Today 17.2

$\boxed{L19}$

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Thursday Review

$\boxed{L19}$

We found that no work was performed by static friction for wheel rolling without slipping

We found that No work was performed by static friction for wheel rolling without slipping <u>On fixed surface</u>

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$$U = \int \vec{F}_F \cdot \vec{v}_c \, dt$$

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$$U = \int \vec{F}_F \cdot \vec{V}_c\, dt$$ But roll No slip

$$\Rightarrow \vec{V}_c = \vec{V}_{surface}$$

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$$\Rightarrow \vec{V}_c = \vec{V}_{surface} = 0$$

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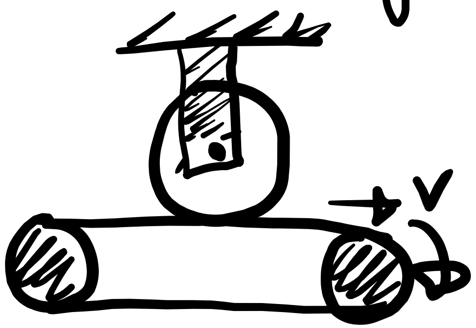

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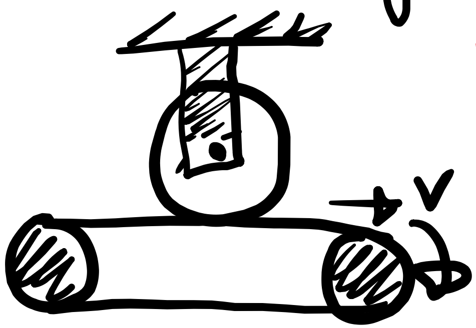

What about case where surface is moving?

We found that No work was performed by static friction for wheel rolling without slipping On fixed surface

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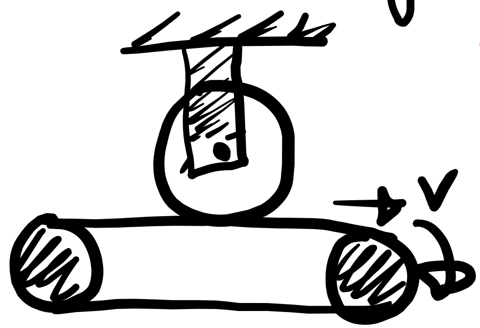

What about case where surface is moving? Here

$\vec{V}_c \neq 0$

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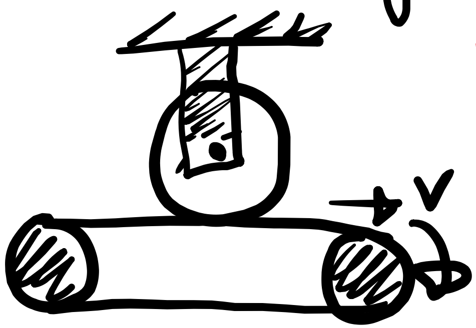

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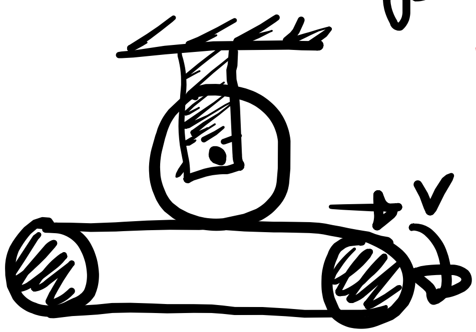

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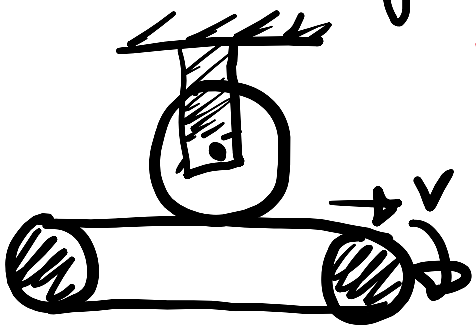

What about case where surface is moving? Here

$\vec{V}_c \neq 0$ &

$$U = \int \vec{F}_F \cdot \vec{V}\, dt \neq 0$$

work performed by this frictional force

Last time we saw that

$$T_I + V_I + U^{NC}_{I\to F} = T_F + V_F$$

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$$T_I + V_I + U^{NC}_{I\to F} = T_F + V_F, \text{ where } T = \frac{1}{2}\bar{I}\,\omega^2 + \frac{1}{2}M\bar{v}^2$$

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$T_I + V_I + U^{NC}_{I\to F} = T_F + V_F$, where $T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}M\bar{v}^2$

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and if there is a fixed point of rotation about P, we could write

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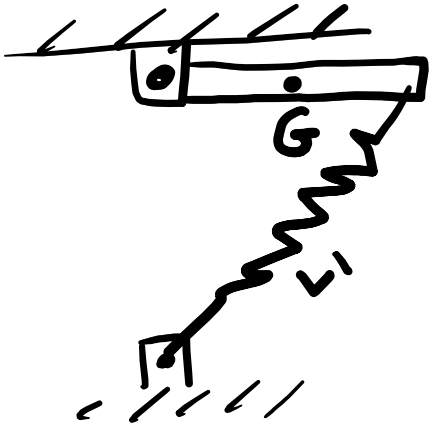
and if there is a fixed point of rotation about P, we could write

$T = \frac{1}{2}I_P\omega^2$, where $I = \bar{I} + m r^2_{G/P}$

Example : Find K.E. when rod is vertical



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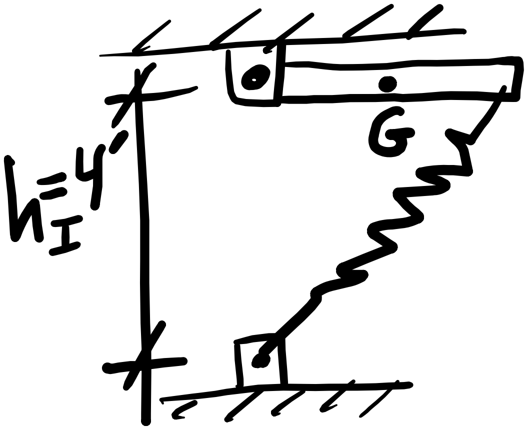

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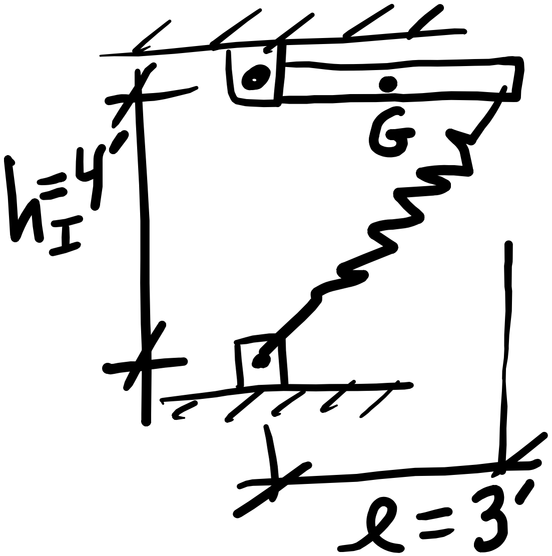

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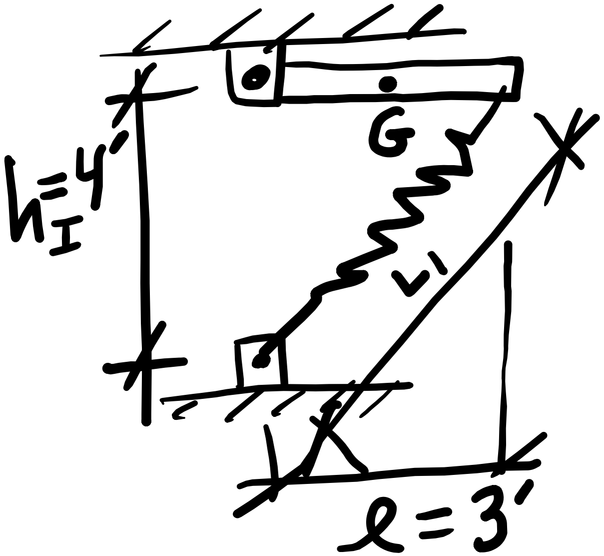

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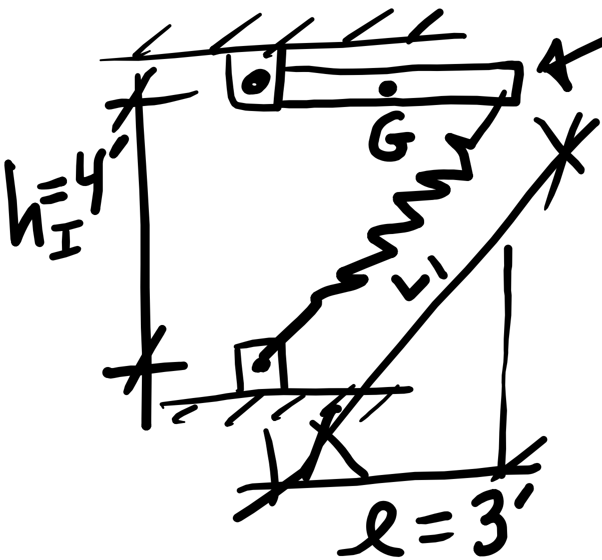
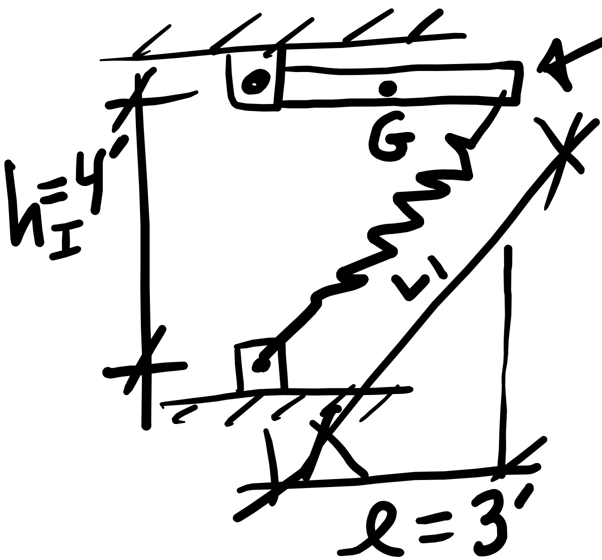


Slender rod of weight $w = 2\,lb$ rotates about fixed point A.

Example: Find K.E. when rod is vertical



Slender rod of weight $W = 2\,lb$ rotates about fixed point A.
Natural length of spring $L_0 = 2\,ft$

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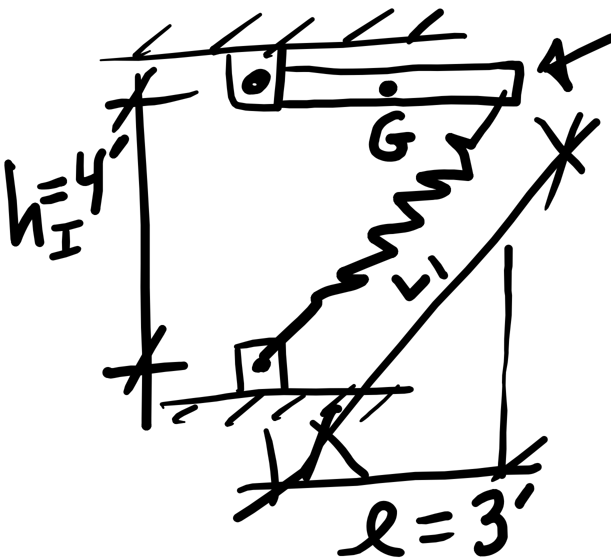
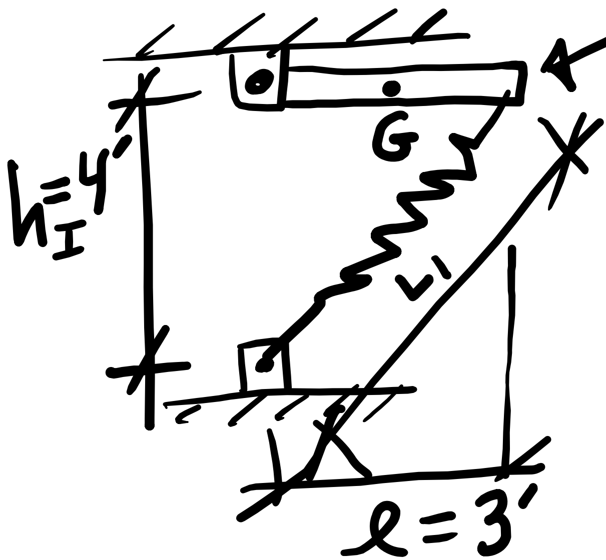


Slender rod of weight $w = 2\,lb$ rotates about fixed point A.

Natural length of spring $L_0 = 2\,ft$

Spring constant $k = 0.75\,lb/ft$

Example : Find K.E. when rod is vertical



Slender rod of weight $w = 2 lb$ rotates about fixed point A. Natural length of spring $L_0 = 2 ft$. Spring constant $k = 0.75 lb/ft$. No friction.

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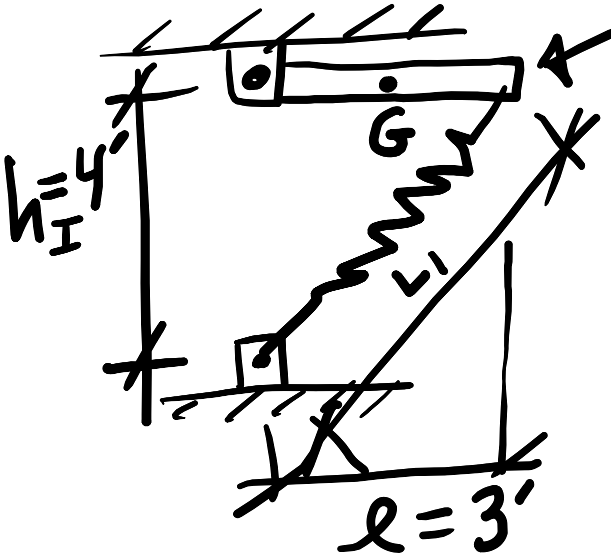
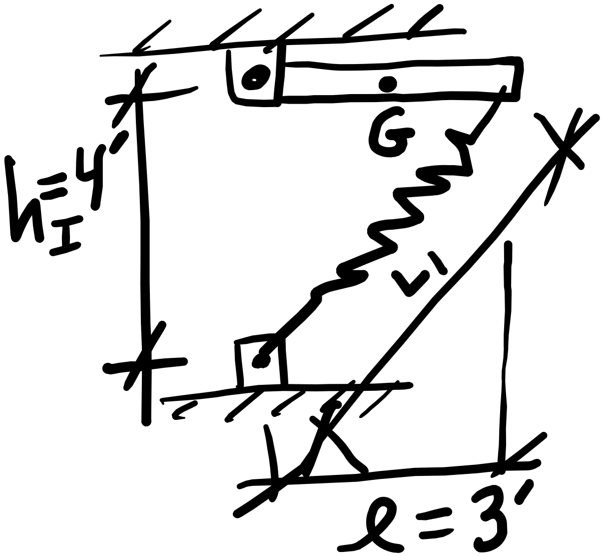


Slender rod of weight $w = 2\,lb$ rotates about fixed point A. Natural length of spring $l_0 = 2\,ft$. Spring constant $k = 0.75\,lb/ft$. No friction. Starts at rest

Example: Find K.E. when rod is vertical

$W = 2\,lb,\ L_0 = 2\,ft,\ k = 0.75\,lb/ft$



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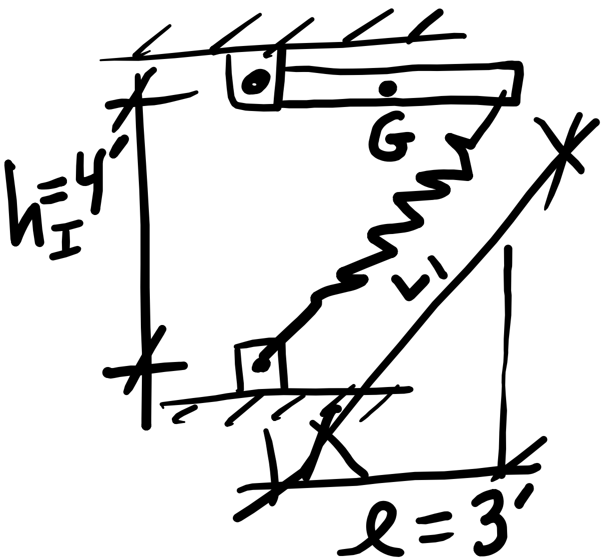

$W = 2\,lb$, $L_0 = 2\,ft$, $k = 0.75\,lb/ft$

$$T_1 + V_1 + U^{nc}_{1\to 2} = T_2 + V_2$$

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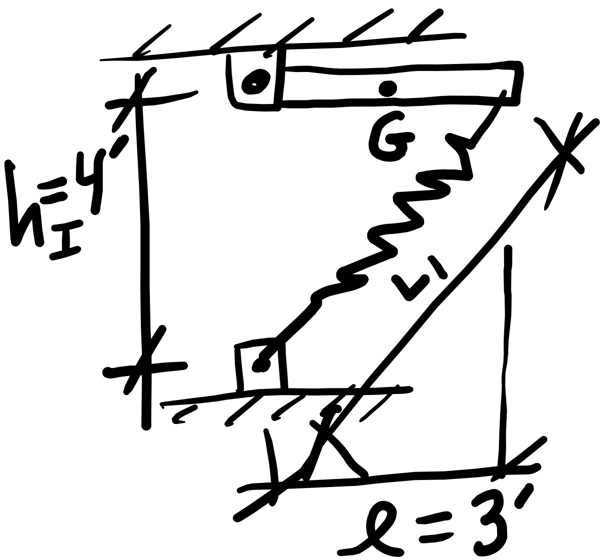

$W = 2\,lb, \; L_0 = 2\,ft, \; k = 0.75\,lb/ft$

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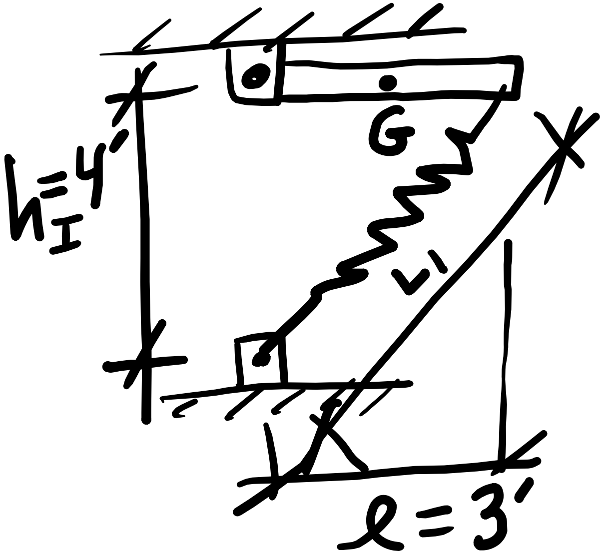

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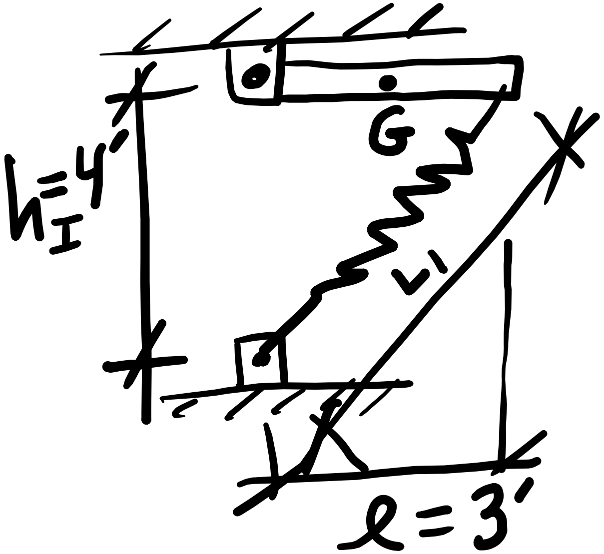

$W = 2\,lb,\ L_0 = 2\,ft,\ k = 0.75\,lb/ft$

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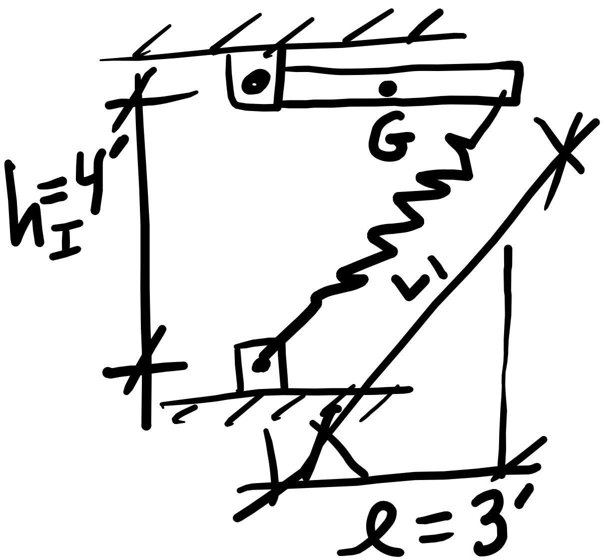

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where $V_1 = \frac{1}{2}kx_1^2 + mgy_1$

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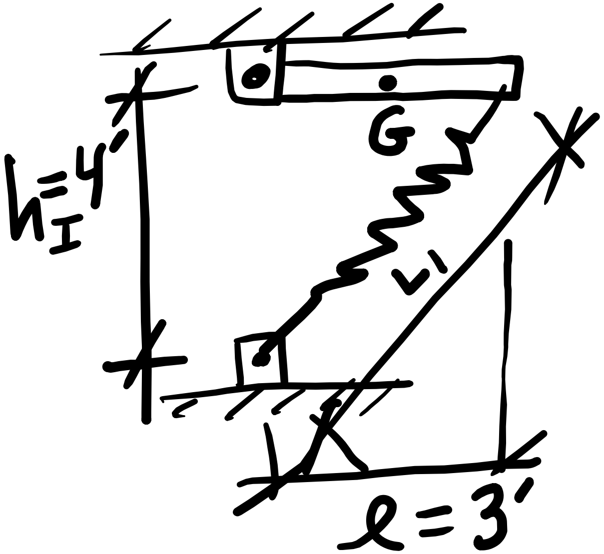

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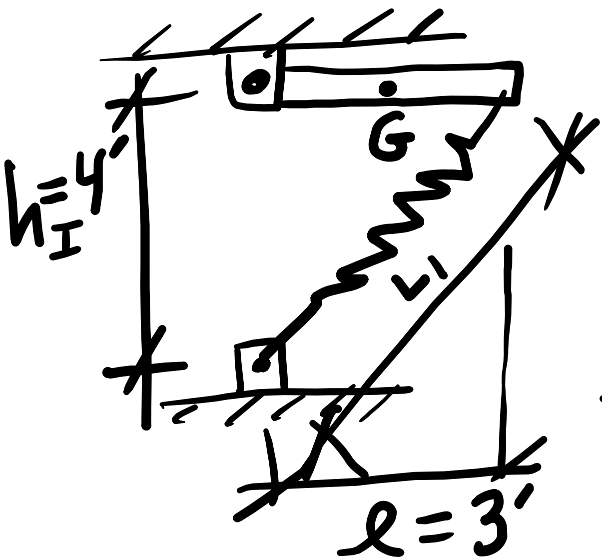

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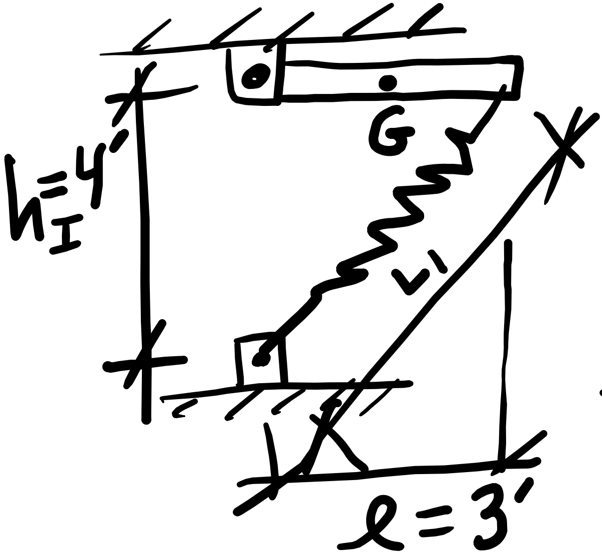

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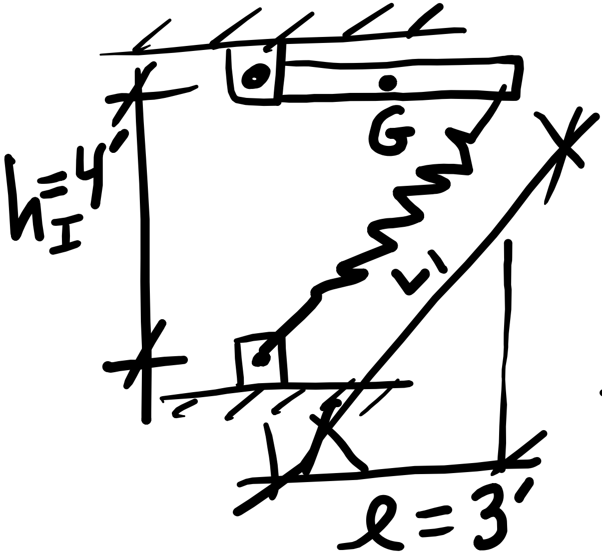
$\Rightarrow T_2 = \frac{1}{2}k(x_1^2 - x_2^2) + mg(y_1 - y_2)$, here

$y_1 - y_2 = l/2 = \frac{3}{2}\,ft$ & $L_1 = L_0 + x_1$ &

$L_1 = \sqrt{3^2+4^2}\,ft = 5\,ft$

Example: Find K.E. when rod is vertical



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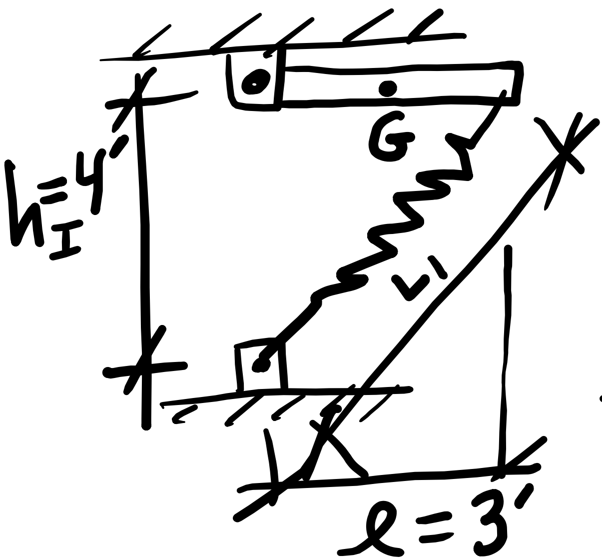
Example: Find K.E. when rod is vertical



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$L_2 = h - l = (4-3)\,ft = 1\,ft$

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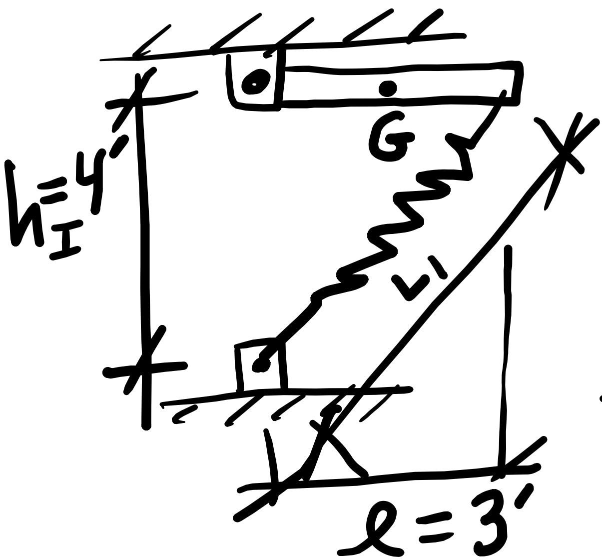

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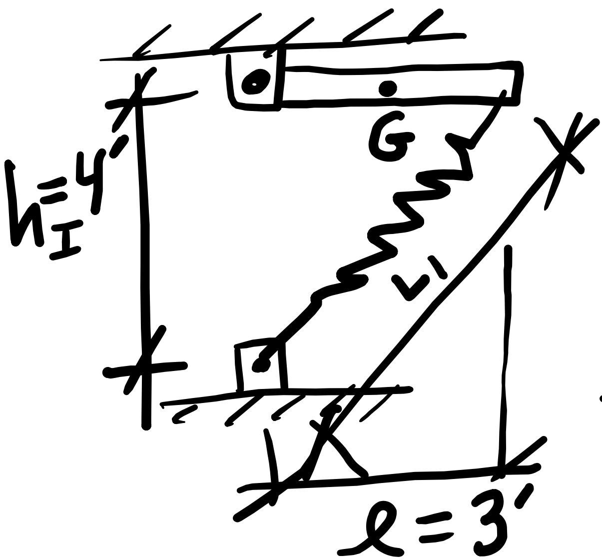

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So $T_2 = \left[\frac{1}{2}\left(\frac{3}{4}\right)(9-1) + 2\left(\frac{3}{2}\right)\right] lb\cdot ft$

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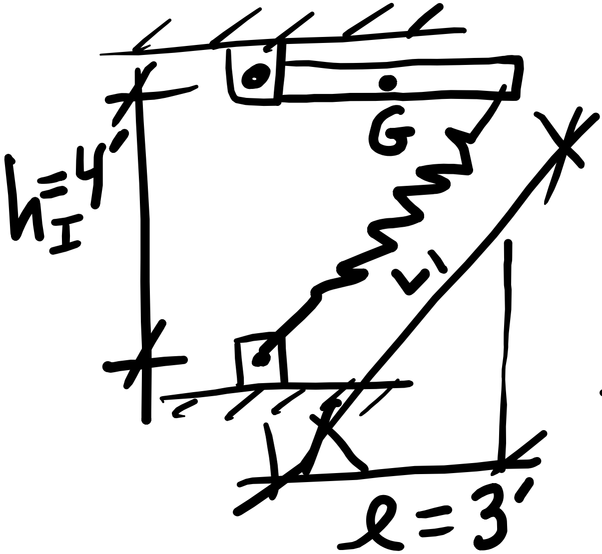

$W = 2\,lb$, $L_0 = 2\,ft$, $k = 0.75\,lb/ft$

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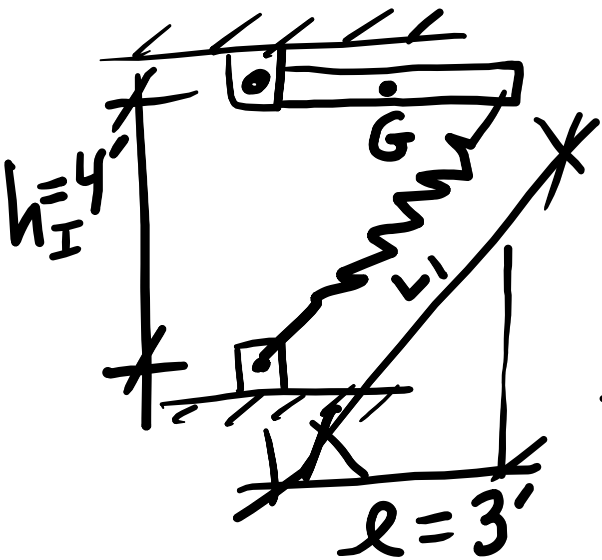
So $T_2 = \left[\frac{1}{2}\left(\frac{3}{4}\right)(9-1) + 2\left(\frac{3}{2}\right)\right] lb\cdot ft = \left[\frac{3}{8} * 8 + 3\right] lb\cdot ft = \boxed{6\,lb\cdot ft}$

Example: Find K.E. when rod is vertical



$W = 2\,lb,\ L_0 = 2\,ft,\ k = 0.75\,lb/ft$

$\cancel{T_1} + V_1 + \cancel{U_{1\to 2}^{NC}} = T_2 + V_2 \Rightarrow T_2 = V_1 - V_2,$

where $V_1 = \frac{1}{2}kx_1^2 + mgy_1$ & $V_2 = \frac{1}{2}kx_2^2 + mgy_2$

$\Rightarrow T_2 = \frac{1}{2}k(x_1^2 - x_2^2) + mg(y_1 - y_2)$, here

$y_1 - y_2 = l/2 = \frac{3}{2}\,ft$ & $L_1 = L_0 + x_1$ &

$L_1 = \sqrt{3^2+4^2}\,ft = 5\,ft \Rightarrow x_1 = L_1 - L_0 = 5\,ft - 2\,ft = 3\,ft$ &

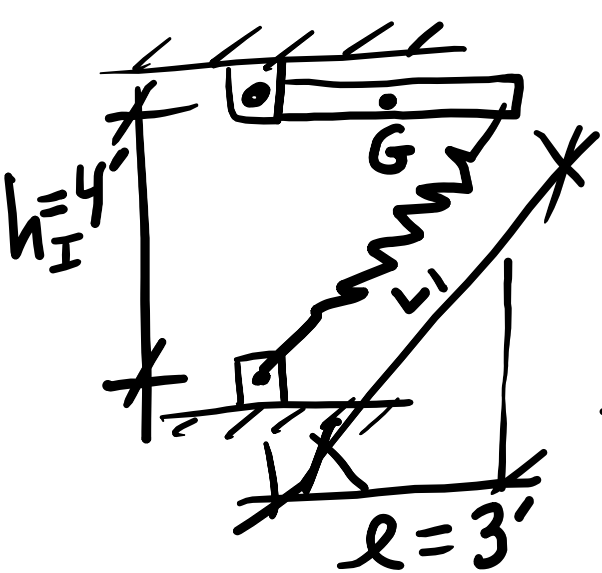
$L_2 = h - l = (4-3)\,ft = 1\,ft \Rightarrow x_2 = L_2 - L_0 = (1-2)\,ft = -1\,ft$

So $T_2 = \left[\frac{1}{2}\left(\frac{3}{4}\right)(9-1) + 2\left(\frac{3}{2}\right)\right] lb\cdot ft = \left[\frac{3}{8} * 8 + 3\right] lb\cdot ft = \boxed{6\,lb\cdot ft}$

If we wanted to find ω_2 we could use

$T_2 = \frac{1}{2}I_A\omega_2^2$

Example: Find K.E. when rod is vertical



$W = 2\,lb$, $L_0 = 2\,ft$, $k = 0.75\,lb/ft$

$$\cancel{T_1}^{\,0} + V_1 + \cancel{U_{1\to2}^{nc}}^{\,0} = T_2 + V_2 \Rightarrow T_2 = V_1 - V_2,$$

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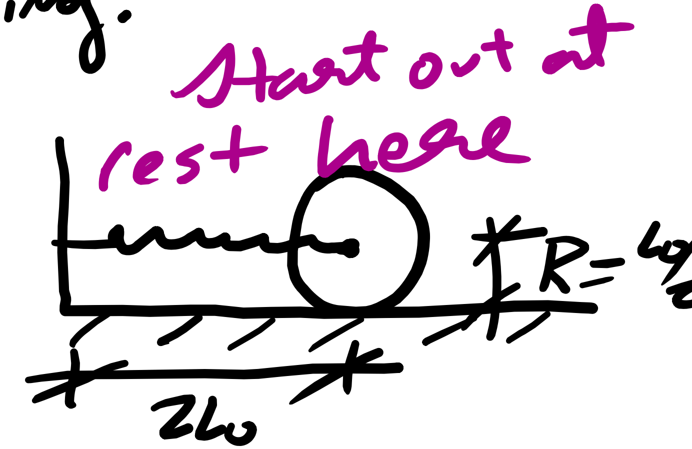
If we wanted to find ω_2 we could use

$$T_2 = \frac{1}{2}I_A\omega_2^2 \Rightarrow \omega_2 = \sqrt{\frac{2T_2}{I_A}}$$

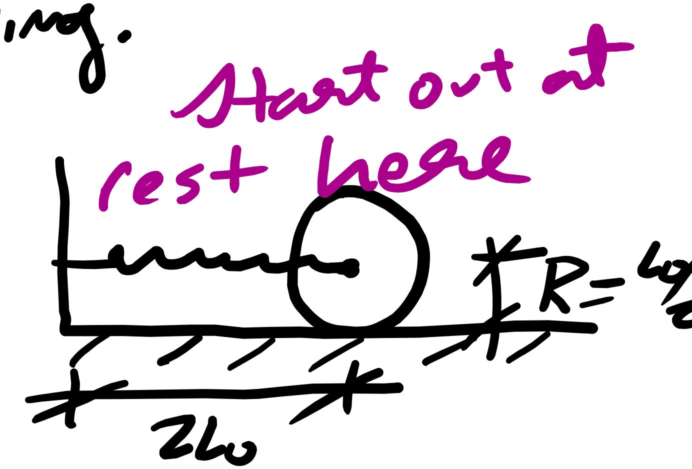
Example: Spring attached to axel of disk that rolls w/out slipping.

Example: Spring attached to axel of disk that rolls w/out slipping.



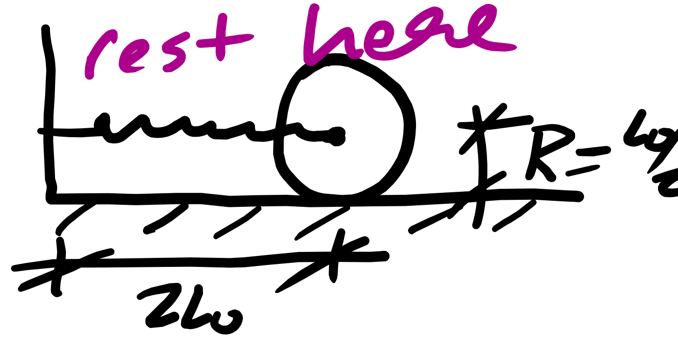
Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}_x$ when disk hits wall?



Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?

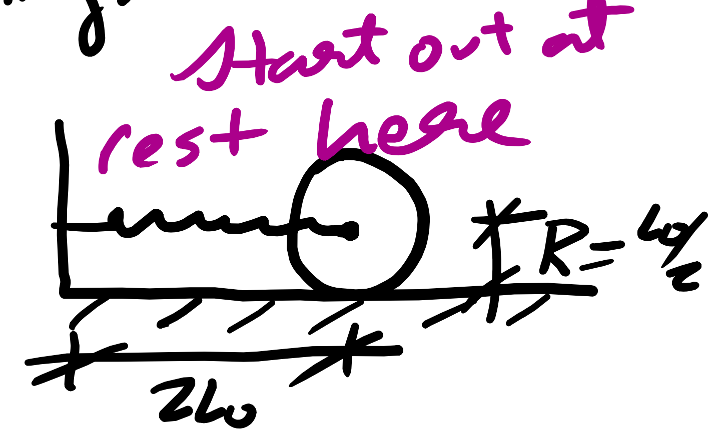


$$T_1 + V_1 + U_{1 \to 2}^{nc} = T_2 + V_2$$

Example: Spring attached to axel of disk that rolls w/out slipping.

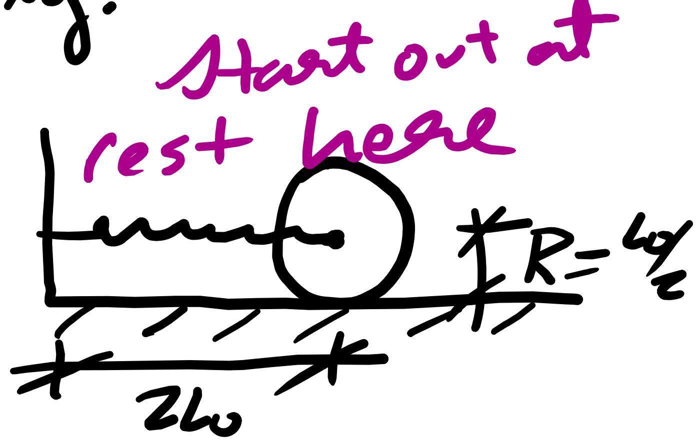
What is $\bar{v}$ when disk hits wall?$_x$

$$\cancel{T_1}^{\,0} + V_1 + U_{1\to2}^{nc} = T_2 + V_2$$



Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancel{T_1}^{0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{0} = T_2 + V_2$$

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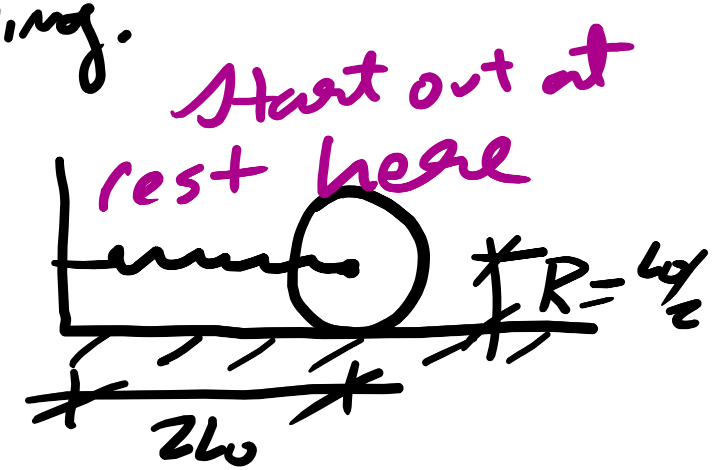
What is $\bar{v}$ when disk hits wall?



$$\cancel{T_1}^{=0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{=0} = T_2 + V_2, \quad V_1 = \tfrac{1}{2}kx_1^2$$

Example: Spring attached to axel of disk that rolls w/out slipping.

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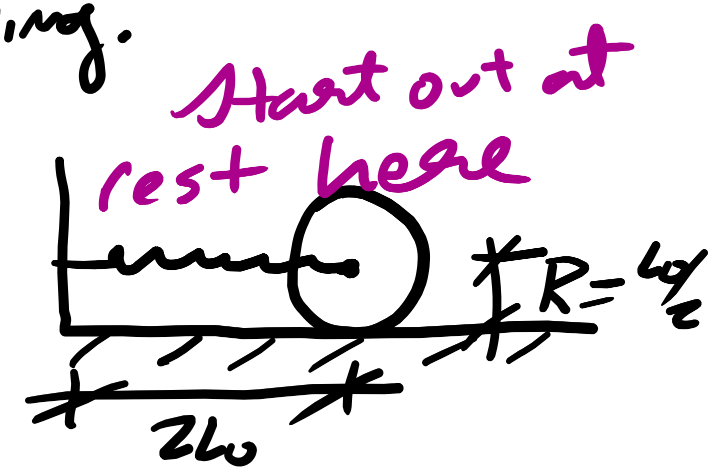

$$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{\,0} = T_2 + V_2\ , \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2$$

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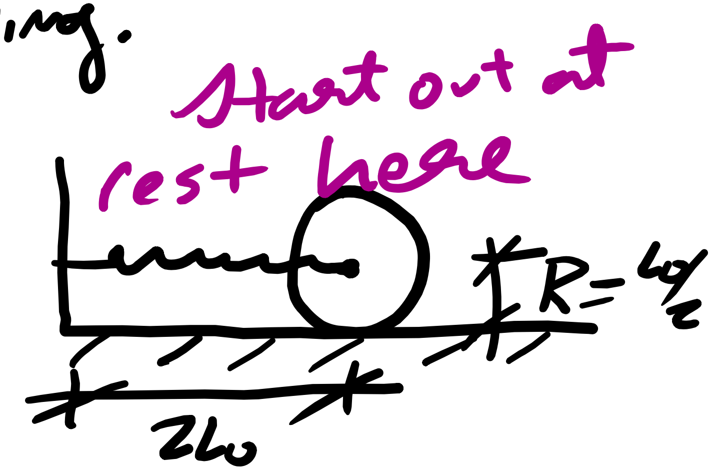

$$\cancel{T_1}^{\,0} + V_1 + \cancel{U_{1\to2}^{nc}}^{\,0} = T_2 + V_2\ , \quad V_1 = \tfrac{1}{2}kx_1^2$$

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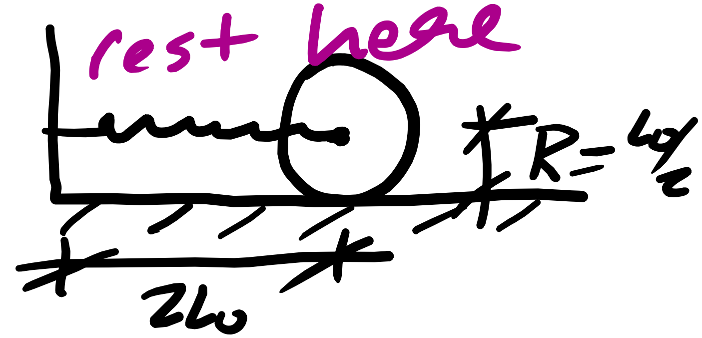
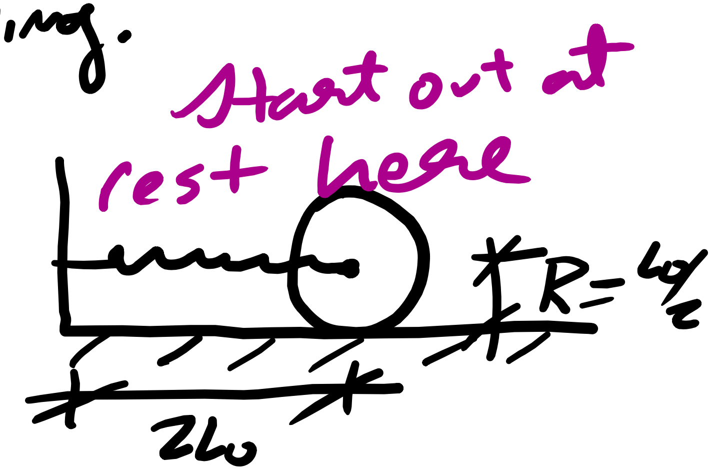


$$\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to 2}^{nc}}^{0} = T_2 + V_2\,,\quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2,\quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U^{nc}_{1\to 2}} = T_2 + V_2\ , \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2,\ V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

But $\bar{I} = \tfrac{1}{2}mR^2$

Example: Spring attached to axel of disk that rolls w/out slipping.

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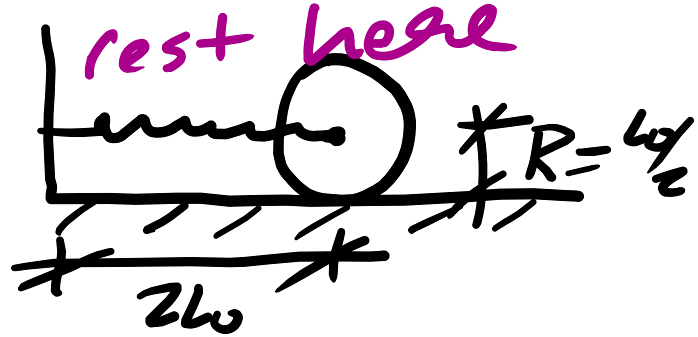
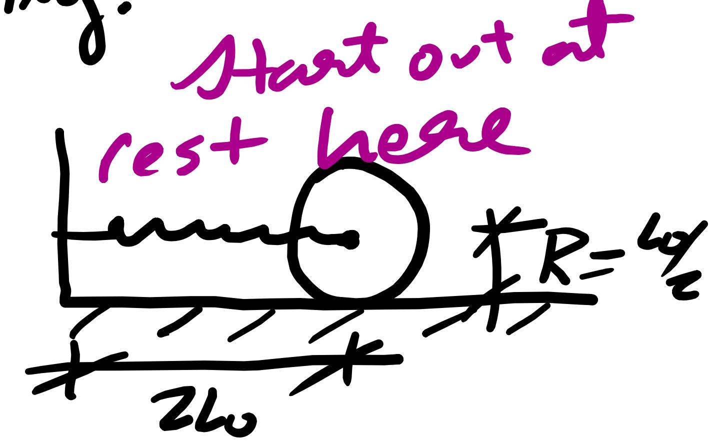


$$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{\,0} = T_2 + V_2 \,, \quad V_1 = \tfrac{1}{2} k x_1^2$$

$$T_2 = \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 , \quad V_2 = \tfrac{1}{2} k x_2^2 \Rightarrow \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 = \tfrac{1}{2} k (x_1^2 - x_2^2)$$

But $\bar{I} = \tfrac{1}{2} m R^2$ & $R\omega = \bar{v}$

Example: Spring attached to axel of disk that rolls w/out slipping.



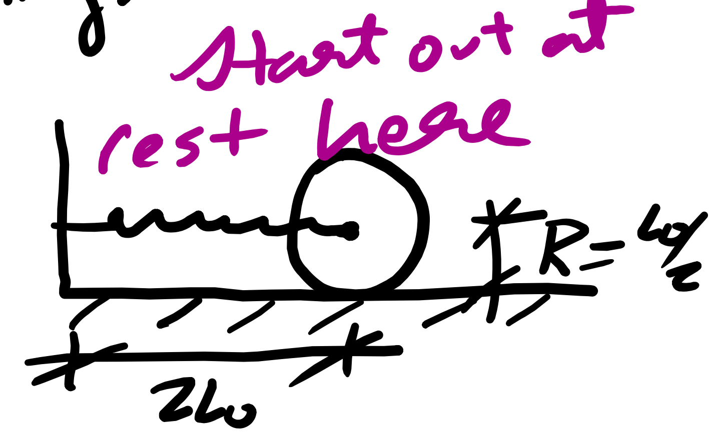
What is $\bar{v}$ when disk hits wall?

$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{\,0} = T_2 + V_2$, $V_1 = \frac{1}{2}kx_1^2$

$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$, $V_2 = \frac{1}{2}kx_2^2 \Rightarrow \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $\bar{I} = \frac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\frac{1}{2}\bar{I}\omega^2 = \frac{1}{4}m\bar{v}^2$

Example: Spring attached to axel of disk that rolls w/out slipping.



What is $\bar{v}$ when disk hits wall?

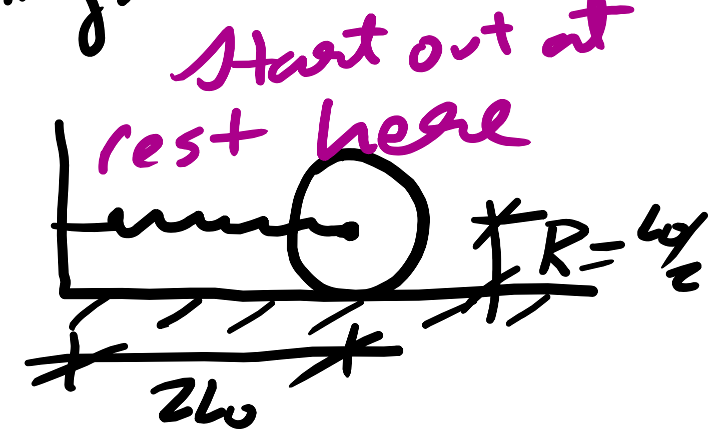
$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U_{1\to 2}^{nc}} = T_2 + V_2 \, , \quad V_1 = \tfrac{1}{2} k x_1^2$$

$$T_2 = \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 , \quad V_2 = \tfrac{1}{2} k x_2^2 \Rightarrow \tfrac{1}{2} m \bar{v}^2 + \tfrac{1}{2} \bar{I} \omega^2 = \tfrac{1}{2} k (x_1^2 - x_2^2)$$

But $\bar{I} = \tfrac{1}{2} m R^2$ & $R\omega = \bar{v}$ so $\tfrac{1}{2} \bar{I} \omega^2 = \tfrac{1}{4} m \bar{v}^2$

Now $\tfrac{1}{4} m \bar{v}^2 + \tfrac{1}{2} m \bar{v}^2 = \tfrac{1}{2} k (x_1^2 - x_2^2)$

Example: Spring attached to axel of disk that rolls w/out slipping.



What is $\bar{v}$ when disk hits wall?

$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U_{1\to 2}^{nc}} = T_2 + V_2, \quad V_1 = \frac{1}{2}kx_1^2$$

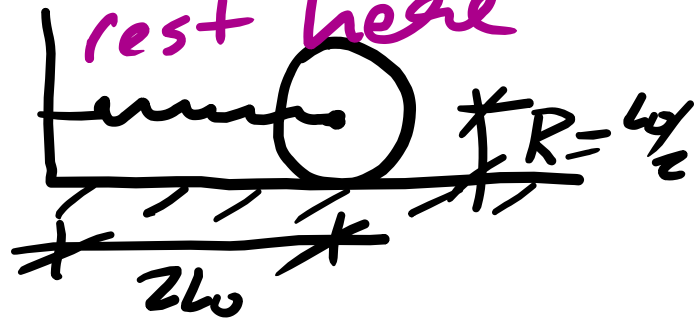
$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2, \quad V_2 = \frac{1}{2}kx_2^2 \Rightarrow \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}k(x_1^2 - x_2^2)$$

But $\bar{I} = \frac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\frac{1}{2}\bar{I}\omega^2 = \frac{1}{4}m\bar{v}^2$

Now $\frac{1}{4}m\bar{v}^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \frac{3}{4}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

Example: Spring attached to axel of disk that rolls w/out slipping.

Start out at rest here



What is $\bar{v}$ when disk hits wall?

$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U_{1\to 2}^{nc}} = T_2 + V_2\,, \quad V_1 = \tfrac{1}{2}kx_1^2$$

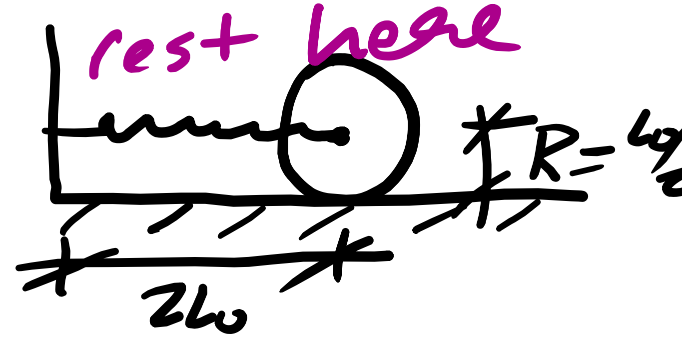
$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2,\quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

But $\bar{I} = \tfrac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{4}m\bar{v}^2$

Now $\tfrac{1}{4}m\bar{v}^2 + \tfrac{1}{2}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \tfrac{3}{4}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0$

Example: Spring attached to axel of disk that rolls w/out slipping.



What is $\bar{v}$ when disk hits wall?

$$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{nc}_{1\to2}}^{\,0} = T_2 + V_2\,, \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2\,, \quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

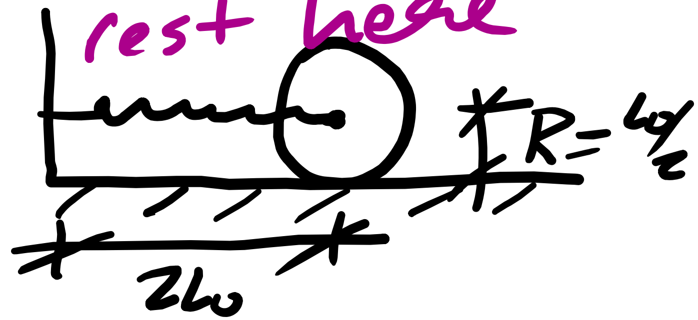
But $\bar{I} = \tfrac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{4}m\bar{v}^2$

Now $\tfrac{1}{4}m\bar{v}^2 + \tfrac{1}{2}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \tfrac{3}{4}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U_{1\to2}^{nc}} = T_2 + V_2, \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}M\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2, \quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}M\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

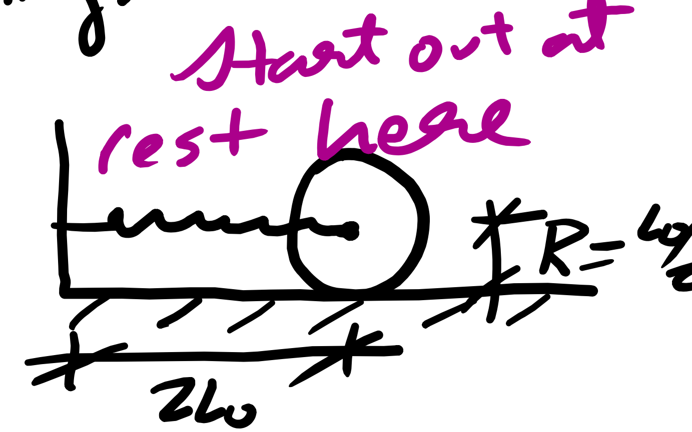
But $\bar{I} = \frac{1}{2}MR^2$ & $R\omega = \bar{v}$ so $\frac{1}{2}\bar{I}\omega^2 = \frac{1}{4}M\bar{v}^2$

Now $\frac{1}{4}M\bar{v}^2 + \frac{1}{2}M\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \frac{3}{4}M\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \frac{1}{2}L_0 - L_0$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{\,0} = T_2 + V_2, \quad V_1 = \tfrac{1}{2}kx_1^2$$

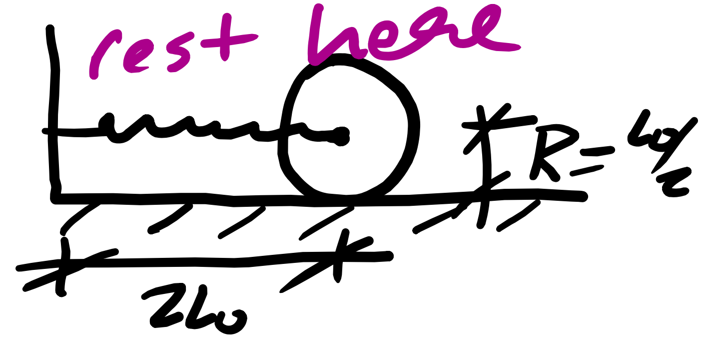
$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2, \quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

But $\bar{I} = \tfrac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{4}m\bar{v}^2$

Now $\tfrac{1}{4}m\bar{v}^2 + \tfrac{1}{2}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \tfrac{3}{4}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \tfrac{1}{2}L_0 - L_0 = -\tfrac{1}{2}L_0$

Example: Spring attached to axel of disk that rolls w/out slipping.



What is $\bar{v}$ when disk hits wall?

$\cancel{T_1}^{\,0} + V_1 + \cancel{U^{NC}_{1\to2}}^{\,0} = T_2 + V_2$, $V_1 = \frac{1}{2}kx_1^2$

$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$, $V_2 = \frac{1}{2}kx_2^2 \Rightarrow \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $\bar{I} = \frac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\frac{1}{2}\bar{I}\omega^2 = \frac{1}{4}m\bar{v}^2$

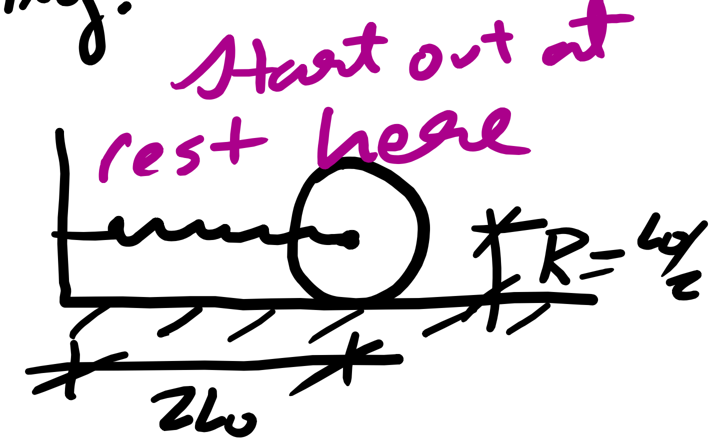
Now $\frac{1}{4}m\bar{v}^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \frac{3}{4}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \frac{1}{2}L_0 - L_0 = -\frac{1}{2}L_0$ so

$$\bar{v}^2 = \left(\frac{2}{3}\right)\left(\frac{k}{m}\right)\left(L_0^2 - \frac{1}{4}L_0^2\right)$$

Example: Spring attached to axel of disk that rolls w/out slipping.

What is $\bar{v}$ when disk hits wall?



$$\cancel{T_1}^{0} + V_1 + \cancel{U^{nc}_{1\to 2}}^{0} = T_2 + V_2, \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2, \quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

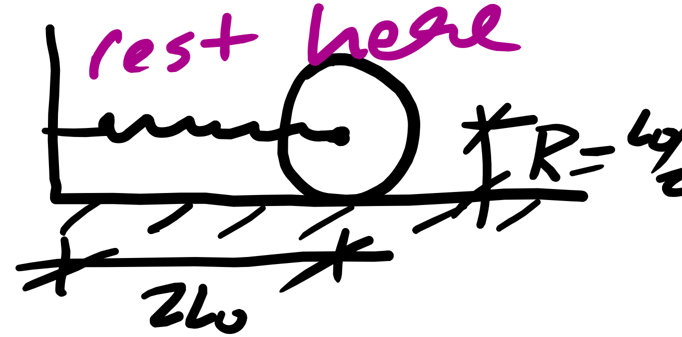
But $\bar{I} = \tfrac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{4}m\bar{v}^2$

Now $\tfrac{1}{4}m\bar{v}^2 + \tfrac{1}{2}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \tfrac{3}{4}m\bar{v}^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \tfrac{1}{2}L_0 - L_0 = -\tfrac{1}{2}L_0$ so

$$\bar{v}^2 = \left(\tfrac{2}{3}\right)\left(\tfrac{k}{m}\right)\left(L_0^2 - \tfrac{1}{4}L_0^2\right) = \tfrac{1}{2}\left(\tfrac{k}{m}\right)L_0^2$$

Example: Spring attached to axel of disk that rolls w/out slipping.



What is $\bar{v}$ when disk hits wall?

$$\cancelto{0}{T_1} + V_1 + \cancelto{0}{U_{1\to 2}^{nc}} = T_2 + V_2 \,, \quad V_1 = \tfrac{1}{2}kx_1^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 \,, \quad V_2 = \tfrac{1}{2}kx_2^2 \Rightarrow \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2}k(x_1^2 - x_2^2)$$

But $\bar{I} = \frac{1}{2}mR^2$ & $R\omega = \bar{v}$ so $\frac{1}{2}\bar{I}\omega^2 = \frac{1}{4}m\bar{v}^2$

Now $\frac{1}{4}m\bar{v}^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2) \Rightarrow \frac{3}{4}m\bar{v}^2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $x_1 = 2L_0 - L_0 = L_0$ & $x_2 = \frac{1}{2}L_0 - L_0 = -\frac{1}{2}L_0$ so

$$\bar{v}^2 = \left(\tfrac{2}{3}\right)\left(\tfrac{k}{m}\right)\left(L_0^2 - \tfrac{1}{4}L_0^2\right) = \tfrac{1}{2}\left(\tfrac{k}{m}\right)L_0^2 \Rightarrow \boxed{\bar{v} = L_0\sqrt{\tfrac{k}{2m}}}$$

ASU

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We want the same sort of relationship for torque and angular momentum that we have for force & linear momentum.

We have $\sum \vec{M}_G = \bar{I}\,\vec{\alpha}$ from the previous chapter

We have $\Sigma \vec{M}_G = \bar{I}\vec{\alpha}$ from the previous chapter & just need to integrate this with respect to time to obtain

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So angular momentum is conserved when no resulting torque is applied to the rigid body

<u>Example</u>: Uniform sphere has horizontal velocity $\bar{v}_1$ along rough horizontal surface.

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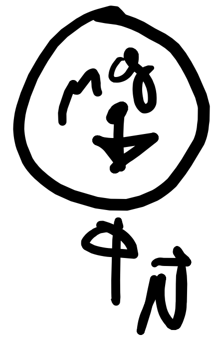
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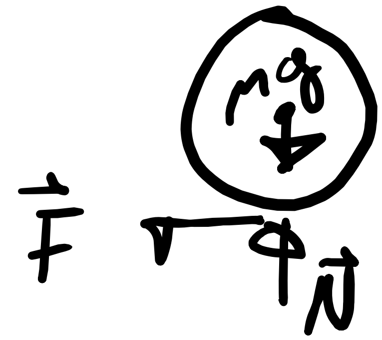
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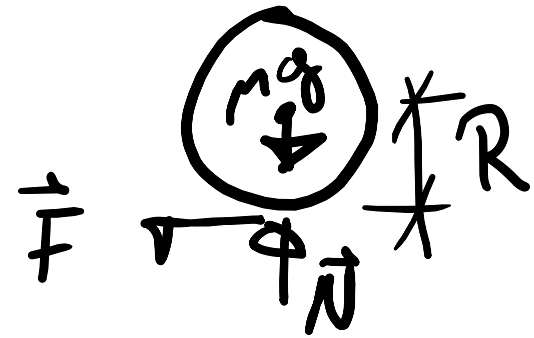
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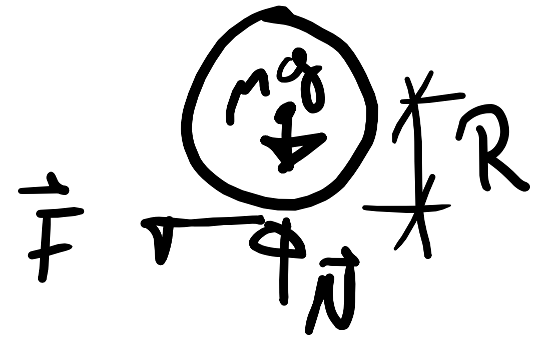
$F = N\mu_k$



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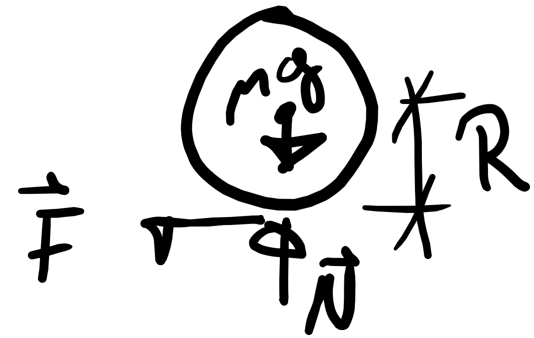

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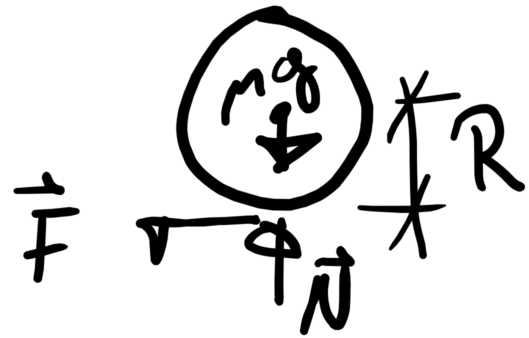

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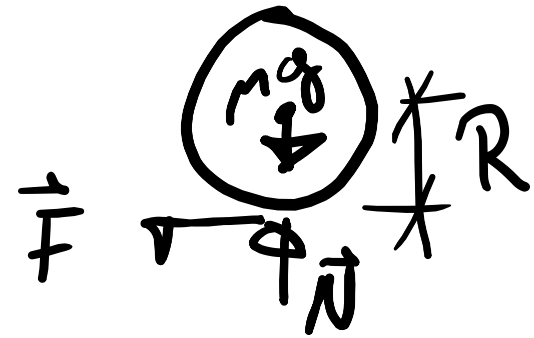

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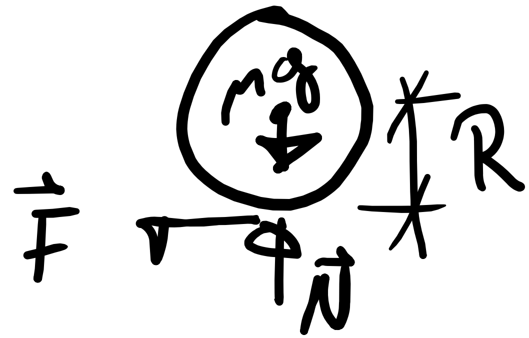

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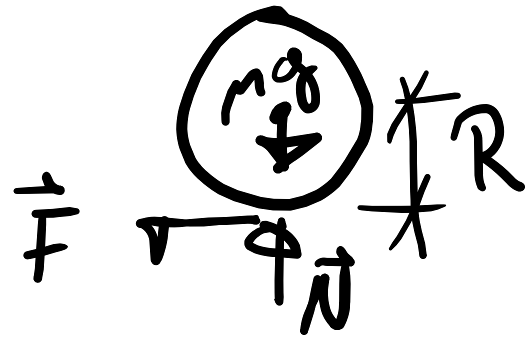

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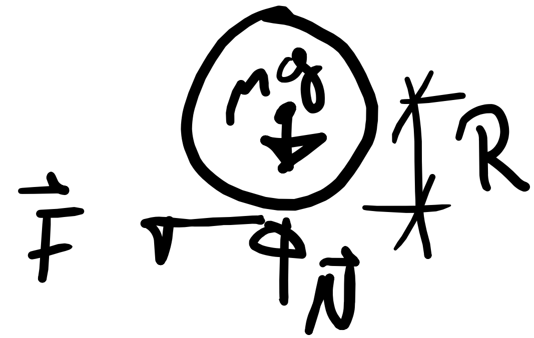

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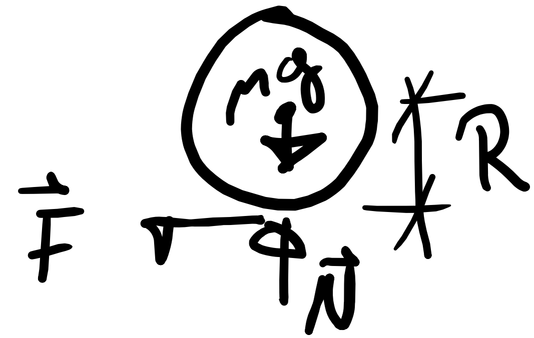

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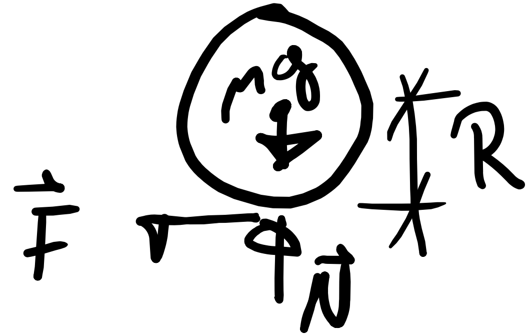

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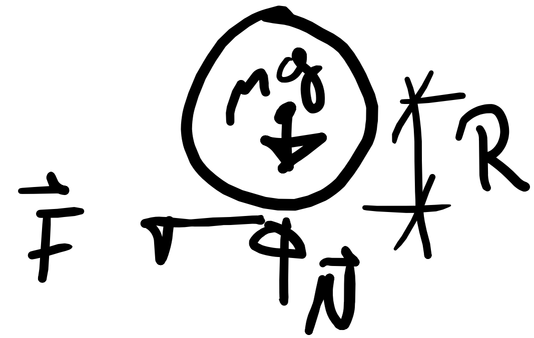

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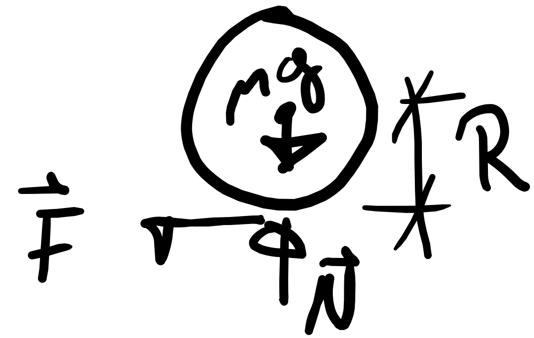

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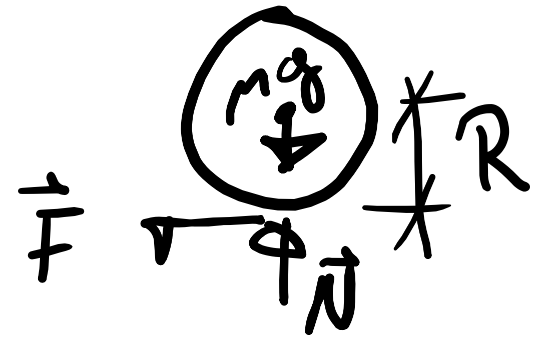

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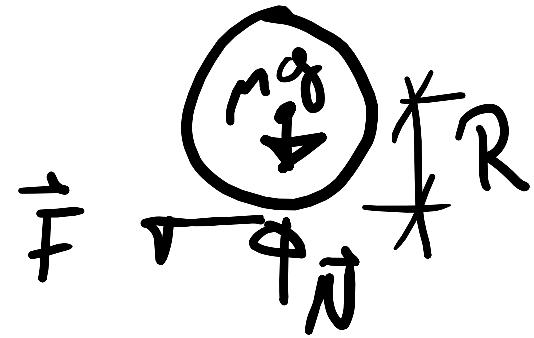

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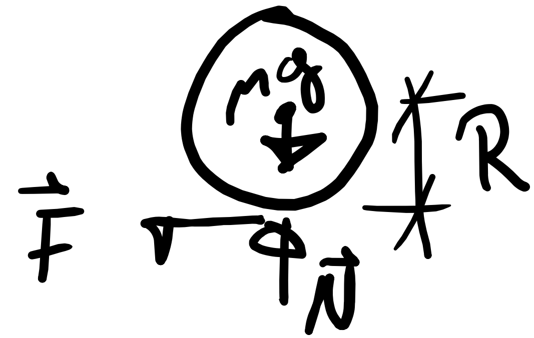

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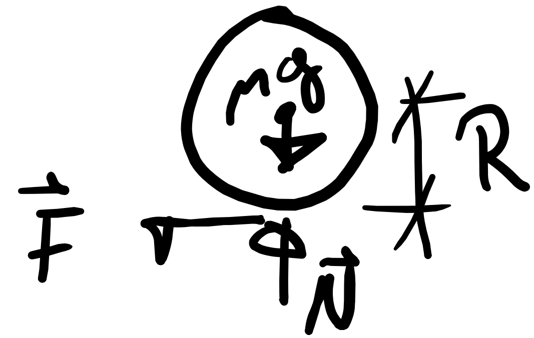

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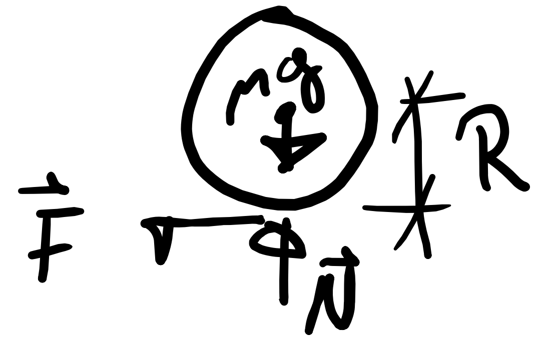

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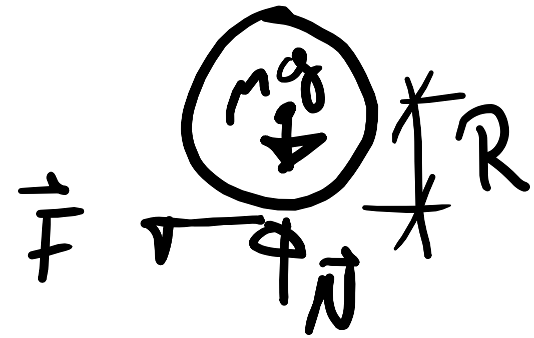

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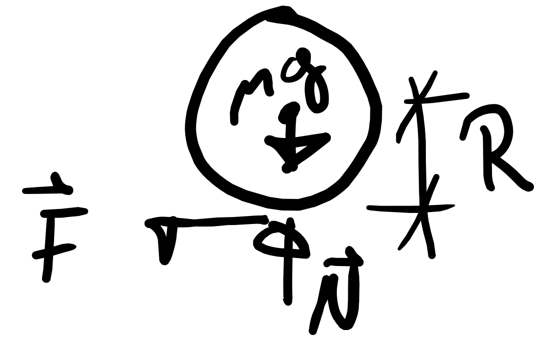

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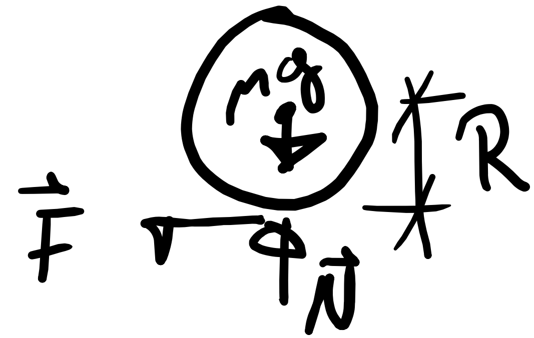

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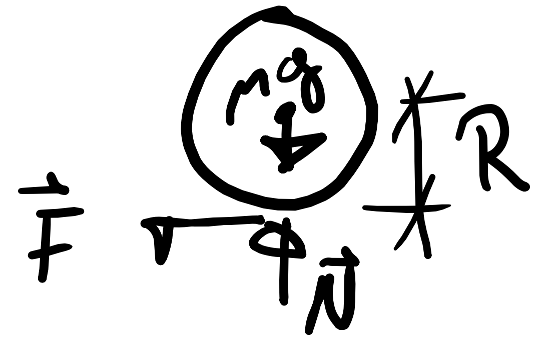

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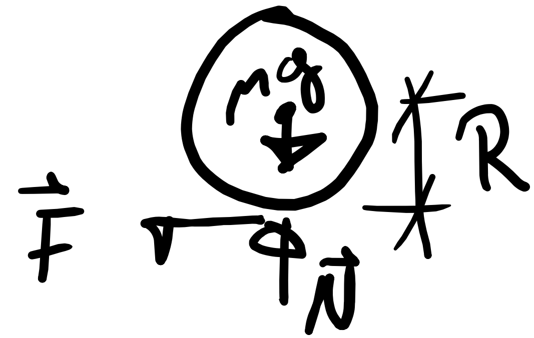

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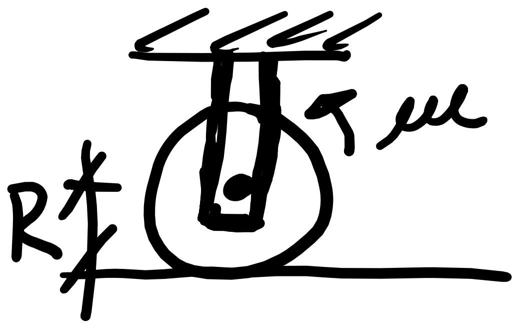
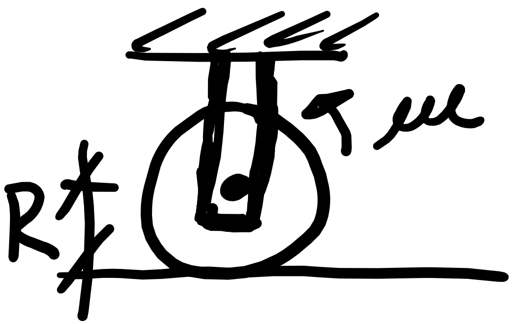
ASU

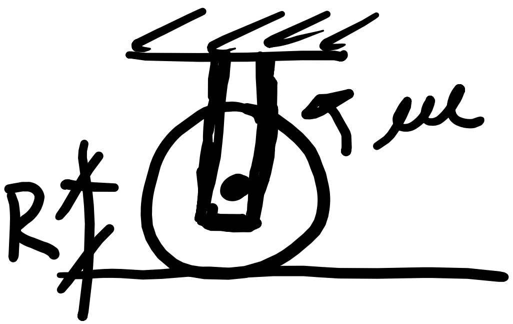
Example: Rotating disk brought into contact with rough surface.



Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$



Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?



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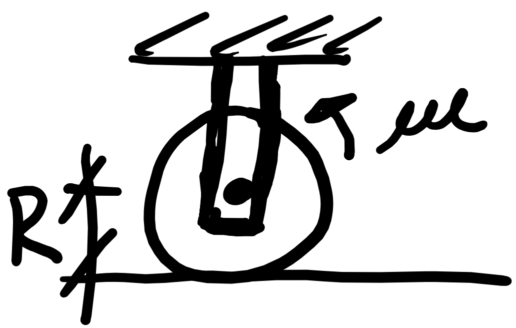
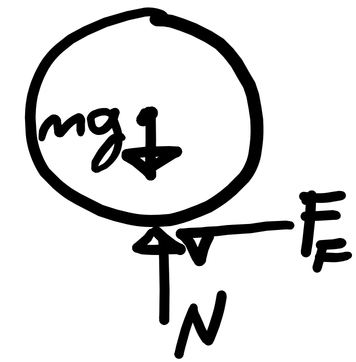
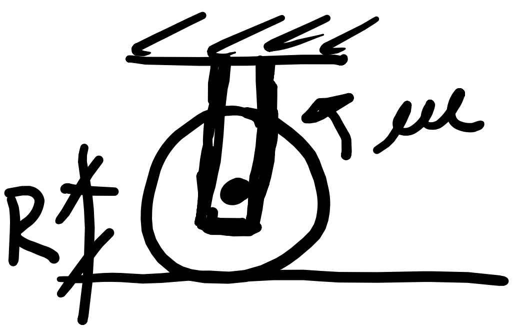
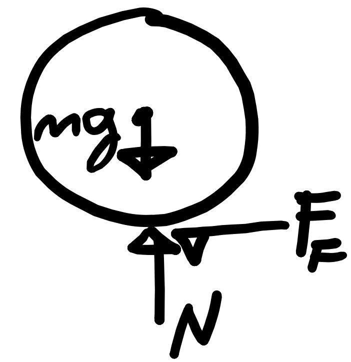




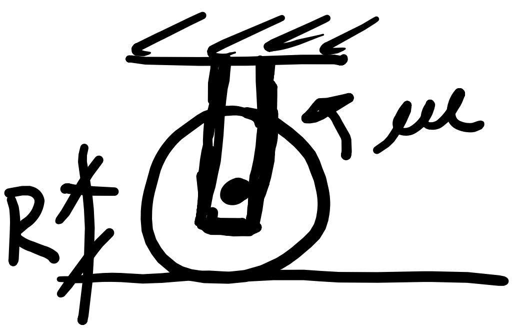
Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?



$F_F = N\mu_K$

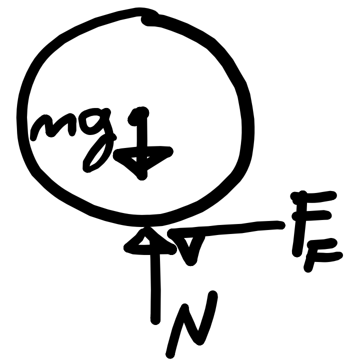


Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?

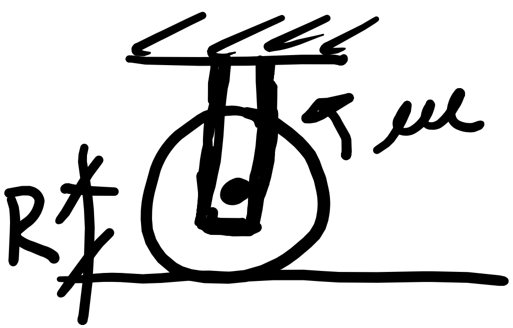


$F_F = N\mu_K$

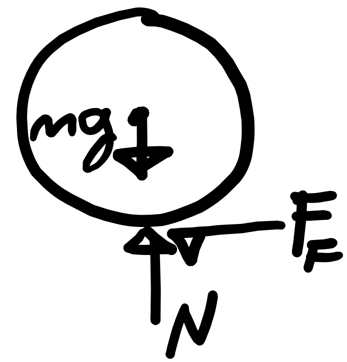
$\& \; N = mg$



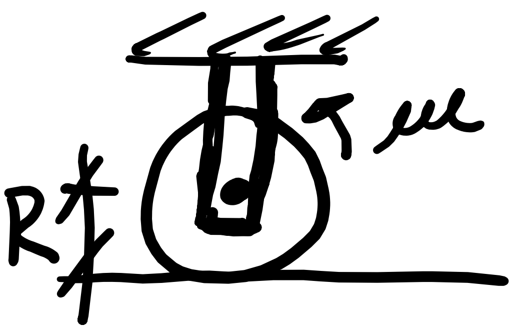
Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?



$$\left.\begin{aligned} F_F &= N\mu_k \\ \& \; N &= mg \end{aligned}\right\} \Rightarrow F_F = mg\mu_k$$

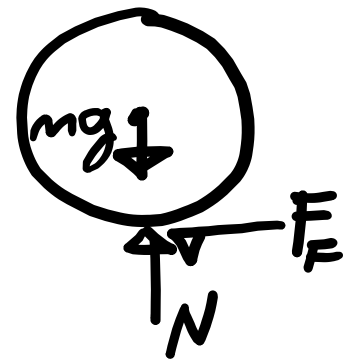


<u>Example</u>: Rotating disk brought into contact with rough surface. If moment of contact is $t_1 = 0$, at what time does the disk stop rotating?

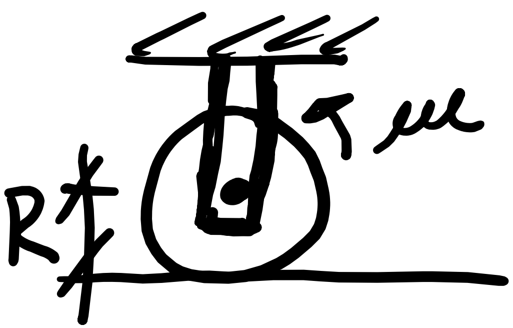


$$\left.\begin{aligned} F_F &= N\mu_k \\ \&\ N &= mg \end{aligned}\right\} \Rightarrow F_F = mg\mu_k$$

$$\&\ \vec{M}_G = R F_F$$

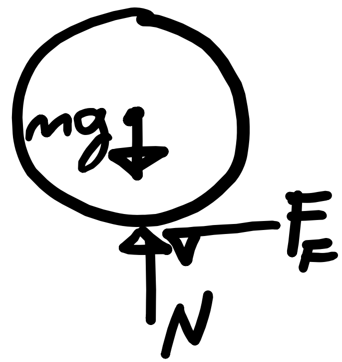


Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?

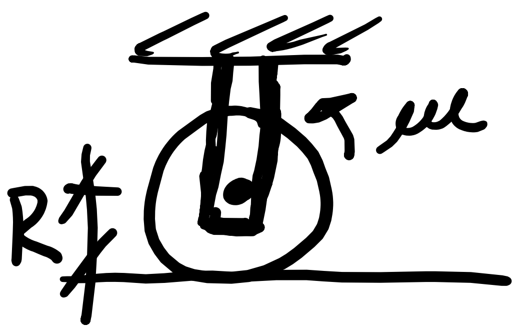


$$\left.\begin{aligned} F_F &= N\mu_k \\ \text{\& } N &= mg \end{aligned}\right\} \Rightarrow F_F = mg\mu_k$$

$$\text{\& } \vec{M}_G = R F_F \circlearrowright = R m g \mu_k \circlearrowright$$

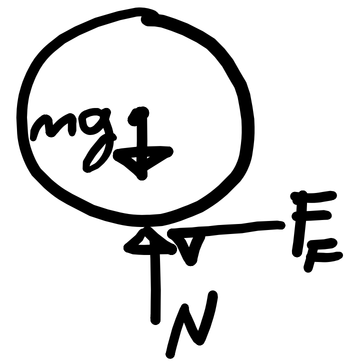


Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?



NOW

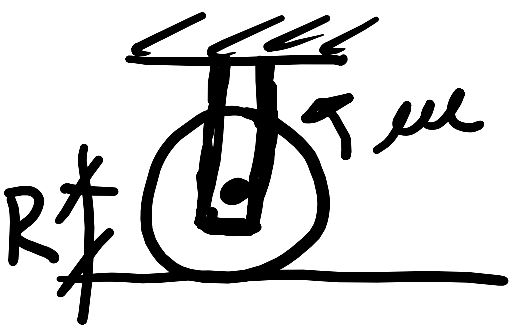
$$\left.\begin{array}{l} F_F = N\mu_k \\ \& \; N = mg \end{array}\right\} \Rightarrow F_F = mg\mu_k$$



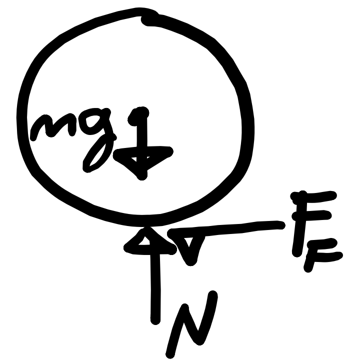
$$\& \; \vec{M}_G = R F_F \circlearrowright = R m g \mu_k \circlearrowright$$

$$\int_{t_I}^{t_F} \Sigma M_G \, dt = \bar{I}\vec{\omega}_F - \bar{I}\vec{\omega}_I$$

Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?



Now

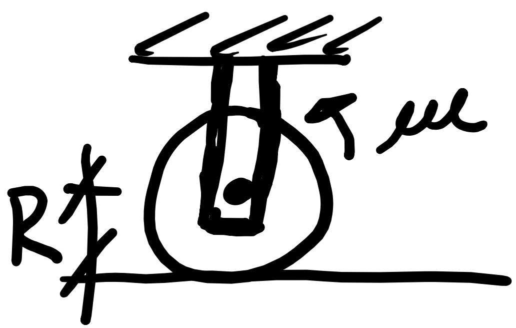


$$\left.\begin{array}{l} F_F = N\mu_k \\ \& \; N = mg \end{array}\right\} \Rightarrow F_F = mg\mu_k$$

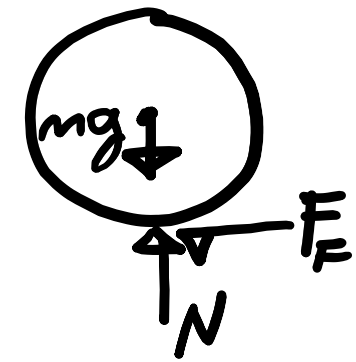
$$\& \; \vec{M}_G = R F_F \circlearrowright = R m g \mu_k \circlearrowright$$

$$\int_{t_I}^{t_F} \Sigma M_G \, dt = \bar{I}\cancel{\vec{\omega}_F}^{\,0} - \bar{I}\vec{\omega}_I$$

Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?



Now

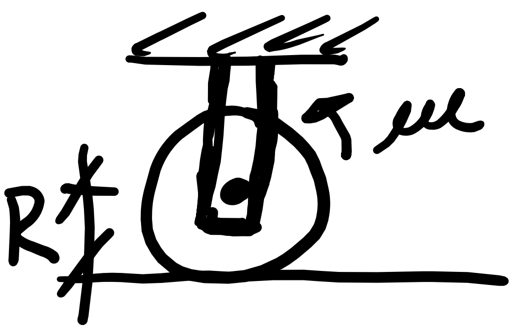


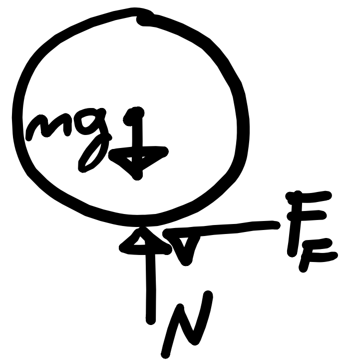
$$\left.\begin{array}{l} F_F = N\mu_k \\ \& \ N = mg \end{array}\right\} \Rightarrow F_F = mg\mu_k$$

$$\& \ \vec{M}_G = RF_F \circlearrowright = Rmg\mu_k \circlearrowright$$

$$\int_{t_I = 0}^{t_F} \Sigma M_G \, dt = \bar{I}\cancel{\omega_F}^{\,0} - \bar{I}\omega_I$$

Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?





$$\left.\begin{array}{l} F_F = N\mu_k \\ \& \; N = mg \end{array}\right\} \Rightarrow F_F = mg\mu_k$$

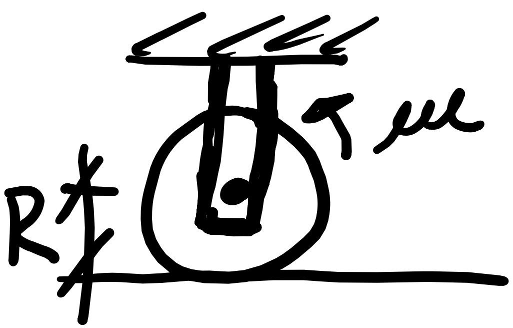
$$\& \; \vec{M}_G = R F_F \circlearrowright = R m g \mu_k \circlearrowright$$

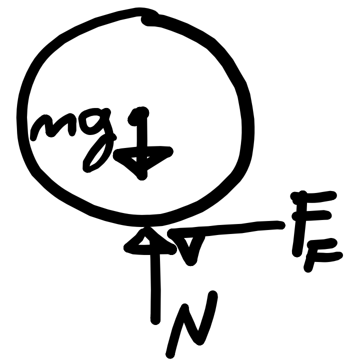
Now

$$\int_{t_I = 0}^{t_F} \Sigma M_G \, dt = \bar{I}\omega_F - \bar{I}\omega_I \Rightarrow \quad \omega_F = 0$$

$$R m g \mu_k t_F = \bar{I}\omega_I$$

Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?





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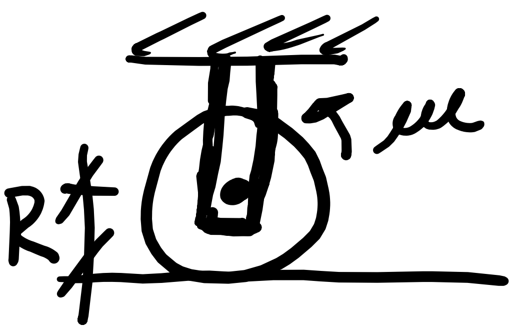
$$\& \; \vec{M}_G = R F_F \circlearrowright = Rmg\mu_k \circlearrowright$$

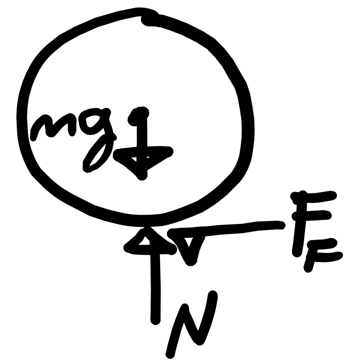
Now

$$\int_{\cancel{t_I}\,0}^{t_F} \Sigma M_G \, dt = \bar{I}\cancel{\omega_F}\,0 - \bar{I}\omega_I \Rightarrow$$

$$Rmg\mu_k t_F = \bar{I}\omega_I \quad \text{But } \bar{I} = \tfrac{1}{2}MR^2$$

Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?





$$\left.\begin{array}{l} F_F = N\mu_k \\ \& \; N = mg \end{array}\right\} \Rightarrow F_F = mg\mu_k$$

$$\& \; \vec{M}_G = RF_F \circlearrowright = Rmg\mu_k \circlearrowright$$

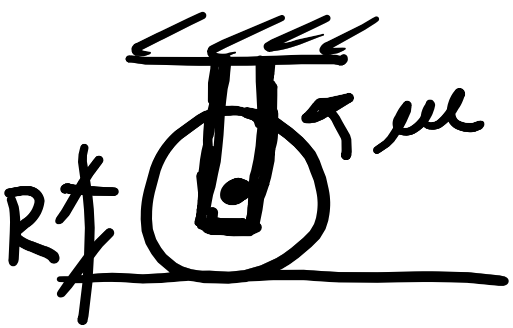
Now

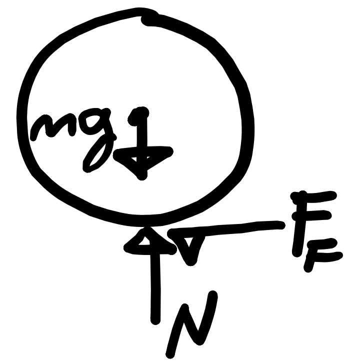
$$\int_{t_I = 0}^{t_F} \Sigma M_G \, dt = \bar{I}\cancel{\omega_F}^{\,0} - \bar{I}\omega_I \Rightarrow$$

$$Rmg\mu_k t_F = \bar{I}\omega_I \quad \text{But } \bar{I} = \tfrac{1}{2}MR^2 \text{ so}$$

$$Rmg\mu_k t_F = \tfrac{1}{2}MR^2\omega_I$$

Example: Rotating disk brought into contact with rough surface. If moment of contact is $t_I = 0$, at what time does the disk stop rotating?





$$\left.\begin{array}{l} F_F = N\mu_k \\ \&\ N = mg \end{array}\right\} \Rightarrow F_F = mg\mu_k$$

$$\&\ \vec{M}_G = RF_F \circlearrowright = Rmg\mu_k \circlearrowright$$

Now

$$\int_{t_I}^{t_F} \Sigma M_G \, dt = \bar{I}\cancel{\omega_F} - \bar{I}\omega_I \Rightarrow$$

$$Rmg\mu_k t_F = \bar{I}\omega_I$$

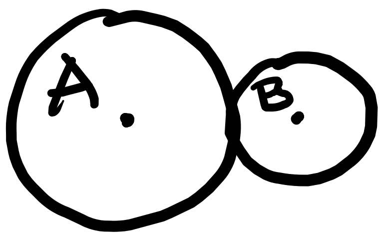
But $\bar{I} = \frac{1}{2}MR^2$ so

$$Rmg\mu_k t_F = \frac{1}{2}MR^2\omega_I \Rightarrow \boxed{t_F = \frac{R\omega_I}{2g\mu_k}}$$

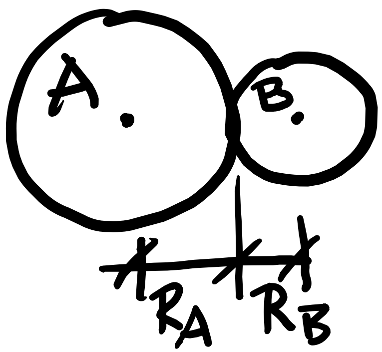
<u>Example</u> Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt.

<u>Example</u> Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



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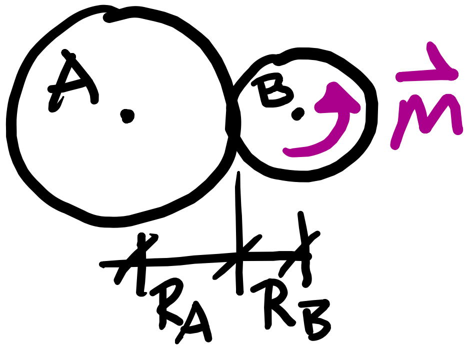
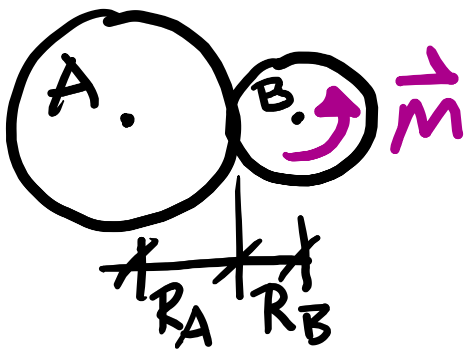
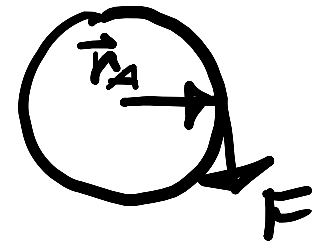
<u>Example</u> Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



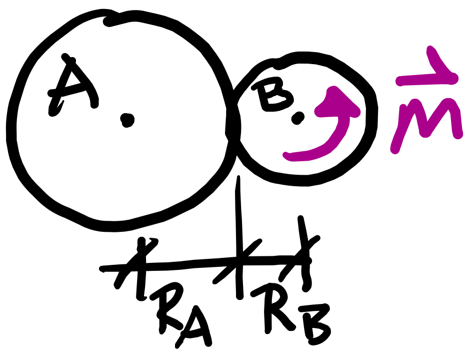
Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



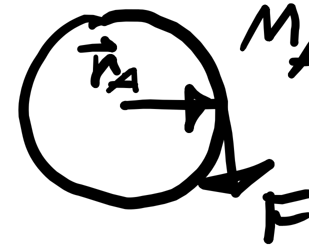
Gear A



Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find α_{BF}

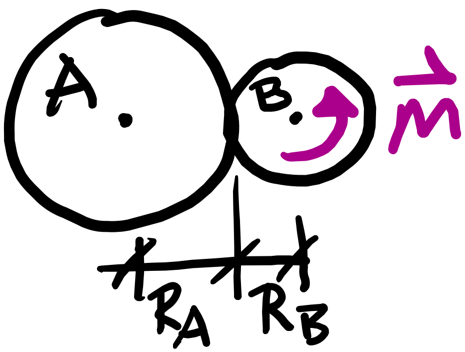


Gear A



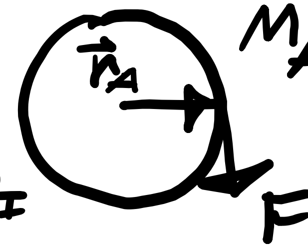
$\vec{M}_A = R_A F$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



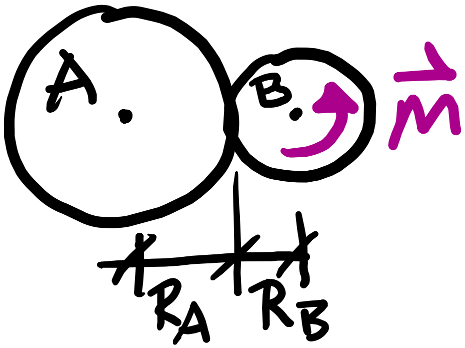
Gear A

$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = \bar{I}\vec{\omega}_F - \bar{I}\vec{\omega}_I$$



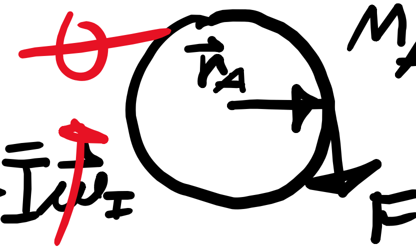
$\vec{M}_A = R_A F$ ↺

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



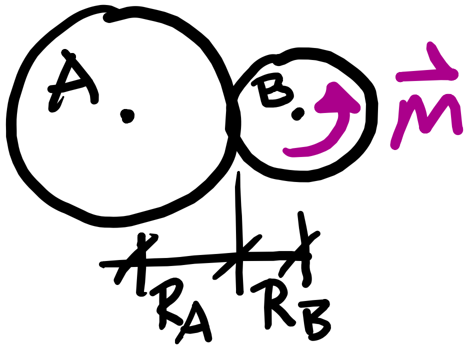
Gear A

$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = I\vec{\omega}_F - \cancel{I\vec{\omega}_I}$$

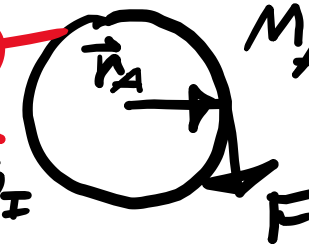


$\vec{M}_A = R_A F \circlearrowright$

__Example__ Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



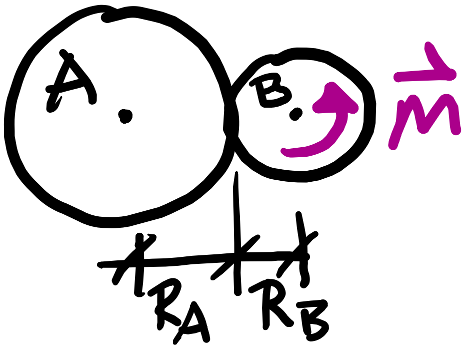
__Gear A__



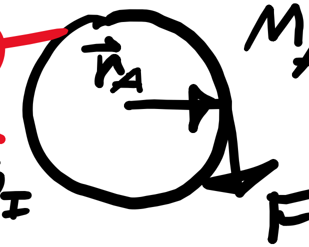
$$\Rightarrow \int_{t_I}^{t_F} \Sigma \vec{M}_A \, dt = \bar{I}\vec{\omega}_F - \cancelto{0}{\bar{I}\vec{\omega}_I}$$

$$\Rightarrow R_A F t = I_A \omega_F$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A

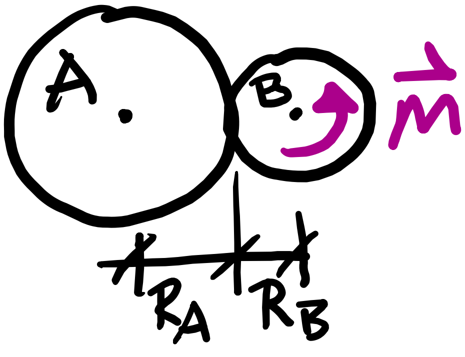


$\vec{M}_A = R_A F$

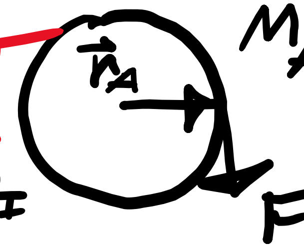
$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = I\vec{\omega}_F - \cancel{I\vec{\omega}_I}$$

$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



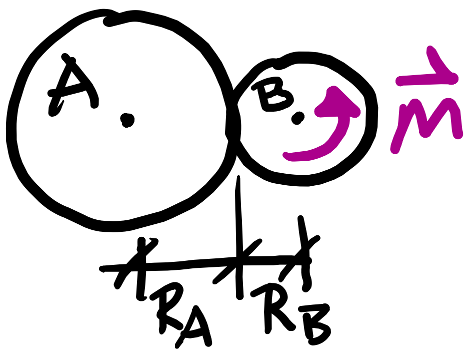
$\vec{M}_A = R_A F$

$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = I\vec{\omega}_F - \cancel{I\vec{\omega}_I}$$

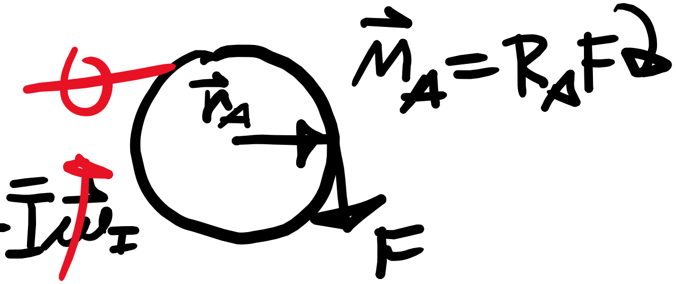
$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

Gear B

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A

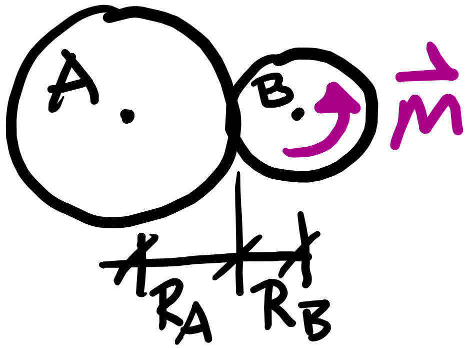


$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = I\vec{\omega}_F - \cancel{I\vec{\omega}_I}$$

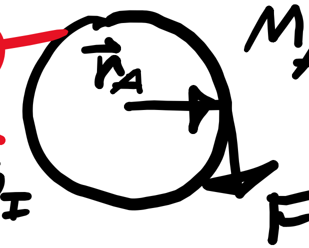
$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

Gear B

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$\vec{M}_A = R_A F$ ↻

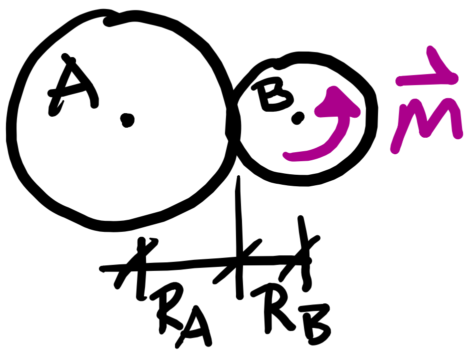
$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = I\vec{\omega}_F - \cancelto{0}{I\vec{\omega}_I}$$

$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

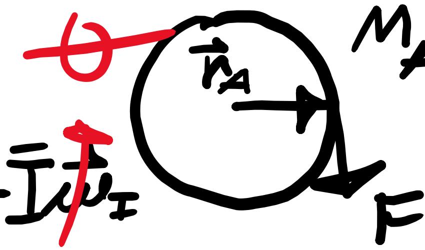
Gear B

$$\sum \vec{M}_B = (M - R_B F) \circlearrowleft$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$\vec{M}_A = R_A F$ ↻

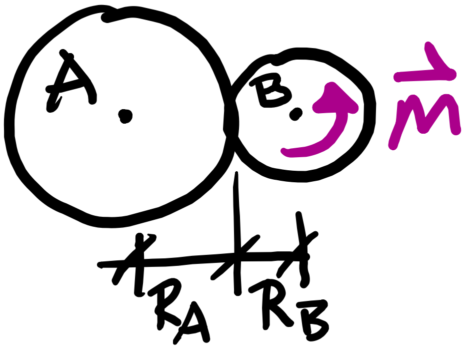
$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = \bar{I}\omega_F - \cancel{\bar{I}\omega_I}$$

$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

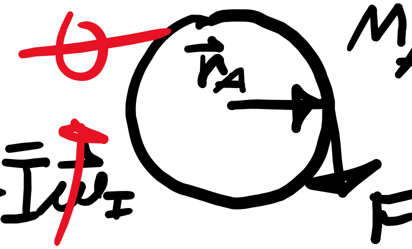
Gear B

$$\sum \vec{M}_B = (M - R_B F) \circlearrowleft = I_B \omega_F$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$\vec{M}_A = R_A F$

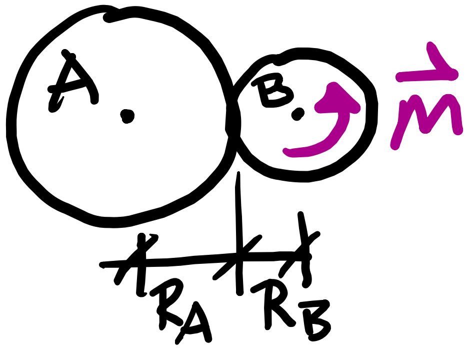
$$\Rightarrow \int_{t_I}^{t_F} \Sigma \vec{M}_A \, dt = \bar{I}\vec{\omega}_F - \cancel{\bar{I}\vec{\omega}_I}$$

$$\Rightarrow R_A F t = I_A \omega_F \qquad (1)$$

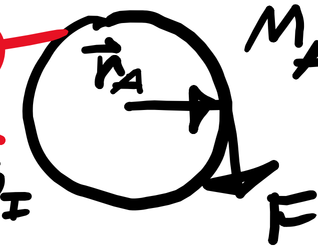
Gear B

$$\Sigma \vec{M}_B = (M - R_B F) = I_B \omega_F \qquad (2)$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



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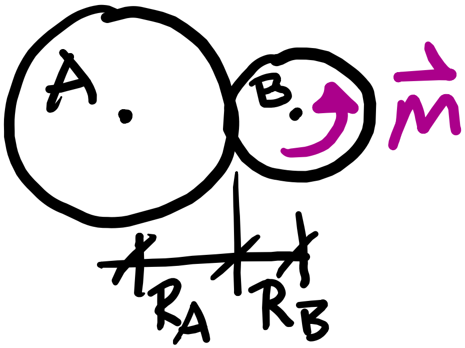
Gear B

$$\sum \vec{M}_B = (M - R_B F) = \bar{I}_B \omega_F \qquad (2)$$

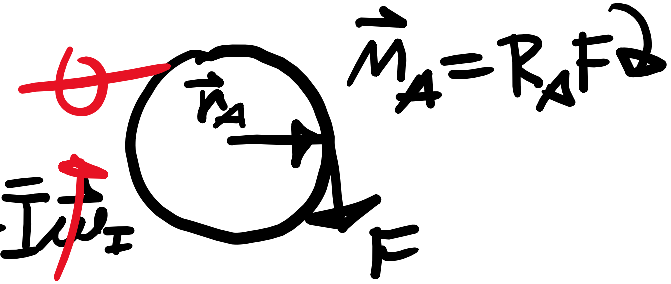
From equation 1 we have

$$F \Delta t = \frac{\bar{I}_A \omega_{AF}}{R_A}$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = \bar{I}\vec{\omega}_F - \cancel{\bar{I}\vec{\omega}_I}$$

$$\Rightarrow R_A F t = \bar{I}_A \omega_F \qquad (1)$$

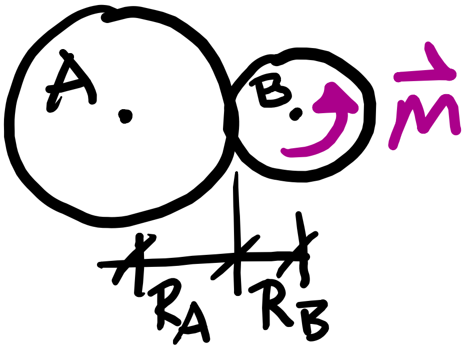
Gear B

$$\sum \vec{M}_B = (M - R_B F) = \bar{I}_B \omega_F \qquad (2)$$

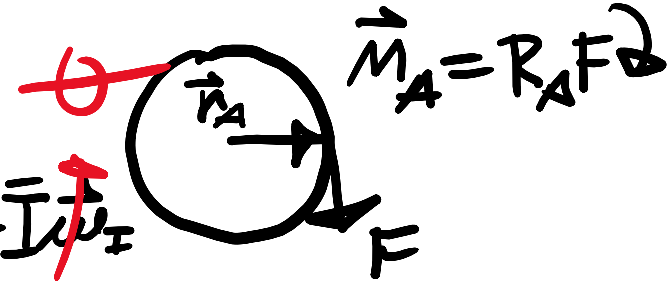
From equation 1 we have

$$F \Delta t = \frac{\bar{I}_A \omega_{AF}}{R_A} \qquad (3)$$

Example Gear A in contact with gear B. Starts at rest and torque applied to gear B for a time Δt. Find ω_{BF}



Gear A



$$\Rightarrow \int_{t_I}^{t_F} \sum \vec{M}_A \, dt = \bar{I}\vec{\omega}_F - \cancel{\bar{I}\vec{\omega}_I}$$

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From previous slide

From previous slide

$$R_A F \Delta t = \bar{I}_A \omega_{AF} \quad (1) \quad \& \quad \Delta t(M - R_B F) = \bar{I}_B \omega_{BF} \quad (2)$$

From previous slide

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Eqn 3 into eqn 2

From previous slide

$R_A F\Delta t = \bar{I}_A \omega_{AF}$ (1) & $\Delta t(M - R_B F) = \bar{I}_B \omega_{BF}$ (2)

& $F\Delta t = \boxed{\bar{I}_A \omega_{AF}/R_A}$ (3) Eqn 3 into eqn 2 gives us $M\Delta t - \bar{I}_A \omega_{AF}\left(\frac{R_B}{R_A}\right) = \bar{I}_B \omega_{BF}$

ASU

From previous slide

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But $R_A \omega_A = R_B \omega_B$

From previous slide

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But $R_A \omega_A = R_B \omega_B$ so $\omega_{AF} = \frac{R_B}{R_A} \omega_{BF}$

From previous slide

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From previous slide

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$$M\Delta t = \left[\bar{I}_A\left(\frac{R_B}{R_A}\right)^2 + \bar{I}_B\right]\omega_{BF}$$

From previous slide

$$R_A F \Delta t = \bar{I}_A \omega_{AF} \quad (1) \quad \& \quad \Delta t (M - R_B F) = \bar{I}_B \omega_{BF} \quad (2)$$

$$\& \quad F\Delta t = \boxed{\bar{I}_A \omega_{AF} / R_A} \quad (3)$$

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$$M\Delta t = \left[\bar{I}_A \left(\frac{R_B}{R_A}\right)^2 + \bar{I}_B\right] \omega_{BF} = \left[\frac{\bar{I}_A R_B^2 + \bar{I}_B R_A^2}{R_A^2}\right] \omega_{BF}$$

ASU

From previous slide

$$R_A F\Delta t = \bar{I}_A \omega_{AF} \quad (1) \quad \& \quad \Delta t(M - R_B F) = \bar{I}_B \omega_{BF} \quad (2)$$

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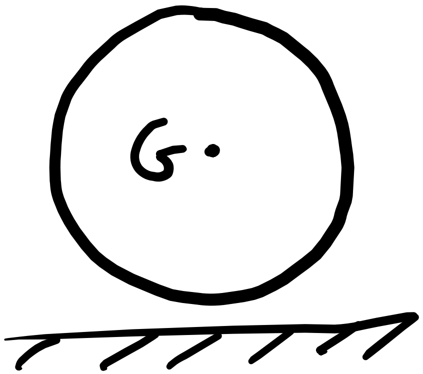
$$M\Delta t = \left[\bar{I}_A\left(\frac{R_B}{R_A}\right)^2 + \bar{I}_B\right]\omega_{BF} = \left[\frac{\bar{I}_A R_B^2 + \bar{I}_B R_A^2}{R_A^2}\right]\omega_{BF}$$

$$\Rightarrow \quad \boxed{\omega_{BF} = \left[\frac{M\Delta t R_A^2}{\bar{I}_A R_B^2 + \bar{I}_B R_A^2}\right]}$$

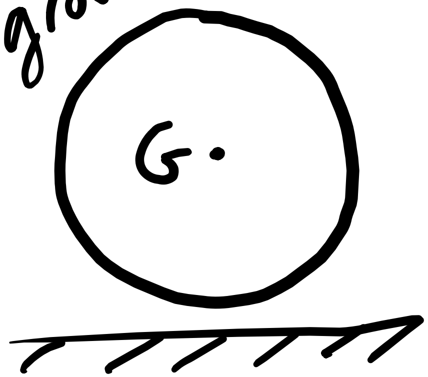
ASU

$\boxed{17.54}$



$\boxed{17.54}$

Not on ground



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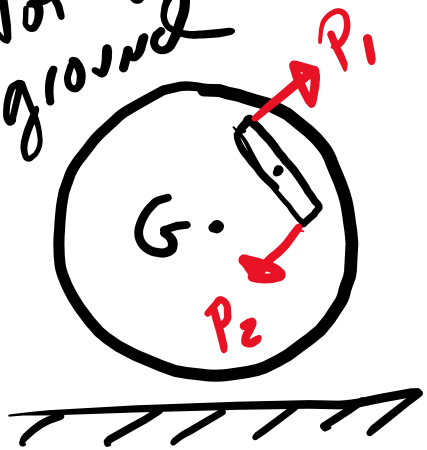

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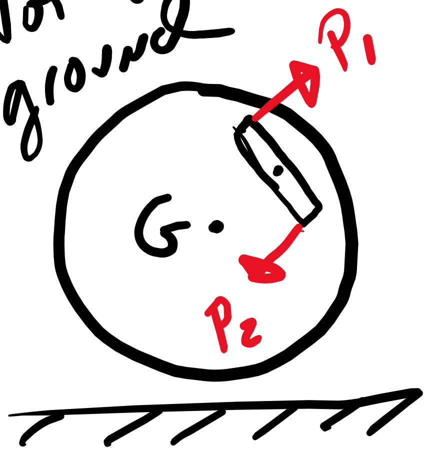
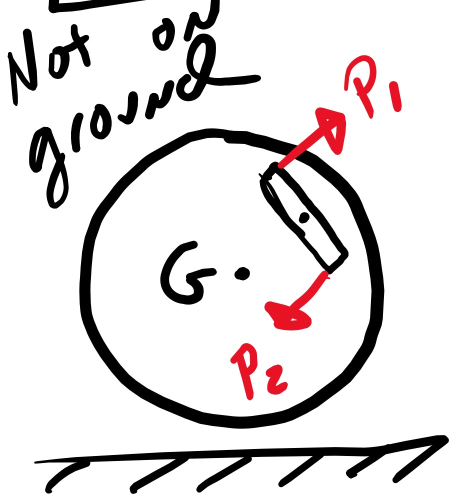


$$\vec{P}_1 = -\vec{P}_2$$

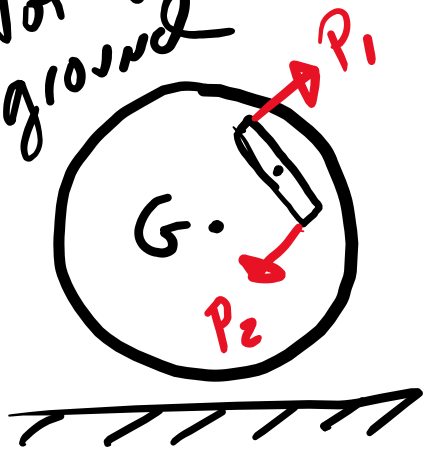
$\boxed{17.54}$



$\vec{P}_1 = -\vec{P}_2$ Let $P \equiv |\vec{P}_1|$

$\boxed{17.54}$

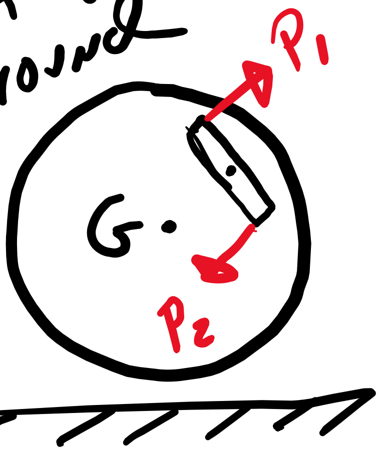
Not on ground



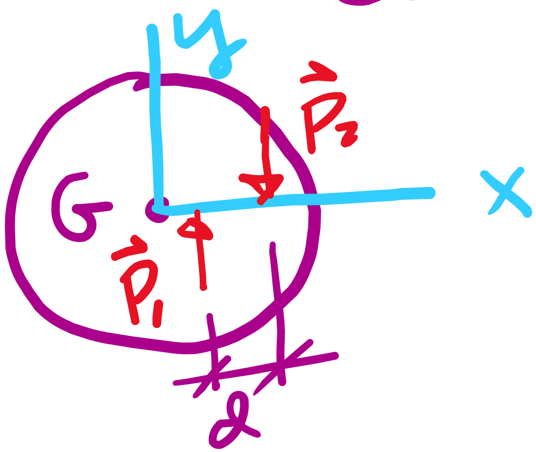
$\vec{P}_1 = -\vec{P}_2$ Let $P \equiv |\vec{P}_1|$ Free to rotate about G

$\boxed{17.54}$

Not on ground



$\vec{P}_1 = -\vec{P}_2$ Let $P \equiv |\vec{P}_1|$ Free to rotate about G. First, look at an easier problem:



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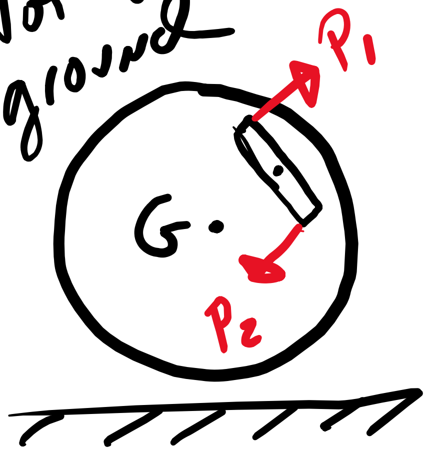

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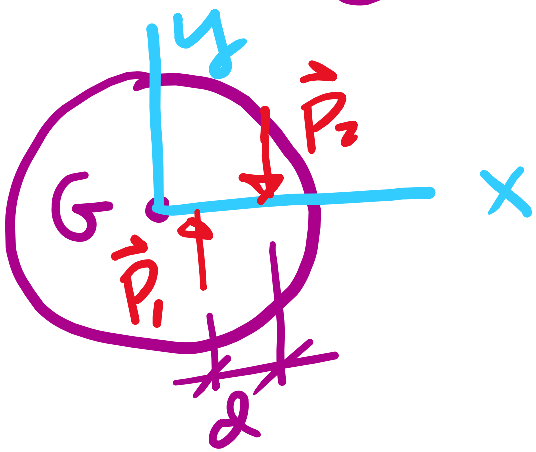
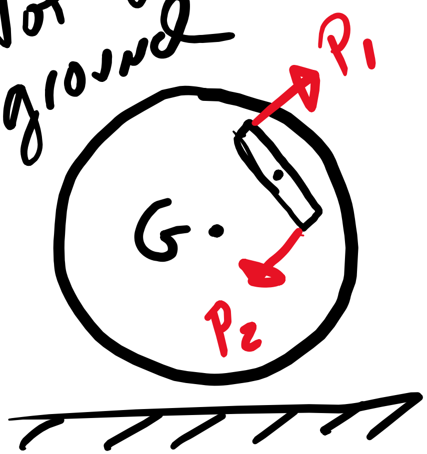


$$\Sigma \vec{M}_G = r_1 P \hat{z} - r_2 P \hat{z}$$

$\boxed{17.54}$

Not on ground



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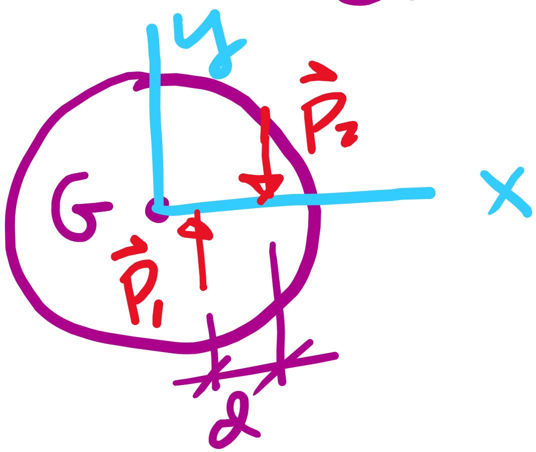
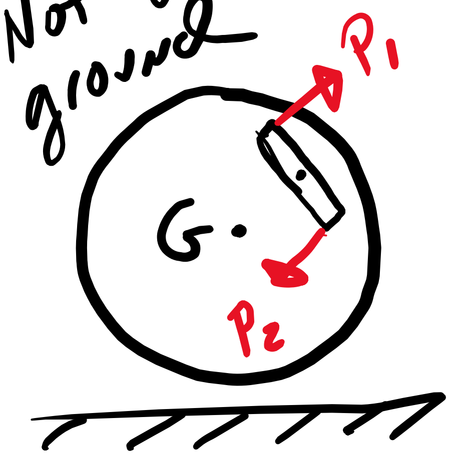


$$\sum \vec{M}_G = r_1 P \hat{z} - r_2 P \hat{z}$$
$$= P(r_1 - r_2)\hat{z}$$

$\boxed{17.54}$

Not on ground



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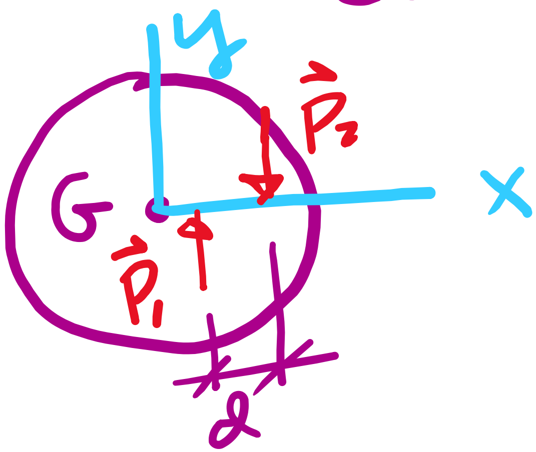

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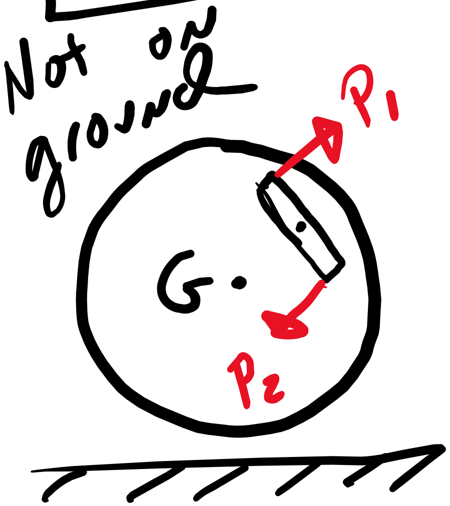

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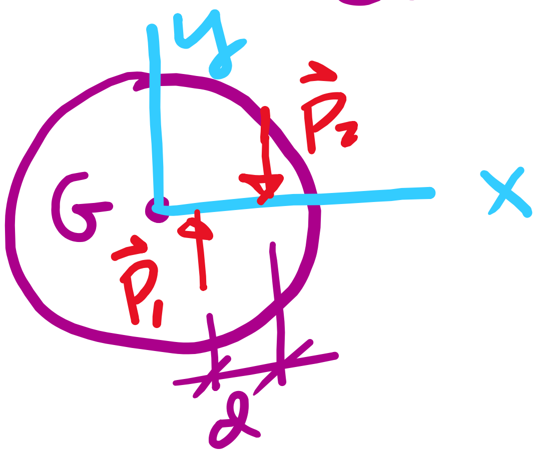

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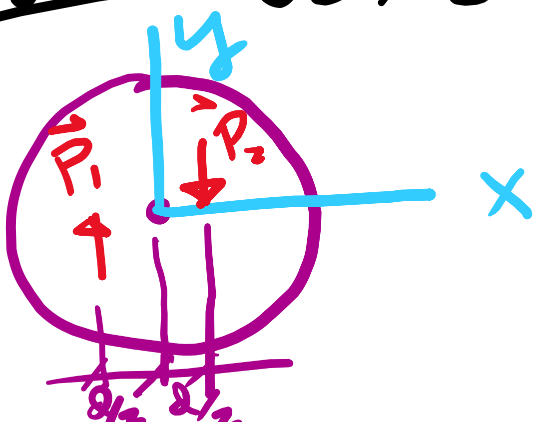

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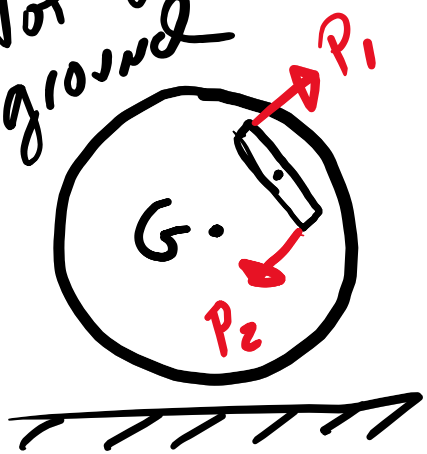

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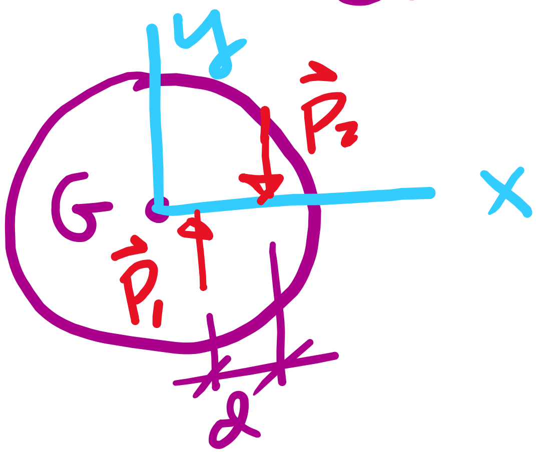

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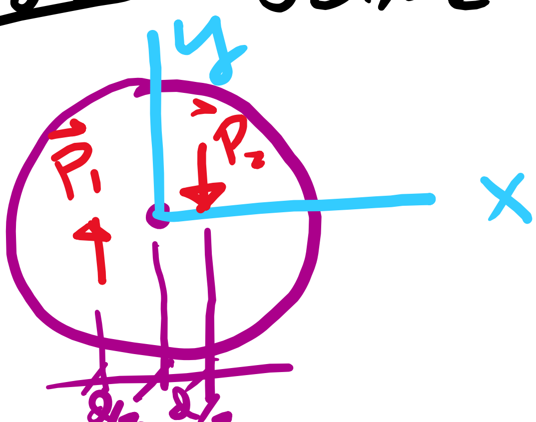
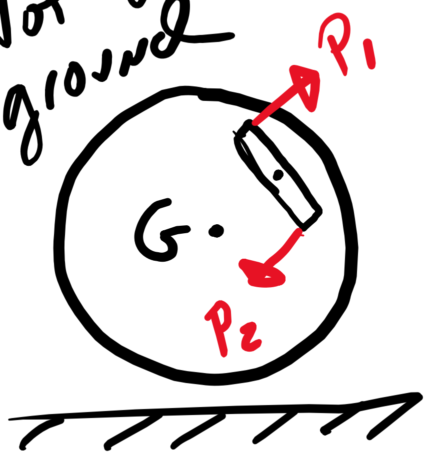


$$\sum \vec{M}_G = -\frac{d}{2}P\hat{z} - \frac{d}{2}P\hat{z}$$

$\boxed{17.54}$

Not on ground



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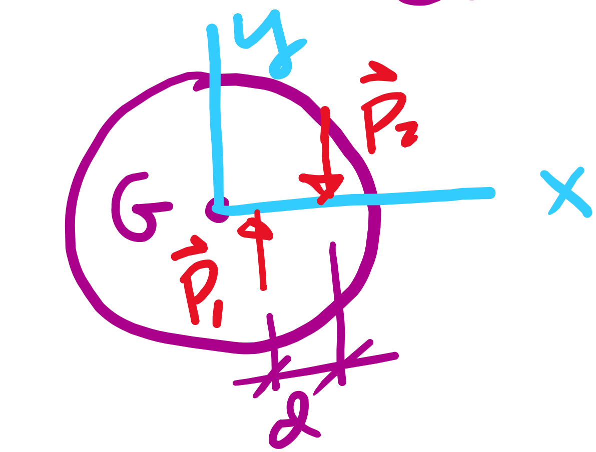

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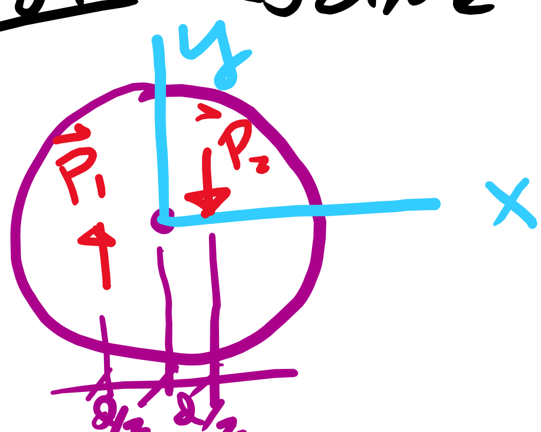

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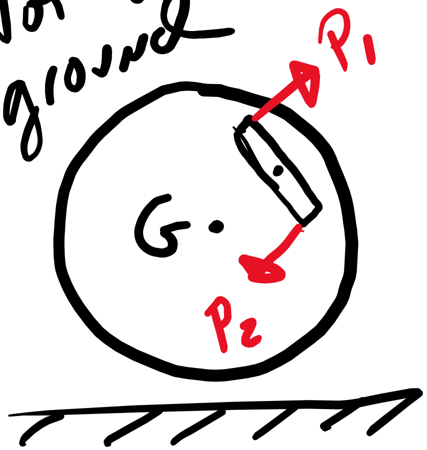
17.54

Not on ground



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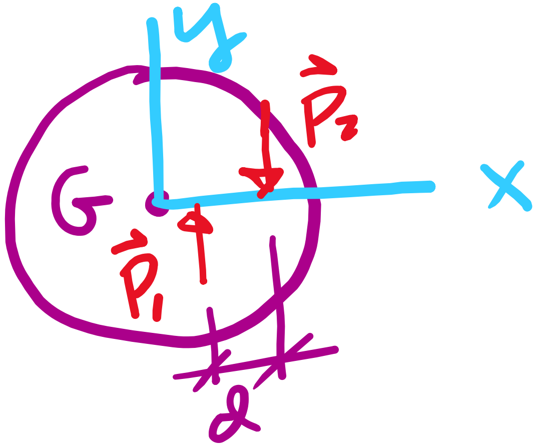

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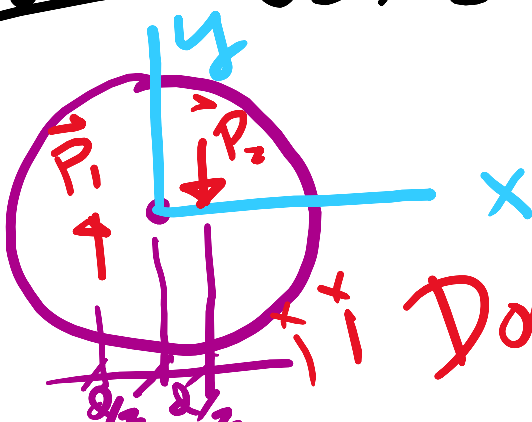

$$\sum \vec{M}_G = -\frac{d}{2}P\hat{z} - \frac{d}{2}P\hat{z} = -Pd\hat{z}$$

!! Does not seem to matter where couple is located on rigid body !!

A more general case : $\vec{P}_A = -\vec{P}_B$ with geometry as shown

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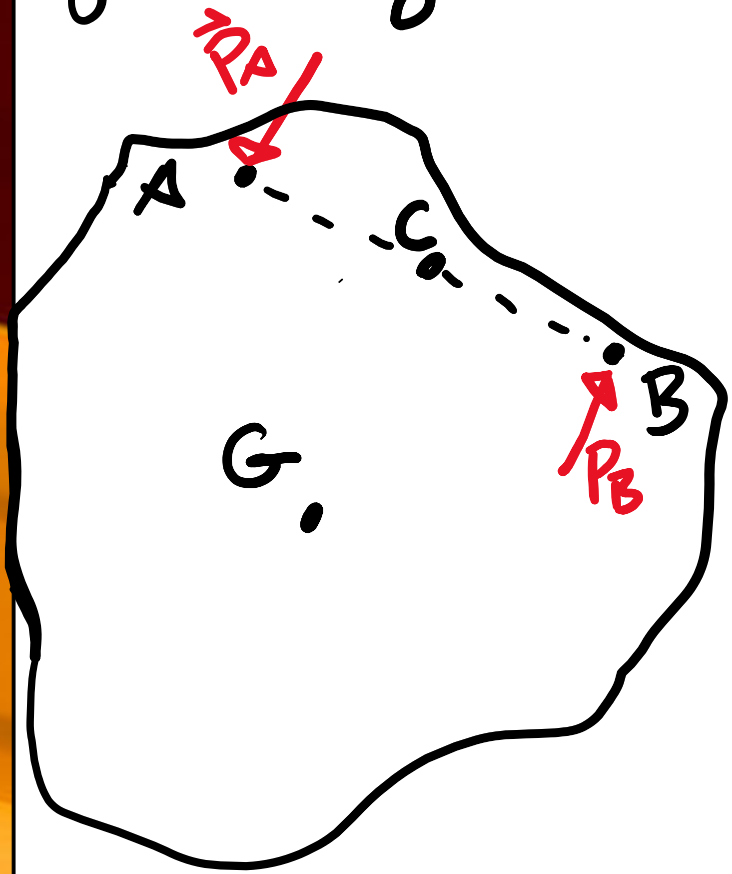

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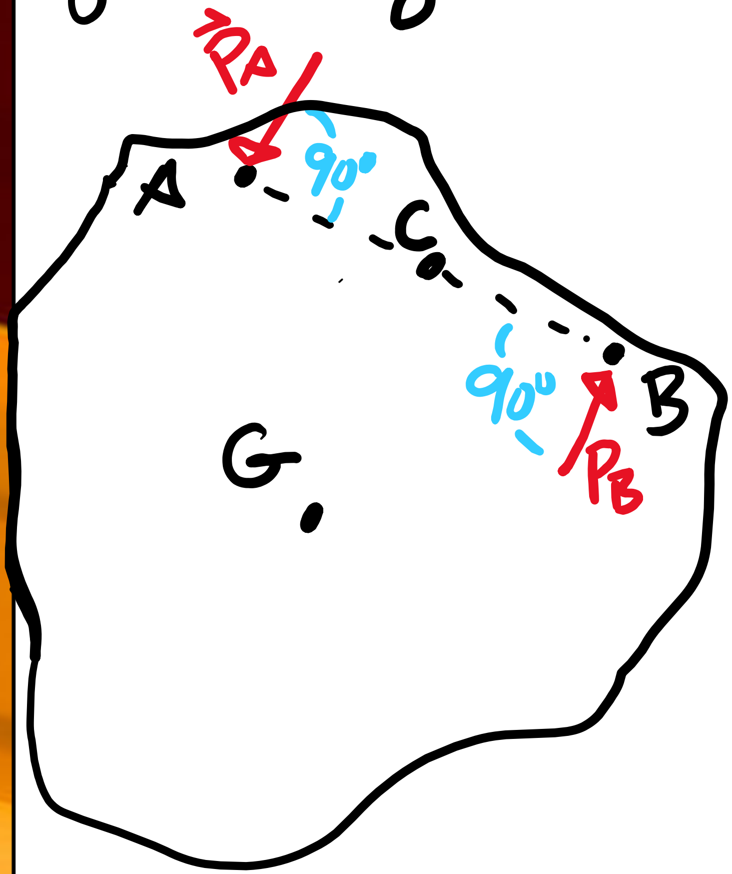

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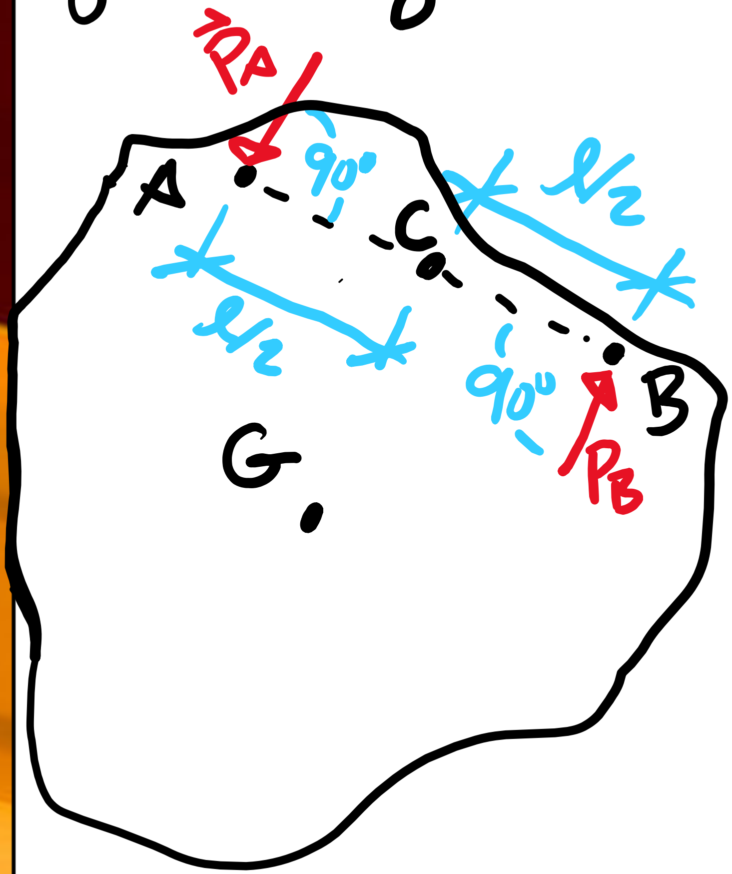

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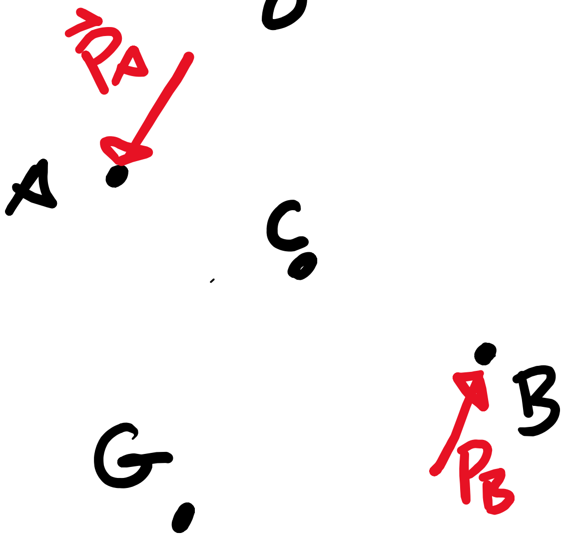

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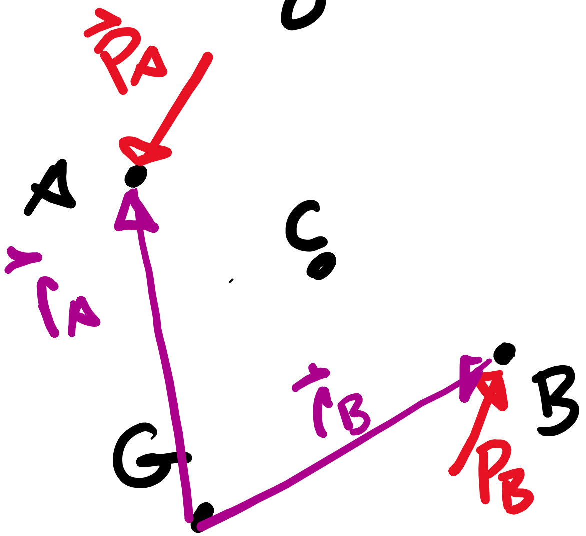
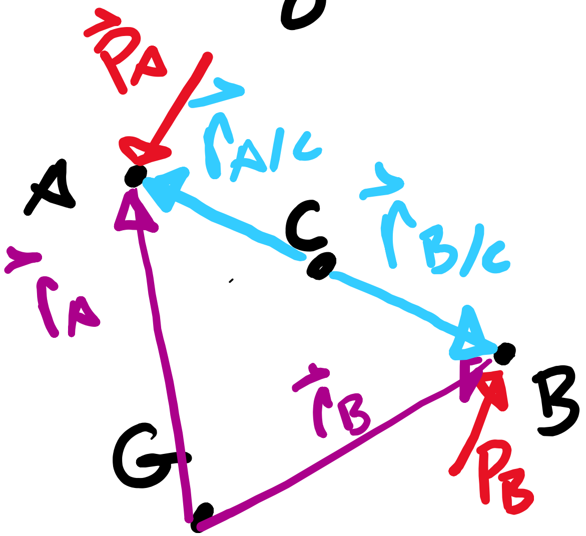


$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B$$

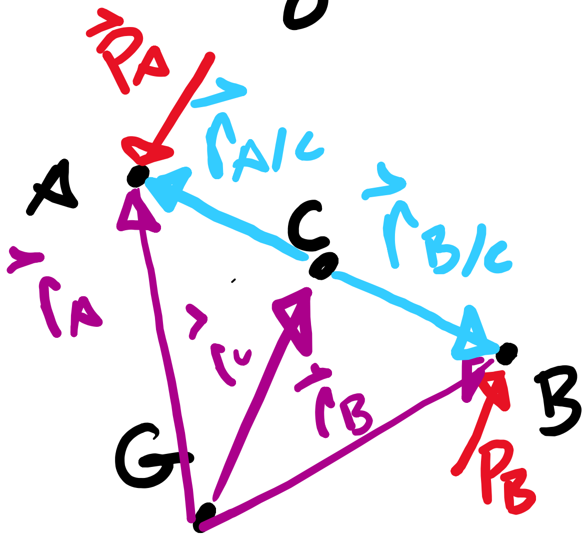
A more general case : $\vec{P}_A = -\vec{P}_B$ with geometry as shown



$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \quad \&$$

$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

A more general case : $\vec{P}_A = -\vec{P}_B$ with geometry as shown

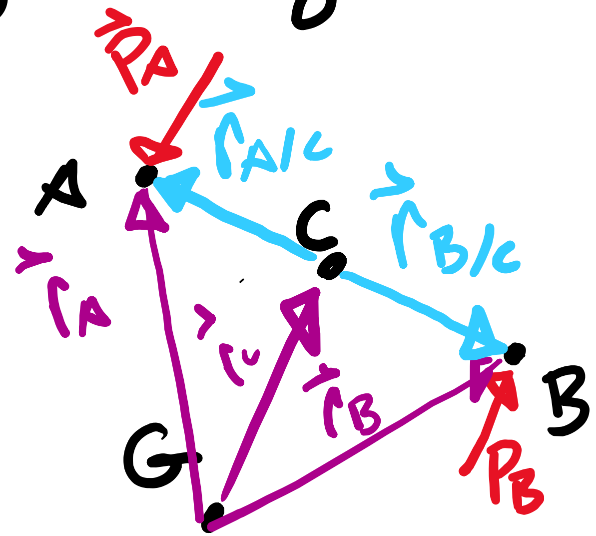


$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \quad \&$$

$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C$

A more general case: $\vec{P}_A = -\vec{P}_B$ with geometry as shown



$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \;\; \&$$

$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C$ & $\vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

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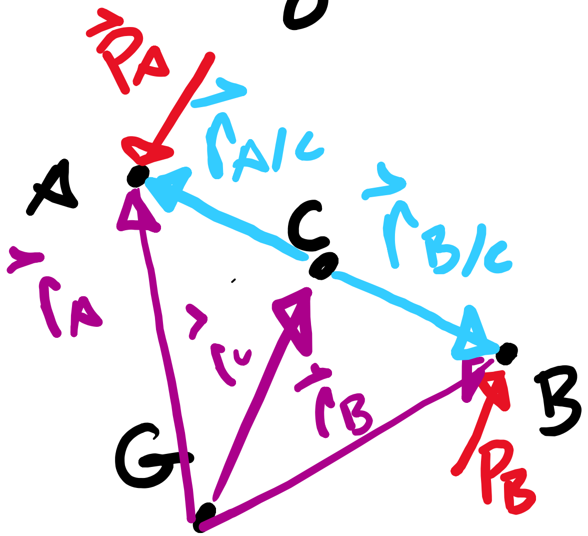


$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \ \&$$

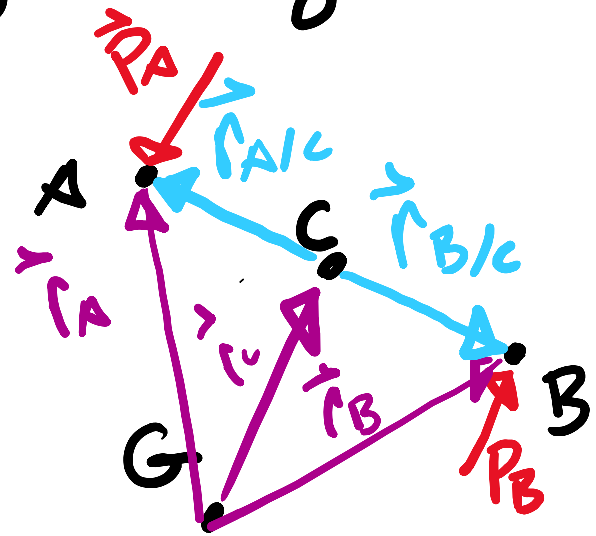
$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C \ \& \ \vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

So

$$\Sigma \vec{M}_C = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B - \vec{r}_C \times \vec{P}_A - \vec{r}_C \times \vec{P}_B$$

A more general case: $\vec{P}_A = -\vec{P}_B$ with geometry as shown



$$\sum \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \;\;\&$$

$$\sum \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

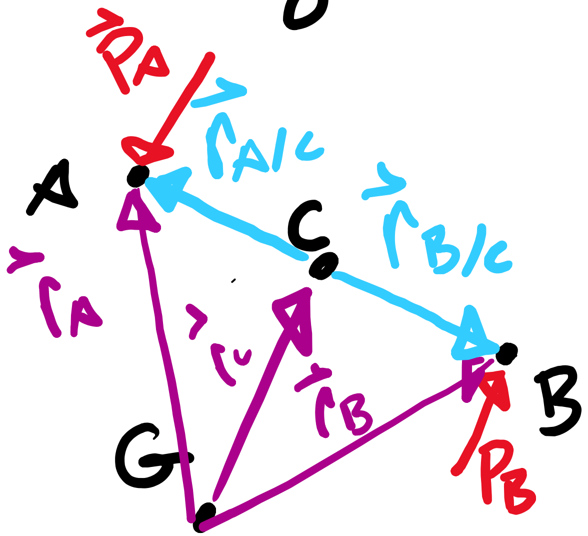
But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C \;\&\; \vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

so

$$\sum \vec{M}_C = \underbrace{\vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B}_{} - \vec{r}_C \times \vec{P}_A - \vec{r}_C \times \vec{P}_B \Rightarrow$$

$$\sum \vec{M}_C = \vec{M}_G - \vec{r}_C \times (\vec{P}_A + \vec{P}_B)$$

A more general case: $\vec{P}_A = -\vec{P}_B$ with geometry as shown



$$\Sigma \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \;\; \&$$

$$\Sigma \vec{M}_C = \vec{r}_{A/C} \times \vec{P}_A + \vec{r}_{B/C} \times \vec{P}_B$$

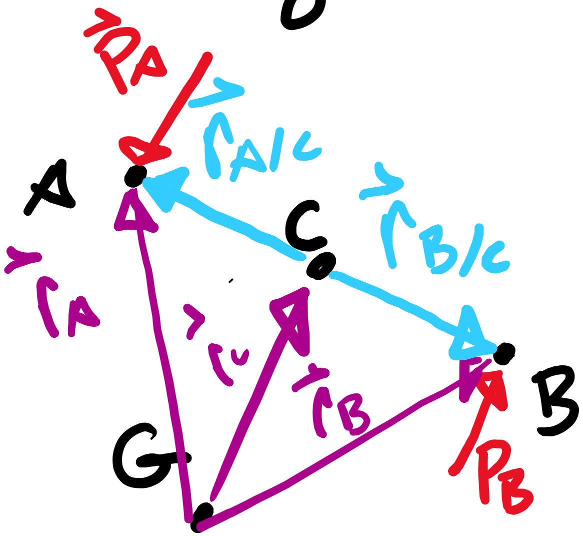
But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C$ & $\vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

So

$$\Sigma \vec{M}_C = \underbrace{\vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B}_{} - \vec{r}_C \times \vec{P}_A - \vec{r}_C \times \vec{P}_B \Rightarrow$$

$$\Sigma \vec{M}_C = \vec{M}_G - \vec{r}_C \times (\vec{P}_A + \vec{P}_B)$$ But $\vec{P}_A = -\vec{P}_B$

A more general case: $\vec{P}_A = -\vec{P}_B$ with geometry as shown



$$\sum \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \quad \&$$

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But $\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C$ & $\vec{r}_{B/C} = \vec{r}_B - \vec{r}_C$

so

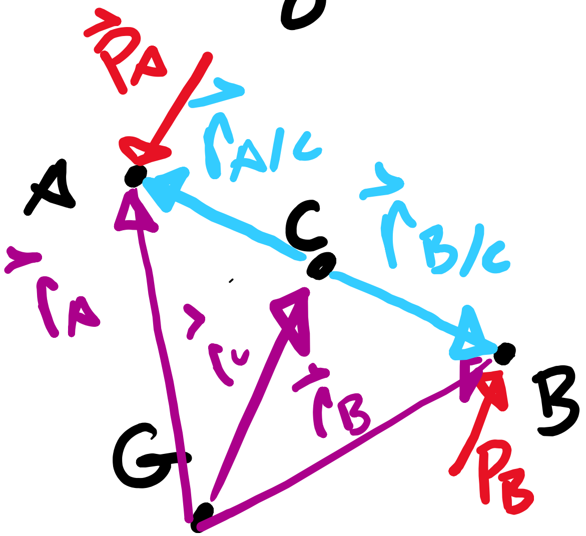
$$\sum \vec{M}_C = \underbrace{\vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B} - \vec{r}_C \times \vec{P}_A - \vec{r}_C \times \vec{P}_B \Rightarrow$$

$$\sum \vec{M}_C = \vec{M}_G - \vec{r}_C \times \cancel{(\vec{P}_A + \vec{P}_B)}$$ But $\vec{P}_A = -\vec{P}_B$

so

$$\sum \vec{M}_C = \vec{M}_G$$

A more general case: $\vec{P}_A = -\vec{P}_B$ with geometry as shown



$$\sum \vec{M}_G = \vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B \quad \&$$

$$\sum \vec{M}_c = \vec{r}_{A/c} \times \vec{P}_A + \vec{r}_{B/c} \times \vec{P}_B$$

But $\vec{r}_{A/c} = \vec{r}_A - \vec{r}_c$ & $\vec{r}_{B/c} = \vec{r}_B - \vec{r}_c$

So

$$\sum \vec{M}_c = \underbrace{\vec{r}_A \times \vec{P}_A + \vec{r}_B \times \vec{P}_B} - \vec{r}_c \times \vec{P}_A - \vec{r}_c \times \vec{P}_B \Rightarrow$$

$$\sum \vec{M}_c = \vec{M}_G - \cancel{\vec{r}_c \times (\vec{P}_A + \vec{P}_B)}\ 0$$

But $\vec{P}_A = -\vec{P}_B$

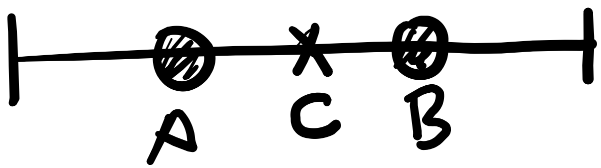
So

$$\sum \vec{M}_c = \vec{M}_G$$

Does not matter where couple is located on rigid body

Example Rod free to rotate about center

__Example__ Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod



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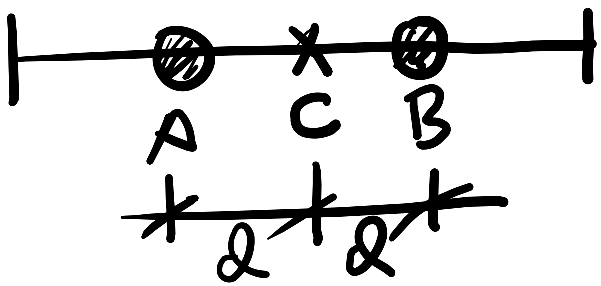

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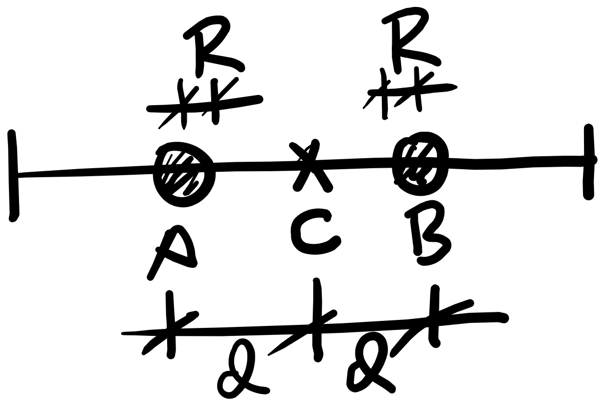

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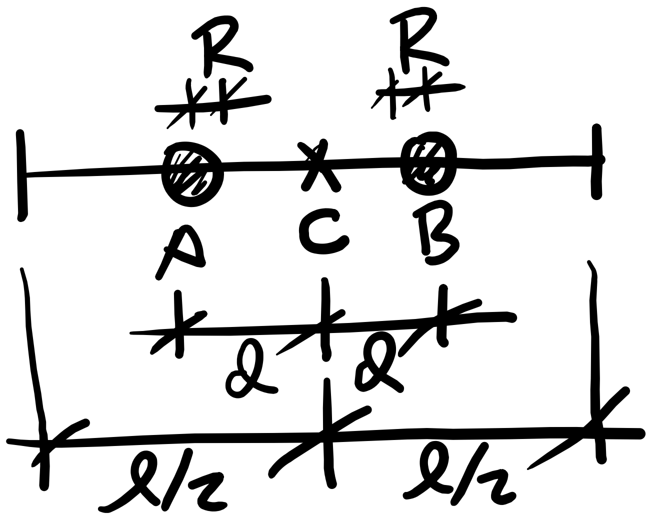
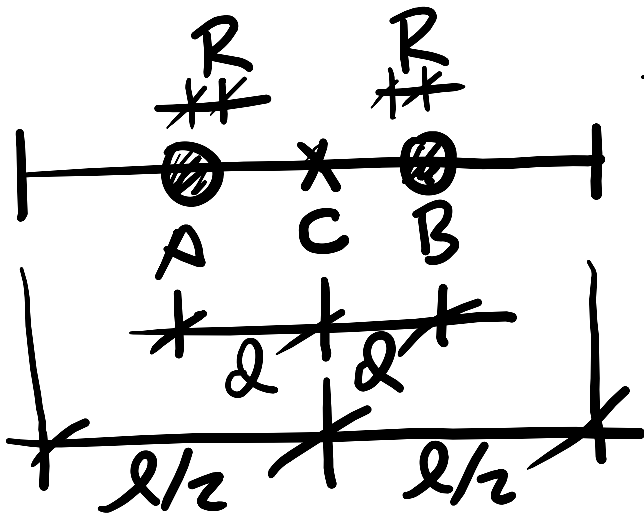


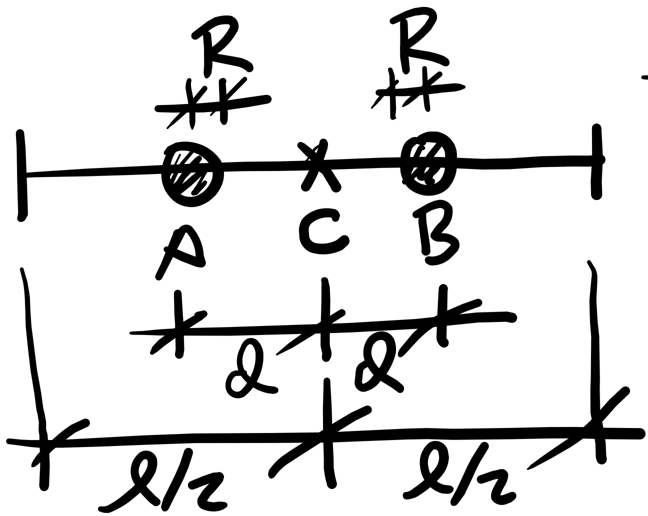
Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



ω_I,

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given

$\omega_I, I_{rod},$



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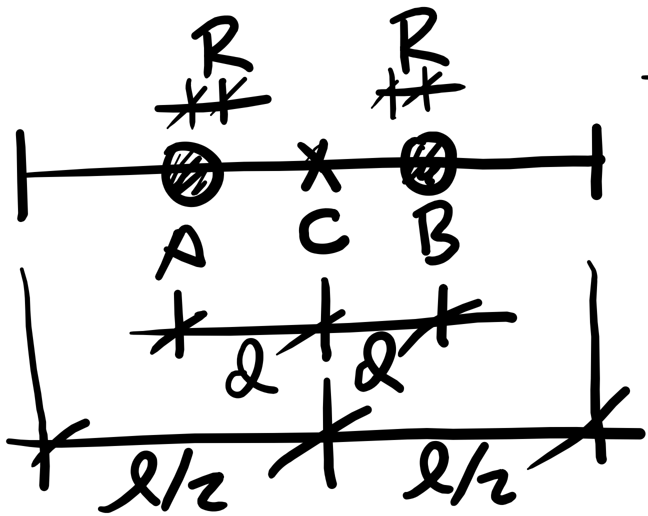
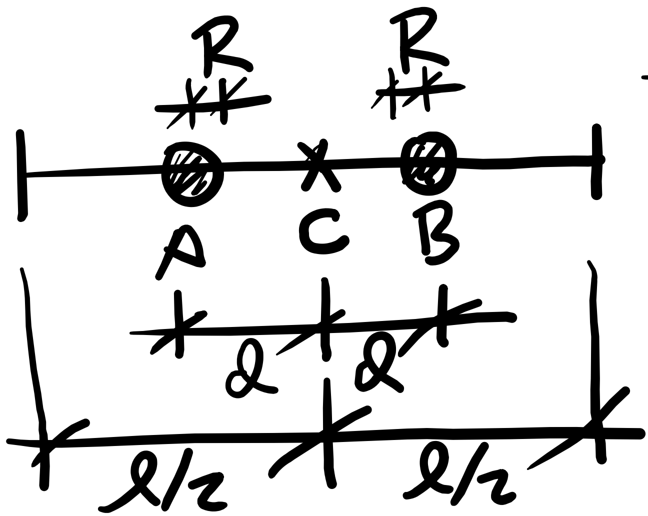


$\ell \ell_I, \bar{I}_{rod}, R,$

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given

$\ell\, m_I,\ I_{rod},\ R,\ d,$



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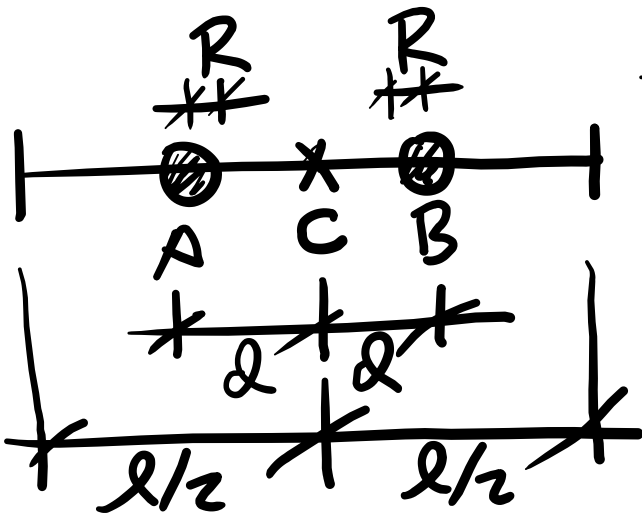
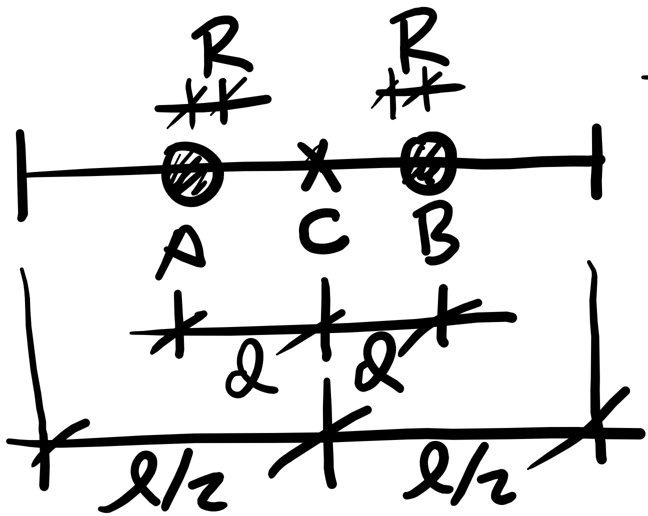


ω_I, $\bar{I}_{rod}$, R, d, l,

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given

ω_I, I_{rod}, R, d, l, m_A & $m_A = m_B$



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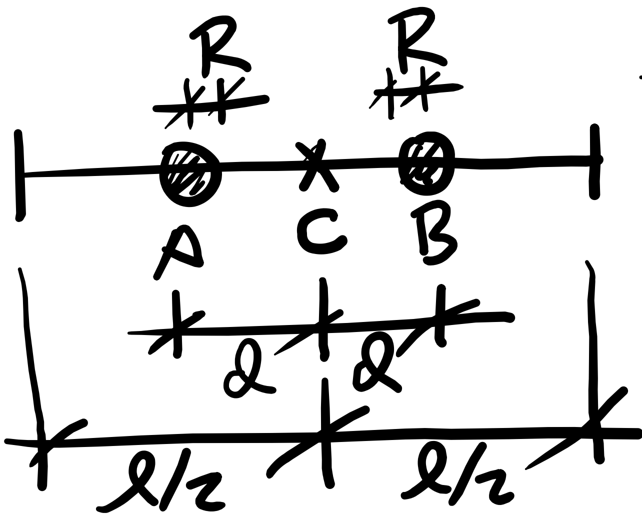
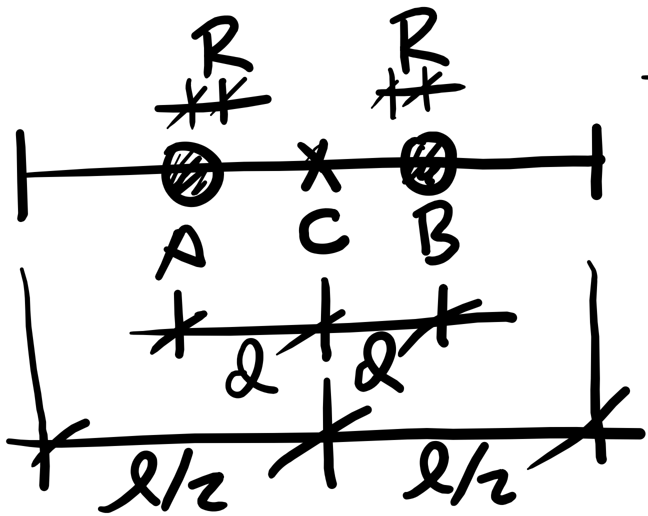


ω_I, I_{rod}, R, d, l, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



ω_I, $\bar{I}_{rod}$, R, d, l, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod $\int \Sigma M\,dt = 0$

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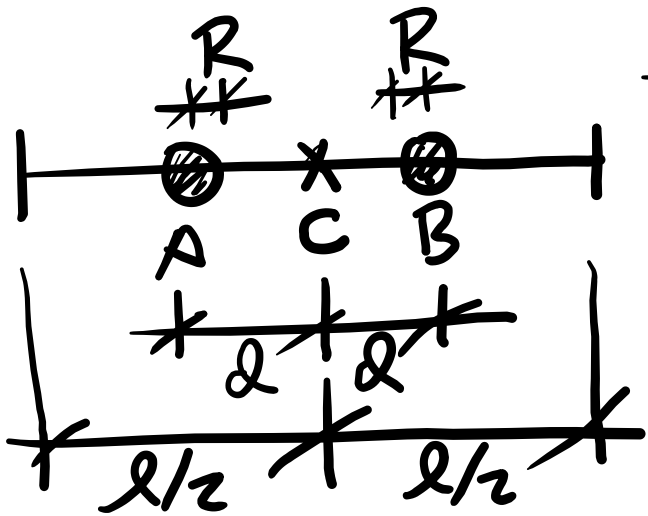

$\omega_I, \bar{I}_{rod}, R, d, l, m_A$ & $m_A = m_B$

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So angular momentum is conserved

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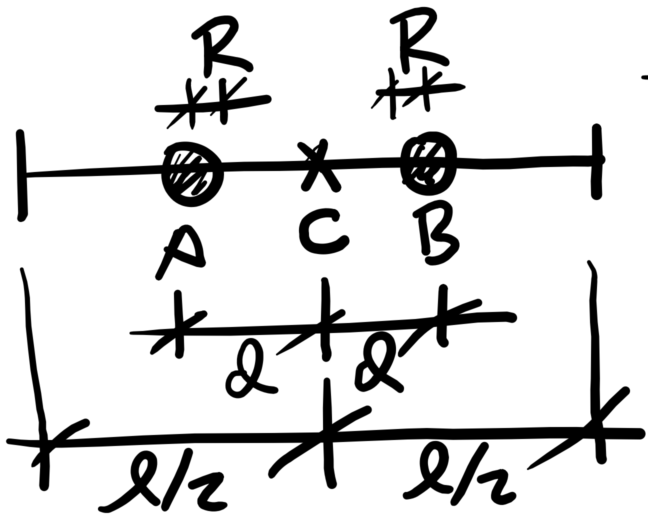

ω_I, $\bar{I}_{rod}$, R, d, l, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod $\int \Sigma M dt = 0$

So angular momentum is conserved $\Rightarrow \vec{H}_{CF} = \vec{H}_{CI}$

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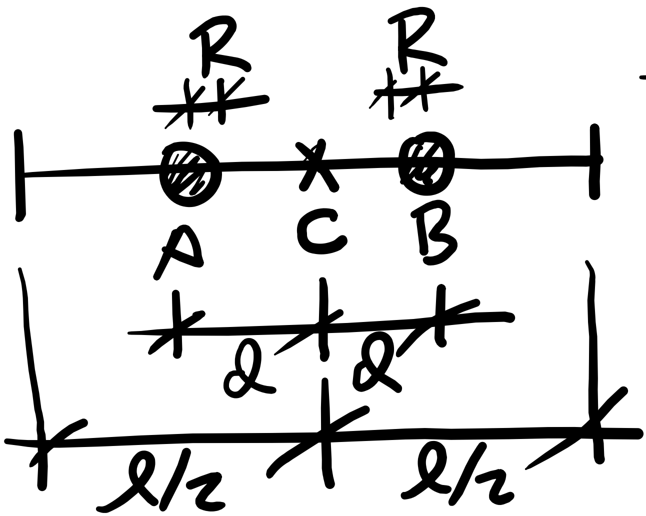
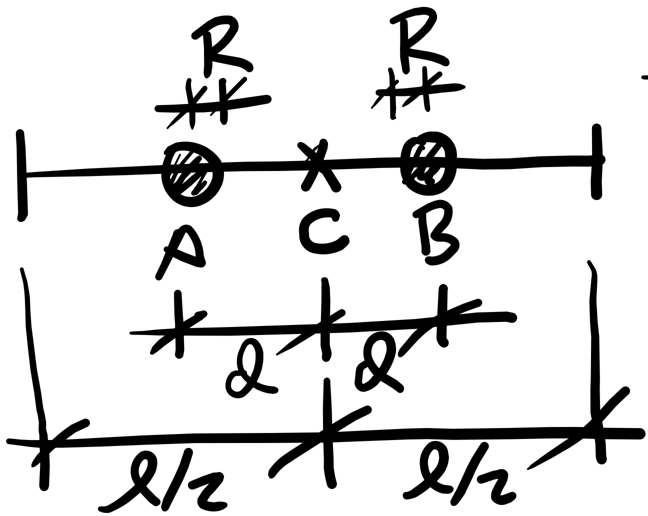


ω_I, $\bar{I}_{rod}$, R, d, l, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod $\int \Sigma M\,dt = 0$ so angular momentum is conserved $\Rightarrow \vec{H}_{CF} = \vec{H}_{CI}$

& $H_C = (\bar{I}_{rod} + I_{CA} + I_{CB})\,\omega$

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



ω_I, $\bar{I}_{rod}$, R, d, l, m_A & $m_A = m_B$

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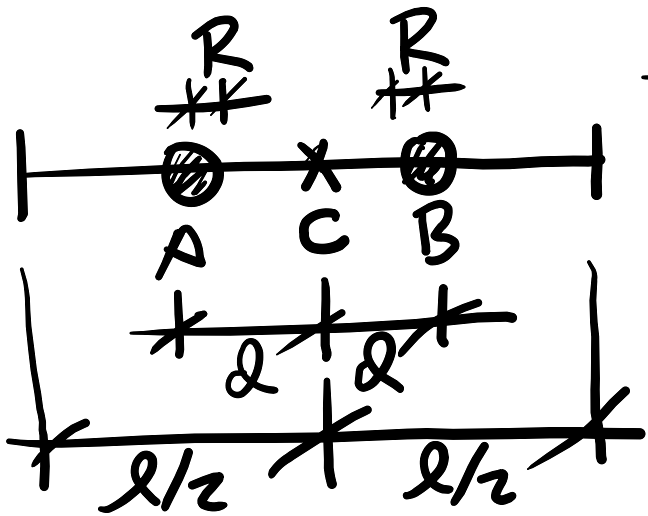

ω_I, $\bar{I}_{rod}$, R, d, l, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod. $\int \Sigma M\,dt = 0$ so angular momentum is conserved $\Rightarrow \vec{H}_{CF} = \vec{H}_{CI}$

& $H_C = (\bar{I}_{rod} + I_{CA} + I_{CB})\omega$ & $I_{CA} = I_{CB}$ so

$$H_C = (\bar{I}_{rod} + 2I_{CA})\omega$$

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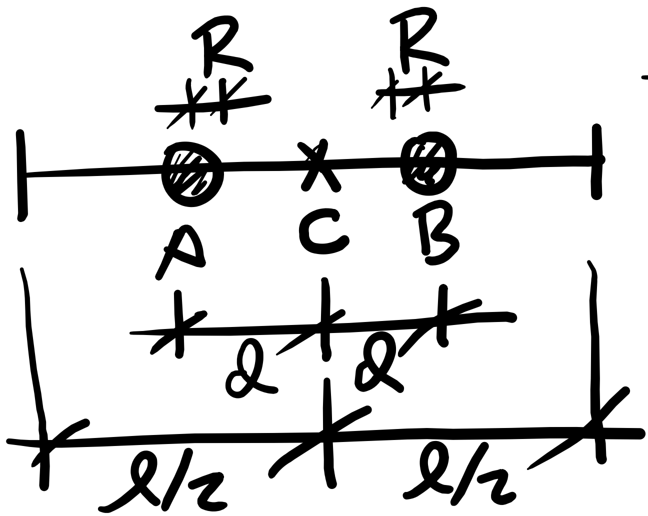

ω_I, $\bar{I}_{rod}$, R, d, l, m_A & $m_A = m_B$

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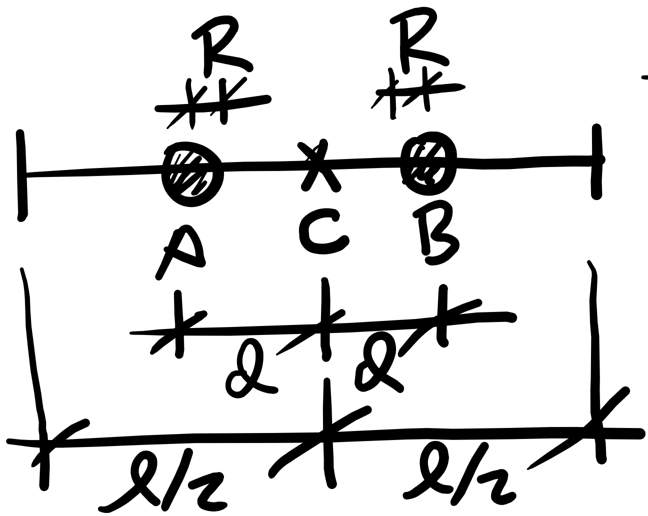

ω_I, $\bar{I}_{rod}$, R, d, l, m_A & $m_A = m_B$

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$H_C = (\bar{I}_{rod} + 2I_{CA})\omega$ & $I_{CA} = \bar{I}_A + m_A d^2 \Rightarrow$

$H_C = (\bar{I}_{rod} + 2\bar{I}_A + 2m_A d^2)\omega$

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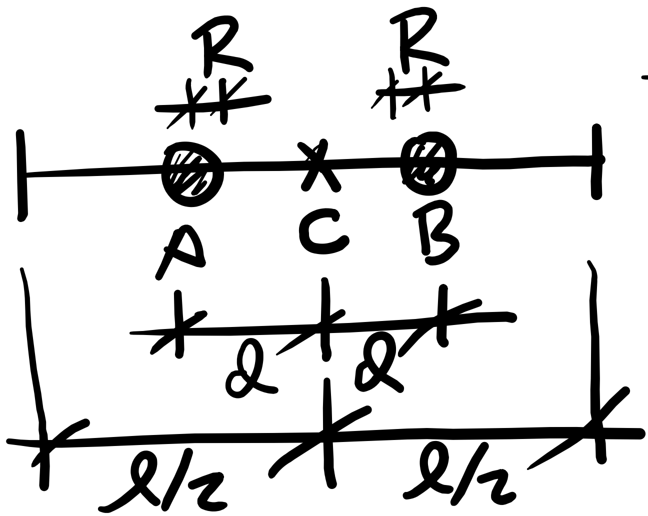

ω_I, $\bar{I}_{rod}$, R, d, l, m_A & $m_A = m_B$

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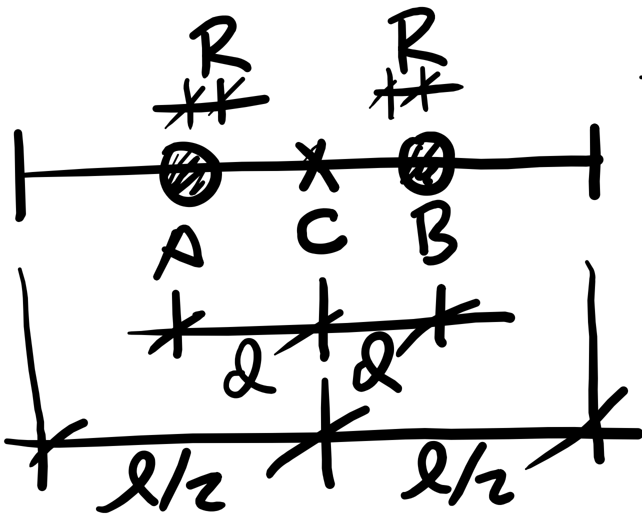
& $H_C = (\bar{I}_{rod} + I_{CA} + I_{CB})\omega$ & $I_{CA} = I_{CB}$ so

$H_C = (\bar{I}_{rod} + 2I_{CA})\omega$ & $I_{CA} = \bar{I}_A + m_A d^2 \Rightarrow$

$H_C = (\bar{I}_{rod} + 2\bar{I}_A + 2m_A d^2)\omega$ Now $H_{CF} = H_{CI} \Rightarrow$

Example Rod free to rotate about center, has two balls (A & B) that are allowed to move along rod. Given



ω_I, $\bar{I}_{rod}$, R, d, ℓ, m_A & $m_A = m_B$

Find ω_F when balls hit ends of rod. $\int \Sigma M \, dt = 0$ so angular momentum is conserved $\Rightarrow \vec{H}_{CF} = \vec{H}_{CI}$

& $H_C = (\bar{I}_{rod} + I_{CA} + I_{CB})\omega$ & $I_{CA} = I_{CB}$ so

$H_C = (\bar{I}_{rod} + 2I_{CA})\omega$ & $I_{CA} = \bar{I}_A + m_A d^2 \Rightarrow$

$H_C = (\bar{I}_{rod} + 2\bar{I}_A + 2m_A d^2)\omega$ Now $H_{CF} = H_{CI} \Rightarrow$

$(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2)\omega_F = (\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2)\omega_I$

From previous slide

$$\left(\bar{I}_{rod} + 2\bar{I}_{s} + 2m_s d_F^2\right)\omega_F =$$

$$\left(\bar{I}_{rod} + 2\bar{I}_{s} + 2m_s d_I^2\right)\omega_I$$

From previous slide

$$(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2)\omega_F =$$

$$(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2)\omega_I \Rightarrow$$

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2}\right]\omega_I$$

From previous slide

$$(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_F^2)\omega_F =$$

$$(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2)\omega_I \Rightarrow$$

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_F^2}\right]\omega_I \quad \$$$

since $d_F + R = \ell/2$

From previous slide

$$(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2)\omega_F =$$

$$(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2)\omega_I \Rightarrow$$

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2}\right]\omega_I \quad \&$$

since $d_F + R = \ell/2 \Rightarrow d_F = \frac{\ell}{2} - R$

From previous slide

$$\left(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2\right)\omega_F = \left(\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2\right)\omega_I \Rightarrow$$

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_F^2}\right]\omega_I \quad \&$$

since $d_F + R = \ell/2 \Rightarrow d_F = \frac{\ell}{2} - R$

then

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2m_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2m_A\left(\frac{\ell}{2} - R\right)^2}\right]\omega_I,$$

From previous slide

$$(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_F^2)\omega_F =$$

$$(\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2)\omega_I \Rightarrow$$

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_F^2}\right]\omega_I \quad \&$$

since $d_F + R = \ell/2 \Rightarrow d_F = \frac{\ell}{2} - R$

then

$$\omega_F = \left[\frac{\bar{I}_{rod} + 2\bar{I}_A + 2M_A d_I^2}{\bar{I}_{rod} + 2\bar{I}_A + 2M_A(\frac{\ell}{2} - R)^2}\right]\omega_I, \text{ where}$$

$$\bar{I}_A = \frac{2}{5}M_A R^2$$

ASU