

Today: 11.2

L2



Today: 11.2

L2



Special  
cases &  
relative  
motion

Today: 11.2  
Tuesday: 11.4

L2

Today: 11.2

Tuesday: 11.4

L2

↓  
Curvilinear  
motion of  
particles

Previously:

Previously:  $v = \frac{dx}{dt}$

Previously:  $v = \frac{dx}{dt}$  &  $a = \frac{dv}{dt}$

Previously:  $v = \frac{dx}{dt}$  &  $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

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If  $a = \text{const.}$

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If  $a = \text{const.}$   $\frac{dv}{dt} = a \Rightarrow \int \frac{dv}{dt} dt = a \int dt$

Previously:  $v = \frac{dx}{dt}$  &  $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

If  $a = \text{const.}$   $\frac{dv}{dt} = a \Rightarrow \int \frac{dv}{dt} dt = a \int dt$

$\Rightarrow \int dv = a \int dt$

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If  $a = \text{const.}$   $\frac{dv}{dt} = a \Rightarrow \int \frac{dv}{dt} dt = a \int dt$

$\Rightarrow \int dv = a \int dt \Rightarrow v - v_0 = at$

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$\Rightarrow \int dv = a \int dt \Rightarrow v - v_0 = at \Rightarrow v = at + v_0$

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But  $v = \frac{dx}{dt}$

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But  $v = \frac{dx}{dt}$  so  $\int \frac{dx}{dt} dt = \int (at + v_0) dt$

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But  $v = \frac{dx}{dt}$  so  $\int \frac{dx}{dt} dt = \int (at + v_0) dt$

$\Rightarrow \int dx = \int (at + v_0) dt \Rightarrow x - x_0 = \frac{1}{2}at^2 + v_0t$

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Also  $a = \frac{dv}{dt}$  ‡

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$\neq$

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Also  $a = \frac{dv}{dt}$   $\neq$   $\frac{dv}{dt} = \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right)$

$\neq v = \frac{dx}{dt} \Rightarrow a = v \frac{dv}{dx}$

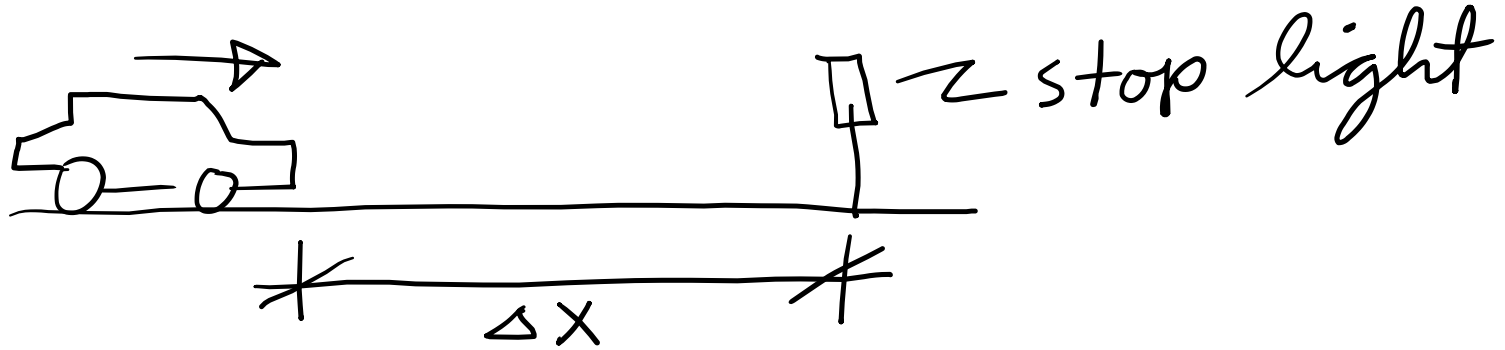
Also  $a = \frac{dv}{dt}$  ‡  $\frac{dv}{dt} = \left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right)$

‡  $v = \frac{dx}{dt} \Rightarrow \underline{a = v \frac{dv}{dx}} \rightarrow \text{Important}$

Form for work/energy  
relation

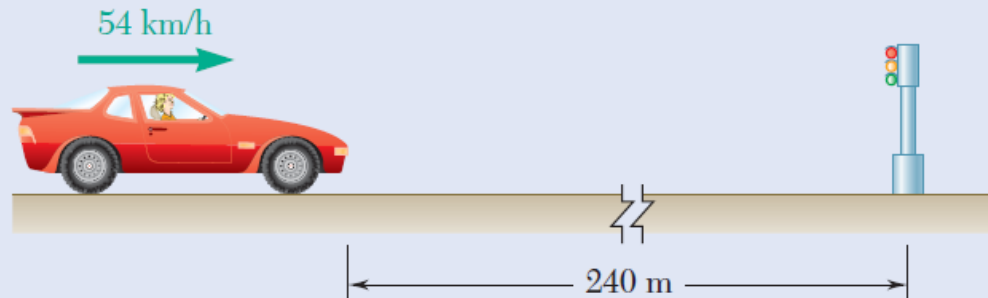
# Notes on problems

# Notes on problems : 11.34

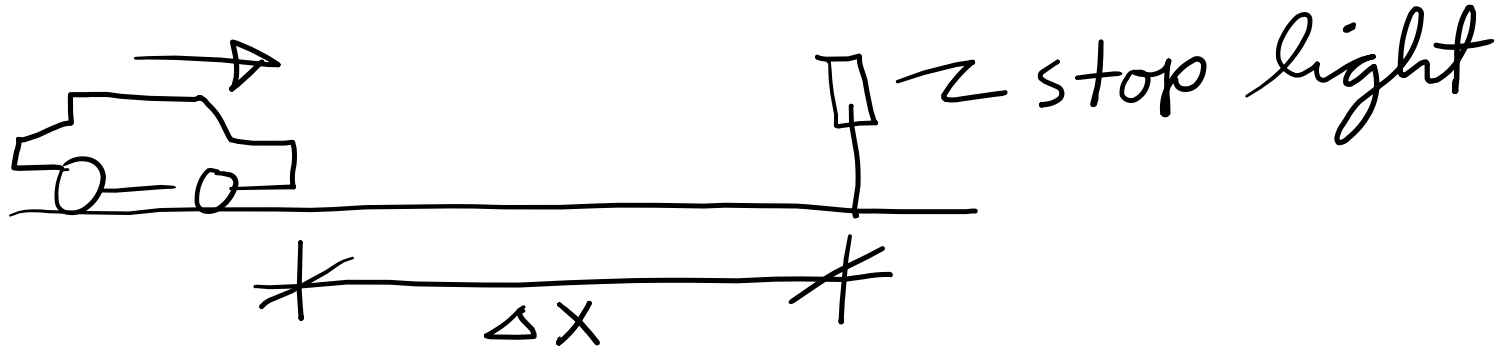


Given:  $V_0 = 54 \frac{\text{km}}{\text{hr}}$ ,  $\Delta x = 240 \text{m}$ ,  $\Delta t = 24 \text{s}$  &  $a = \text{const.}$

**11.34** A motorist is traveling at 54 km/h when she observes that a traffic light 240 m ahead of her turns red. The traffic light is timed to stay red for 24 s. If the motorist wishes to pass the light without stopping just as it turns green again, determine (a) the required uniform deceleration of the car, (b) the speed of the car as it passes the light.



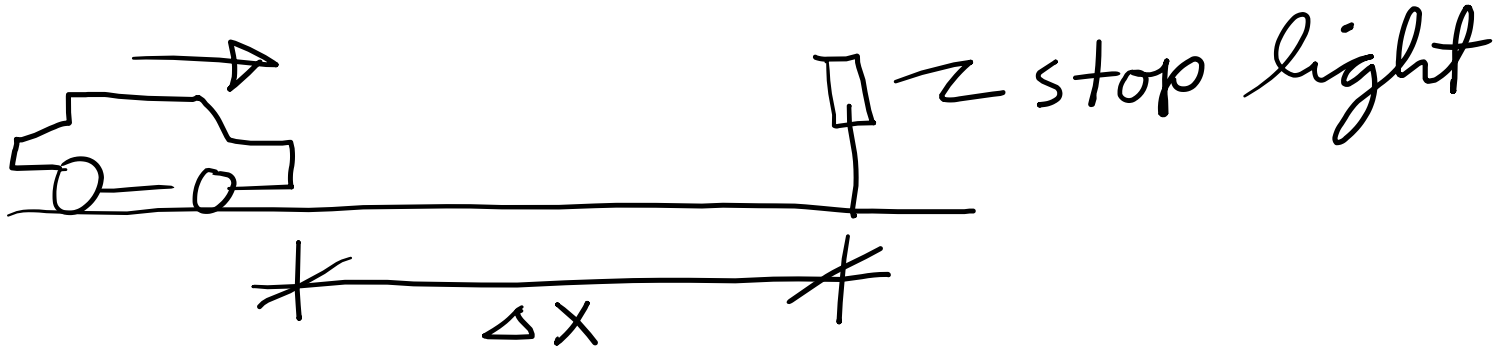
# Notes on problems : 11.34



Given:  $v_0 = 54 \frac{\text{km}}{\text{hr}}$ ,  $\Delta x = 240 \text{m}$ ,  $\Delta t = 24 \text{s}$  &  $a = \text{const.}$

Note units  $\downarrow \downarrow$

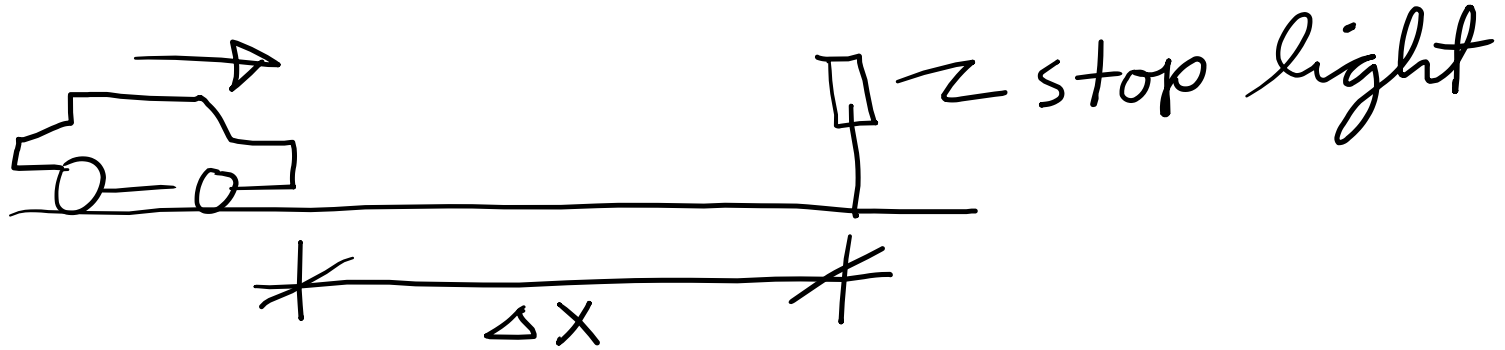
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Given:  $v_0 = 54 \frac{\text{km}}{\text{hr}}$ ,  $\Delta x = 240\text{m}$ ,  $\Delta t = 24\text{s}$  &  $a = \text{const.}$

(a) Find  $a$ :

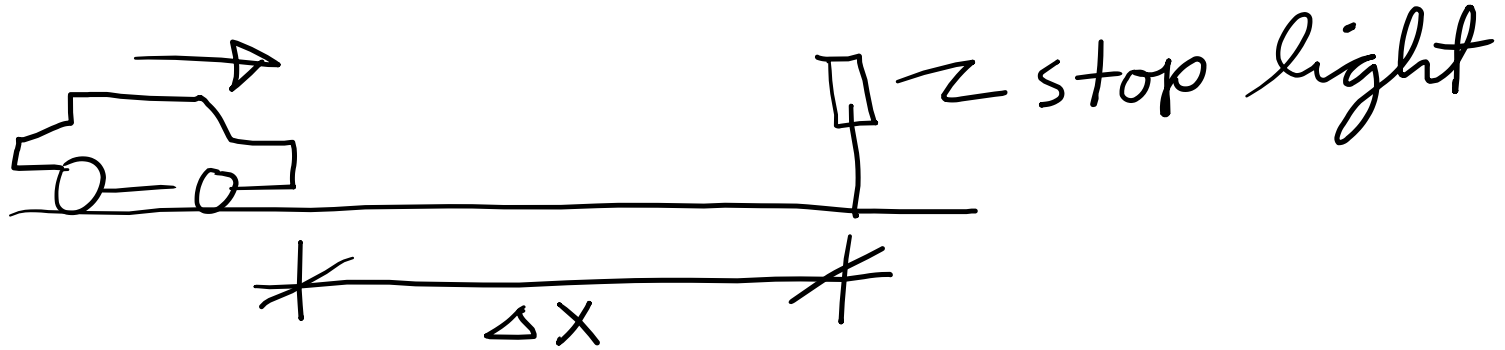
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(a) Find  $a$ : We know  $x = \frac{1}{2}at^2 + v_0t + x_0$   
&  $\Delta x$  &  $\Delta t$

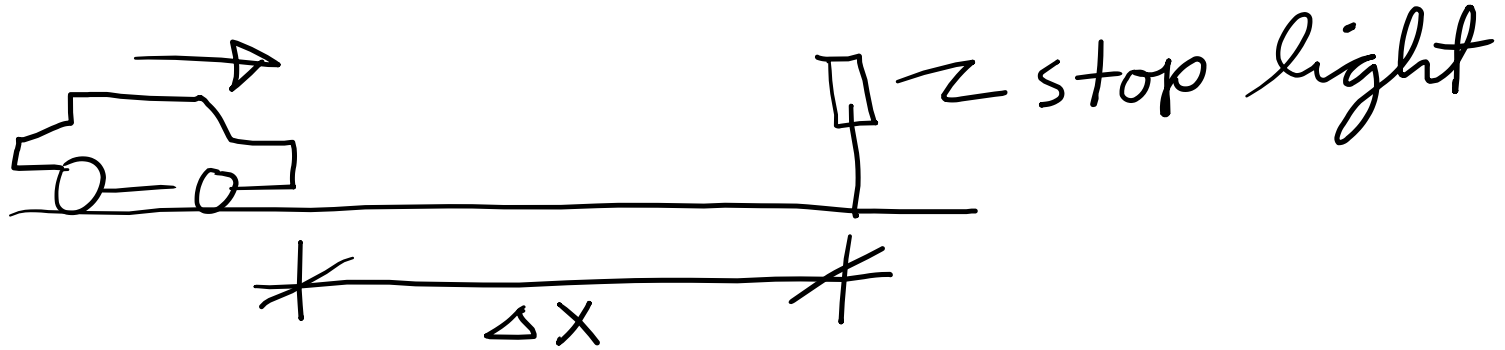
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(a) Find  $a$ : We know  $x = \frac{1}{2}at^2 + v_0t + x_0$   
&  $\Delta x$  &  $\Delta t \Rightarrow$  just need to dig  
out  $a$

# Notes on problems : 11.34

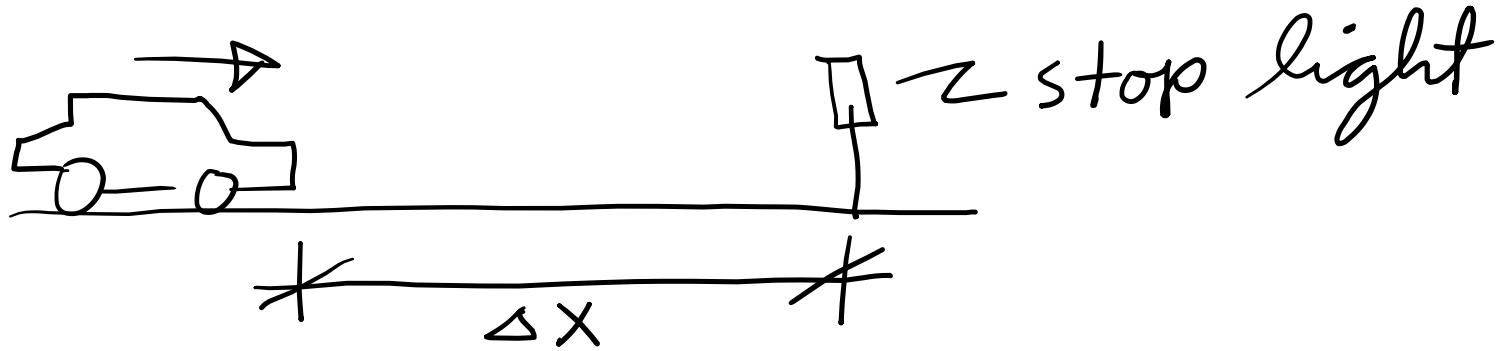


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out  $a$

(b) Find  $v$  at  $t = 24\text{s}$ :

# Notes on problems : 11.34

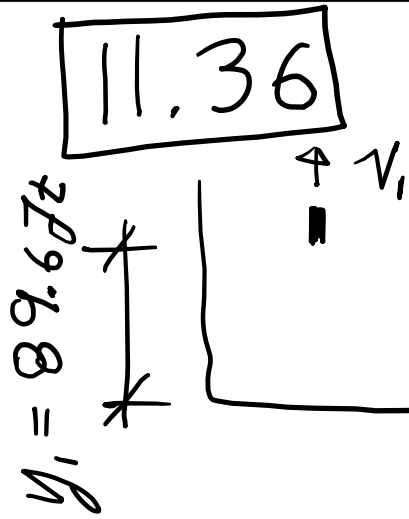


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&  $\Delta x$  &  $\Delta t \Rightarrow$  just need to dig  
out  $a$

(b) Find  $v$  at  $t = 24\text{s}$ :  $v = \frac{dx}{dt} \Rightarrow$

$$v(t=24\text{s}) = \left. \frac{dx}{dt} \right|_{t=24\text{s}}$$



Lands 16 s later

$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

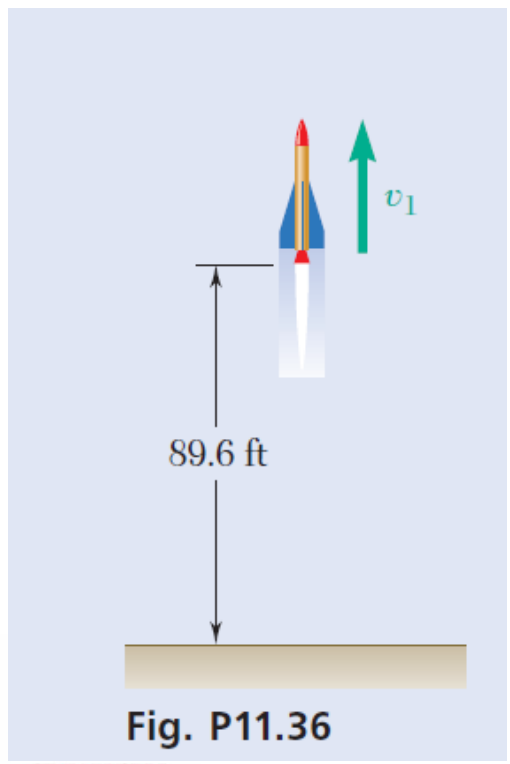
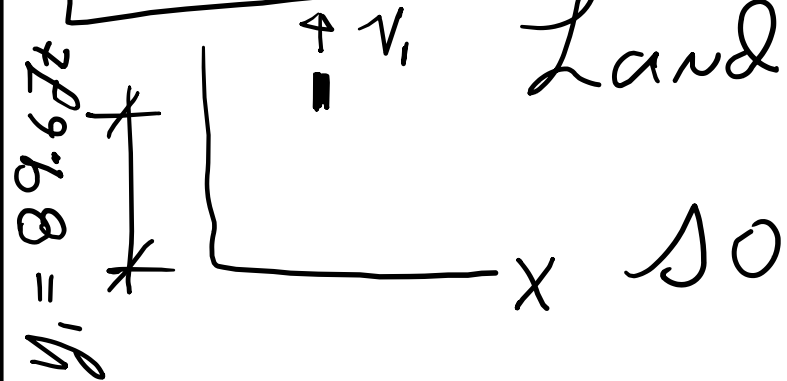


Fig. P11.36

**11.36** A group of students launches a model rocket in the vertical direction. Based on tracking data, they determine that the altitude of the rocket was 89.6 ft at the end of the powered portion of the flight and that the rocket landed 16 s later. Knowing that the descent parachute failed to deploy so that the rocket fell freely to the ground after reaching its maximum altitude and assuming that  $g = 32.2 \text{ ft/s}^2$ , determine (a) the speed  $v_1$  of the rocket at the end of powered flight, (b) the maximum altitude reached by the rocket.

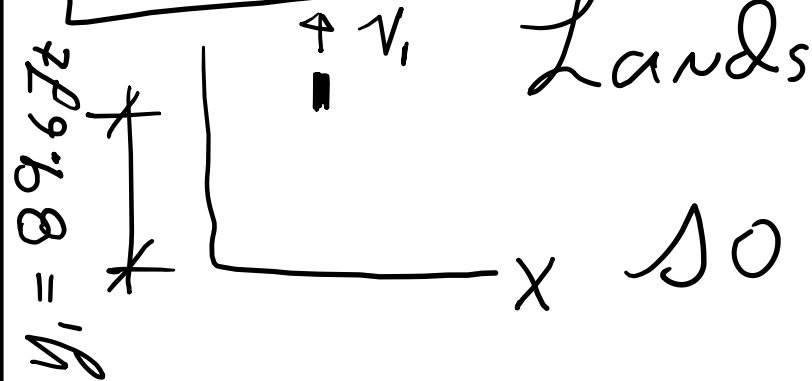
11.36



$$\begin{cases} y_1 = 89.6 \text{ ft}, t_1 = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find  $v_1$

11.36



$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

(a) Find  $v_i$

$$y_f = -\frac{1}{2}gt^2 + v_i t_f + y_i$$

11.36

$y_i = 89.6 \text{ ft}$

$\uparrow v_i$

Lands 16s later

so

$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

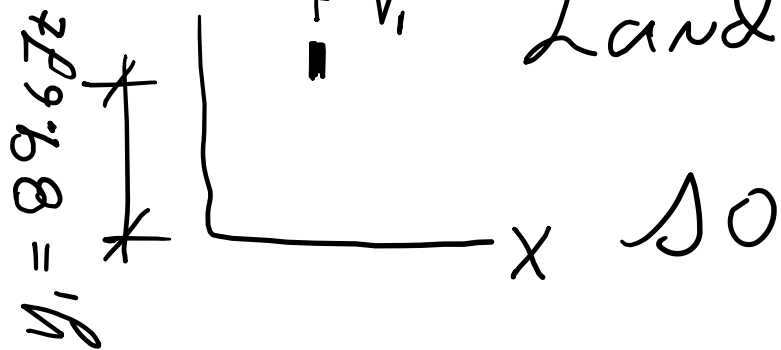
(a) Find  $v_i$

$y_f = -\frac{1}{2}gt^2 + v_i t_f + y_i$  we know  $t_f, y_f \neq y_i$   
so just need to dig out  $v_i$

11.36

$\uparrow v_i$

Lands 16s later



$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

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(b) Find  $y_{\text{max}}$ :

11.36

$y_f = 89.6 \text{ ft}$

$\uparrow v_i$

Lands 16s later

so

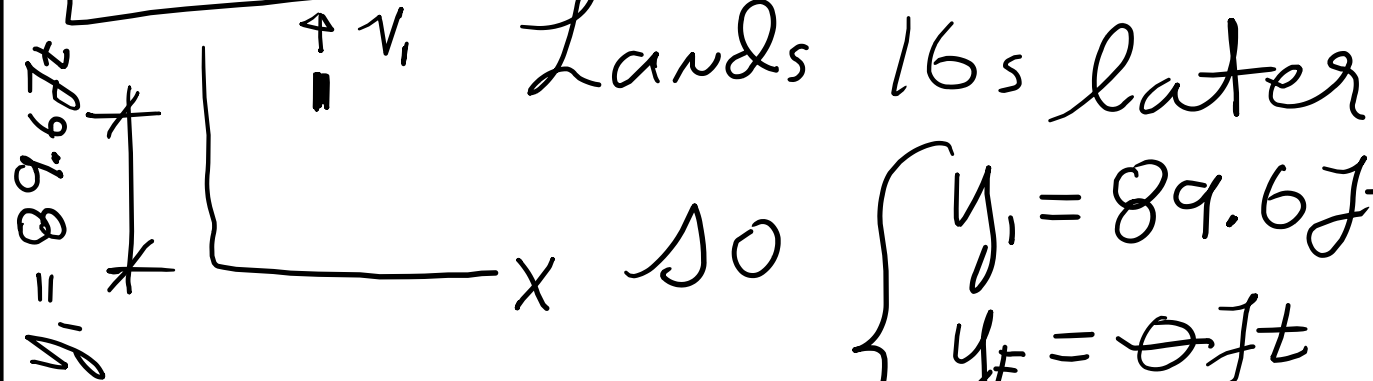
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so just need to dig out  $v_i$

(b) Find  $y_{\text{max}}$  : 2 ways

11.36



Lands 16s later

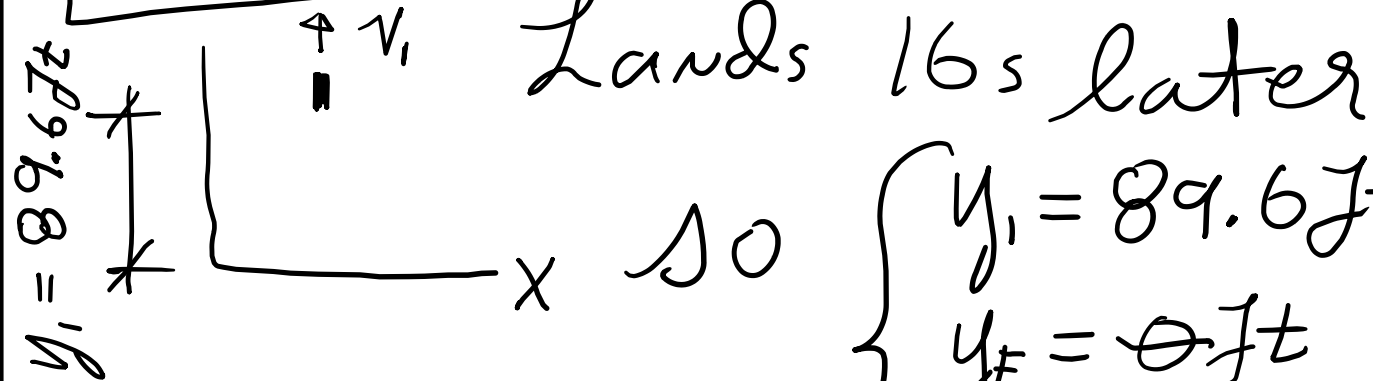
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so just need to dig out  $v_i$

(b) Find  $y_{\text{max}}$ : 2 ways: ①  $y_{\text{max}}$  when  $v = 0$   
 $\Rightarrow$  Find  $t$  at  $v = 0$  then use  $t$  to get  $y_{\text{max}}$

11.36



$$\begin{cases} y_i = 89.6 \text{ ft}, t_i = 0 \\ y_f = 0 \text{ ft}, t_f = 16 \text{ s} \\ a = -g = -32.2 \text{ ft/s}^2 \end{cases}$$

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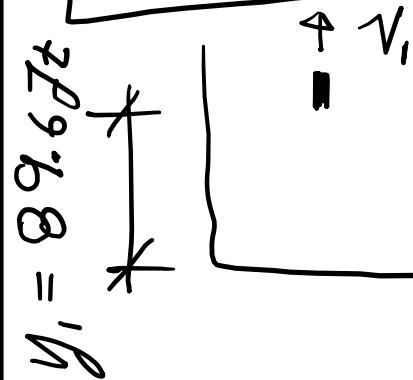
$y_f = -\frac{1}{2}gt^2 + v_1 t_f + y_i$  we know  $t_f, y_f \neq y_i$   
so just need to dig out  $v_1$

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$\Rightarrow$  Find  $t$  at  $v = 0$  then use  $t$  to get  $y_{\max}$  or

② Use  $a = v \frac{dv}{dy} \Rightarrow -g = v \frac{dv}{dy}$

11.36



Lands 16s later

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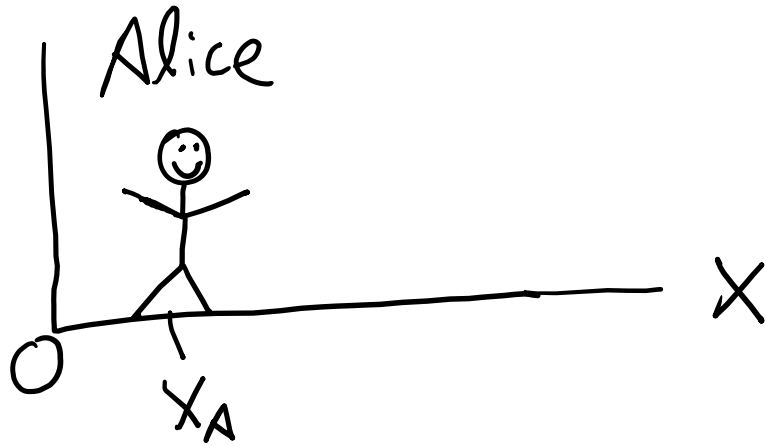
② Use  $a = v \frac{dv}{dy} \Rightarrow -g = v \frac{dv}{dy} \Rightarrow -g dy = v dv$

$$\Rightarrow -g \int_{y_i}^{y_{\text{max}}} dy = \int_{v_i}^{v=0} v dv$$

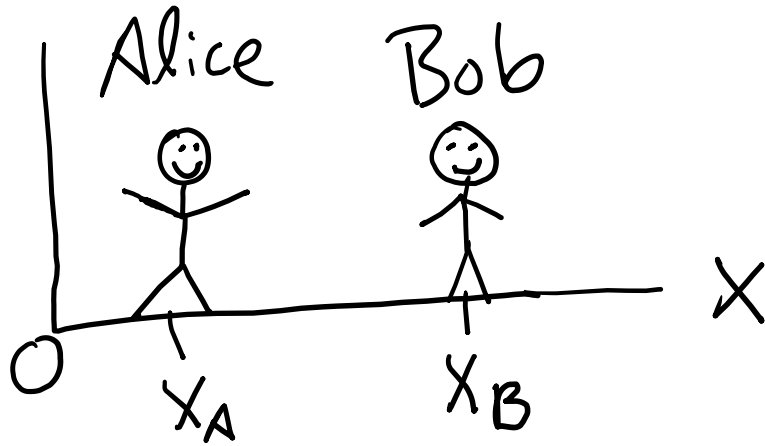


# Relative motion

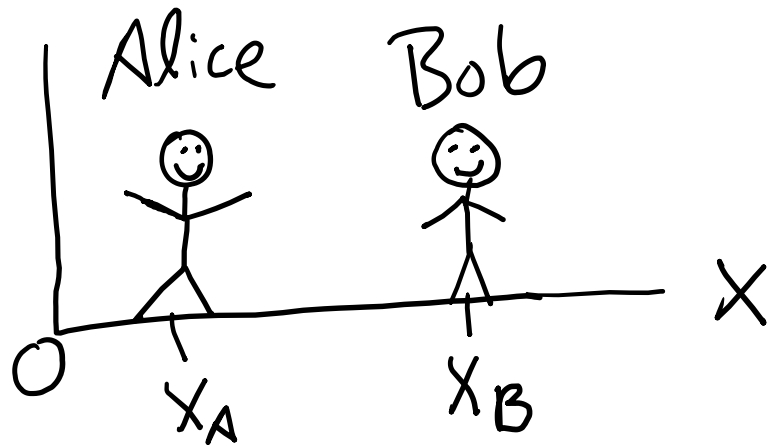
# Relative motion



# Relative motion



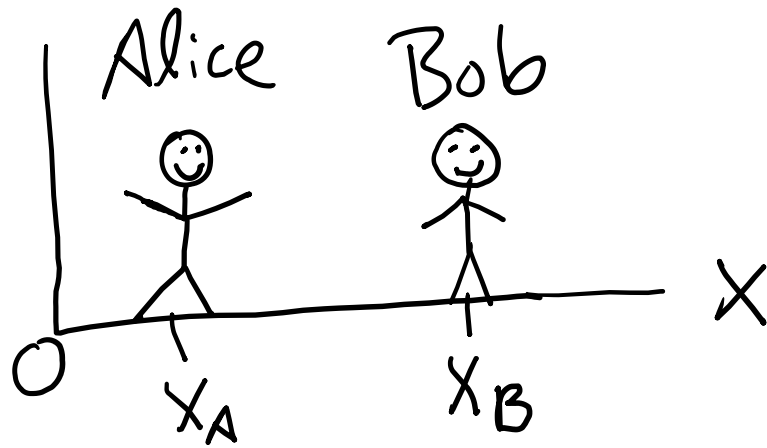
# Relative motion



position:

$x_{B/A} \equiv x$  of B relative to A

# Relative motion

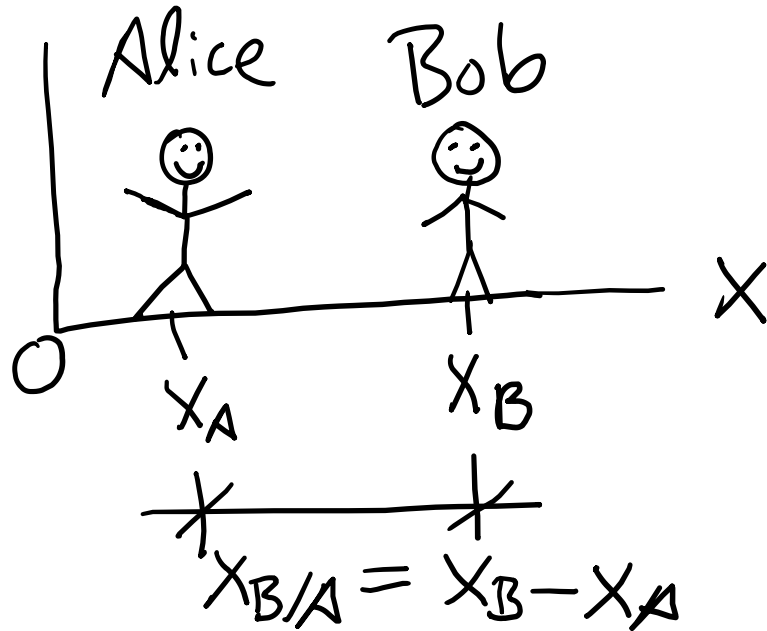


position:

$x_{B/A} \equiv x$  of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

# Relative motion

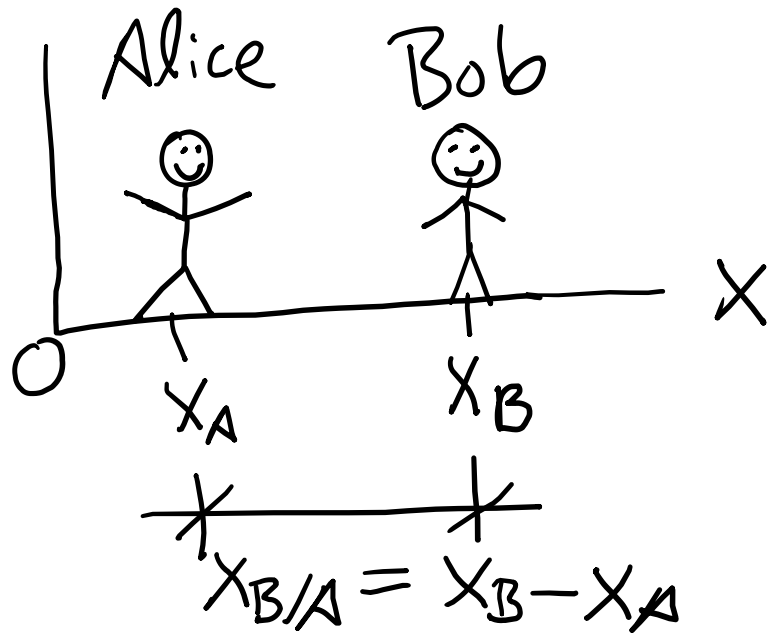


position:

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# Relative motion



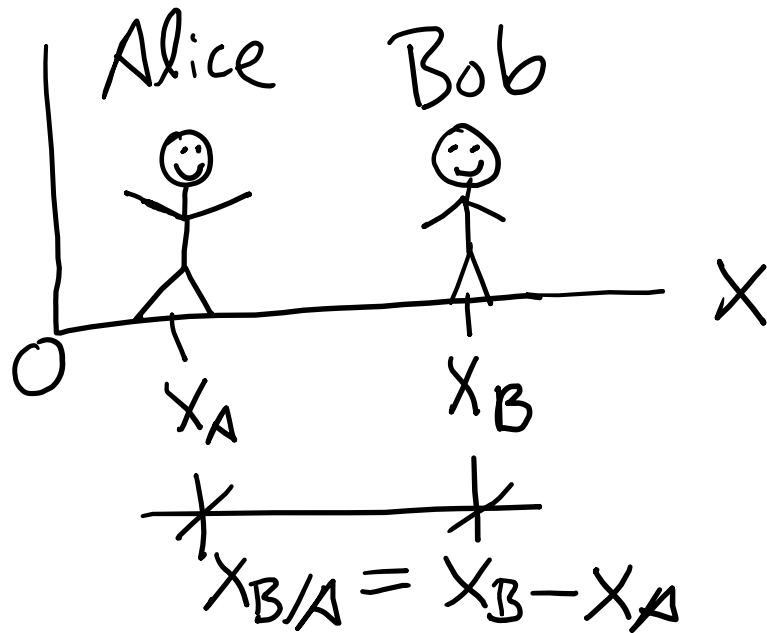
position:

$x_{B/A} \equiv x$  of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity:

# Relative motion



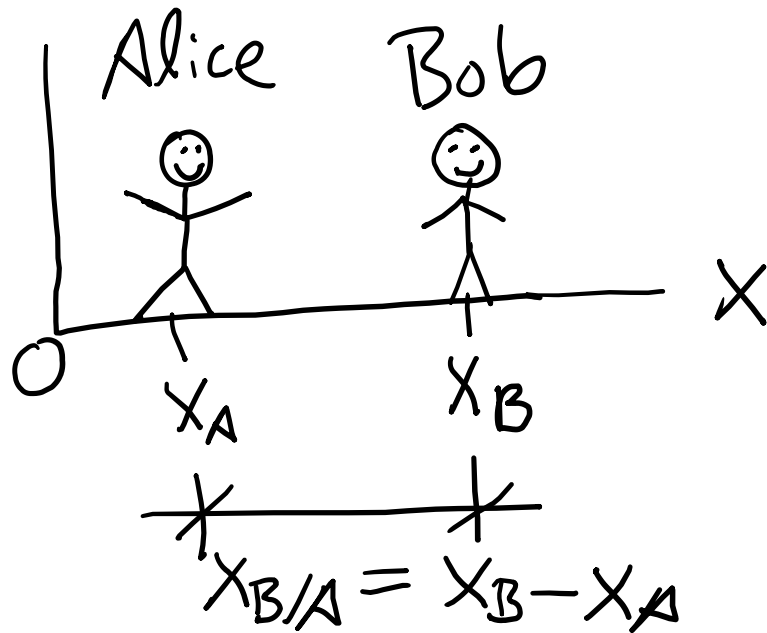
position:

$x_{B/A} \equiv x$  of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity:  $v_{B/A} = v_B - v_A$

# Relative motion



position:

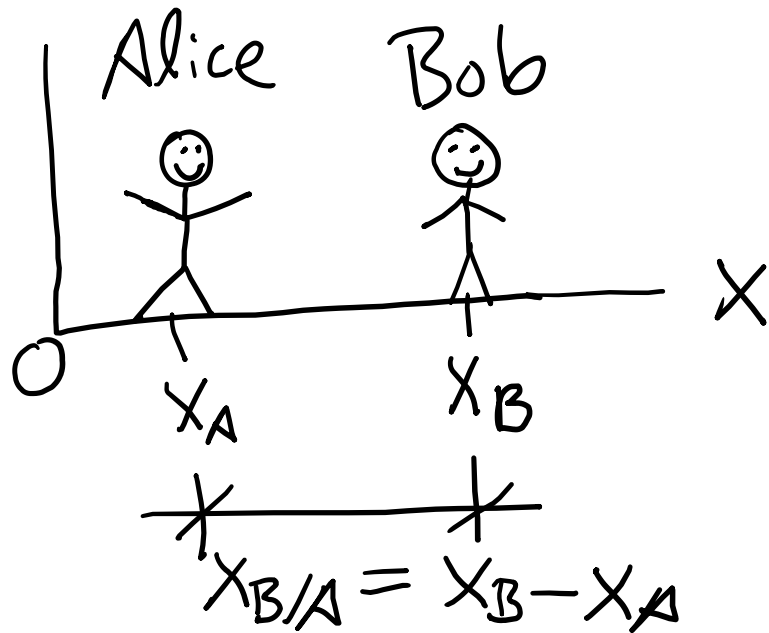
$x_{B/A} \equiv x$  of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity:  $v_{B/A} = v_B - v_A$

acceleration:

# Relative motion



position:

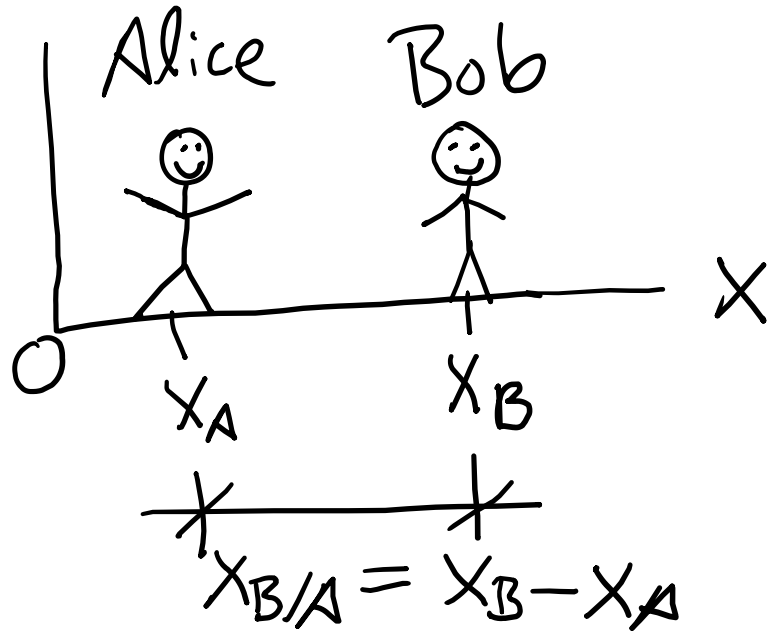
$x_{B/A} \equiv x$  of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity:  $v_{B/A} = v_B - v_A$

acceleration:  $a_{B/A} = a_B - a_A$

# Relative motion



position:

$x_{B/A} \equiv x$  of B relative to A

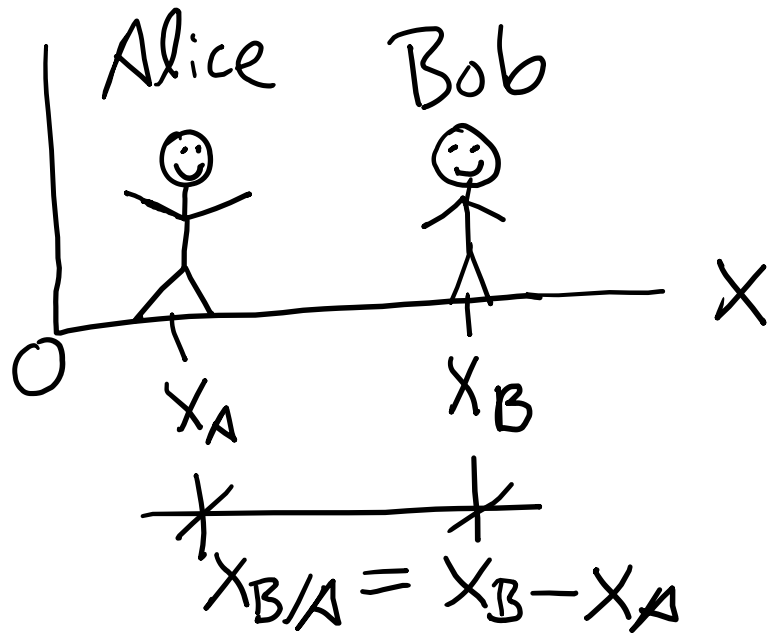
$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity:  $v_{B/A} = v_B - v_A$

acceleration:  $a_{B/A} = a_B - a_A$

Note:  $v_{B/A} = \frac{dx_{B/A}}{dt}$

# Relative motion



position:

$x_{B/A} \equiv x$  of B relative to A

$$\& \quad x_{B/A} \equiv x_B - x_A$$

Velocity:  $v_{B/A} = v_B - v_A$

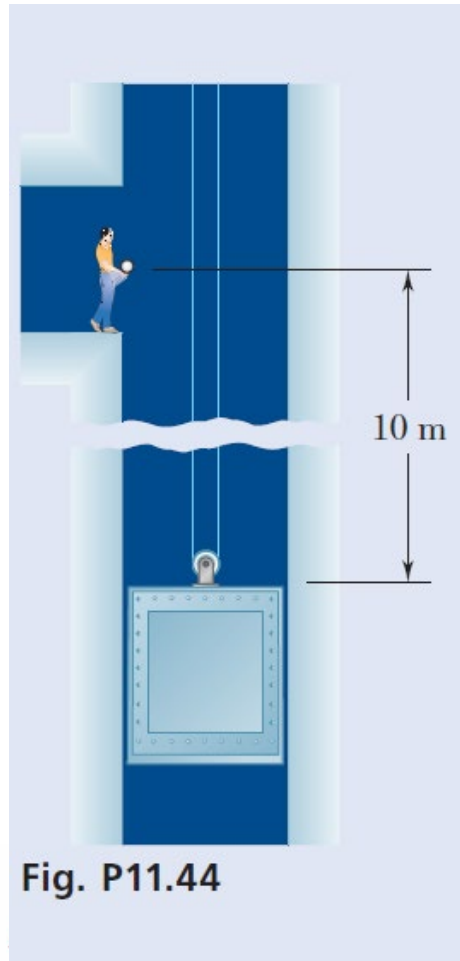
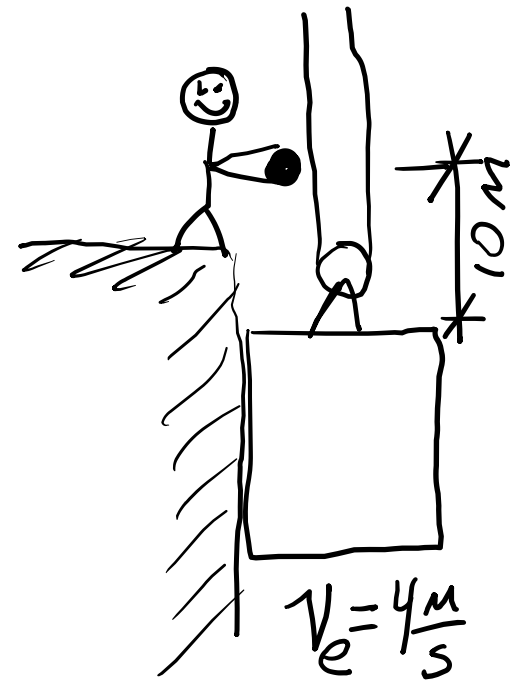
acceleration:  $a_{B/A} = a_B - a_A$

Note:  $v_{B/A} = \frac{dx_{B/A}}{dt}$   $\&$   $a_{B/A} = \frac{dv_{B/A}}{dt}$

# Notes on problem 11.44:

# Notes on problem 11.44:

Man throws ball up with initial velocity of  $v_{bI} = 3 \frac{m}{s}$



- 11.44** An elevator is moving upward at a constant speed of 4 m/s. A man standing 10 m above the top of the elevator throws a ball upward with a speed of 3 m/s. Determine (a) when the ball will hit the elevator, (b) where the ball will hit the elevator with respect to the location of the man.

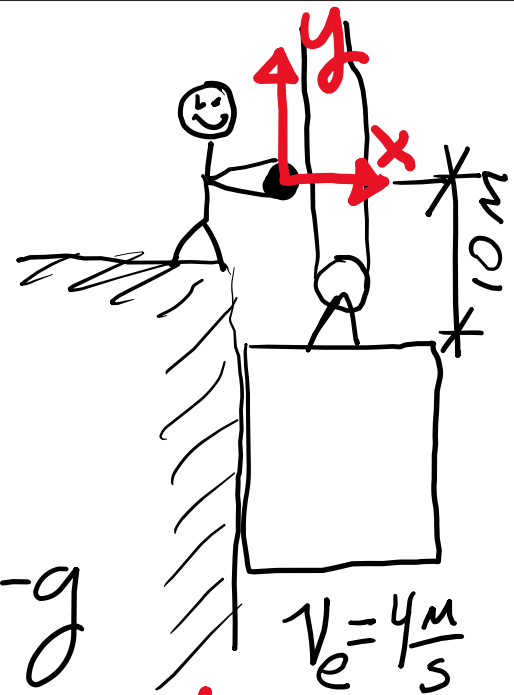
Fig. P11.44

# Notes on problem 11.44:

Man throws ball up with initial velocity of  $v_{bI} = 3 \frac{m}{s}$

$$\text{so } \begin{cases} v_e = 4 \frac{m}{s}, & y_{e0} = -10 \text{ m} \\ v_{b0} = 3 \frac{m}{s}, & y_{b0} = 0 \text{ m} \end{cases} \quad \& \quad a_b = -g$$

Note: Setting coordinate system at initial position of ball system at



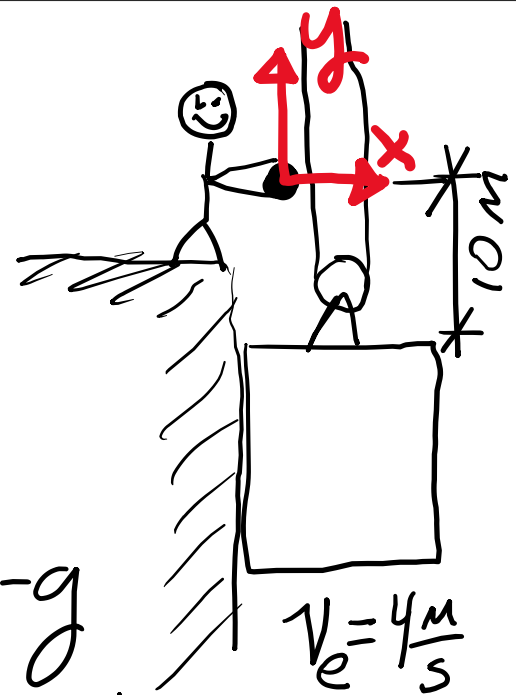


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Can write  $y_b = -\frac{1}{2}gt^2 + v_{b0}t + y_{b0}$  &  $y_e = v_e t + y_{e0}$



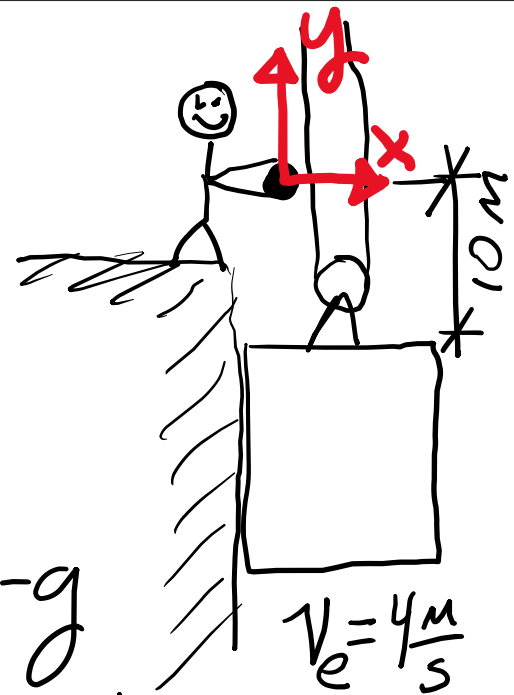
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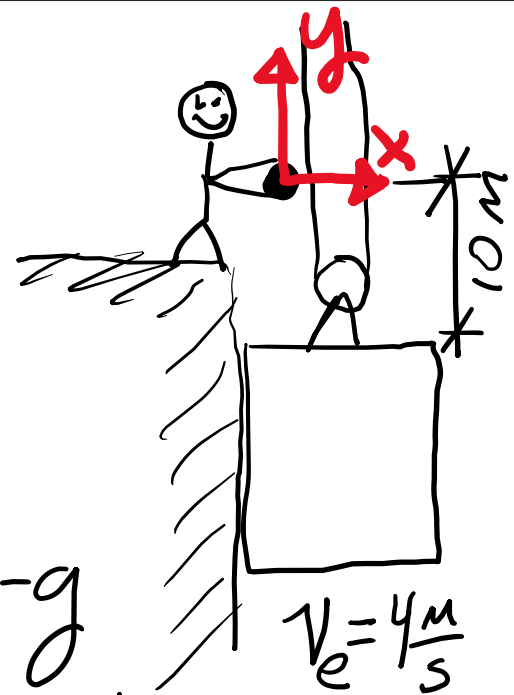
(a) Find  $t$  at hit  $\equiv t_h$ :



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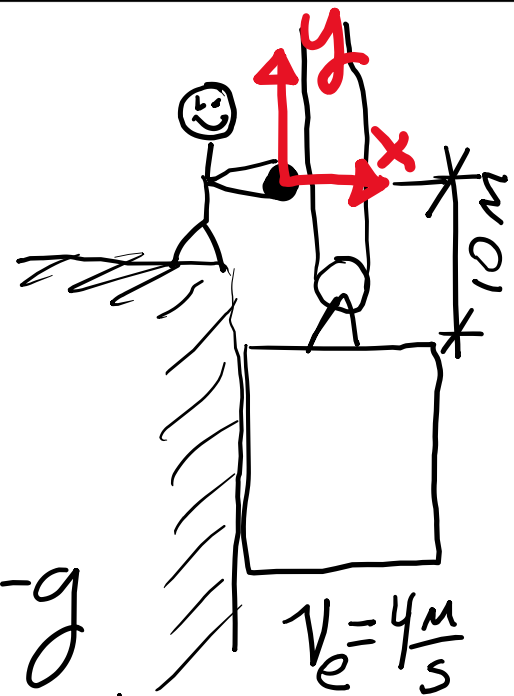
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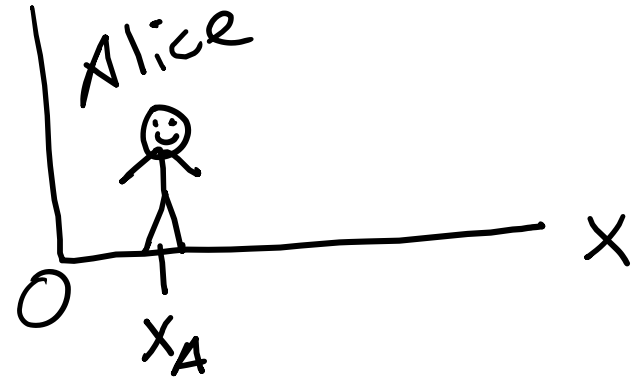
(a) Find  $t$  at hit  $\equiv t_h$ : Note:  $y_{b/e} = 0$

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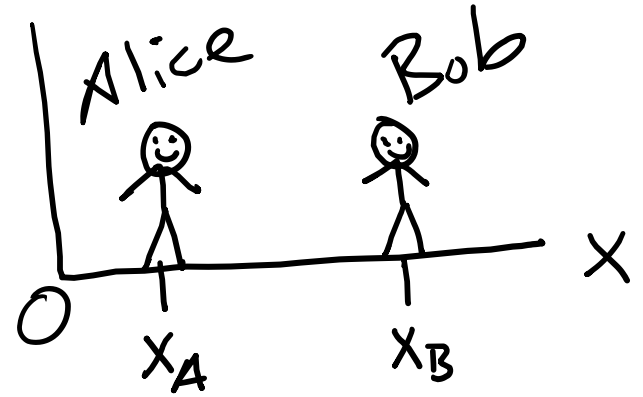
(b) Find  $y_e(t_h)$  or  $y_b(t_h)$ : Since  $y_{b/e} = 0$   
 $\Rightarrow y_b = y_e$

# Relative motion

# Relative motion



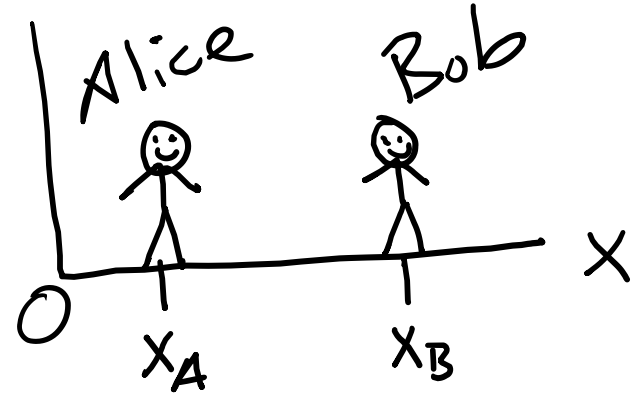
# Relative motion



# Relative motion

position:

$x_{B/A} \equiv$   $x$  of B relative to A

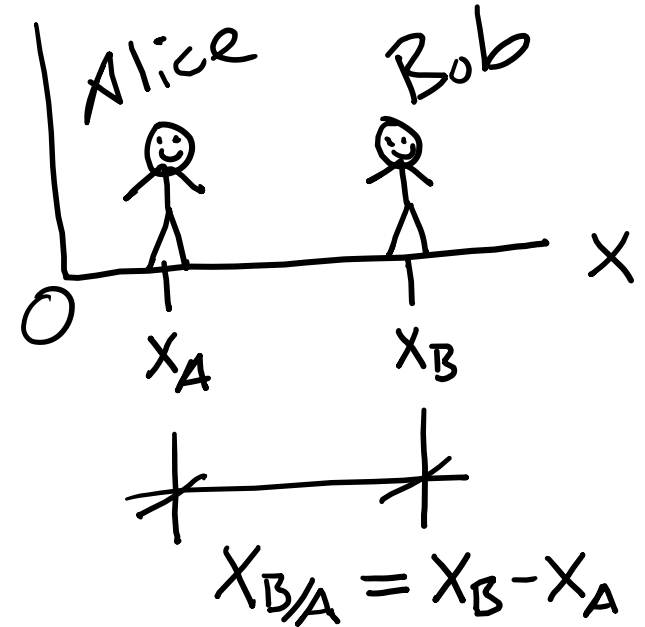


# Relative motion

position:

$X_{B/A} \equiv X$  of B relative to A

$$X_{B/A} \equiv X_B - X_A$$



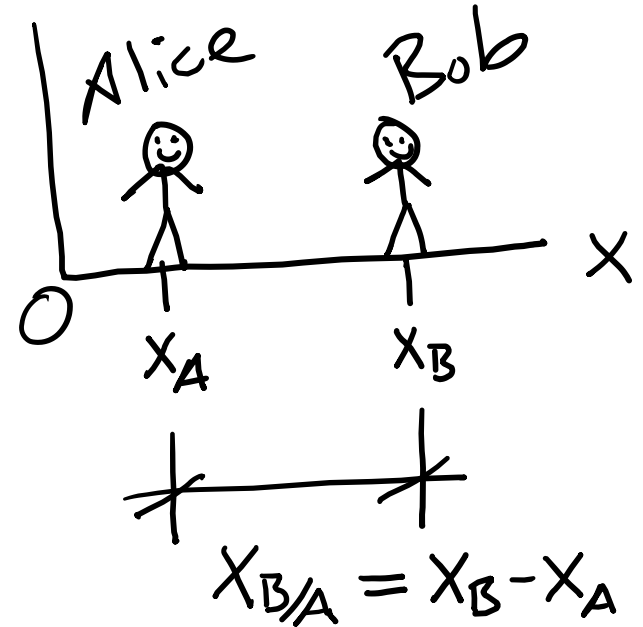
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$X_{B/A}$  is position of Bob  
as measured by Alice



# Relative motion

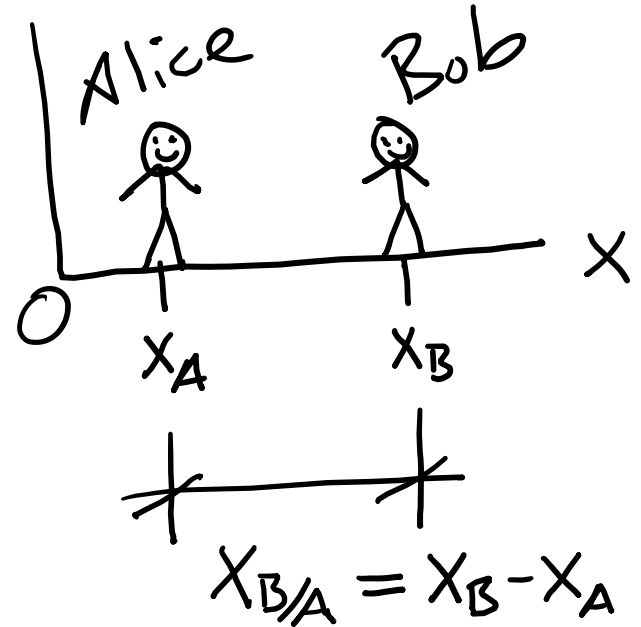
position:

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velocity:  $v_{B/A} = v_B - v_A$



# Relative motion

position:

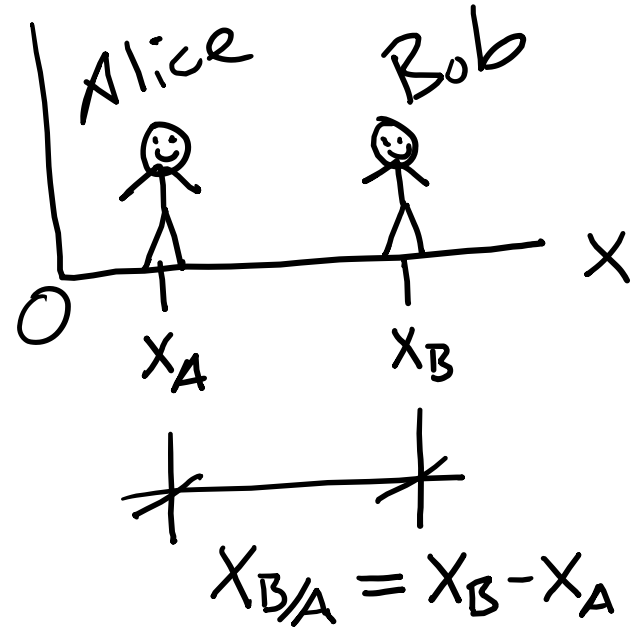
$X_{B/A} \equiv X$  of B relative to A

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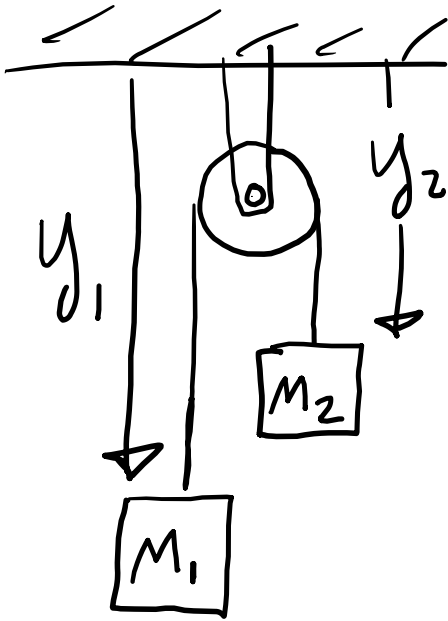
$X_{B/A}$  is position of Bob  
as measured by Alice

velocity:  $v_{B/A} = v_B - v_A$

acceleration:  $a_{B/A} = a_B - a_A$

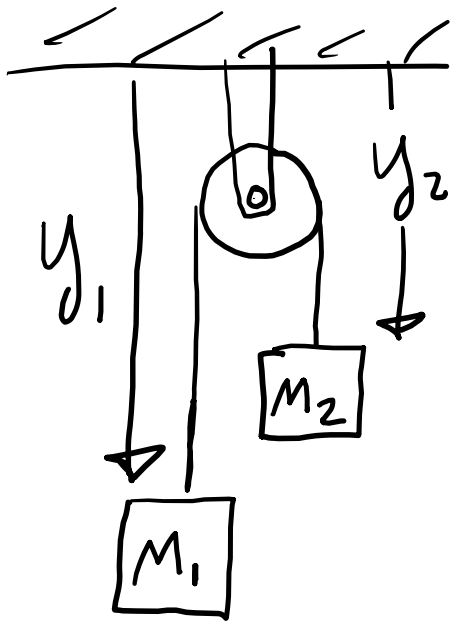


# Pulley problems



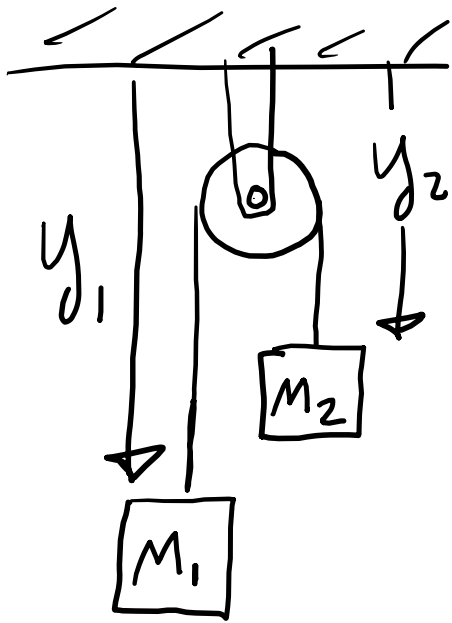
← Super simple example

# Pulley problems



For these problems we tend to want the velocity  $\frac{dy_i}{dt}$  and/or acceleration  $\frac{d^2y_i}{dt^2}$

# Pulley problems

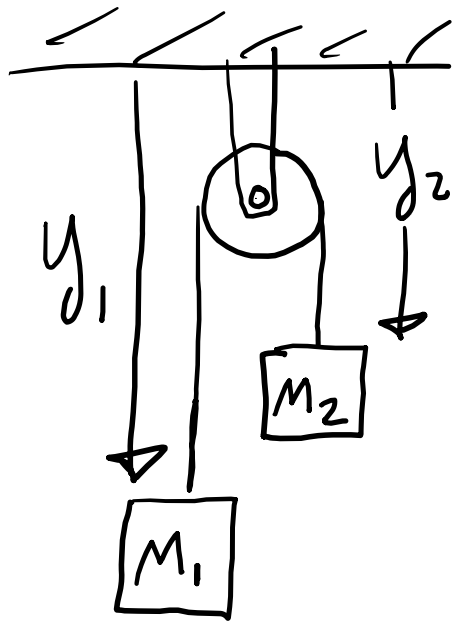


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Note: If length of rope  $\equiv l$

$$\& l = y_1 + y_2 + \text{constant}$$

# Pulley problems



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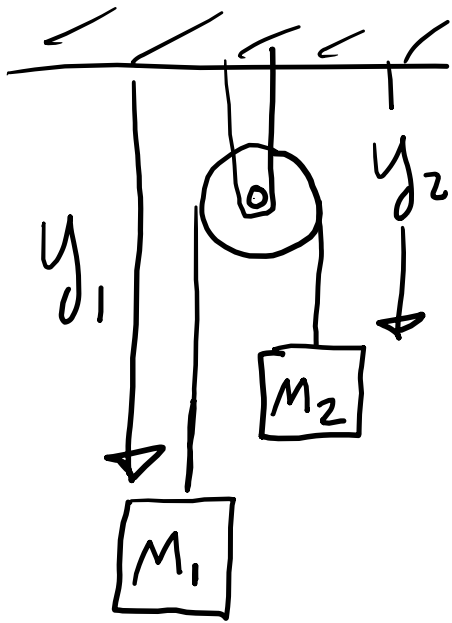
Note: If length of rope  $\equiv l$

$\& l = y_1 + y_2 + \text{constant}$  then

$$\frac{d}{dt} l = \frac{d}{dt} [y_1 + y_2 + \text{constant}] \Rightarrow$$

$$0 = v_1 + v_2$$

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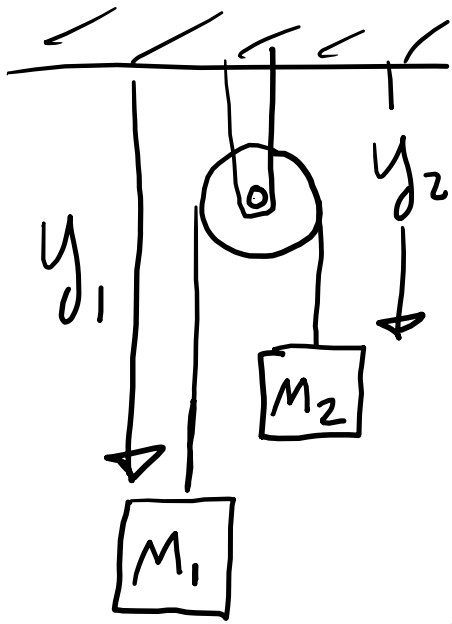
$\& l = y_1 + y_2 + \text{constant}$  then

$$\frac{d}{dt} l = \frac{d}{dt} [y_1 + y_2 + \text{constant}] \Rightarrow$$

$$0 = v_1 + v_2 \Rightarrow v_1 = -v_2$$

As expected 😊

# Pulley problems

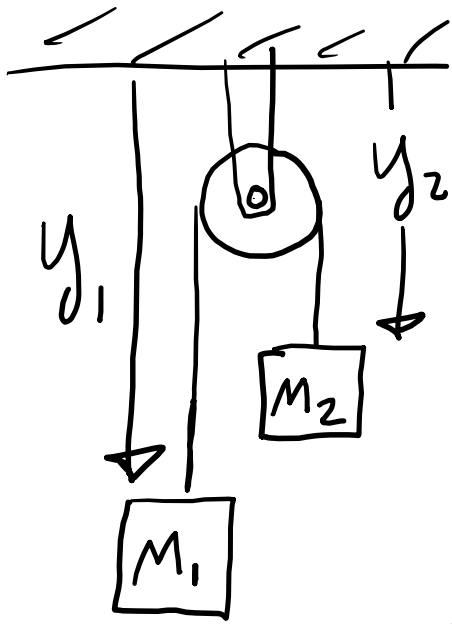


We can relate  $y_1$  &  $y_2$   
to length of rope ( $l$ )  
& take time derivative

to get rid of any  
constant lengths

[like length  $l$  of rope]

# Pulley problems



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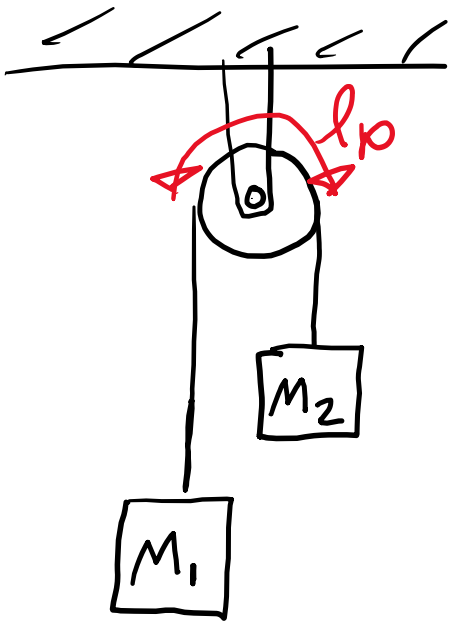
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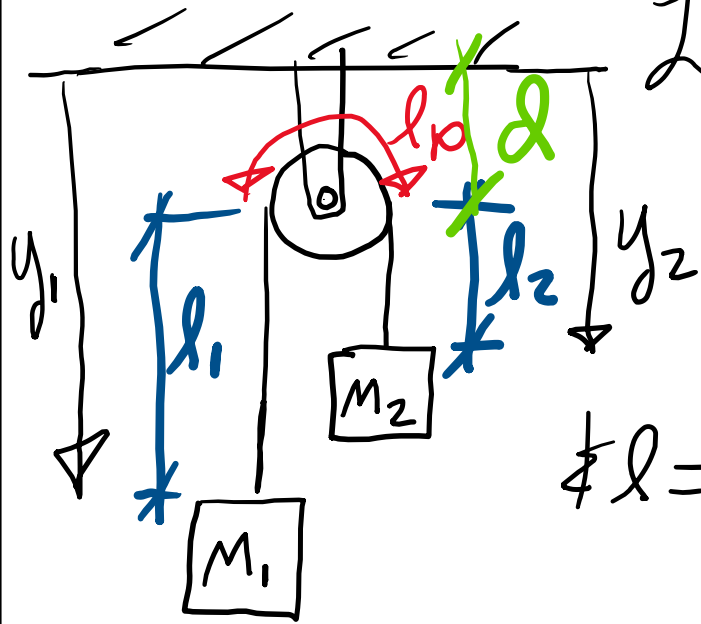
With some practice, we  
can get fairly quick at  
these kind of calculations 😊

# Pulley problems

Let  $l_p \equiv$  length of rope  
along pulley



# Pulley problems

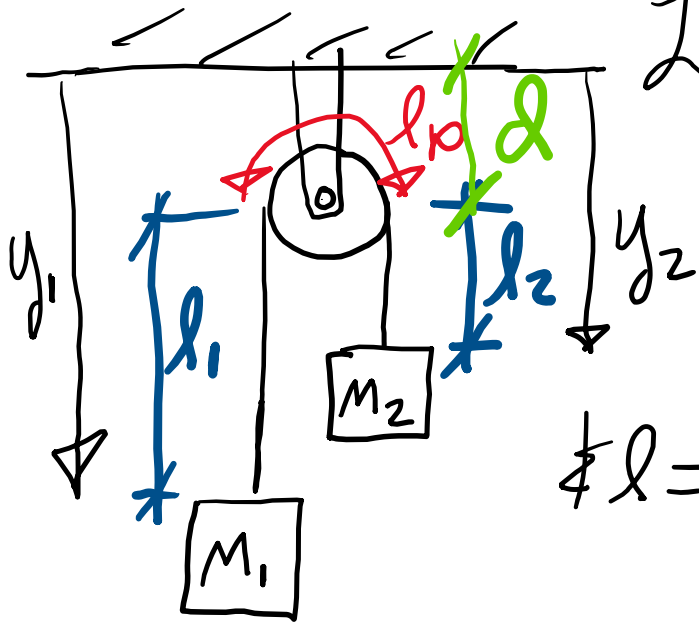


Let  $l_p \equiv$  length of rope  
along pulley

Total length of rope  $\equiv l$

$$l = l_1 + l_p + l_2$$

# Pulley problems

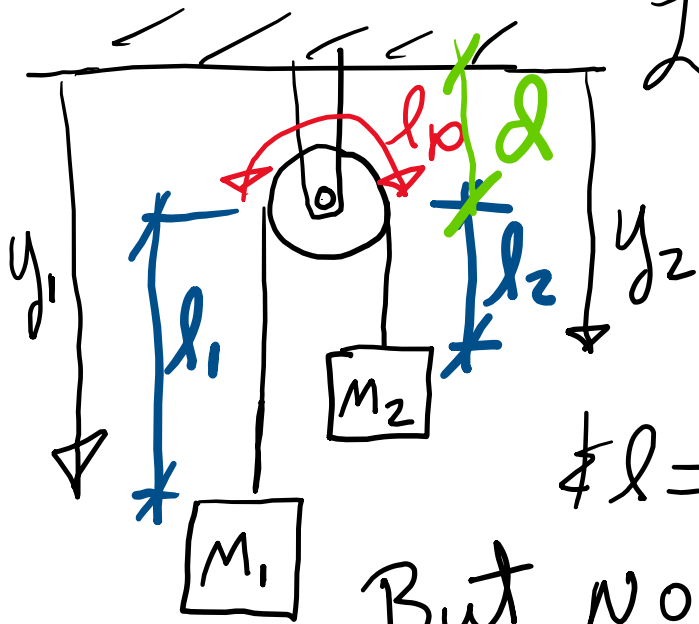


Let  $l_p \equiv$  length of rope  
along pulley

Total length of rope  $\equiv l$

$$l = l_1 + l_p + l_2 \quad \underline{\underline{\text{or}}} \quad l = (y_1 - d) + l_p + (y_2 - d)$$

# Pulley problems



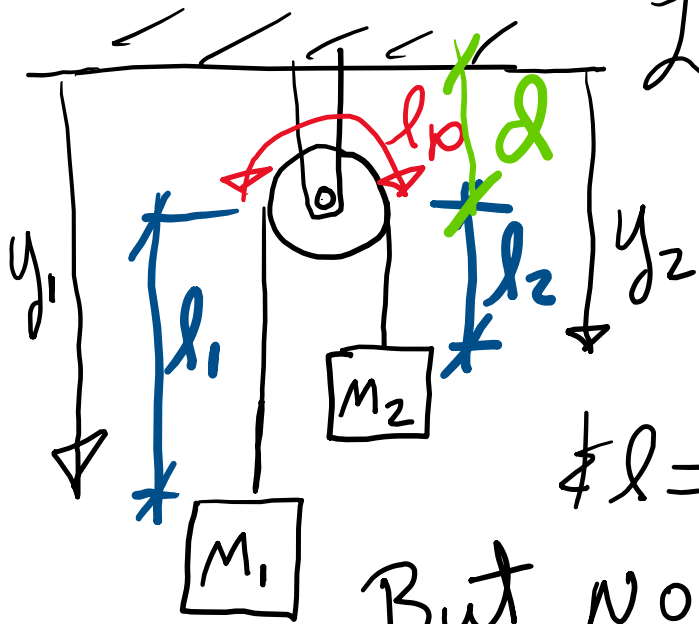
Let  $l_p \equiv$  length of rope along pulley

Total length of rope  $\equiv l$

$$l = l_1 + l_p + l_2 \quad \text{or} \quad l = (y_1 - d) + l_p + (y_2 - d)$$

But no matter how  $m_1$  &  $m_2$  move  
 $l_p$  &  $d$  do not change

# Pulley problems



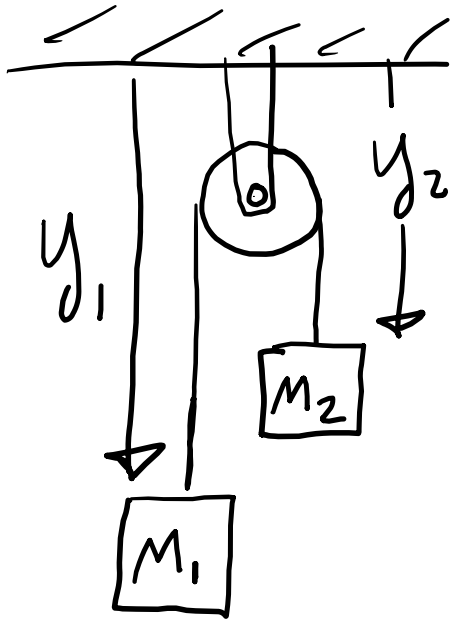
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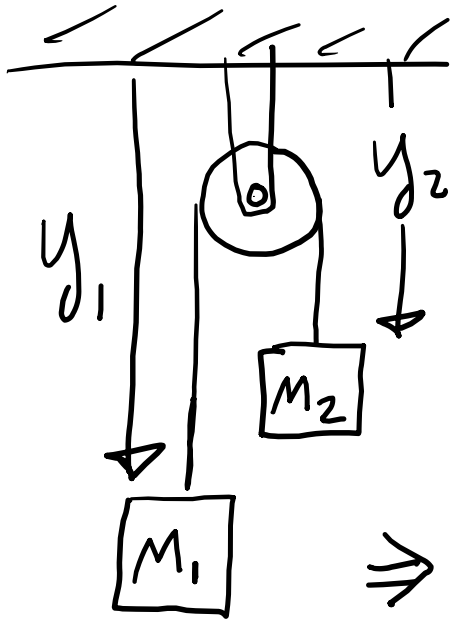
But no matter how  $m_1$  &  $m_2$  move  
 $l_p$  &  $d$  do not change. Could  
simply write  $y_1 + y_2 = \text{Constant}$  😊

# Pulley problems



We simply neglect those parts of rope that are constant.

# Pulley problems

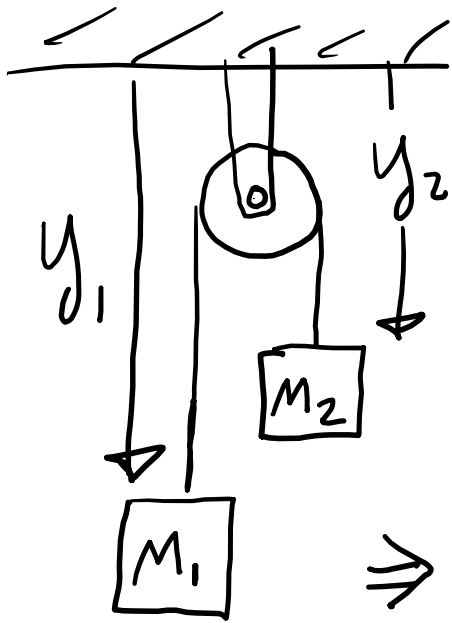


We simply neglect those parts of rope that are constant. As stated

before  $\frac{d}{dt} [y_1 + y_2] = \frac{d}{dt} [\text{const}]$

$$\Rightarrow v_1 = -v_2$$

# Pulley problems



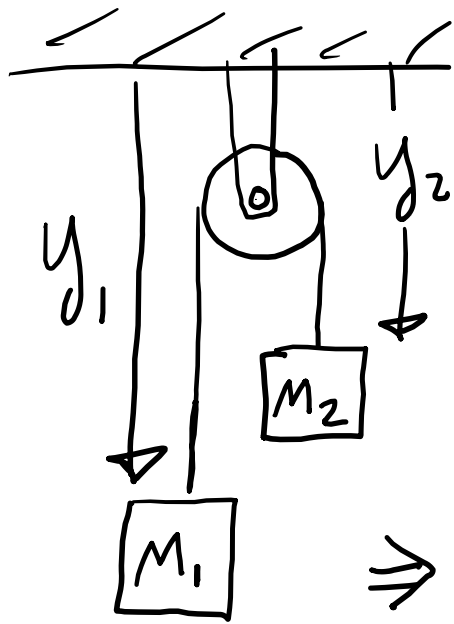
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But we already knew that by just looking at the picture

# Pulley problems



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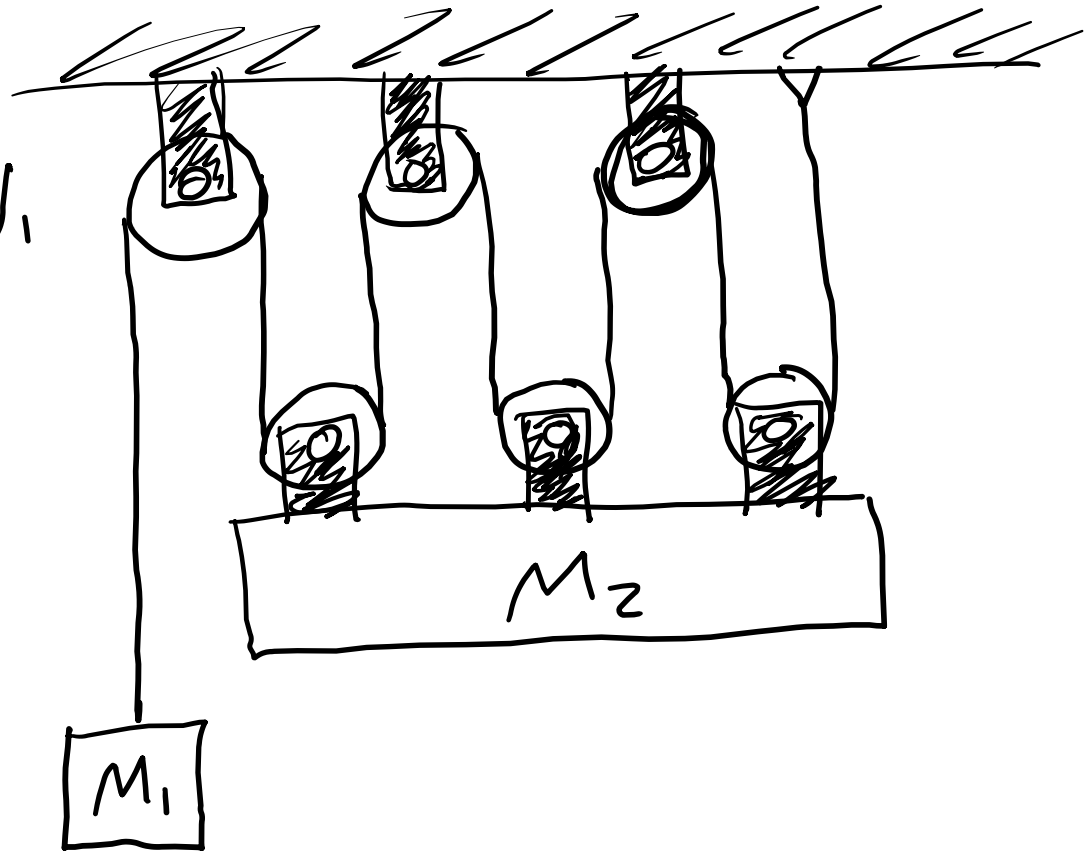
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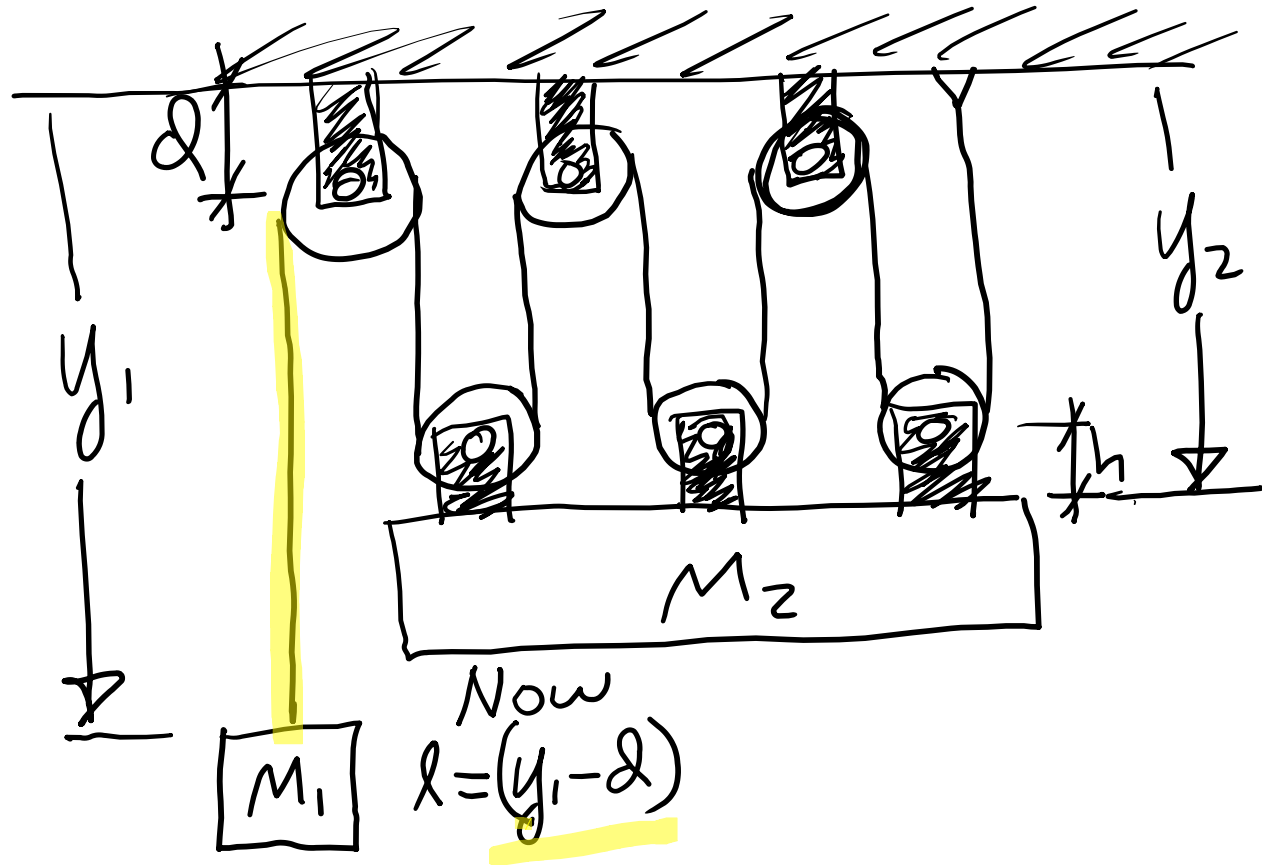
More difficult problem  $\rightarrow$

# Another example

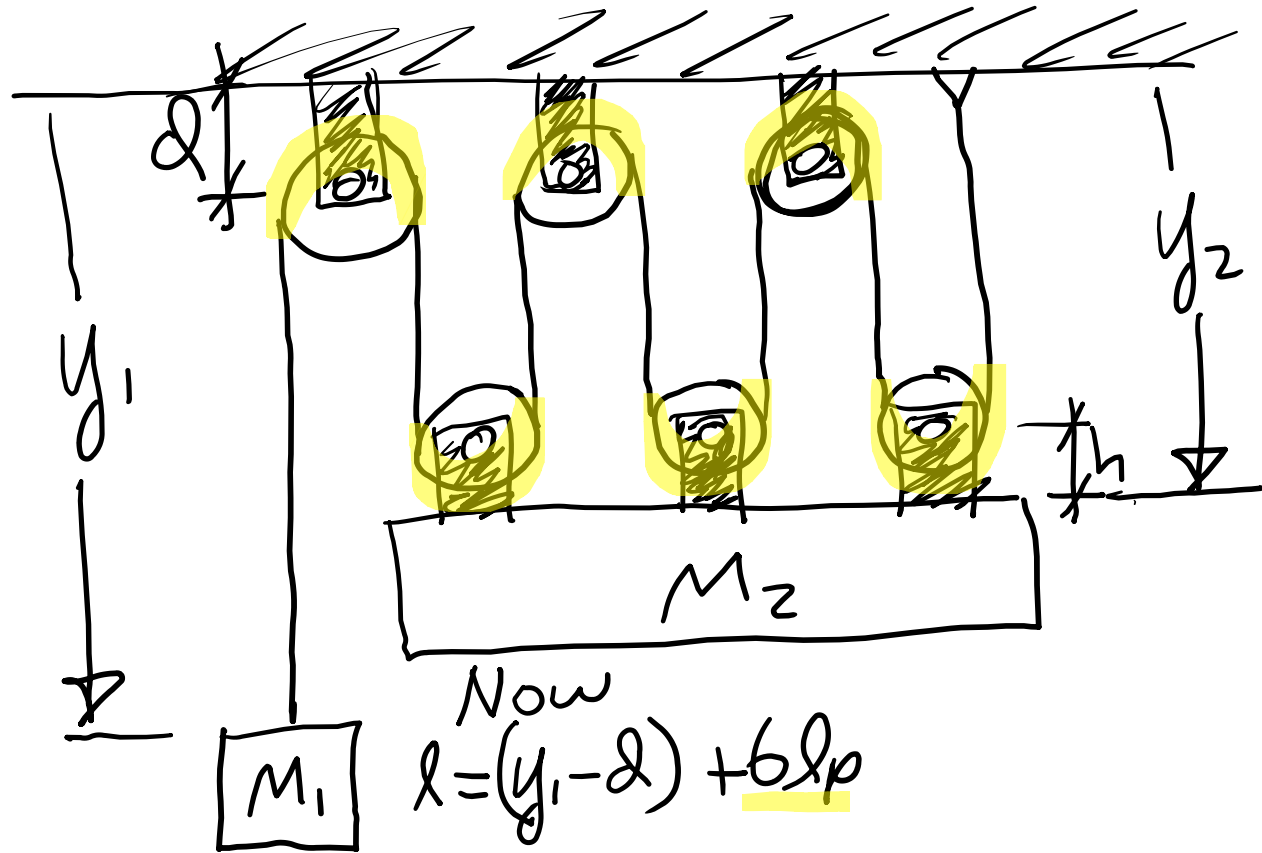
Find  $v_2$  given  $v_1$



# Another example

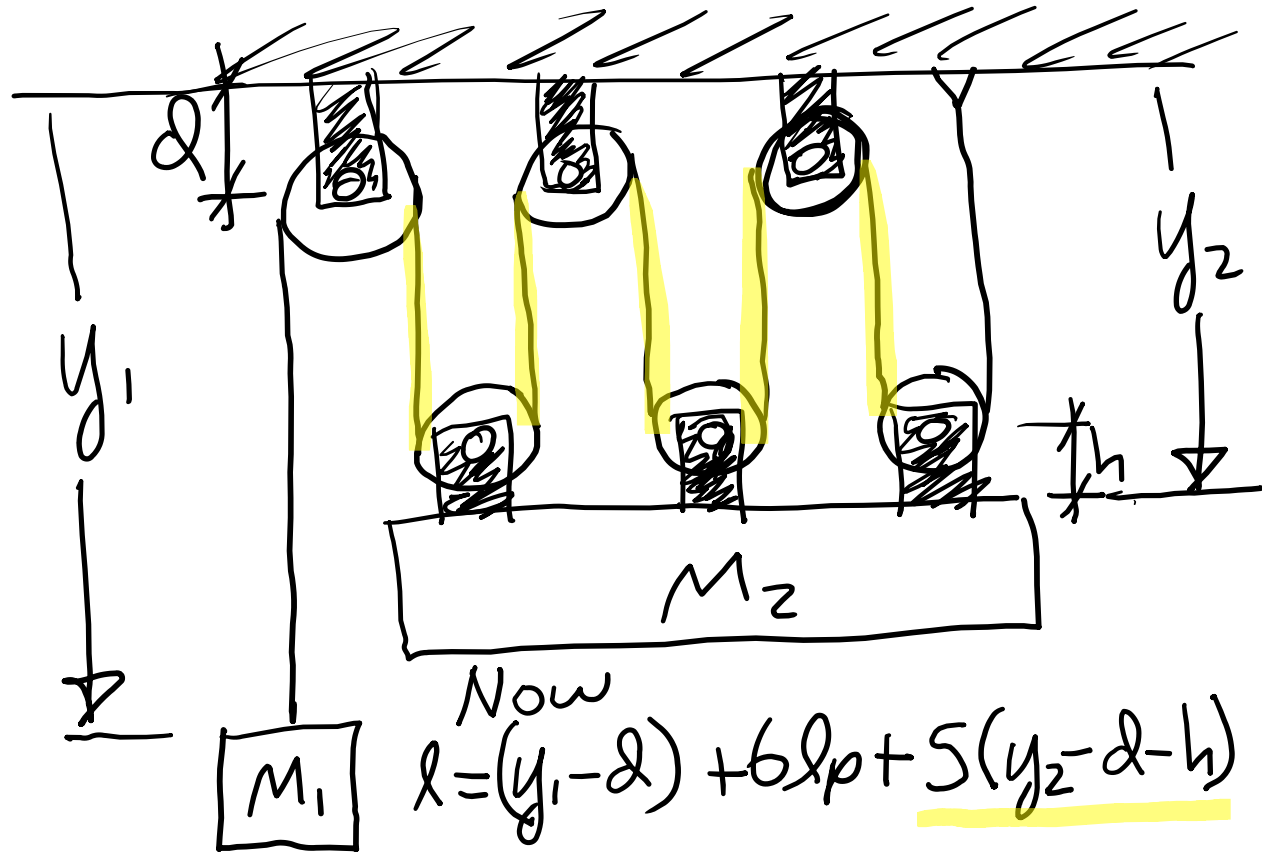


# Another example

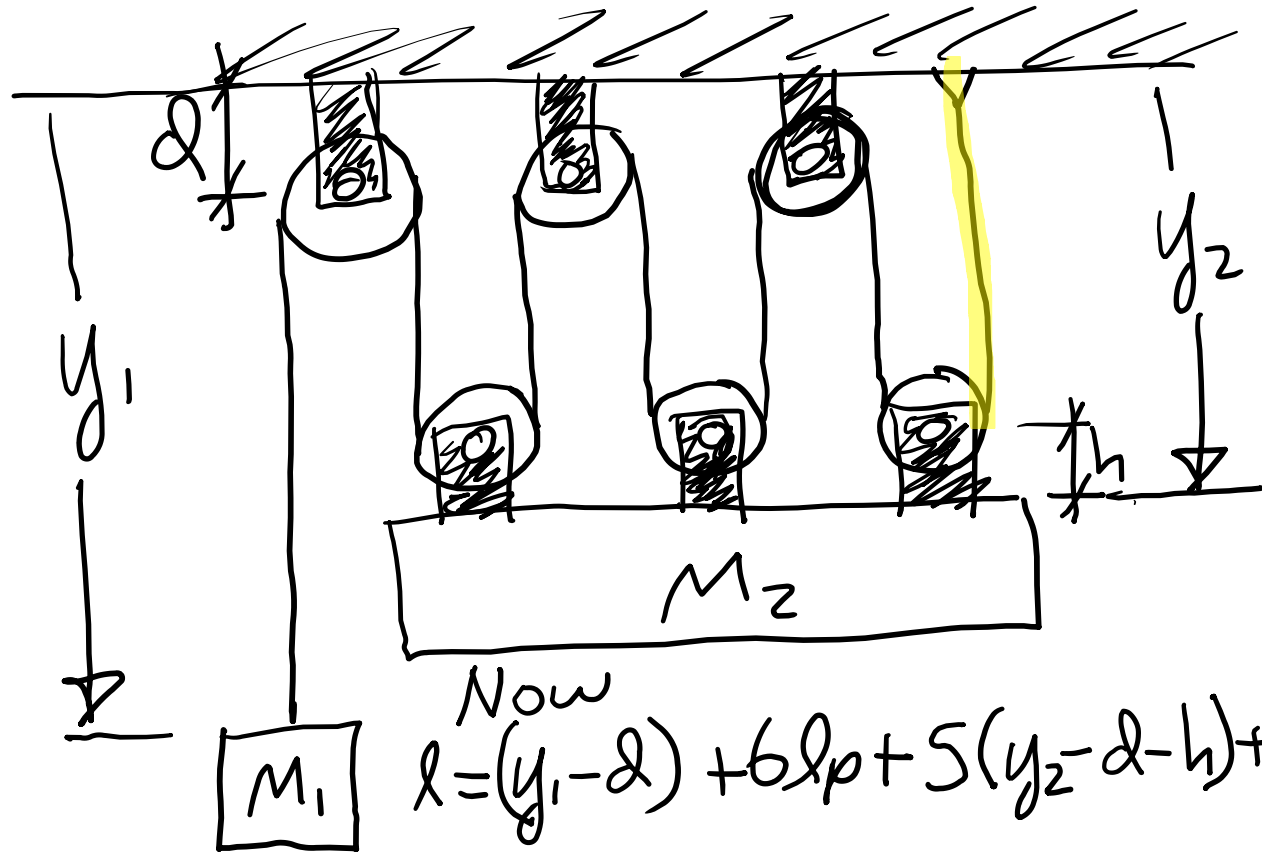


Now  
 $l = (y_1 - d) + 6lp$

# Another example



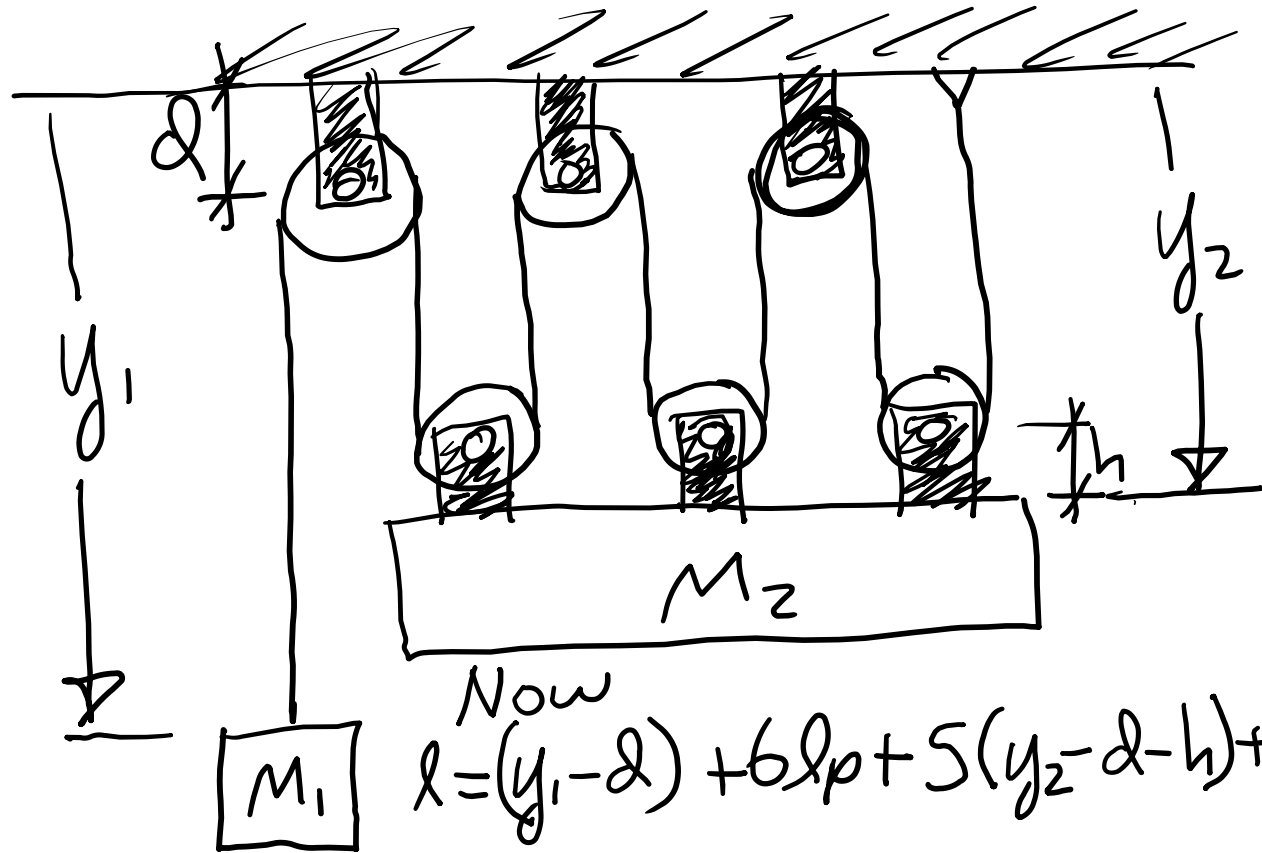
# Another example



Now

$$l = (y_1 - d) + 6lp + 5(y_2 - d - h) + (y_2 - h)$$

# Another example

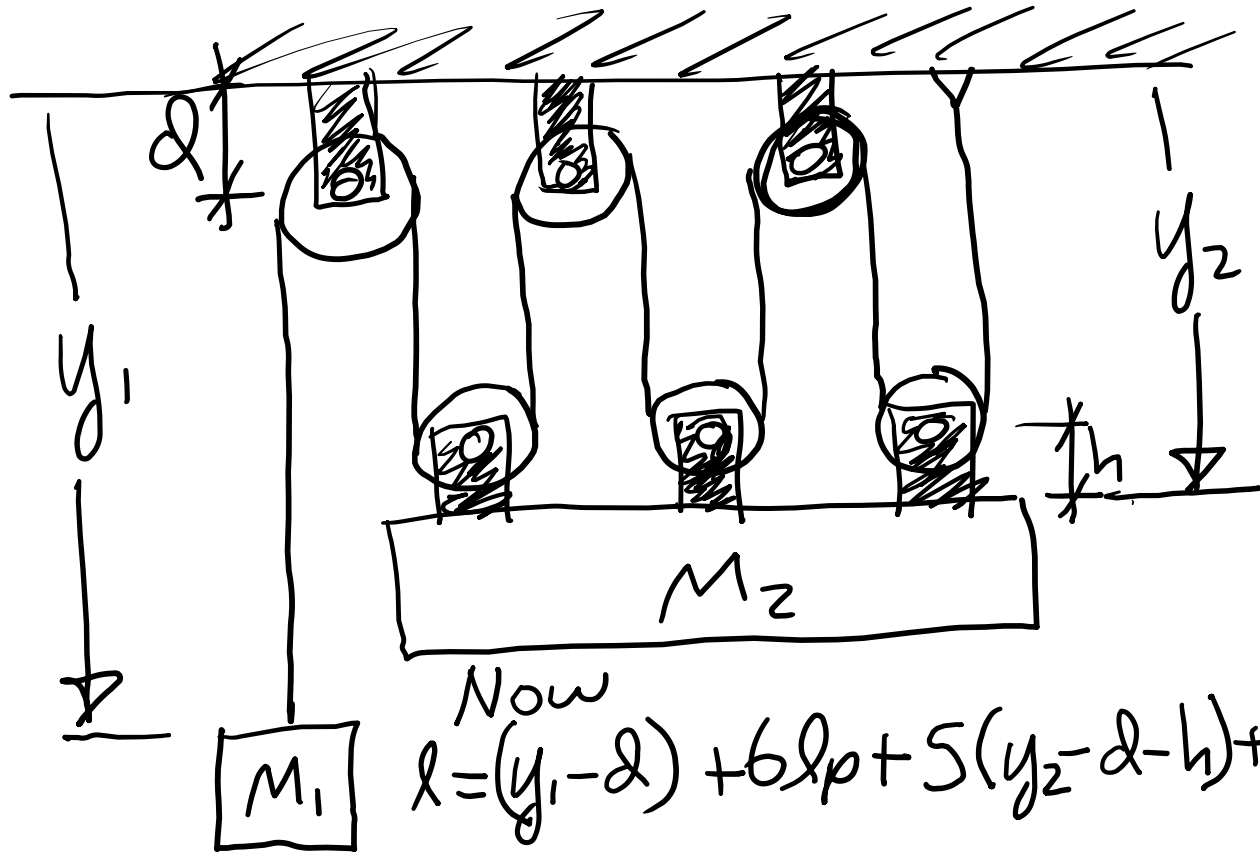


Now

$$l = (y_1 - d) + 6lp + 5(y_2 - d - h) + (y_2 - h)$$

OR simply  $y_1 + 6y_2 = \text{const}$

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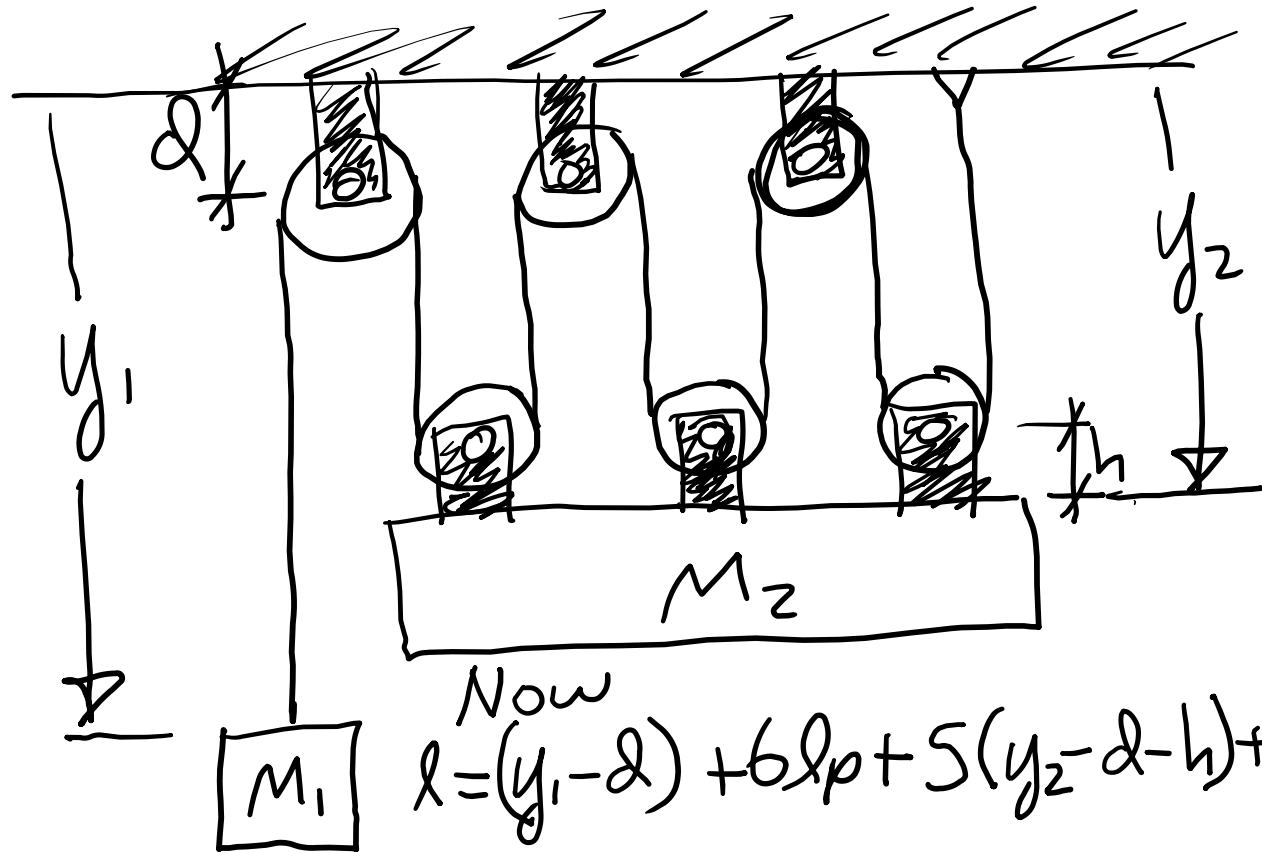
Now

$$l = (y_1 - d) + 6d + 5(y_2 - d - h) + (y_2 - h)$$

OR simply  $y_1 + 6y_2 = \text{const} \Rightarrow$

$$v_1 + 6v_2 = 0$$

# Another example

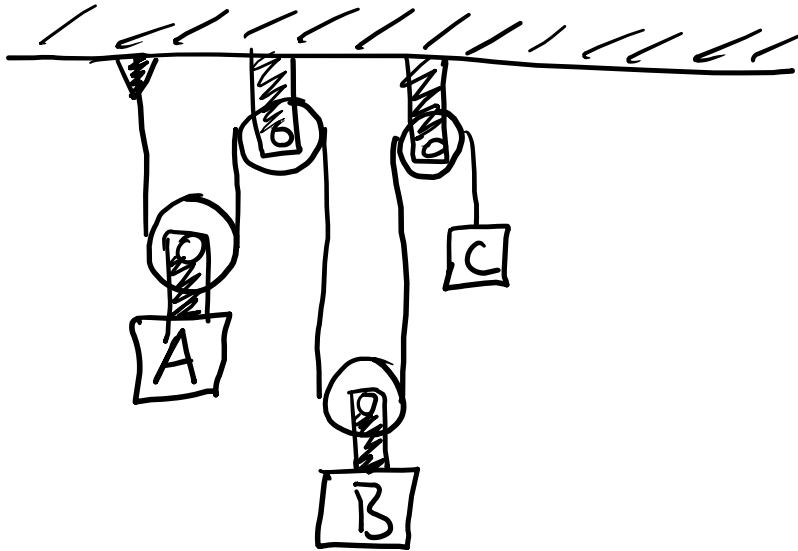


$$l = (y_1 - d) + 6d + 5(y_2 - d - h) + (y_2 - h)$$

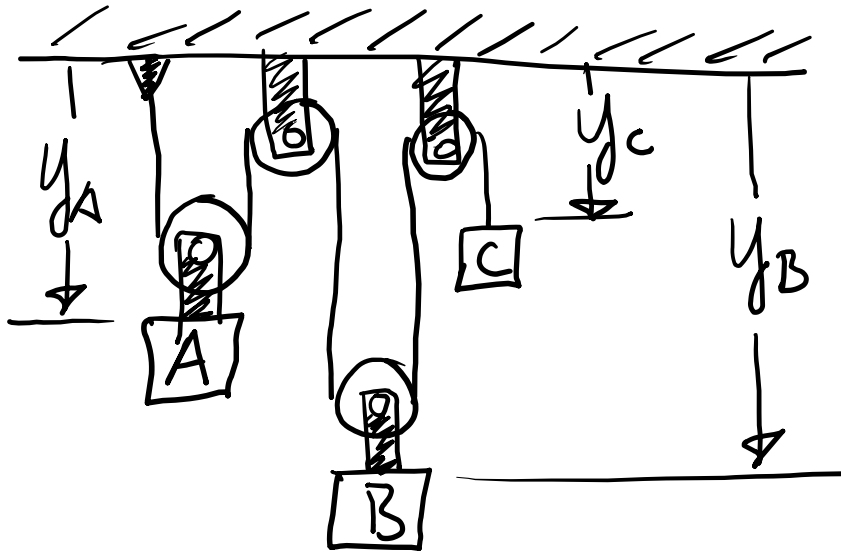
OR simply  $y_1 + 6y_2 = \text{const} \Rightarrow$

$$v_1 + 6v_2 = 0 \Rightarrow v_2 = -\frac{v_1}{6}$$

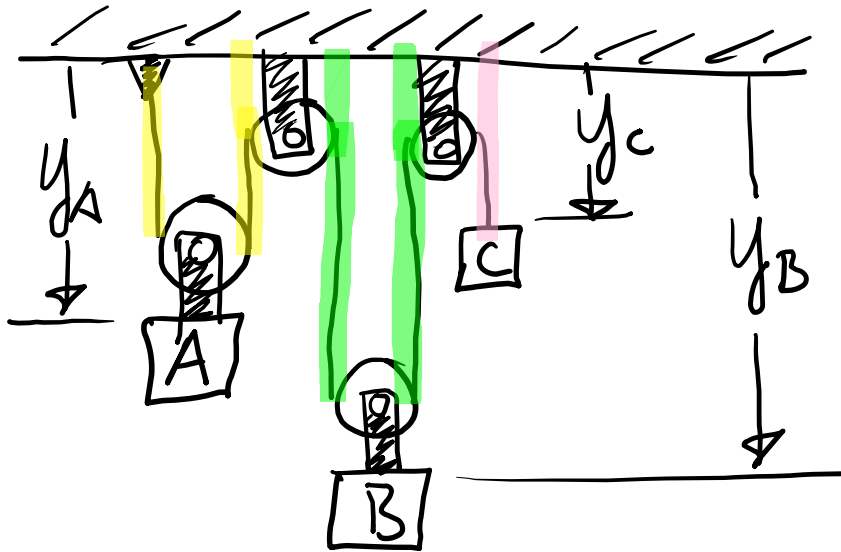
# One more example



# One more example



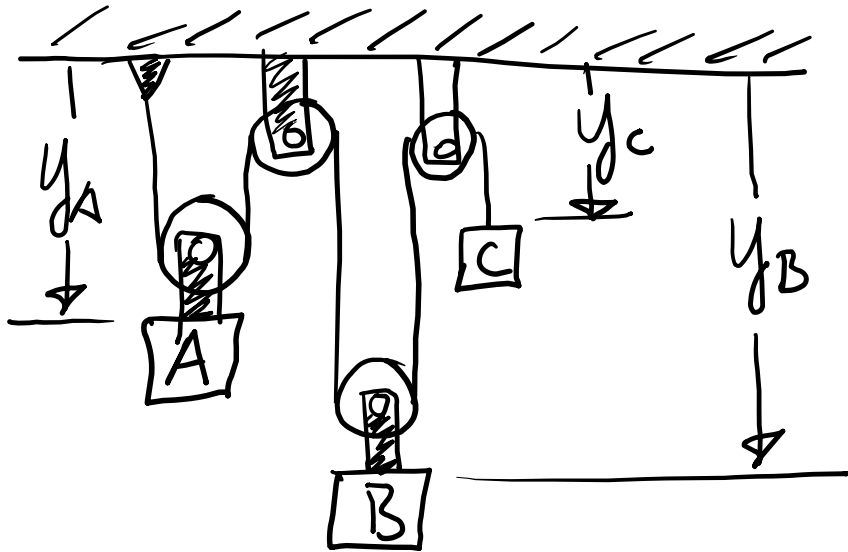
# One more example



Do

$$2y_A + 2y_B + y_C = \text{Const}$$

# One more example

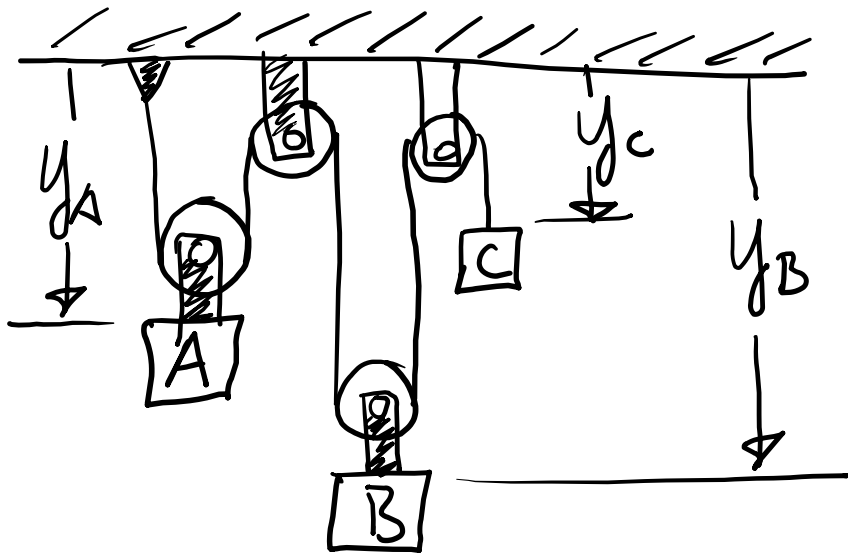


Do

$$2y_A + 2y_B + y_C = \text{const}$$

$$\Rightarrow 2V_A + 2V_B + V_C = 0$$

# One more example



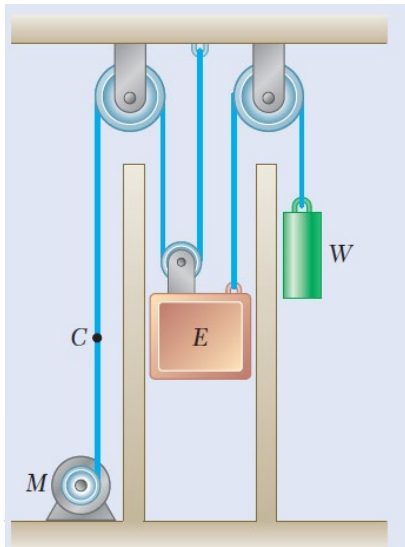
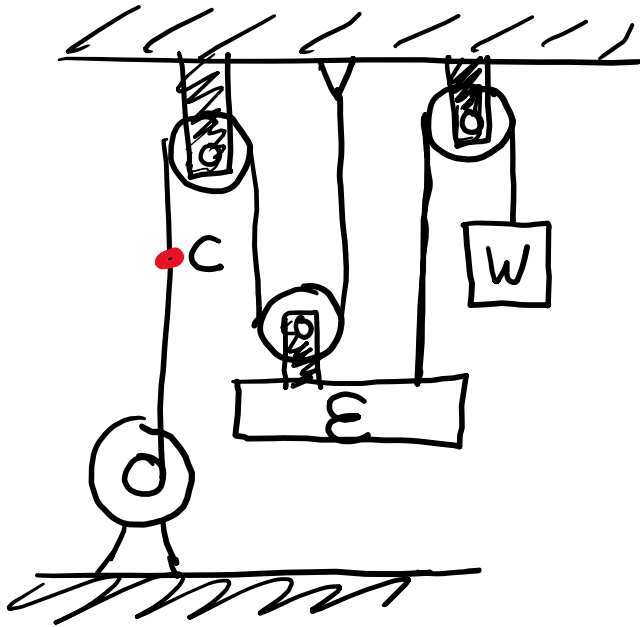
Do

$$2y_A + 2y_B + y_C = \text{Const}$$

$$\Rightarrow 2v_A + 2v_B + v_C = 0$$

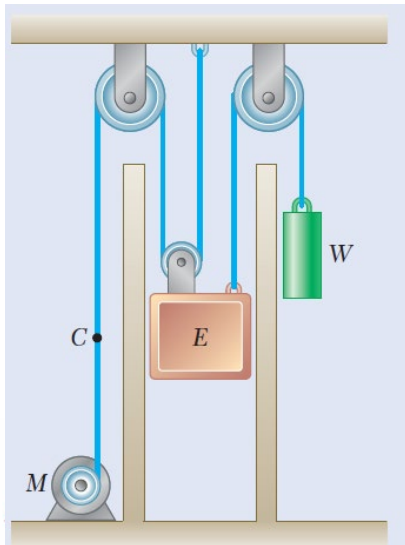
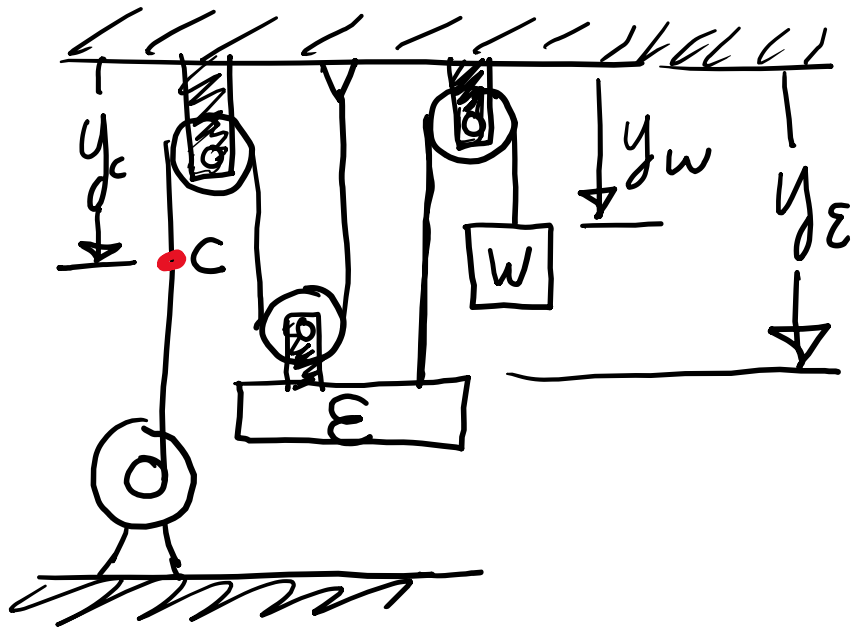
$$\S 2a_A + 2a_B + a_C = 0$$

# Notes on problems: 11.47



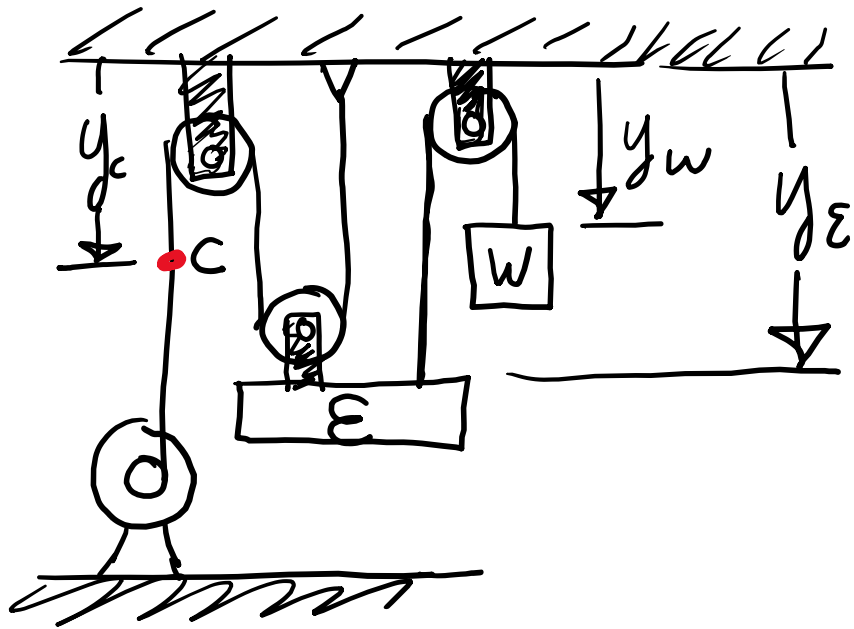
**11.47** The elevator  $E$  shown in the figure moves downward with a constant velocity of 4 m/s. Determine (a) the velocity of the cable  $C$ , (b) the velocity of the counterweight  $W$ , (c) the relative velocity of the cable  $C$  with respect to the elevator, (d) the relative velocity of the counterweight  $W$  with respect to the elevator.

# Notes on problems: 11.47

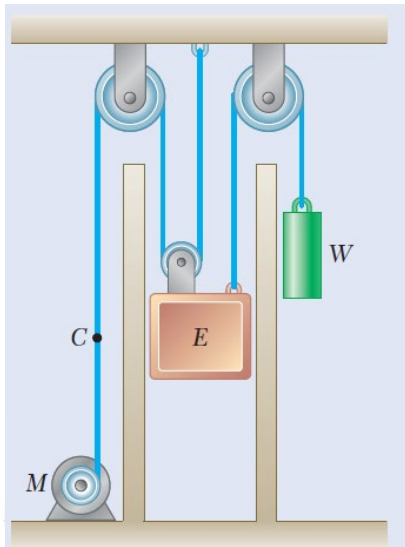


**11.47** The elevator  $E$  shown in the figure moves downward with a constant velocity of 4 m/s. Determine (a) the velocity of the cable  $C$ , (b) the velocity of the counterweight  $W$ , (c) the relative velocity of the cable  $C$  with respect to the elevator, (d) the relative velocity of the counterweight  $W$  with respect to the elevator.

# Notes on problems: 11.47

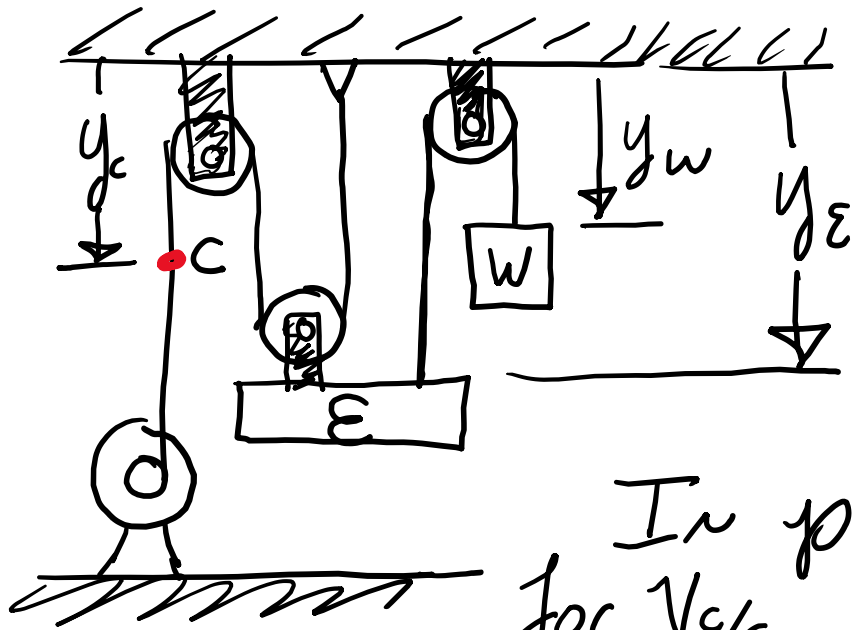


Notice the two ropes  $\Rightarrow$  two equations



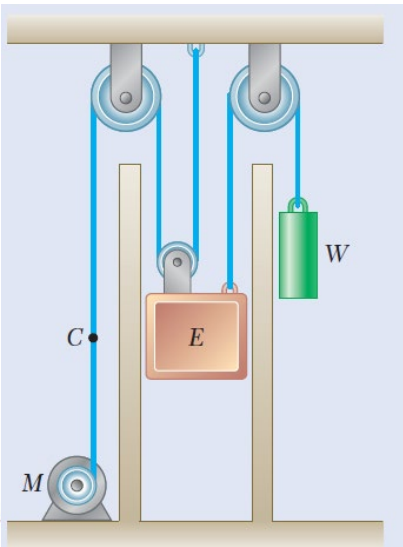
**11.47** The elevator  $E$  shown in the figure moves downward with a constant velocity of 4 m/s. Determine (a) the velocity of the cable  $C$ , (b) the velocity of the counterweight  $W$ , (c) the relative velocity of the cable  $C$  with respect to the elevator, (d) the relative velocity of the counterweight  $W$  with respect to the elevator.

# Notes on problems: 11.47



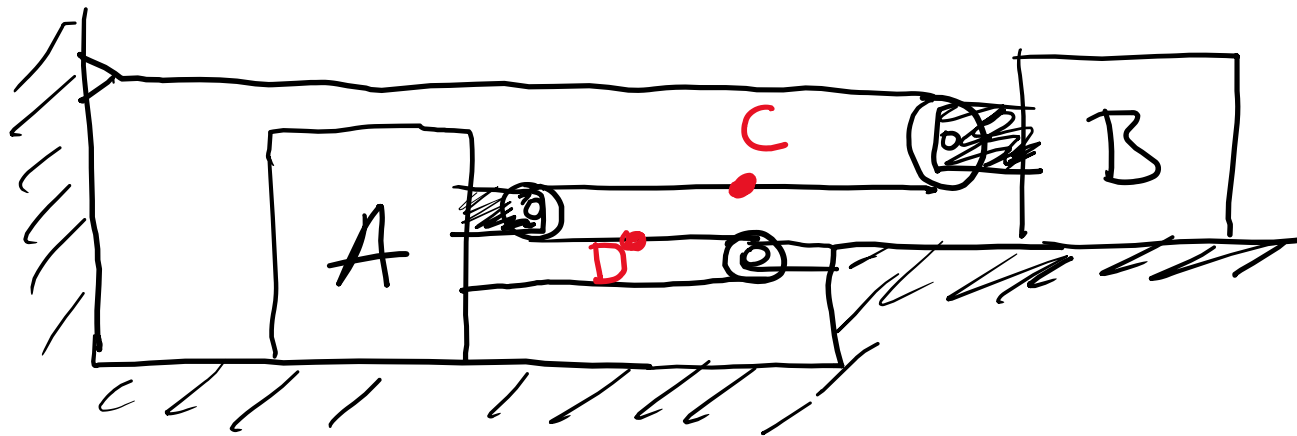
Notice the two ropes  $\Rightarrow$  two equations

In part (c) you are asked for  $v_{C/E}$ . Make sure to include direction in calculation  $\downarrow \downarrow$



- 11.47** The elevator  $E$  shown in the figure moves downward with a constant velocity of 4 m/s. Determine (a) the velocity of the cable  $C$ , (b) the velocity of the counterweight  $W$ , (c) the relative velocity of the cable  $C$  with respect to the elevator, (d) the relative velocity of the counterweight  $W$  with respect to the elevator.

# Notes on problem 11.51



Given  
 $v_B = 300 \frac{\text{mm}}{\text{s}}$

**11.51** Slider block *B* moves to the right with a constant velocity of 300 mm/s. Determine (a) the velocity of slider block *A*, (b) the velocity of portion *C* of the cable, (c) the velocity of portion *D* of the cable, (d) the relative velocity of portion *C* of the cable with respect to slider block *A*.

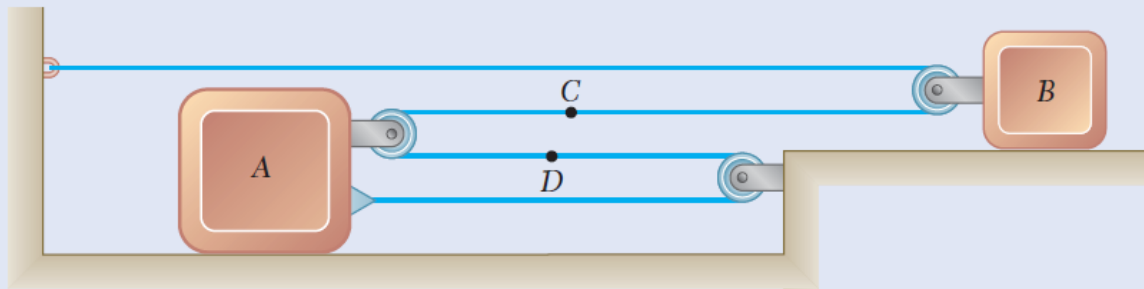
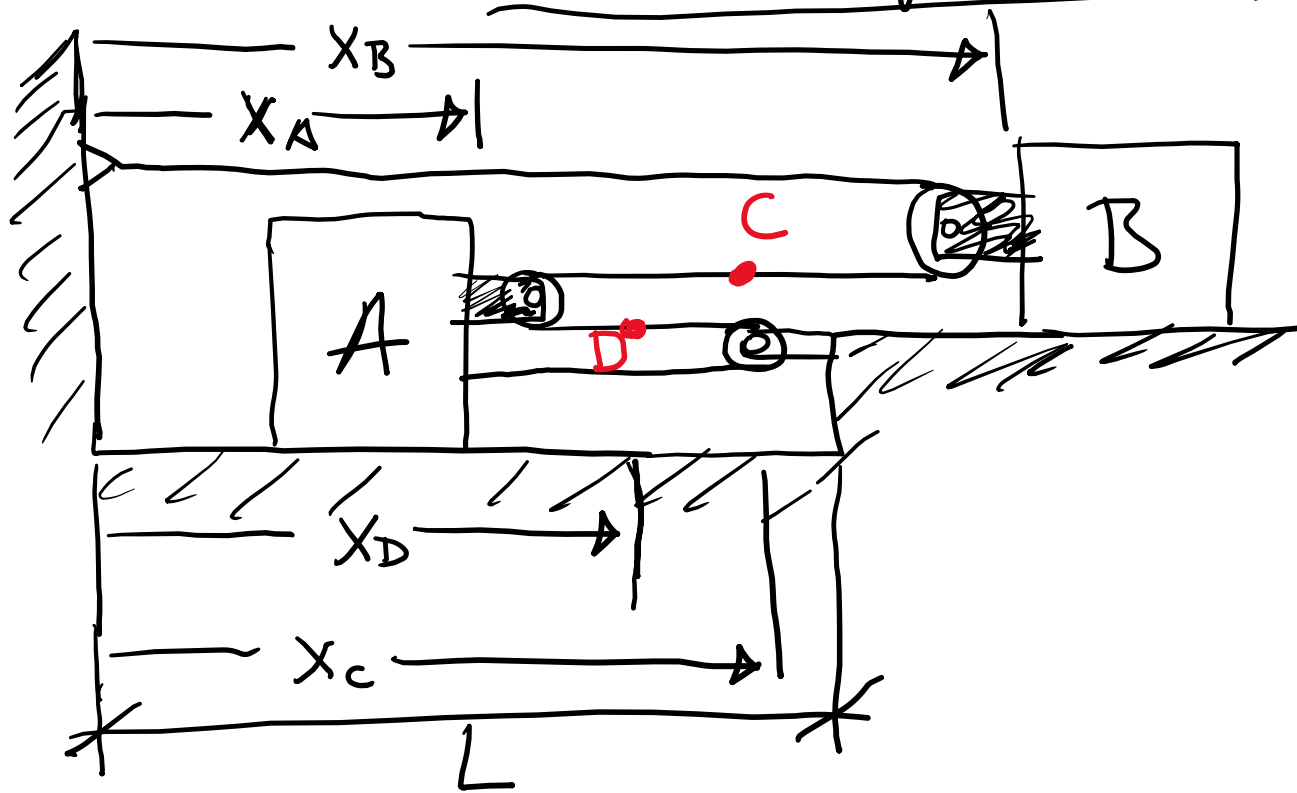


Fig. P11.51 and P11.52

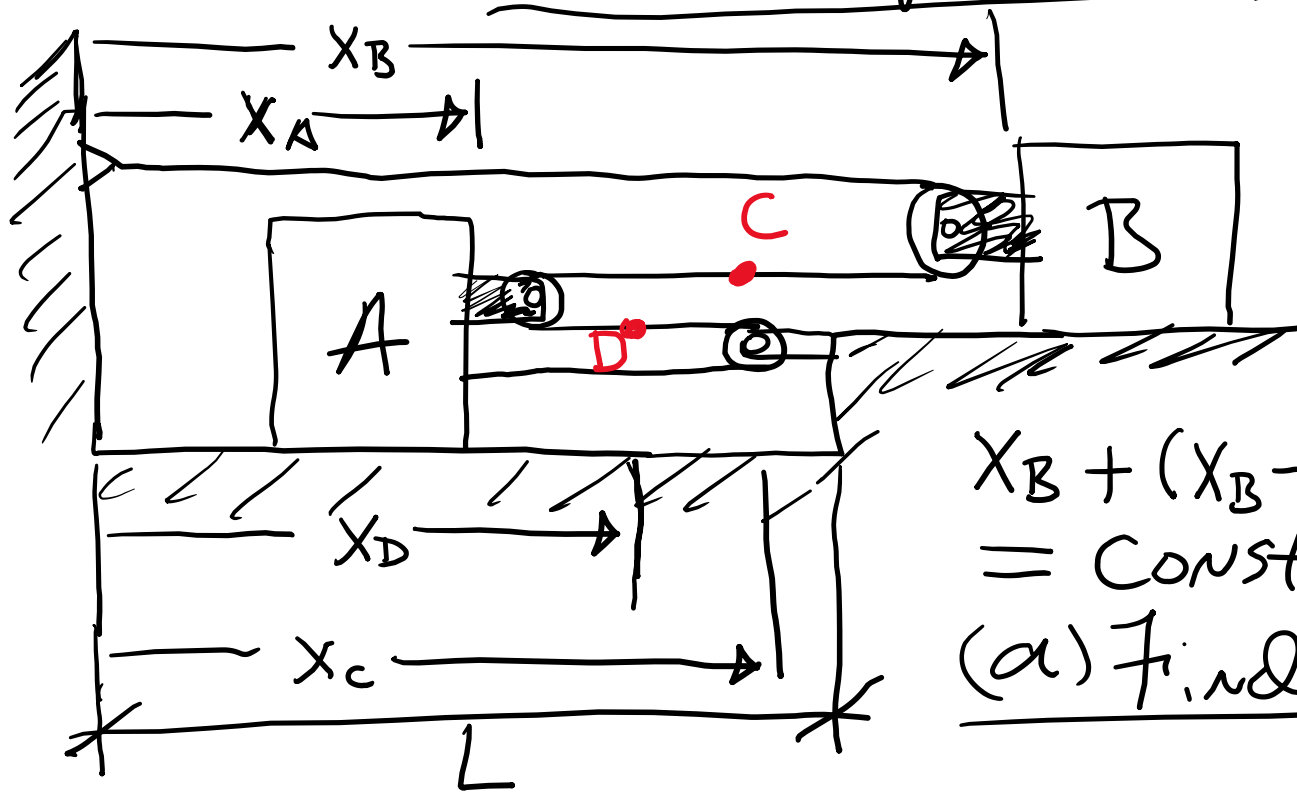
# Notes on problem 11.51



Given  
 $v_B = 300 \frac{\text{mm}}{\text{s}}$



# Notes on problem 11.51



Given  
 $v_B = 300 \frac{\text{mm}}{\text{s}}$

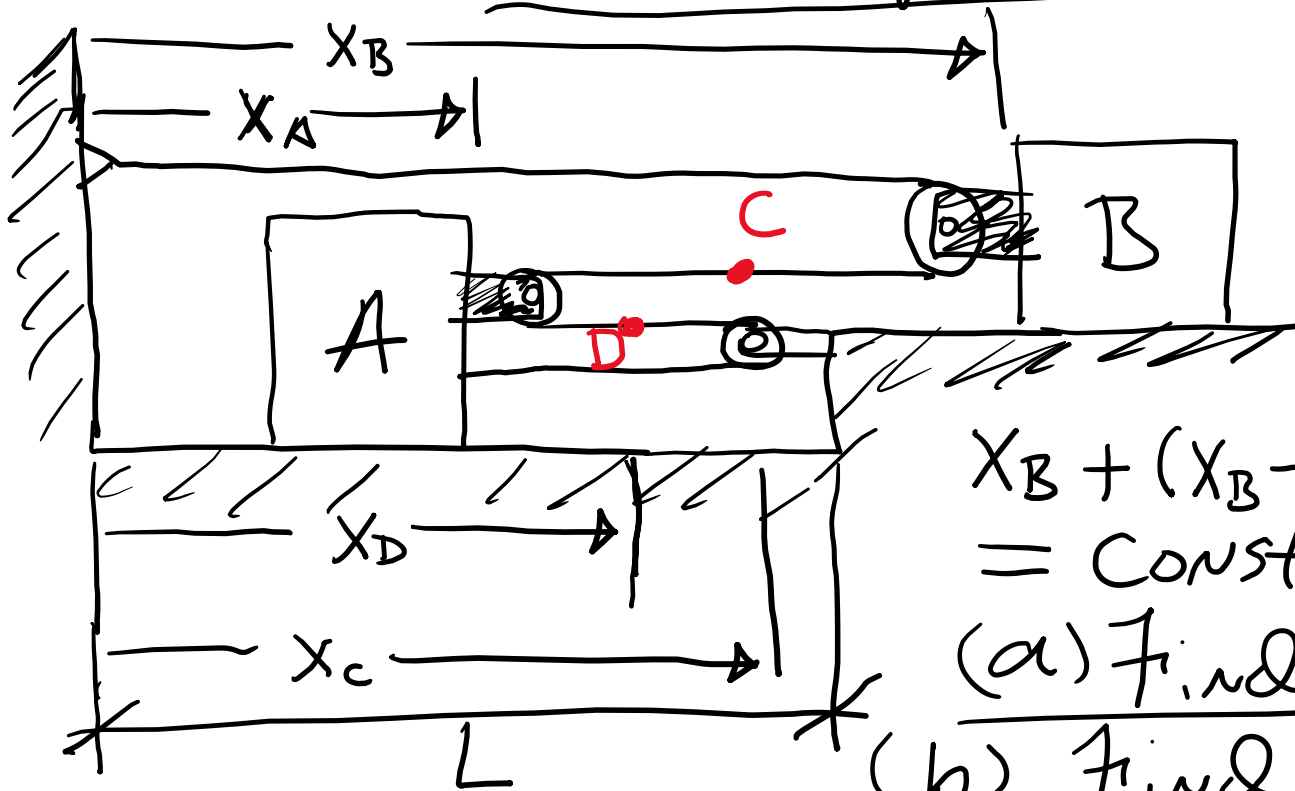
(a)

$$x_B + (x_B - x_A) + (L - x_A)z = \text{constant}$$

(a) Find  $v_A$ : just

take derivatives.

# Notes on problem 11.51



Given

$$v_B = 300 \frac{\text{mm}}{\text{s}}$$

(a)

$$X_B + (X_B - X_A) + (L - X_A) = \text{Constant}$$

(a) Find  $v_A$ : just

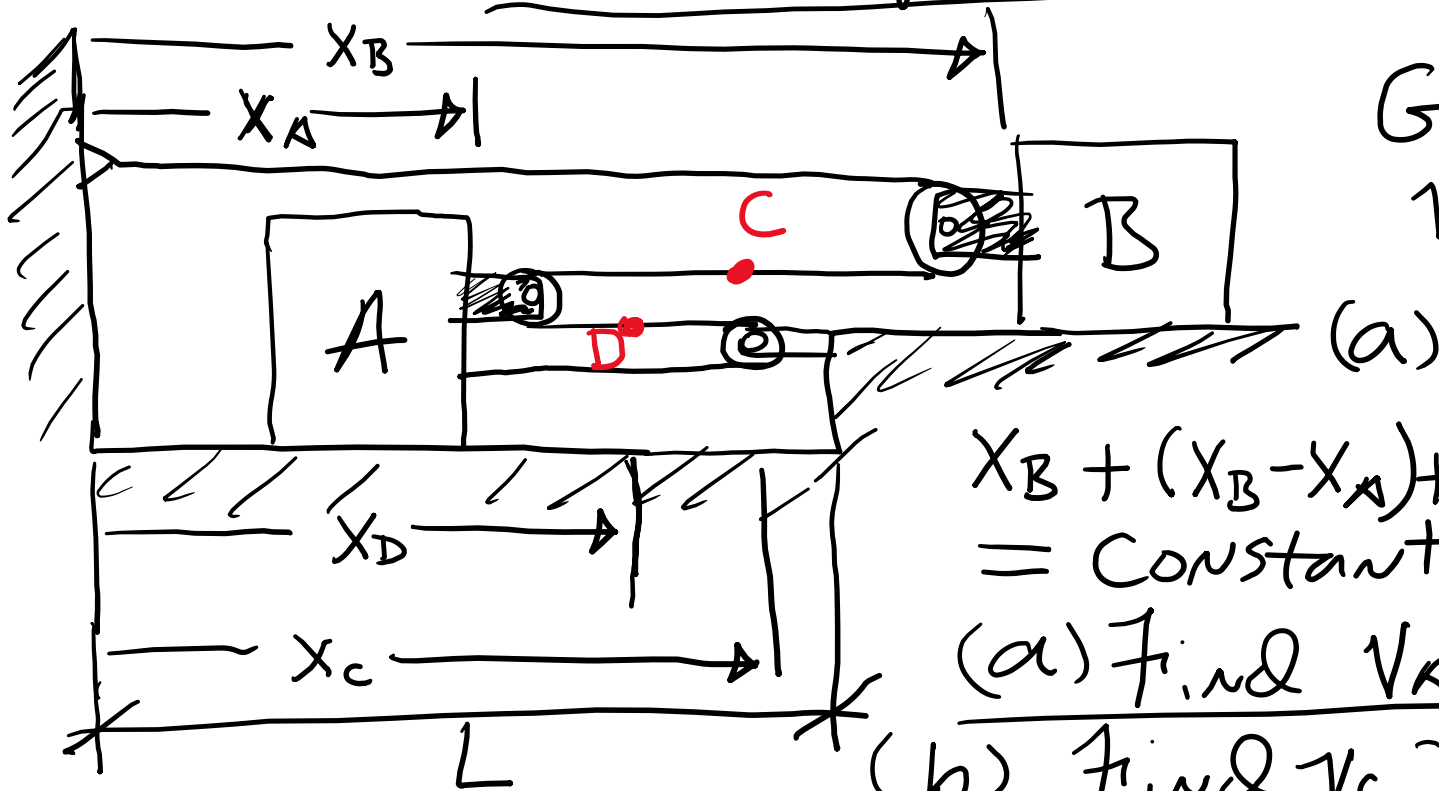
(b) Find  $v_C$

(c) Find  $v_D$

} Follow

take derivatives. rope from one end to point of interest.

# Notes on problem 11.51



Given  
 $v_B = 300 \frac{\text{mm}}{\text{s}}$

$$x_B + (x_B - x_A) + (L - x_A) = \text{constant}$$

(a) Find  $v_A$ : just

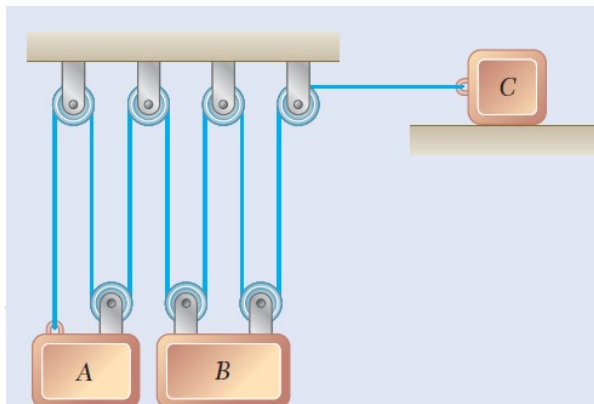
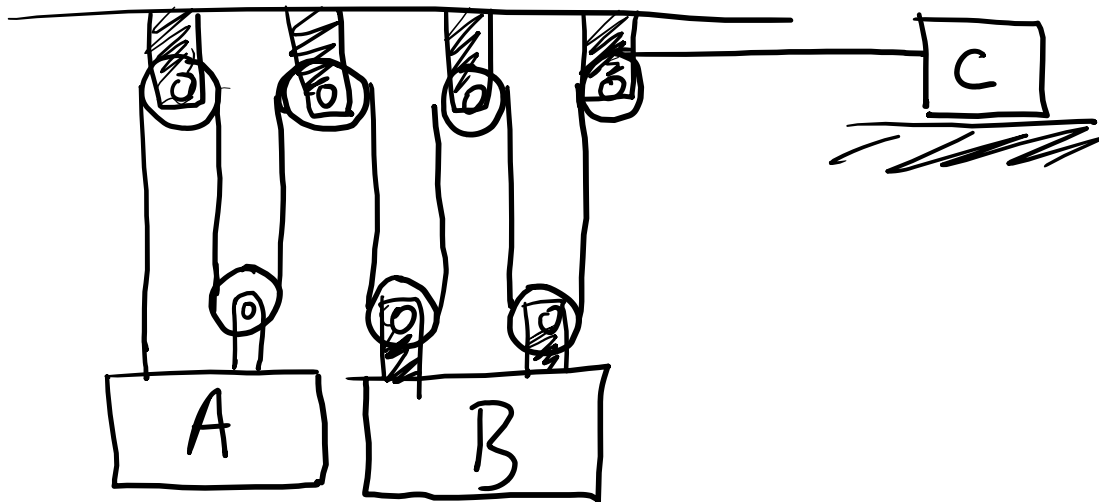
(b) Find  $v_C$   
 (c) Find  $v_D$  } Follow

take derivatives. rope from one end to point of interest.

(d) Find  $v_{C/A}$ : Make sure to note direction

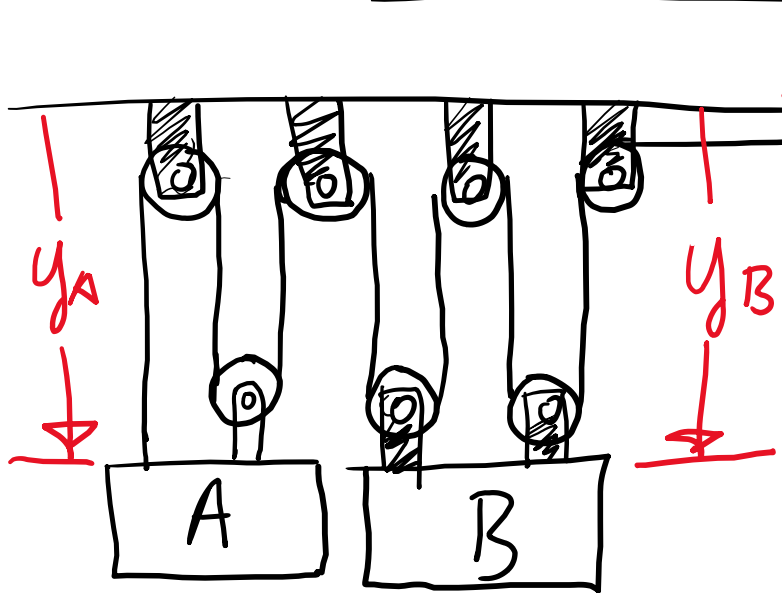


# Notes on problem 11.57



**11.57** Block  $B$  starts from rest, block  $A$  moves with a constant acceleration, and slider block  $C$  moves to the right with a constant acceleration of  $75 \text{ mm/s}^2$ . Knowing that at  $t = 2 \text{ s}$  the velocities of  $B$  and  $C$  are  $480 \text{ mm/s}$  downward and  $280 \text{ mm/s}$  to the right, respectively, determine (a) the accelerations of  $A$  and  $B$ , (b) the initial velocities of  $A$  and  $C$ , (c) the change in position of slider block  $C$  after  $3 \text{ s}$ .

# Notes on problem 11.57

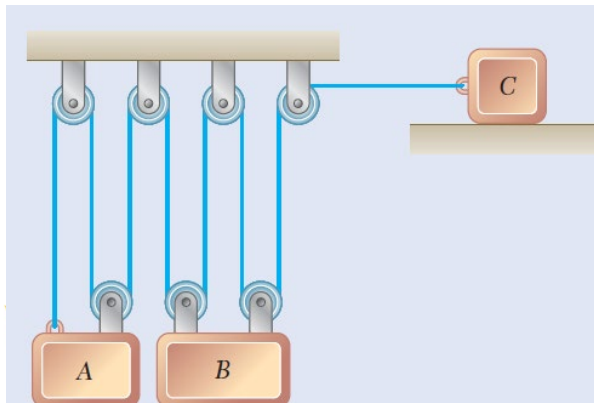


Given:  $v_{0B} = 0$

$$a_A = \text{const.}, a_C = 75 \frac{\text{mm}}{\text{s}^2}$$

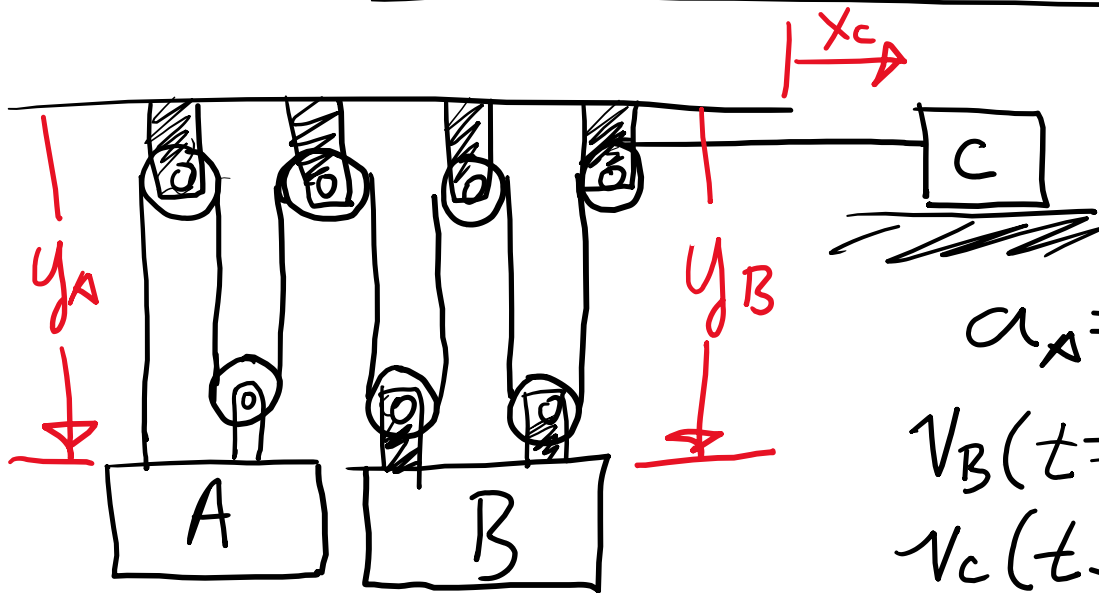
$$v_B(t=2\text{s}) = 480 \frac{\text{mm}}{\text{s}}$$

$$v_C(t=2\text{s}) = 280 \frac{\text{mm}}{\text{s}}$$



**11.57** Block  $B$  starts from rest, block  $A$  moves with a constant acceleration, and slider block  $C$  moves to the right with a constant acceleration of  $75 \text{ mm/s}^2$ . Knowing that at  $t = 2 \text{ s}$  the velocities of  $B$  and  $C$  are  $480 \text{ mm/s}$  downward and  $280 \text{ mm/s}$  to the right, respectively, determine (a) the accelerations of  $A$  and  $B$ , (b) the initial velocities of  $A$  and  $C$ , (c) the change in position of slider block  $C$  after  $3 \text{ s}$ .

# Notes on problem 11.57



Given:  $v_{0B} = 0$

$$a_A = \text{const.}, a_C = 75 \frac{\text{mm}}{\text{s}^2}$$

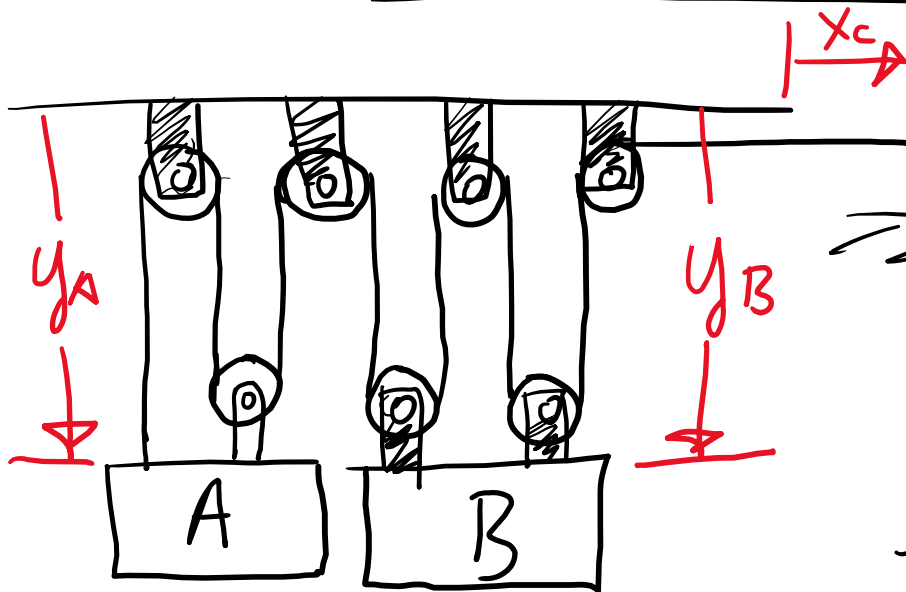
$$v_B(t=2\text{s}) = 480 \frac{\text{mm}}{\text{s}}$$

$$v_C(t=2\text{s}) = 280 \frac{\text{mm}}{\text{s}}$$

(a) Find  $a_A$  &  $a_B$ : Get equation

relating  $y_A$ ,  $y_B$  &  $x_C$ , then  $v_A$ ,  $v_B$  &  $v_C$

# Notes on problem 11.57



Given:  $v_{0B} = 0$

$$a_A = \text{const.}, \quad a_C = 75 \frac{\text{mm}}{\text{s}^2}$$

$$v_B(t=2\text{s}) = 480 \frac{\text{mm}}{\text{s}}$$

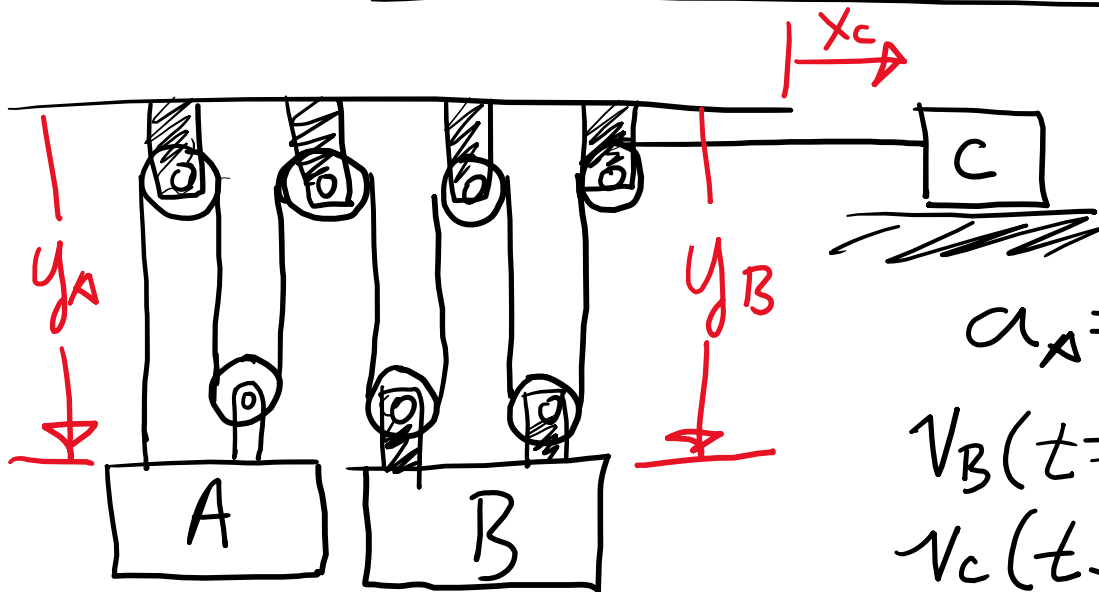
$$v_C(t=2\text{s}) = 280 \frac{\text{mm}}{\text{s}}$$

(a) Find  $a_A$  &  $a_B$  : Get equation

relating  $y_A, y_B$  &  $x_C$ , then  $v_A, v_B$  &  $v_C$

Note:  $a_A = \text{const.}$  &  $a_C = \text{const.} \Rightarrow a_B = \text{const.}$

# Notes on problem 11.57



Given:  $v_{0B} = 0$

$$a_A = \text{const.}, a_C = 75 \frac{\text{mm}}{\text{s}^2}$$

$$v_B(t=2\text{s}) = 480 \frac{\text{mm}}{\text{s}}$$

$$v_C(t=2\text{s}) = 280 \frac{\text{mm}}{\text{s}}$$

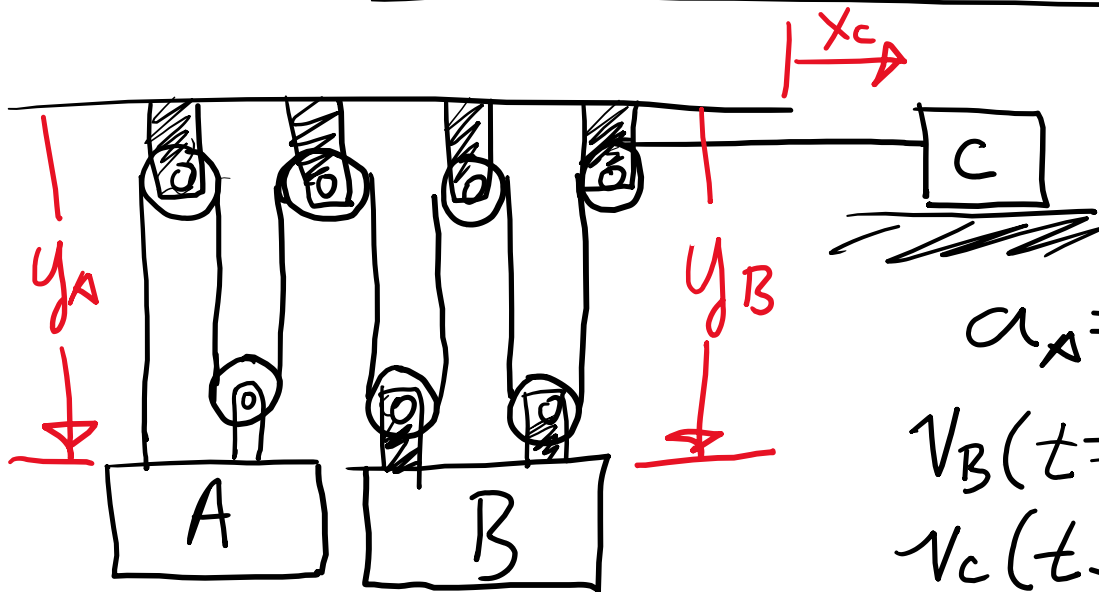
(a) Find  $a_A$  &  $a_B$  : Get equation

relating  $y_A, y_B$  &  $x_C$ , then  $v_A, v_B$  &  $v_C$

Note:  $a_A = \text{const.}$  &  $a_C = \text{const.} \Rightarrow a_B = \text{const.}$

& since  $v_{B0} = 0$  then  $v_B = a_B t$

# Notes on problem 11.57



Given:  $v_{0B} = 0$

$$a_A = \text{const.}, a_C = 75 \frac{\text{mm}}{\text{s}^2}$$

$$v_B(t=2\text{s}) = 480 \frac{\text{mm}}{\text{s}}$$

$$v_C(t=2\text{s}) = 280 \frac{\text{mm}}{\text{s}}$$

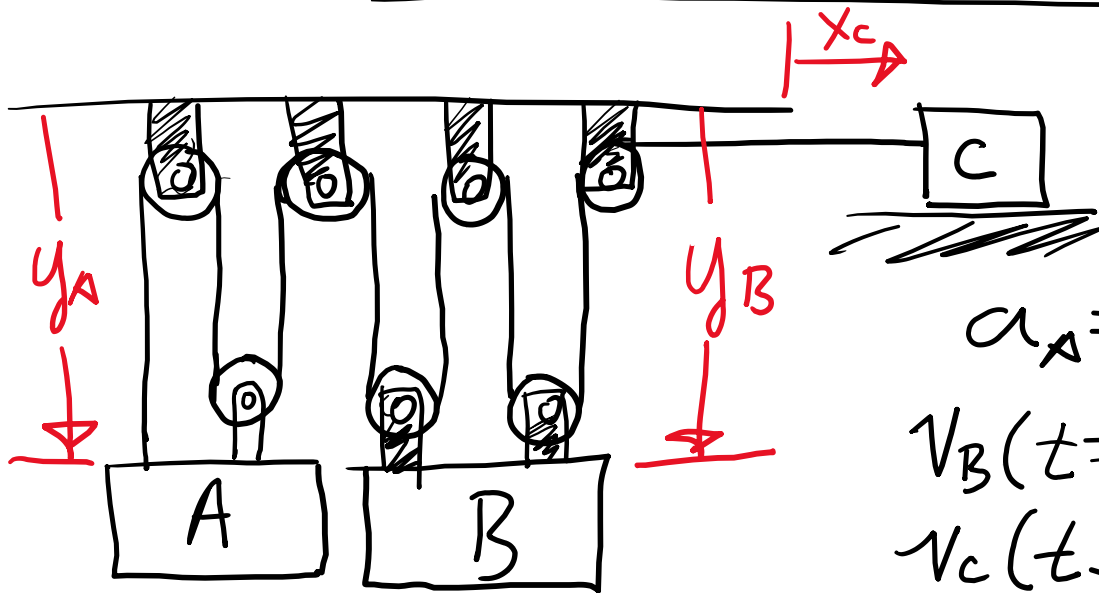
(a) Find  $a_A$  &  $a_B$  : Get equation

relating  $y_A, y_B$  &  $x_C$ , then  $v_A, v_B$  &  $v_C$

Note:  $a_A = \text{const.}$  &  $a_C = \text{const.} \Rightarrow a_B = \text{const.}$

& since  $v_{0B} = 0$  then  $v_B = a_B t$ .  $v_B(t=2\text{s})$  is given. Use system of equations to get  $a_A$  &  $a_B$

# Notes on problem 11.57



Given:  $v_{0B} = 0$

$$a_A = \text{const.}, a_C = 75 \frac{\text{mm}}{\text{s}^2}$$

$$v_B(t=2\text{s}) = 480 \frac{\text{mm}}{\text{s}}$$

$$v_C(t=2\text{s}) = 280 \frac{\text{mm}}{\text{s}}$$

(a) Find  $a_A$  &  $a_B$  : Get equation

relating  $y_A, y_B$  &  $x_C$ , then  $v_A, v_B$  &  $v_C$

Note:  $a_A = \text{const.}$  &  $a_C = \text{const.} \Rightarrow a_B = \text{const.}$

& since  $v_{0B} = 0$  then  $v_B = a_B t$ .  $v_B(t=2\text{s})$  is

given. Use system of equations to get  $a_A$

&  $a_B$ . For rest: kinematics & system of equations

