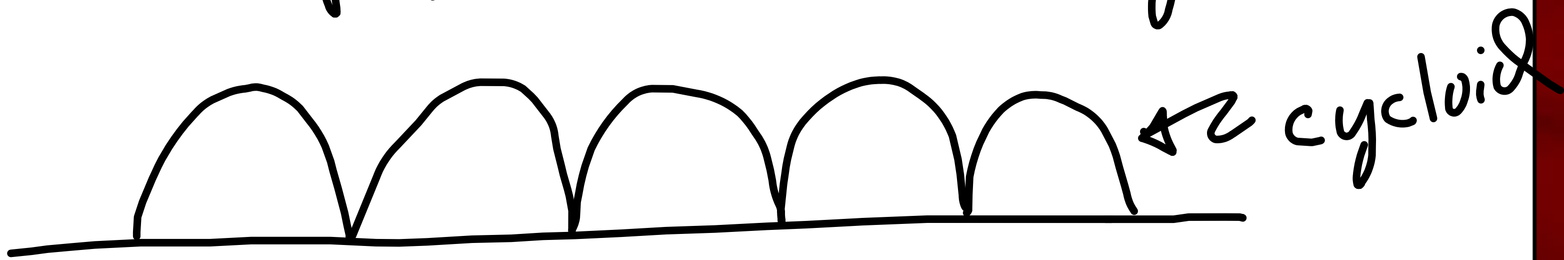


Frictional force applied at point c, but the path of point c is a cycloid

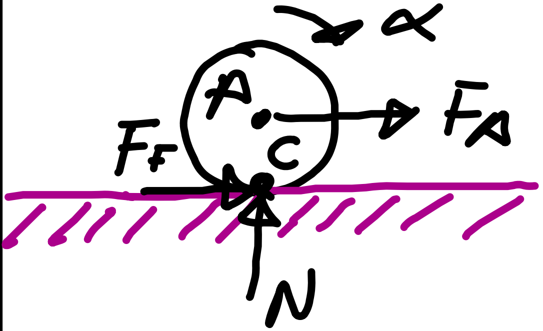




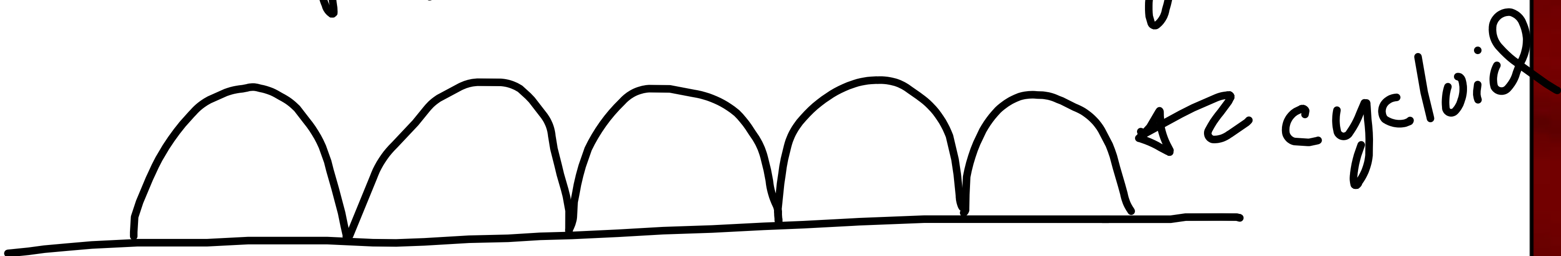
Frictional force applied at point C, but the path of point C is a cycloid



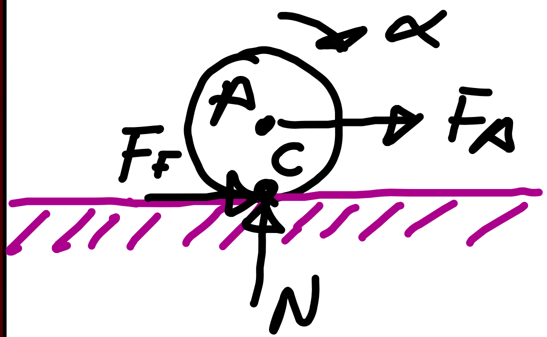
As point C touches the ground and F_F acts on that point.



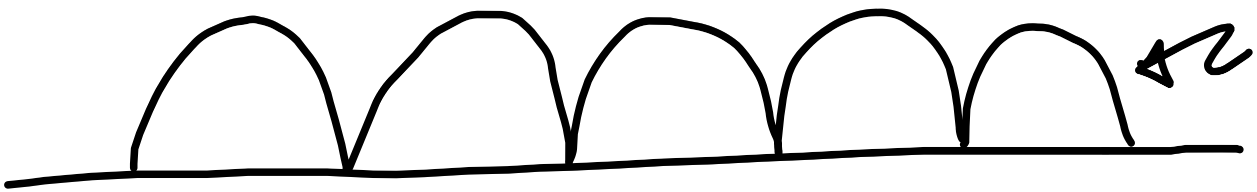
Frictional force applied at point C, but the path of point C is a cycloid



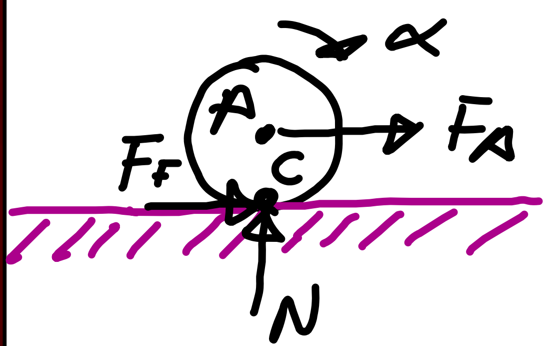
As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_C = 0$.



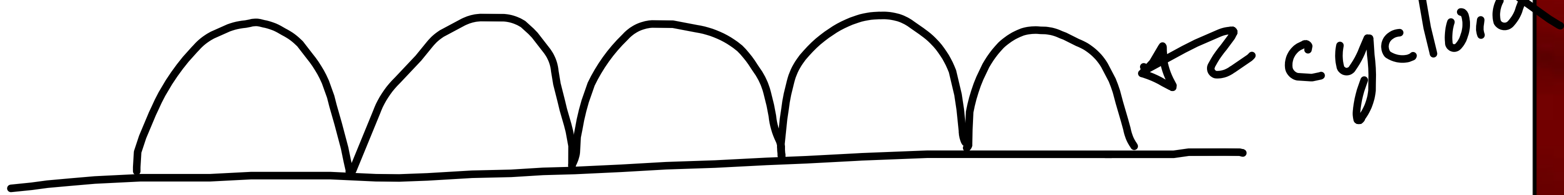
Frictional force applied at point C, but the path of point C is a cycloid



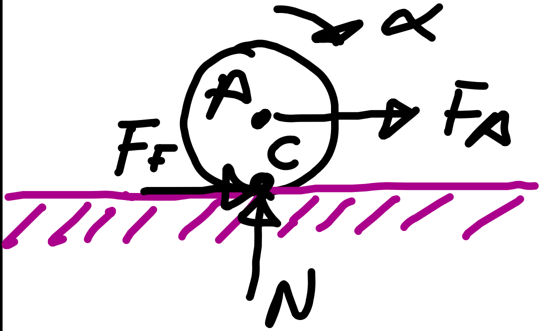
As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_c = 0$. In the past we found that $\vec{V}_c = 0$



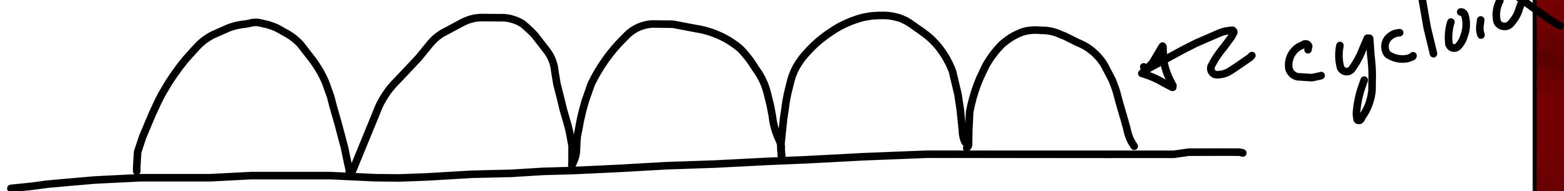
Frictional force applied at point C, but the path of point C is a cycloid



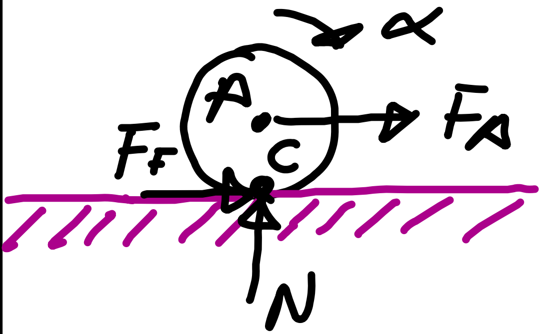
As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_c = 0$. In the past we found that $\vec{V}_c = 0$ but $\vec{V}_c = \frac{d\vec{l}_c}{dt}$



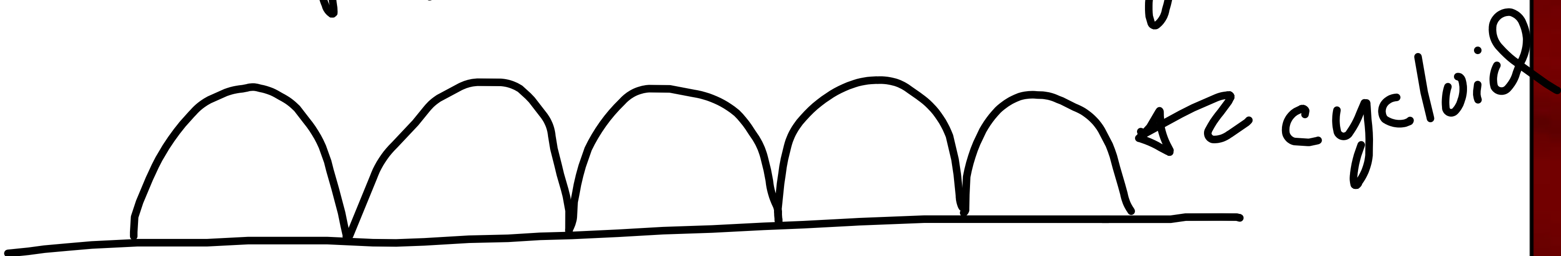
Frictional force applied at point C, but the path of point C is a cycloid



As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_c = 0$. In the past we found that $\vec{V}_c = 0$ but $\vec{V}_c = \frac{d\vec{l}_c}{dt} \Rightarrow d\vec{l} = \frac{d\vec{l}}{dt} dt$

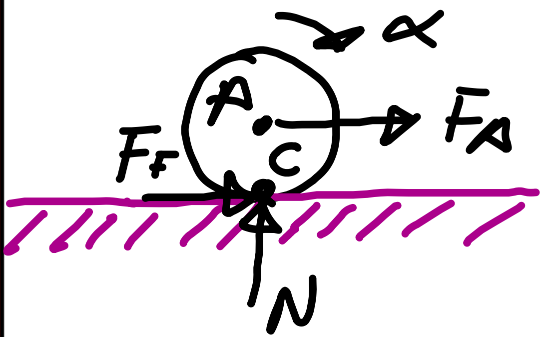


Frictional force applied at point C, but the path of point C is a cycloid

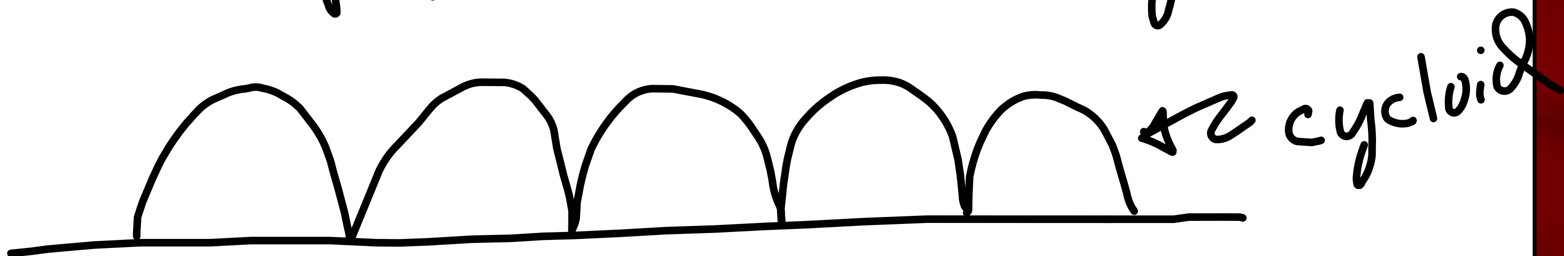


As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_c = 0$. In the past we found that $\vec{V}_c = 0$ but $\vec{V}_c = \frac{d\vec{l}_c}{dt} \Rightarrow d\vec{l} = \frac{d\vec{l}}{dt} dt$ so

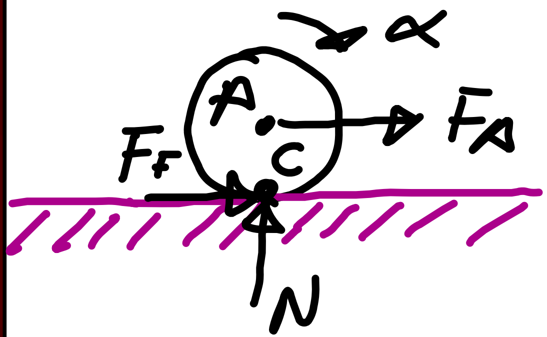
$$\vec{F}_F \cdot d\vec{l}_c = \left(\vec{F}_F \cdot \frac{d\vec{l}_c}{dt}\right) dt = 0.$$



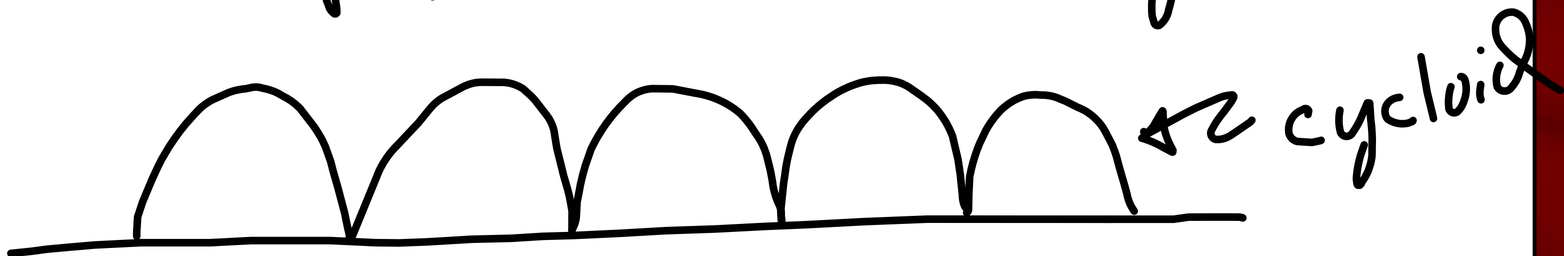
Frictional force applied at point C, but the path of point C is a cycloid



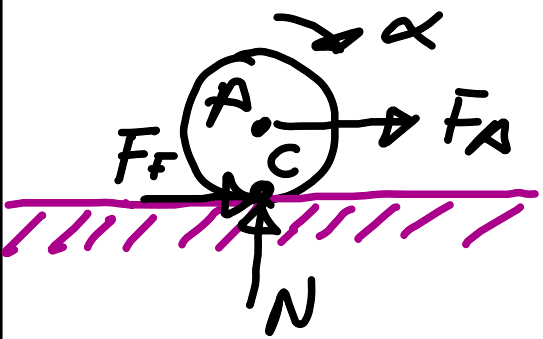
As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_c = 0$. In the past we found that $\vec{V}_c = 0$ but $\vec{V}_c = \frac{d\vec{l}_c}{dt} \Rightarrow d\vec{l} = \frac{d\vec{l}}{dt}\, dt$ so $\vec{F}_F \cdot d\vec{l}_c = \left(\vec{F}_F \cdot \frac{d\vec{l}_c}{dt}\right) dt = 0$.



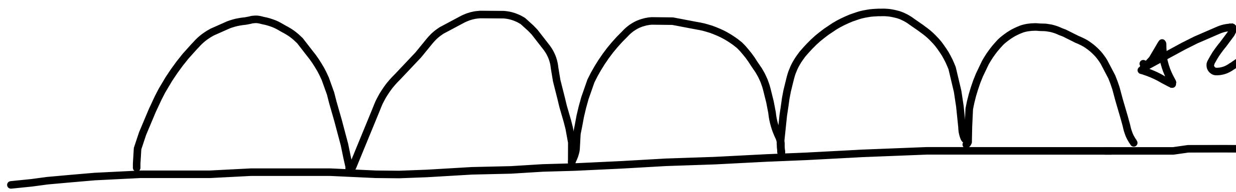
Frictional force applied at point C, but the path of point C is a cycloid



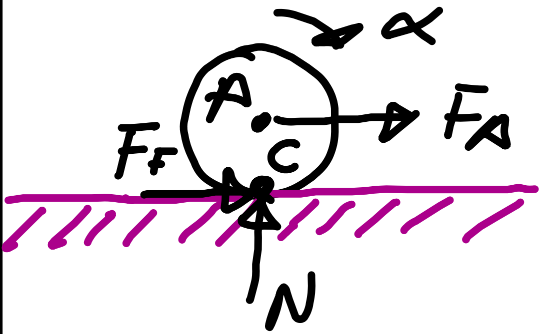
As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_c = 0$. In the past we found that $\vec{V}_c = 0$ but $\vec{V}_c = \frac{d\vec{l}_c}{dt} \Rightarrow d\vec{l} = \frac{d\vec{l}}{dt}dt$ so $\vec{F}_F \cdot d\vec{l}_c = \left(\vec{F}_F \cdot \frac{d\vec{l}_c}{dt}\right)dt = 0$. & No work done by this frictional force



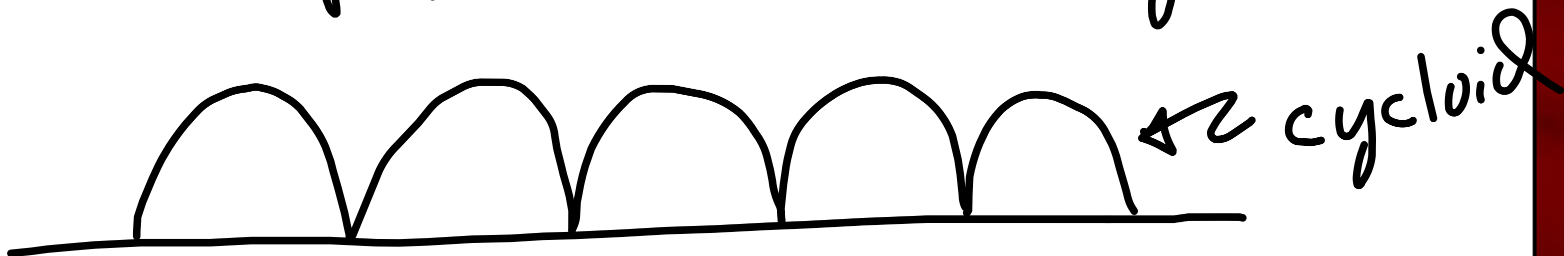
Frictional force applied at point C, but the path of point C is a cycloid



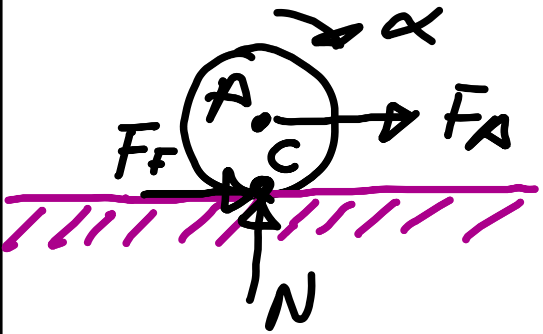
As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_c = 0$. In the past we found that $\vec{V}_c = 0$ but $\vec{V}_c = \frac{d\vec{l}_c}{dt} \Rightarrow d\vec{l} = \frac{d\vec{l}}{dt}dt$ so $\vec{F}_F \cdot d\vec{l}_c = \left(\vec{F}_F \cdot \frac{d\vec{l}_c}{dt}\right)dt = 0$. & No work done by this frictional force. However, if sliding,



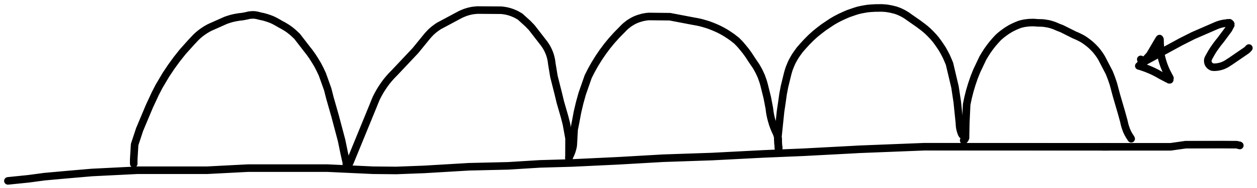
Frictional force applied at point C, but the path of point C is a cycloid



As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_c = 0$. In the past we found that $\vec{V}_c = 0$ but $\vec{V}_c = \frac{d\vec{l}_c}{dt} \Rightarrow d\vec{l} = \frac{d\vec{l}}{dt} dt$ so $\vec{F}_F \cdot d\vec{l}_c = \left(\vec{F}_F \cdot \frac{d\vec{l}_c}{dt}\right) dt = 0$. & No work done by this frictional force. However, if sliding, $W^{NC} \neq 0$



Frictional force applied at point C, but the path of point C is a cycloid



As point C touches the ground and F_F acts on that point, the product $\vec{F}_F \cdot d\vec{l}_c = 0$. In the past we found that $\vec{V}_c = 0$ but $\vec{V}_c = \frac{d\vec{l}_c}{dt} \Rightarrow d\vec{l} = \frac{d\vec{l}}{dt} dt$ so $\vec{F}_F \cdot d\vec{l}_c = \left(\vec{F}_F \cdot \frac{d\vec{l}_c}{dt}\right) dt = 0$. & No work done by this frictional force. However, if sliding, $W^{nc} \neq 0$ & work by that frictional force

For a disk rolling down an incline, starting at rest:

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Potential energy converted into 2 kinds of K.E.

For a disk rolling down an incline, starting at rest:

$$Mgh = \underbrace{\frac{1}{2}\bar{I}\omega^2}_{\text{Rotational K.E.}} + \frac{1}{2}M\bar{v}^2$$

Potential energy converted into 2 kinds of K.E.

For a disk rolling down an incline, starting at rest:

$$Mgh = \underbrace{\frac{1}{2}I\omega^2}_{\text{Rotational K.E.}} + \underbrace{\frac{1}{2}M\bar{v}^2}_{\text{Translational K.E.}}$$

Potential energy converted into 2 kinds of K.E.

Rotational K.E.

Translational K.E.

For a disk rolling down an incline, starting at rest:

$$mgh = \underbrace{\frac{1}{2}\bar{I}\omega^2}_{\text{Rotational K.E.}} + \underbrace{\frac{1}{2}m\bar{v}^2}_{\text{Translational K.E.}}$$

Potential energy converted into 2 kinds of K.E.

Rotational K.E.

Translational K.E.

We can write $\bar{I} = m\bar{k}^2$

For a disk rolling down an incline, starting at rest:

$$mgh = \underbrace{\frac{1}{2}\bar{I}\omega^2}_{\text{Rotational K.E.}} + \underbrace{\frac{1}{2}m\bar{v}^2}_{\text{Translational K.E.}}$$

Potential energy converted into 2 kinds of K.E.

We can write $\bar{I} = m\bar{k}^2 = m\left(\frac{\bar{k}^2}{r^2}\right)r^2$

For a disk rolling down an incline, starting at rest:

$$\underbrace{mgh}_{} = \underbrace{\tfrac{1}{2}\bar{I}\omega^2}_{\text{Rotational K.E.}} + \underbrace{\tfrac{1}{2}m\bar{v}^2}_{\text{Translational K.E.}}$$

Potential energy converted into 2 kinds of K.E.

We can write $\bar{I} = m\bar{k}^2 = m\left(\frac{\bar{k}^2}{r^2}\right)r^2$, Let $\lambda \equiv \bar{k}^2/r^2$

For a disk rolling down an incline, starting at rest:

$$mgh = \underbrace{\tfrac{1}{2}\bar{I}\omega^2}_{\text{Rotational K.E.}} + \underbrace{\tfrac{1}{2}m\bar{v}^2}_{\text{Translational K.E.}}$$

Potential energy converted into 2 kinds of K.E.

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$$\bar{v}^2 = \left(\frac{2gh}{\lambda+1}\right)$$

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$\bar{v}^2 = \left(\frac{2gh}{\lambda+1}\right) \Rightarrow$ the smaller λ is, the greater the velocity of our wheel for a fixed amount of expended potential energy.

For a disk rolling down an incline, starting at rest:

$$mgh = \underbrace{\tfrac{1}{2}\bar{I}\omega^2}_{\text{Rotational K.E.}} + \underbrace{\tfrac{1}{2}m\bar{v}^2}_{\text{Translational K.E.}}$$

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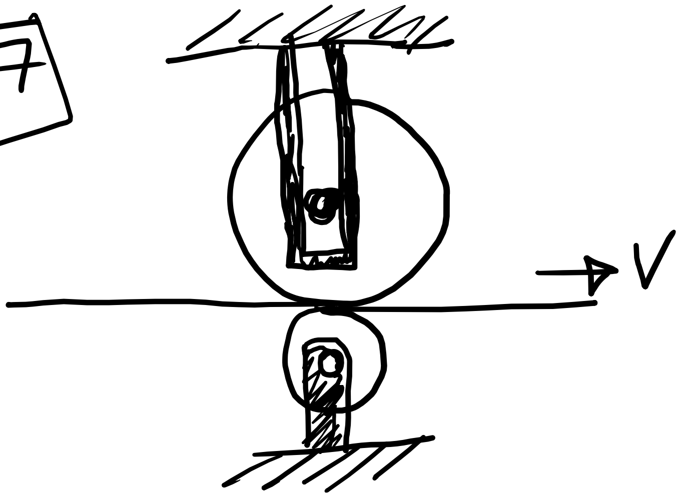
Now $mgh = \frac{1}{2}m\lambda r^2\omega^2 + \frac{1}{2}m\bar{v}^2 = \left(\frac{1}{2}m\bar{v}^2\right)(\lambda+1) \Rightarrow$

$\bar{v}^2 = \left(\frac{2gh}{\lambda+1}\right) \Rightarrow$ the smaller λ is, the greater the velocity of our wheel for a fixed amount of expended potential energy. So, for large $\bar{v}$ we want small $\frac{\bar{k}^2}{r^2} = \bar{I}/(mr^2)$

Notes on problems

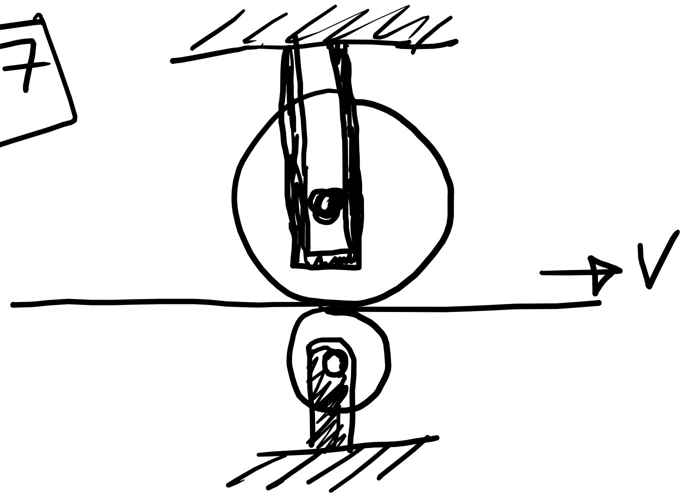
Notes on problems

17.7



Notes on problems

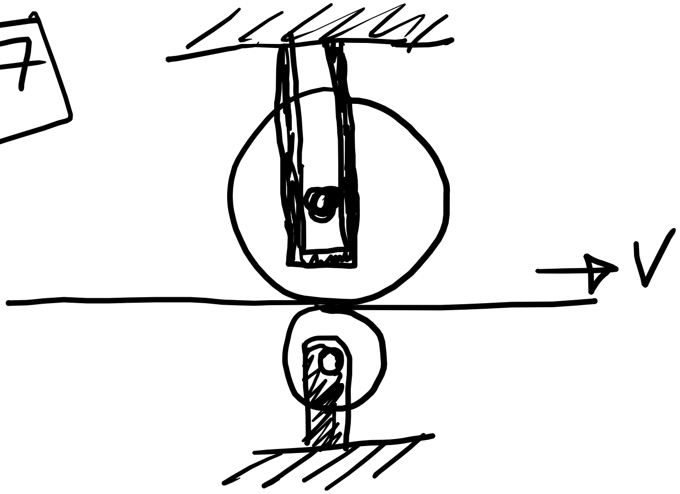
17.7



Given W=10lb,

Notes on problems

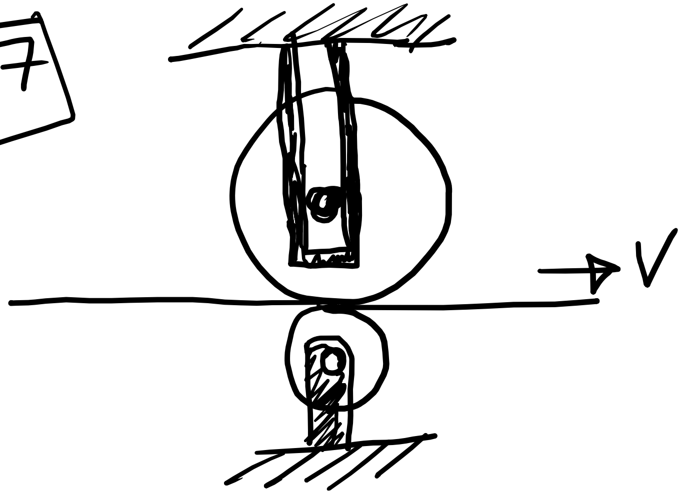
$\boxed{17.7}$



Given $W = 10\,lb$, $r = \frac{1}{2}\,ft$

Notes on problems

$\boxed{17.7}$

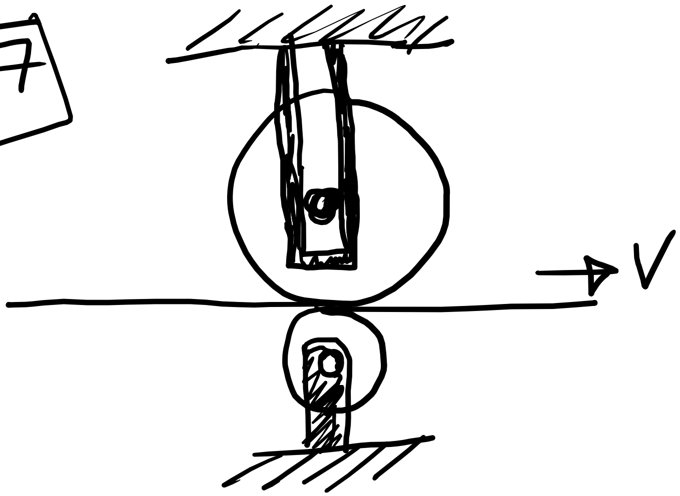


Given $W = 10\,lb$, $r = \frac{1}{2}\,ft$

$\mu_I = \theta$,

Notes on problems

17.7

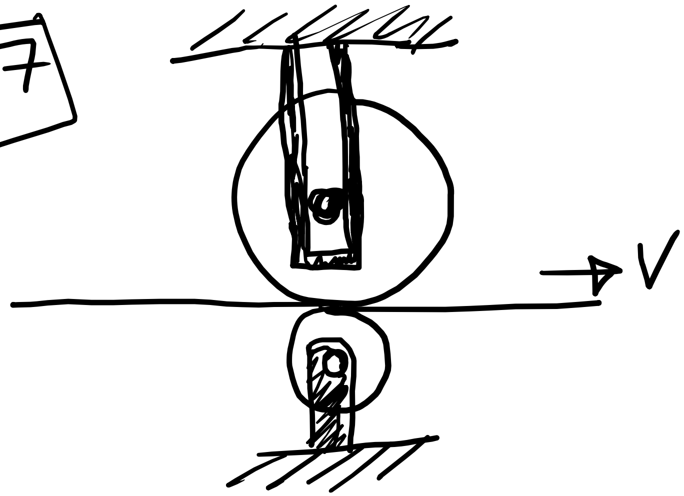


Given $W = 10\,lb$, $r = \frac{1}{2}\,ft$

$\ell\ell_I = \cancel{0}$, $V = 40\,ft/s$

Notes on problems

$\boxed{17.7}$



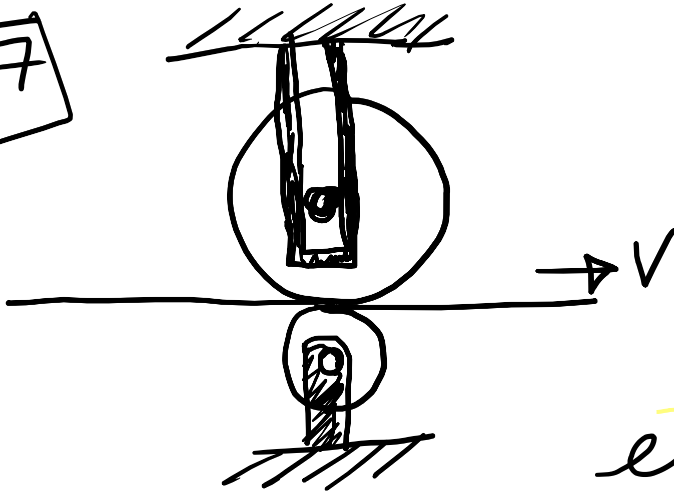
Given $W = 10\,lb$, $r = \frac{1}{2}\,ft$

$\omega_1 = \cancel{0}$, $v = 40\,ft/s$,

$\mu_k = 0.2$

Notes on problems

$\boxed{17.7}$



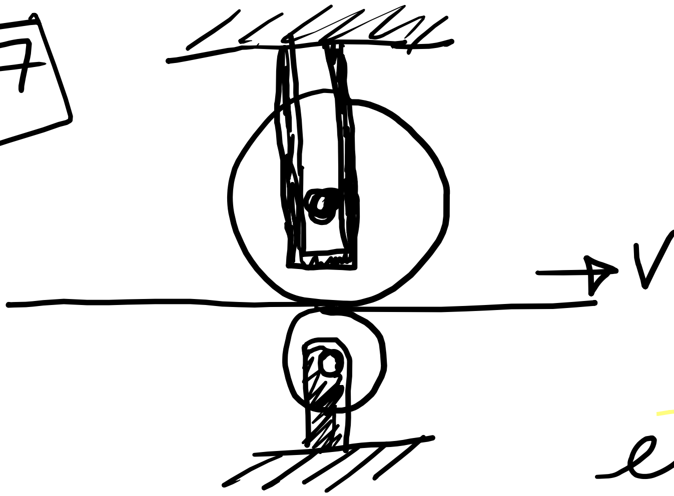
Given $W = 10\,lb$, $r = \frac{1}{2}\,ft$

$\omega_1 = 0$, $v = 40\,ft/s$,

$\mu_k = 0.2$ Find # of revolutions till $\omega = const.$

Notes on problems

17.7



Given $W = 10\,lb$, $r = \frac{1}{2}\,ft$

$\omega_I = \theta$, $v = 40\,ft/s$,

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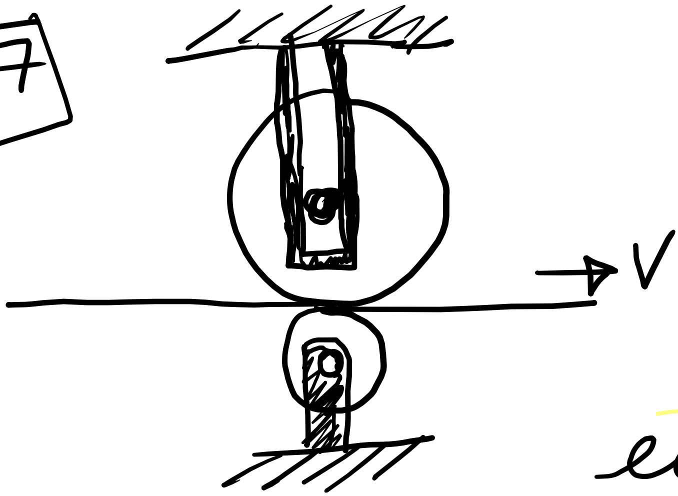
$* \; W = const.$ when $\omega r = v$

Notes on problems

$\boxed{17.7}$



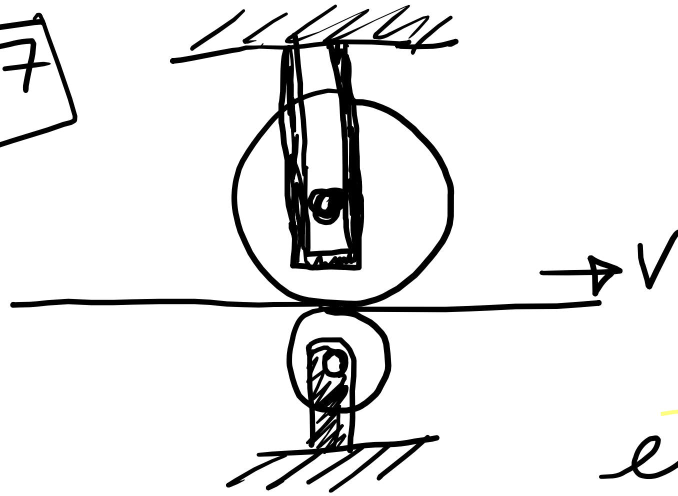
Given $W = 10\,lb$, $r = \frac{1}{2}\,ft$, $\omega_I = \theta$, $v = 40\,ft/s$, $\mu_k = 0.2$. Find # of revolutions till $\omega = $ const.

* $\omega = $ const. when $\omega r = V$ &

* # rev $= \dfrac{\Delta\theta}{2\pi}$

Notes on problems

$\boxed{17.7}$



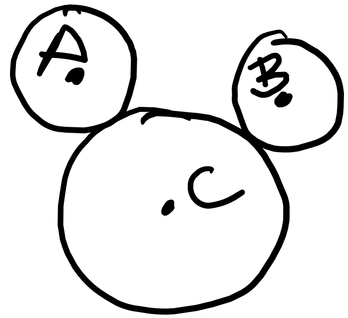
Given $W = 10\,lb$, $r = \frac{1}{2}\,ft$, $\omega_1 = 0$, $v = 40\,ft/s$, $\mu_k = 0.2$. Find # of revolutions till $\omega = \text{const.}$

* $\omega = \text{const.}$ when $\omega r = V$ &

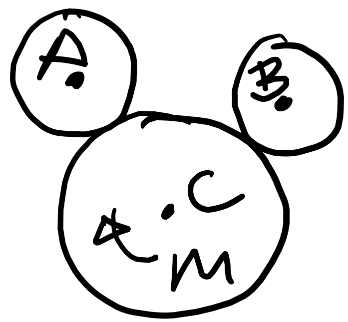
* $\#\,rev = \dfrac{\Delta\theta}{2\pi}$ &

* $\int M\,d\theta = T_2 - T_1$

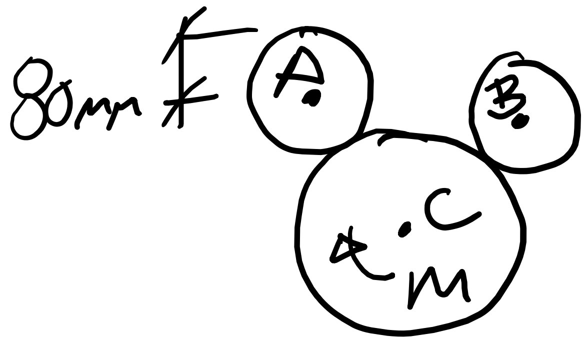
17.11a



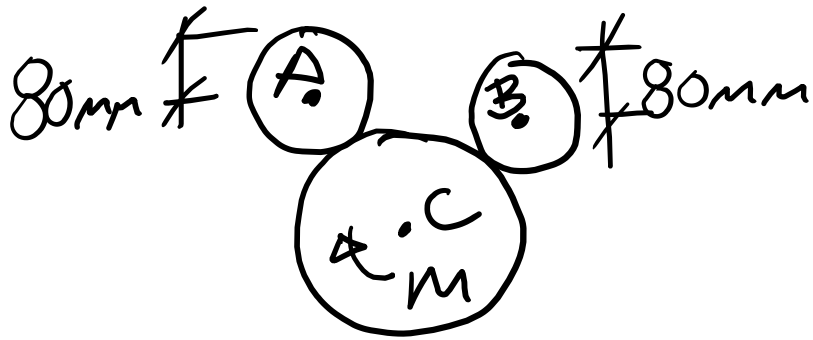
17.11a



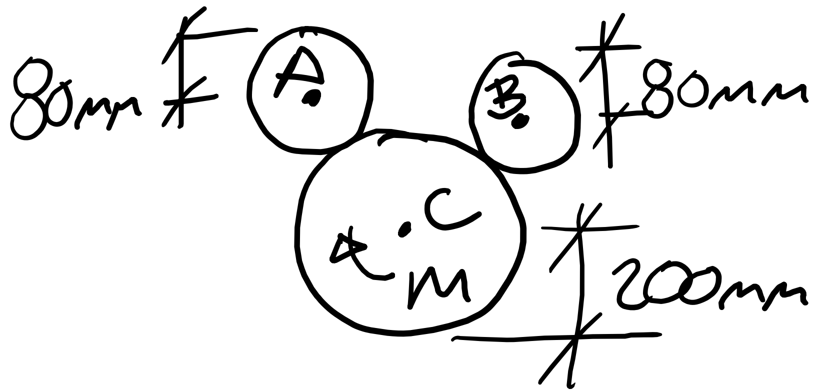
17.11a



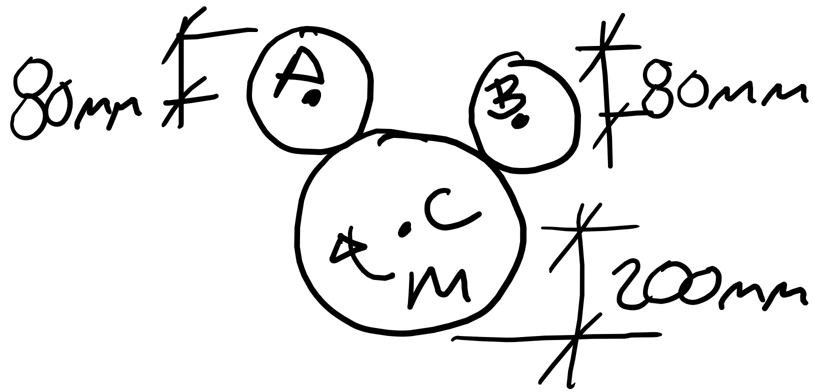
17.11a



17.11a

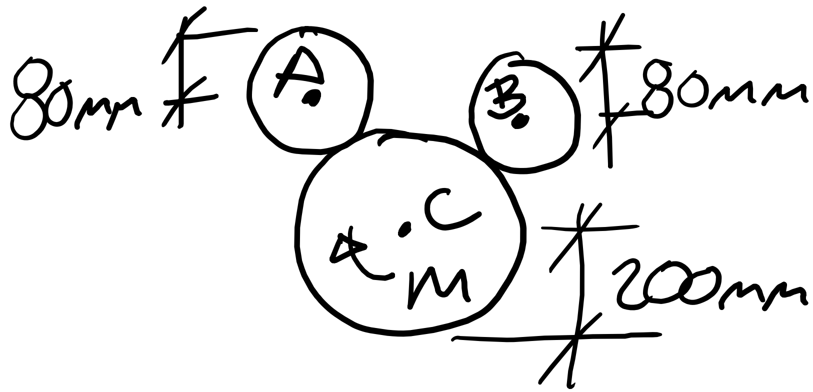


$\boxed{17.11a}$



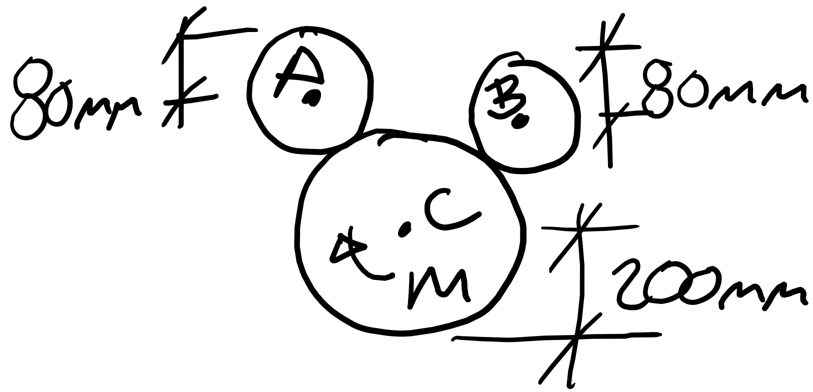
* Notice that $r_A\omega_A = r_B\omega_B$

17.11a



* Notice that $r_A \omega_A = r_B \omega_B = r_C \omega_C$

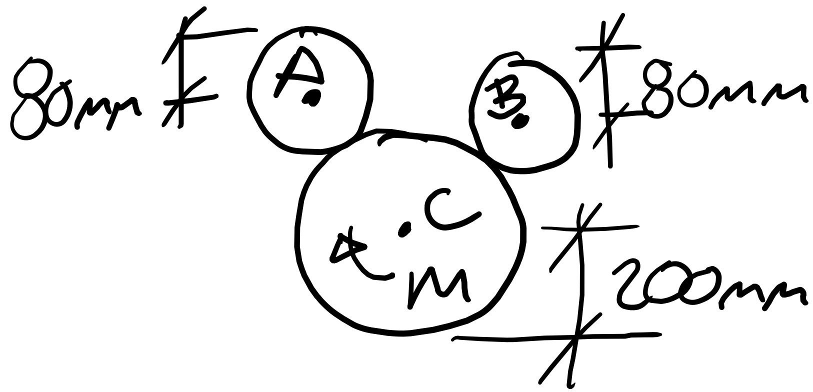
$\boxed{17.11a}$



* Notice that $r_A \omega_A = r_B \omega_B = r_C \omega_C$

* $\int M_C \, d\theta_C = T_2 - T_1$

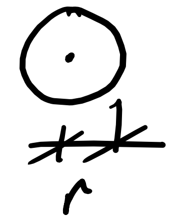
17.11a



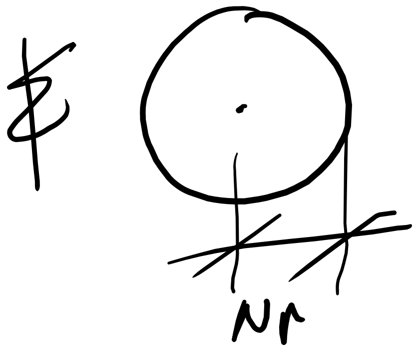
* Notice that $r_A\omega_A = r_B\omega_B = r_C\omega_C$

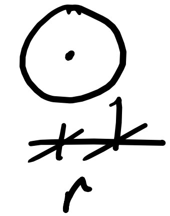
* $\int M_C \, d\theta_C = T_2 - T_1$, where T includes all 3 gears

$\boxed{17.13}$ Given two sized gears



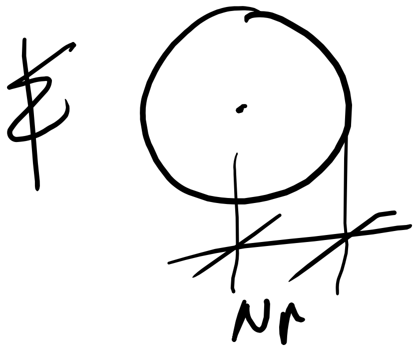
$\boxed{17.13}$ Given two sized gears

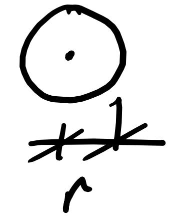




$\boxed{17.13}$

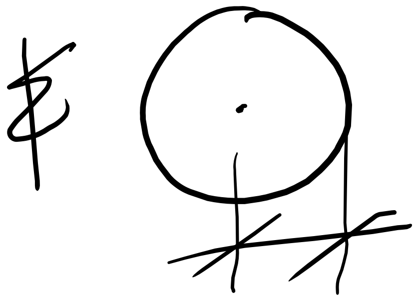
Given two sized gears and that the moment of inertia for gear of radius r is $\bar{I}_o$.



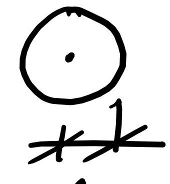


$\boxed{17.13}$

Given two sized gears and that the moment of inertia for gear of radius r is $\bar{I}_0$. You will need to figure out how



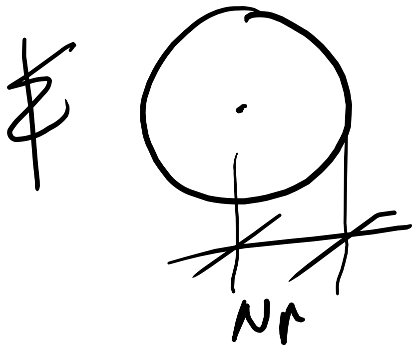
Nr



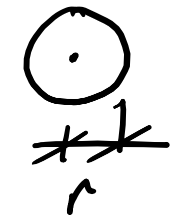
r

$\bar{I}_N$

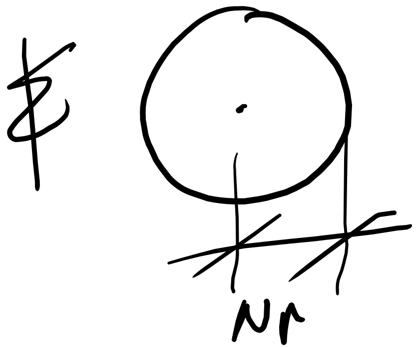
$\boxed{17.13}$



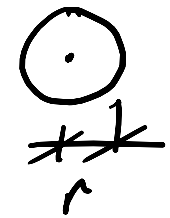
Given two sized gears and that the moment of inertia for gear of radius r is $\bar{I}_o$. You will need to figure out how $\bar{I}_N$ (moment of inertia for gear of radius $= Nr$)



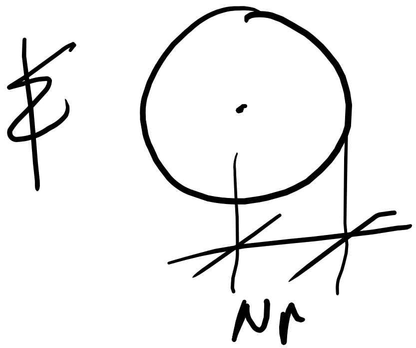
$\boxed{17.13}$

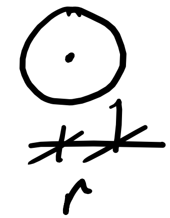


Given two sized gears and that the moment of inertia for gear of radius r is $\bar{I}_o$. You will need to figure out how $\bar{I}_N$ (moment of inertia for gear of radius=Nr) scales with $\bar{I}_o$.



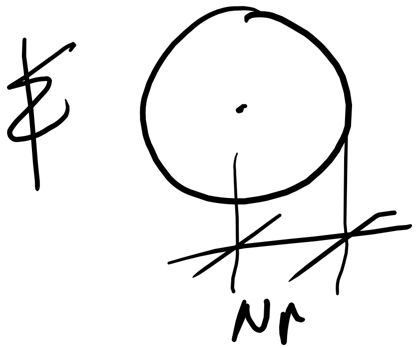
$\boxed{17.13}$ Given two sized gears and that the moment of inertia for gear of radius r is $\bar{I}_o$.

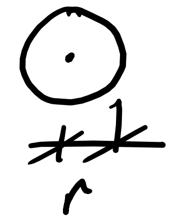




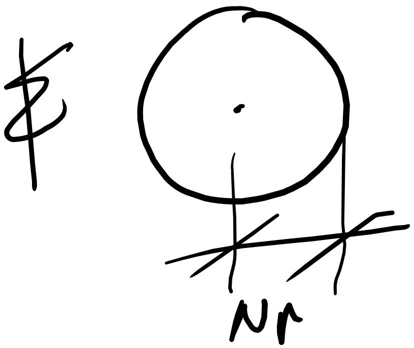
You will need to figure out how $\bar{I}_N$ (moment of inertia for gear of radius $= Nr$) scales with $\bar{I}_o$. Use disk as reference:

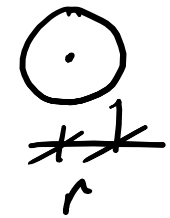
$\boxed{17.13}$ Given two sized gears and that the moment of inertia for gear of radius r is $\bar{I}_o$. You will need to figure out how $\bar{I}_N$ (moment of inertia for gear of radius $= Nr$) scales with $\bar{I}_o$. Use disk as reference: $\bar{I}_{disk} = \frac{1}{2}Mr^2$





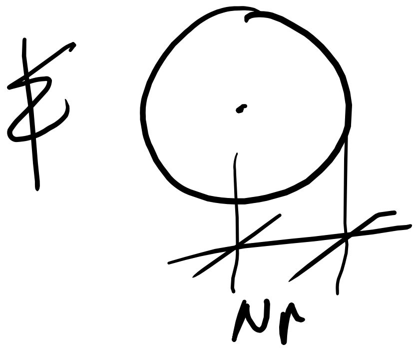
$\boxed{17.13}$

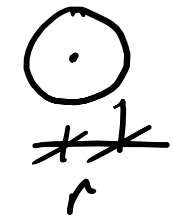




Given two sized gears and that the moment of inertia for gear of radius r is $\bar{I}_0$. You will need to figure out how $\bar{I}_N$ (moment of inertia for gear of radius=Nr) scales with $\bar{I}_0$. Use disk as reference: $\bar{I}_{disk} = \frac{1}{2}Mr^2$ so $r \to Nr$ implies $\bar{I}_{disk} \to \frac{1}{2}M'(Nr)^2$

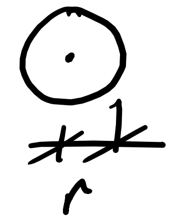
$\boxed{17.13}$



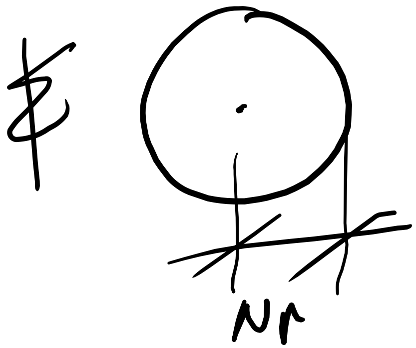


Given two sized gears and that the moment of inertia for gear of radius r is $\bar{I}_0$. You will need to figure out how $\bar{I}_N$ (moment of inertia for gear of radius $= Nr$) scales with $\bar{I}_0$. Use disk as reference: $\bar{I}_{disk} = \frac{1}{2}Mr^2$ so $r \to Nr$ implies $\bar{I}_{disk} \to \frac{1}{2}M'(Nr)^2$ & since $M = \rho\pi r^2$,

$\boxed{17.13}$ Given two sized gears

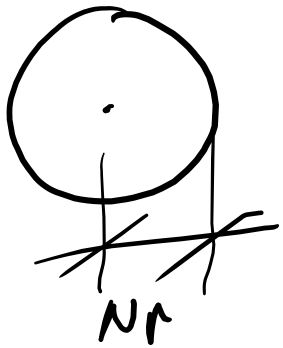


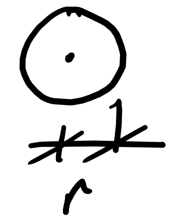
and that the moment of inertia for gear of radius r is $\bar{I}_o$.



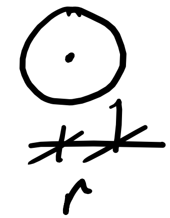
You will need to figure out how $\bar{I}_N$ (moment of inertia for gear of radius $= Nr$) scales with $\bar{I}_o$. Use disk as reference: $\bar{I}_{disk} = \frac{1}{2}Mr^2$ so $r \to Nr$ implies $\bar{I}_{disk} \to \frac{1}{2}M'(Nr)^2$ & since $M = \rho \pi r^2$, where $\rho = \text{mass}/\text{Area}$

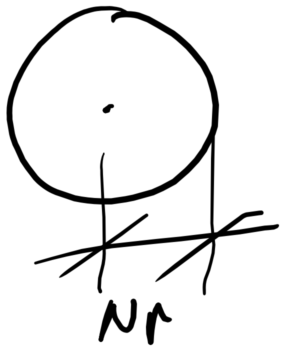
$\boxed{17.13}$ Given two sized gears and that the moment of inertia for gear of radius r is $\bar{I}_0$. You will need to figure out how $\bar{I}_N$ (moment of inertia for gear of radius $= Nr$) scales with $\bar{I}_0$. Use disk as reference: $\bar{I}_{disk} = \frac{1}{2}Mr^2$ so $r \to Nr$ implies $\bar{I}_{disk} \to \frac{1}{2}M'(Nr)^2$ & since $M = \rho\pi r^2$, where ρ = mass/Area then $M' = \rho\pi(Nr^2)$





$\boxed{17.13}$

Given two sized gears 

& 

and that the moment of inertia for gear of radius r is $\bar{I}_o$. You will need to figure out how $\bar{I}_N$ (moment of inertia for gear of radius$=Nr$) scales with $\bar{I}_o$. Use disk as reference: $\bar{I}_{disk} = \frac{1}{2}Mr^2$ so $r \to Nr$ implies $\bar{I}_{disk} \to \frac{1}{2}M'(Nr)^2$ & since $M=\rho\pi r^2$, where $\rho = $ mass/Area then $M' = \rho\pi(Nr^2)$

$$\Rightarrow \quad \bar{I}_N = \bar{I}_o N^4$$

$\boxed{17.16a}$ → Conservation of energy

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$L \equiv$ length of spring

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$L \equiv$ length of spring, $L_0 \equiv$ natural length of spring

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$\boxed{17.18}$ → Conserve energy, remember that $L = L_0 + x$, where $L \equiv$ length of spring, $L_0 \equiv$ natural length of spring & $V = \frac{1}{2}kx^2$