

# Today Review

L20



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Tuesday Exam 3

L20



Angular velocity  $\equiv \omega$

Angular velocity  $\equiv \omega$  &  $\omega = \frac{d\theta}{dt}$

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Also  $\alpha = \omega \frac{d\omega}{d\theta}$

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We can write the velocity of a point on rigid body undergoing a pure rotation as  $\vec{v} = \vec{\omega} \times \vec{r}$ ,

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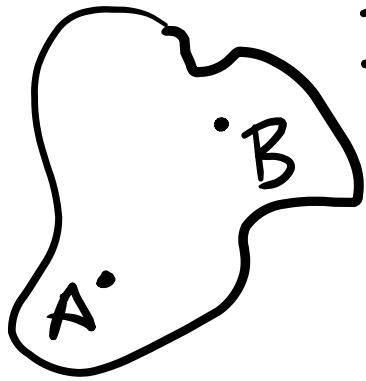
Angular acceleration  $\equiv \alpha$  &  $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

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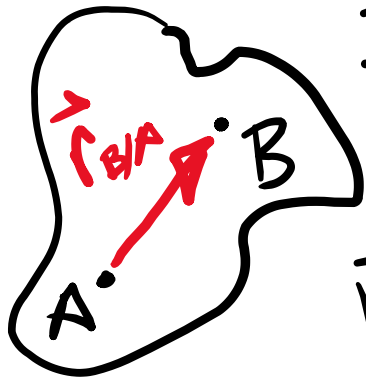
We can write the velocity of a point on rigid body undergoing a pure rotation as  $\vec{v} = \vec{\omega} \times \vec{r}$ , where  $\vec{r}$  begins on the axis of rotation & ends at the point of interest.

The acceleration can be written as

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times [\vec{\omega} \times \vec{r}]$$

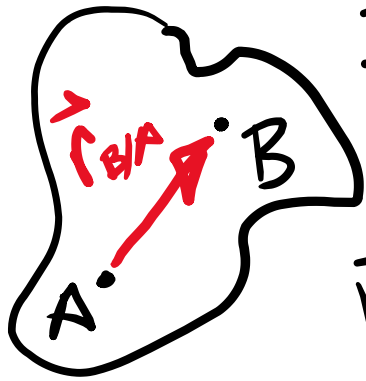


If points A & B are on a rigid body in plane motion



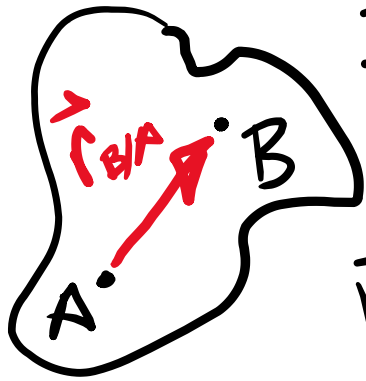
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$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$



If points A & B are on a rigid body in plane motion

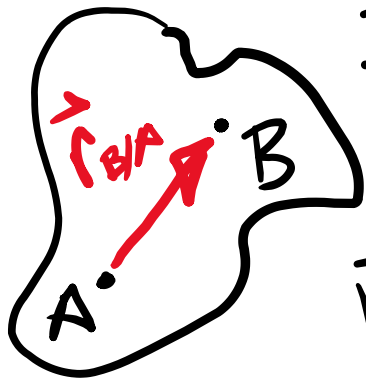
$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$



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Rotating disk

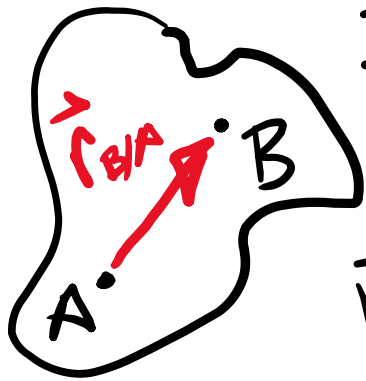


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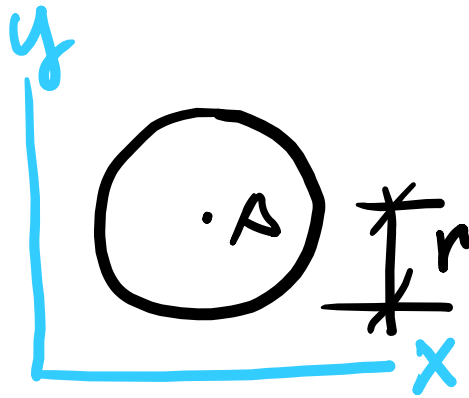


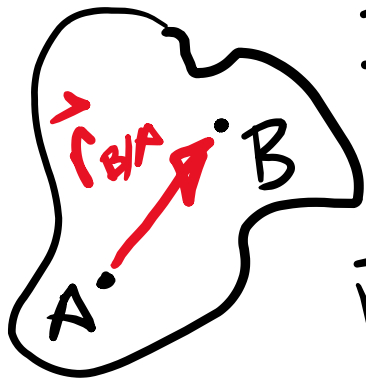


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Rotating disk

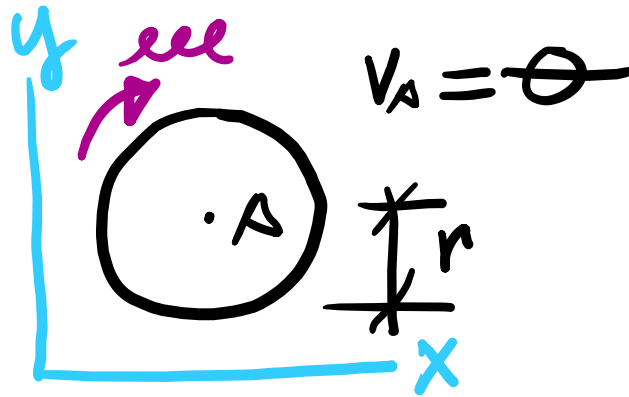


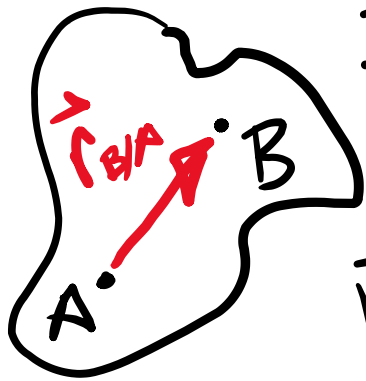


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Rotating disk

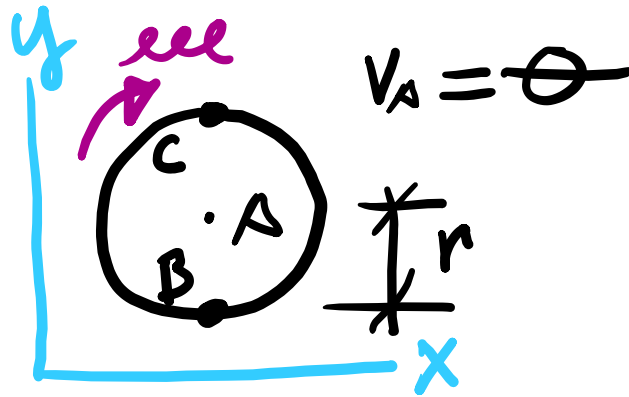


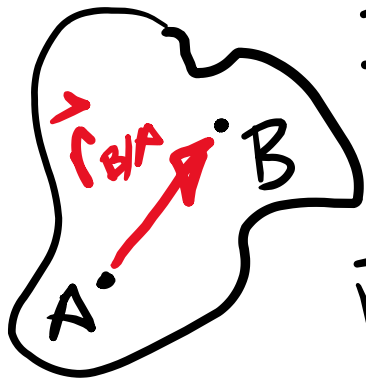


If points A & B are on a rigid body in plane motion

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Rotating disk

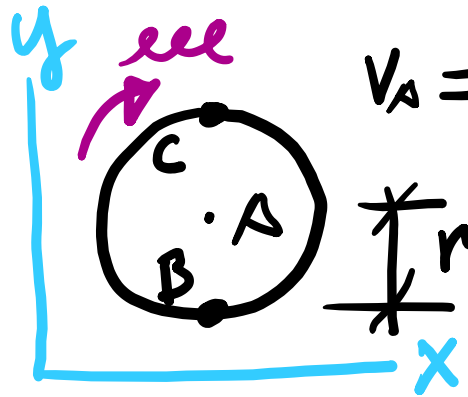




If points A & B are on a rigid body in plane motion

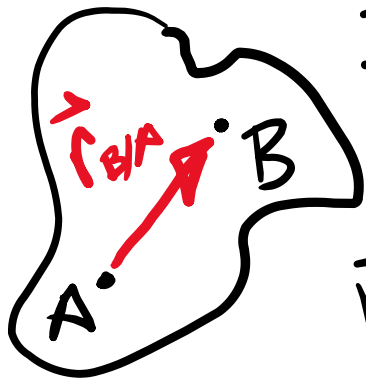
$$\vec{v}_{B/A} = \vec{e}_e \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = e e r_{B/A}$$

Rotating disk



$$v_A = 0$$

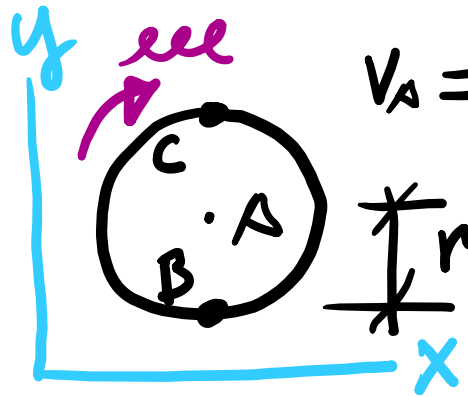
$$\Rightarrow \vec{v}_{B/A} = r e e (-\hat{x})$$



If points A & B are on a rigid body in plane motion

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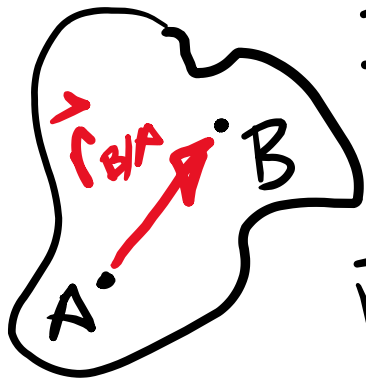
Rotating disk



$$v_A = 0$$

$$\Rightarrow \vec{v}_{B/A} = r \omega (-\hat{x})$$

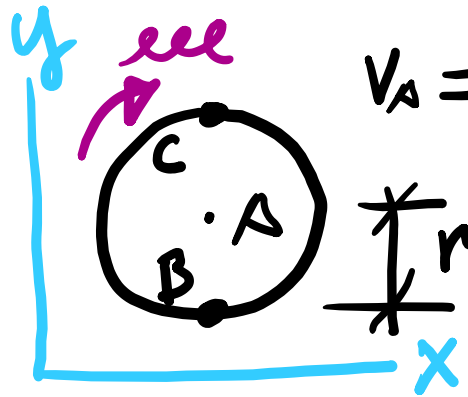
$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$



If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{e}_e \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = e |\vec{r}_{B/A}|$$

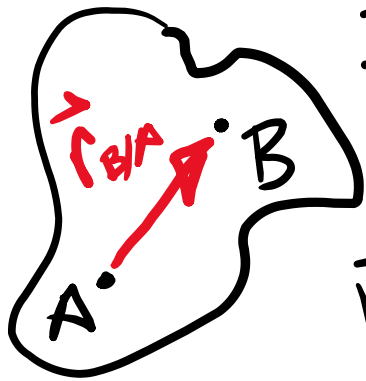
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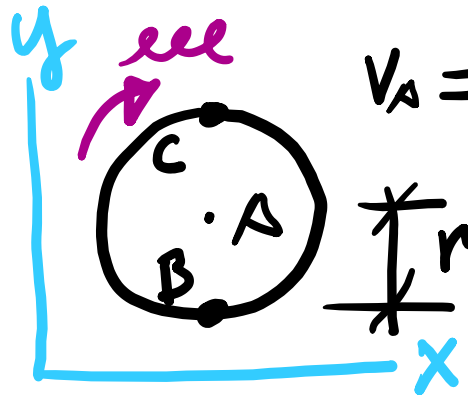
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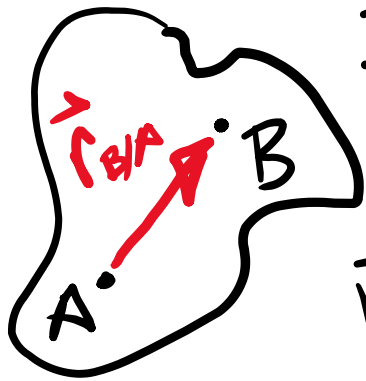


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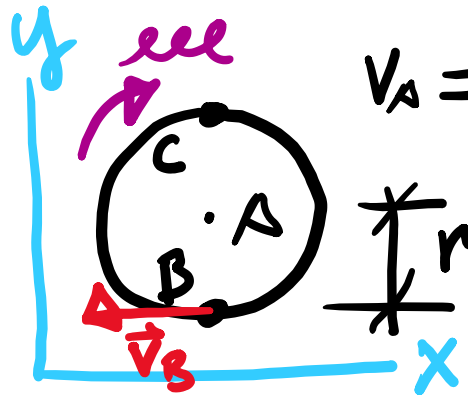
$$\vec{v}_B = \omega e_e (-\hat{x})$$



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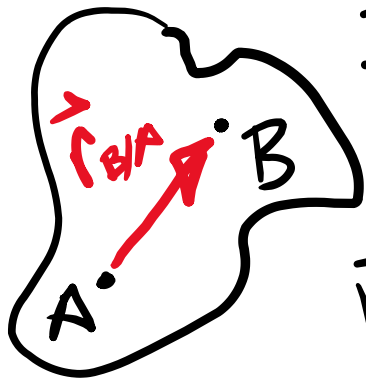


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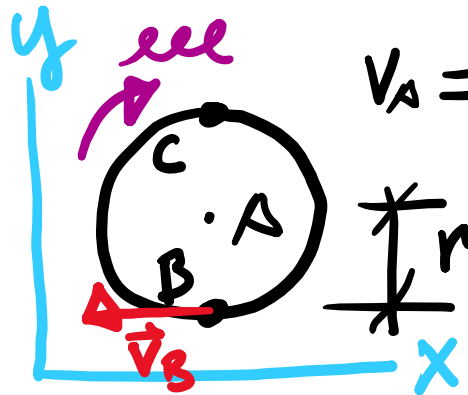
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Rotating disk



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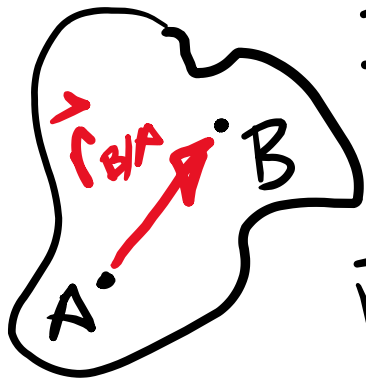
$$\Rightarrow \vec{v}_{B/A} = r\omega (-\hat{x})$$

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If disk is rolling without slipping

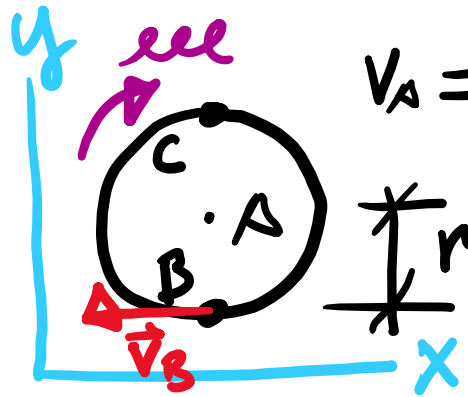
$$\vec{v}_A = r\omega \hat{x}$$



If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

Rotating disk



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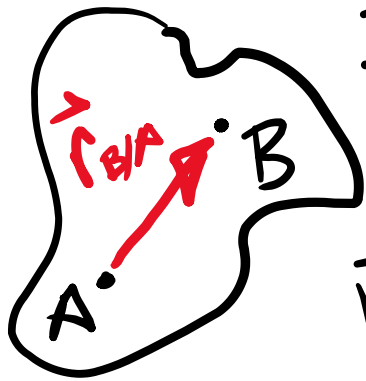
$$\Rightarrow \vec{v}_{B/A} = r\omega(-\hat{x})$$

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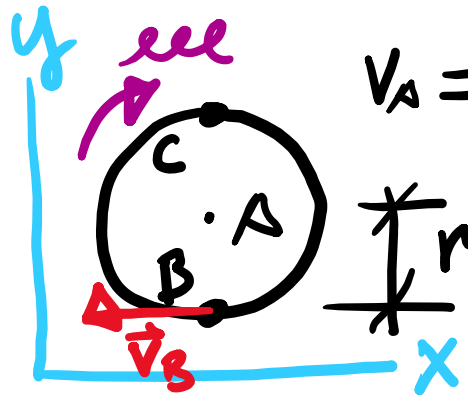
$\vec{v}_A = r\omega\hat{x}$  & we still have  $\vec{v}_{B/A} = r\omega(-\hat{x})$



If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

Rotating disk



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$$\Rightarrow \vec{v}_{B/A} = r\omega(-\hat{x})$$

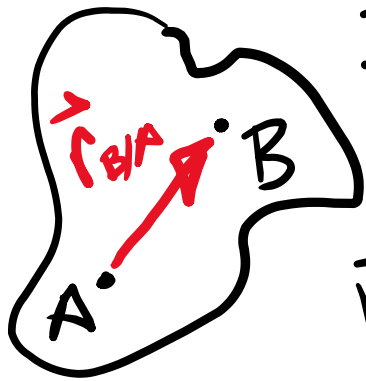
$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$

$$\vec{v}_B = r\omega(-\hat{x})$$

If disk is rolling without slipping

$$\vec{v}_A = r\omega\hat{x} \text{ \& we still have } \vec{v}_{B/A} = r\omega(-\hat{x})$$

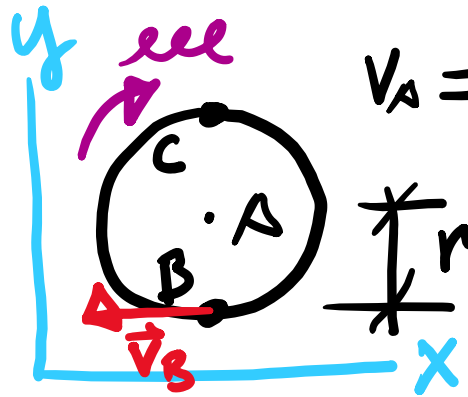
$$\text{And this gives us } \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$



If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{e}_e \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

Rotating disk



$$v_A = 0$$

$$\Rightarrow \vec{v}_{B/A} = r \omega \hat{x}$$

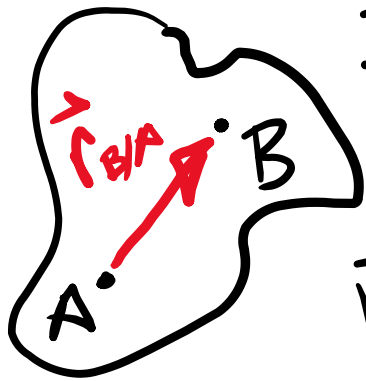
$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$

$$\vec{v}_B = r \omega \hat{x}$$

If disk is rolling without slipping

$\vec{v}_A = r \omega \hat{x}$  & we still have  $\vec{v}_{B/A} = r \omega \hat{x}$

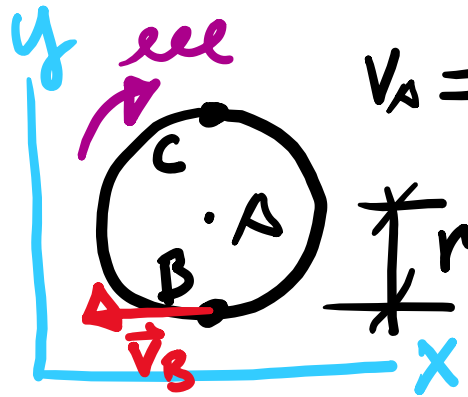
And this gives us  $\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A = r \omega \hat{x} - r \omega \hat{x}$



If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{e}_e \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

Rotating disk



$$v_A = 0$$

$$\Rightarrow \vec{v}_{B/A} = r \omega (-\hat{x})$$

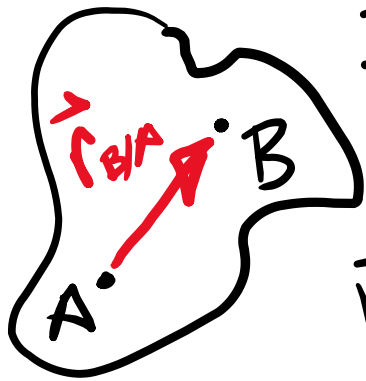
$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$

$$\vec{v}_B = r \omega (-\hat{x})$$

If disk is rolling without slipping

$\vec{v}_A = r \omega \hat{x}$  & we still have  $\vec{v}_{B/A} = r \omega (-\hat{x})$

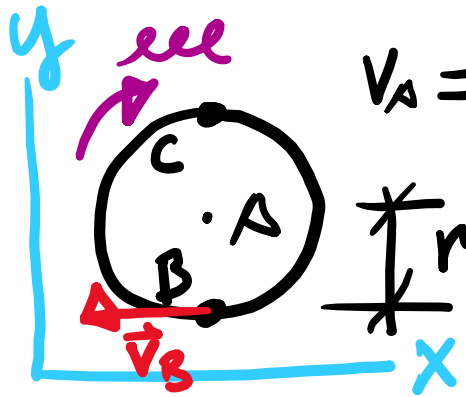
And this gives us  $\vec{v}_B = \vec{v}_{B/A} + \vec{v}_A = r \omega (-\hat{x}) + r \omega \hat{x} = 0$



If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

Rotating disk



$$v_A = 0$$

$$\Rightarrow \vec{v}_{B/A} = r\omega(-\hat{x})$$

$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$

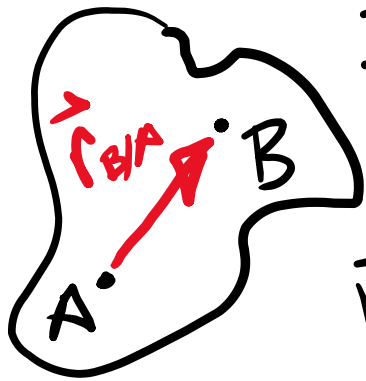
$$\vec{v}_B = r\omega(-\hat{x})$$

If disk is rolling without slipping

$$\vec{v}_A = r\omega\hat{x} \text{ \& we still have } \vec{v}_{B/A} = r\omega(-\hat{x})$$

$$\text{And this gives us } \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A = r\omega(-\hat{x}) + r\omega\hat{x} = 0$$

So  $\vec{v}_B$  is at rest at this moment in time

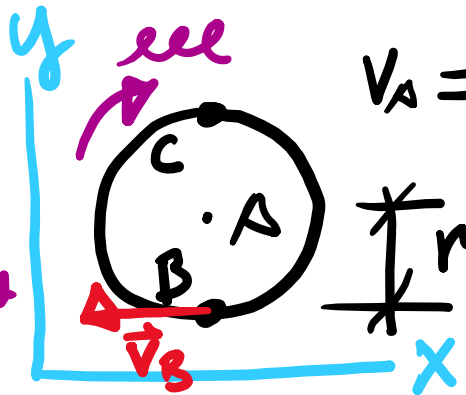


If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

## Rotating disk

What about point C at top of disk?

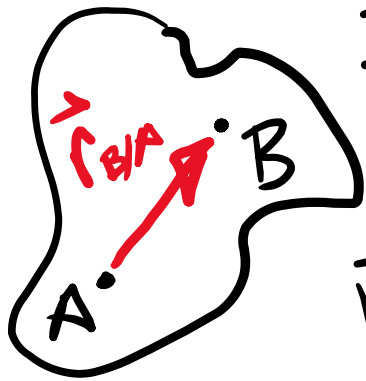


$$v_A = 0$$

$$\Rightarrow \vec{v}_{B/A} = r \omega (-\hat{x})$$

$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A$$

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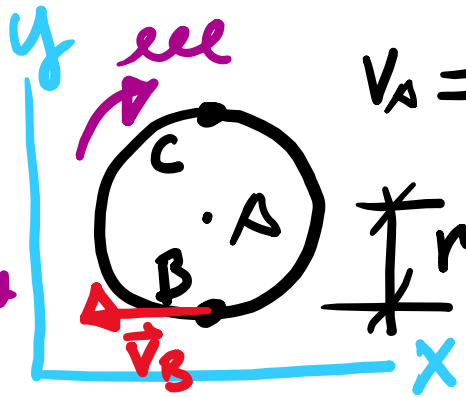


If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

## Rotating disk

What about point C at top of disk?



$$v_A = 0$$

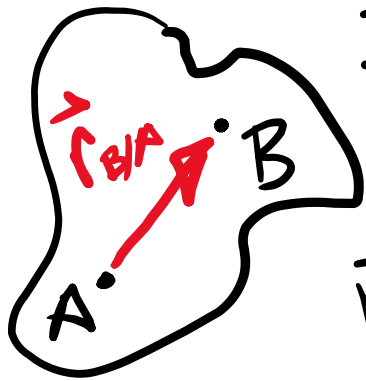
$$\Rightarrow \vec{v}_{B/A} = r\omega(-\hat{x})$$

$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A \Rightarrow$$

$$\vec{v}_B = r\omega(-\hat{x})$$

If disk is rolling without slipping

$$v_C = \vec{v}_{C/A} + \vec{v}_A$$

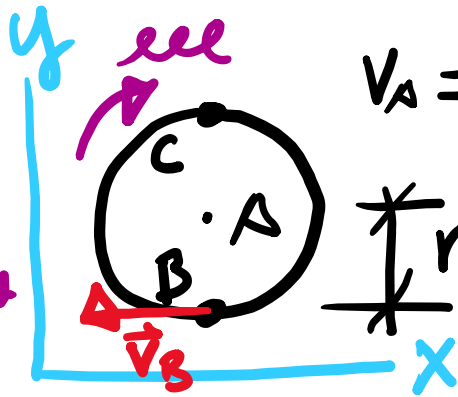


If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

## Rotating disk

What about point C at top of disk?



$$v_A = 0$$

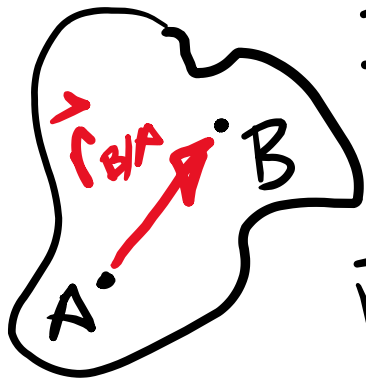
$$\Rightarrow \vec{v}_{B/A} = r\omega(-\hat{x})$$

$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A \Rightarrow$$

$$\vec{v}_B = r\omega(-\hat{x})$$

If disk is rolling without slipping

$$v_C = \vec{v}_{C/A} + \vec{v}_A, \text{ but } \vec{v}_{C/A} = r\omega\hat{x}$$

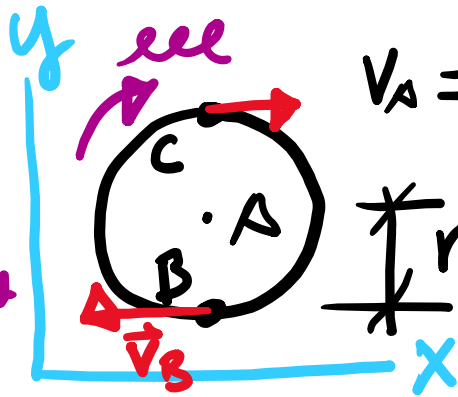


If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

## Rotating disk

What about point C at top of disk?



$$v_A = 0$$

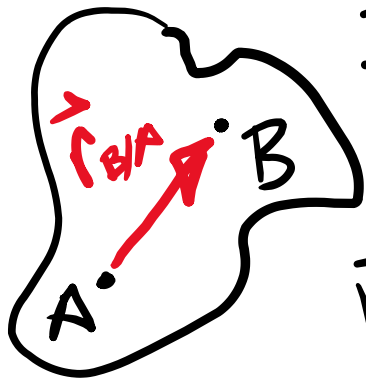
$$\Rightarrow \vec{v}_{B/A} = r\omega(-\hat{x})$$

$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A \Rightarrow$$

$$\vec{v}_B = r\omega(-\hat{x})$$

If disk is rolling without slipping

$$v_C = \vec{v}_{C/A} + \vec{v}_A, \text{ but } \vec{v}_{C/A} = r\omega\hat{x}$$

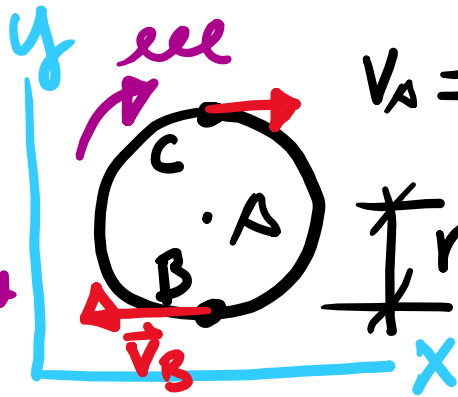


If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

## Rotating disk

What about point C at top of disk?



$$v_A = 0$$

$$\Rightarrow \vec{v}_{B/A} = r\omega(-\hat{x})$$

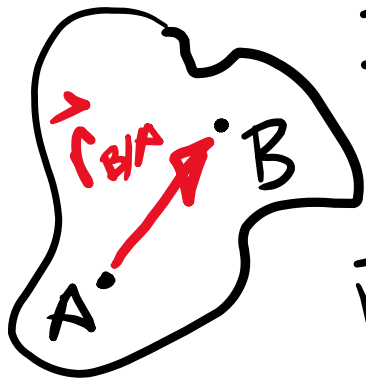
$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A \Rightarrow$$

$$\vec{v}_B = r\omega(-\hat{x})$$

If disk is rolling without slipping

$$\vec{v}_C = \vec{v}_{C/A} + \vec{v}_A, \text{ but } \vec{v}_{C/A} = r\omega\hat{x} \text{ so}$$

$$\vec{v}_C = r\omega\hat{x} + r\omega\hat{x}$$

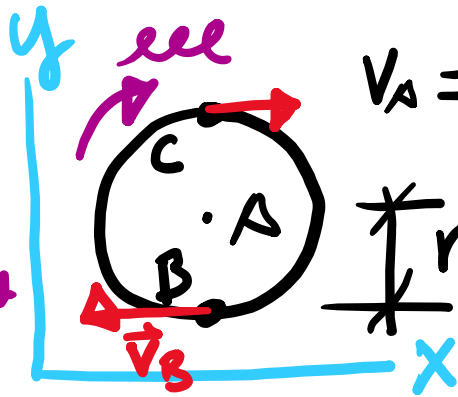


If points A & B are on a rigid body in plane motion

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} \Rightarrow |\vec{v}_{B/A}| = \omega r_{B/A}$$

## Rotating disk

What about point C at top of disk?



$$v_A = 0$$

$$\Rightarrow \vec{v}_{B/A} = r\omega(-\hat{x})$$

$$\Rightarrow \vec{v}_B = \vec{v}_{B/A} + \vec{v}_A \Rightarrow$$

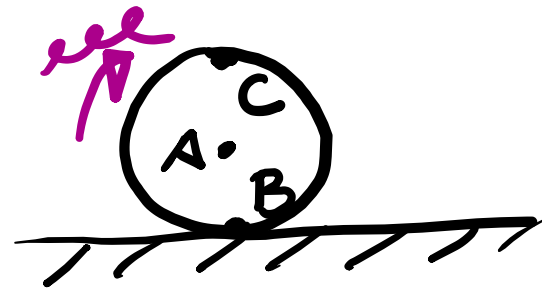
$$\vec{v}_B = r\omega(-\hat{x})$$

If disk is rolling without slipping

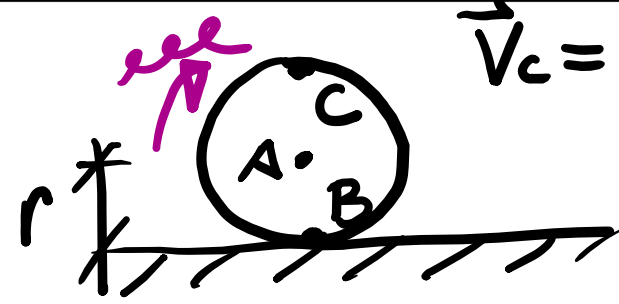
$$\vec{v}_C = \vec{v}_{C/A} + \vec{v}_A, \text{ but } \vec{v}_{C/A} = r\omega\hat{x} \text{ so}$$

$$\vec{v}_C = r\omega\hat{x} + r\omega\hat{x} \Rightarrow \vec{v}_C = 2r\omega\hat{x}$$

Wheel rolling with  
no slipping

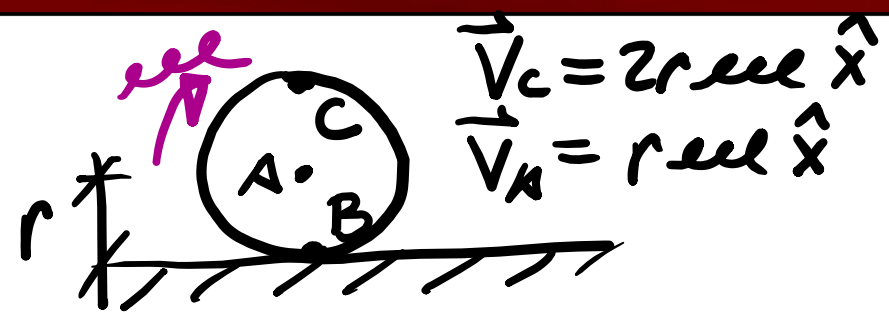


Wheel rolling with  
no slipping

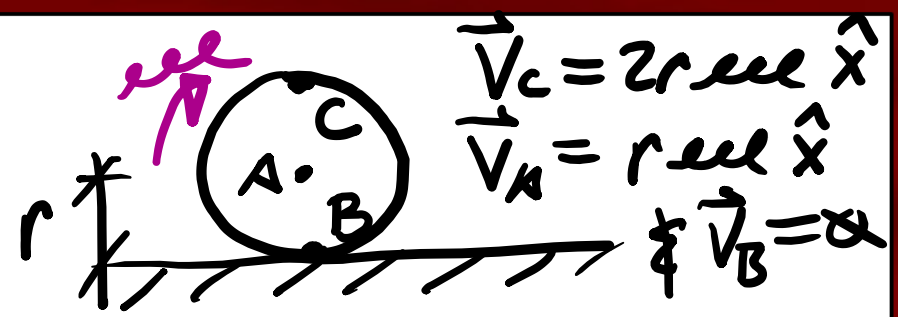


$$\vec{V}_C = 2r\omega \hat{x}$$

Wheel rolling with  
no slipping



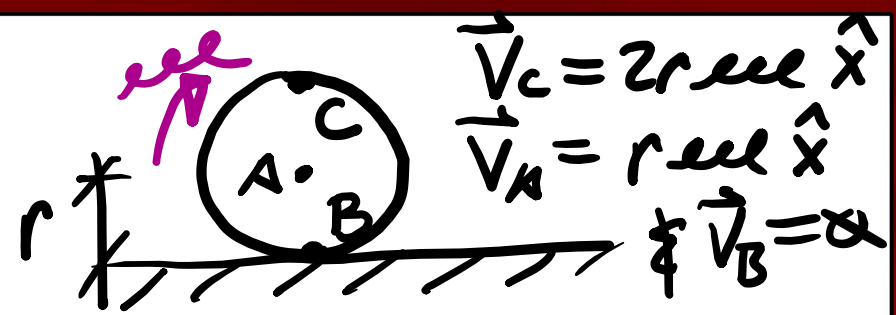
Wheel rolling with  
no slipping



Wheel rolling with  
no slipping

We can now write

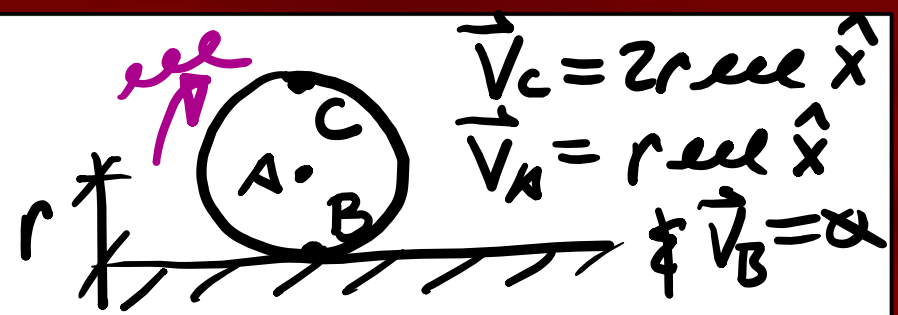
$$\vec{v}_{C/B} = \vec{v}_C - \vec{v}_B$$



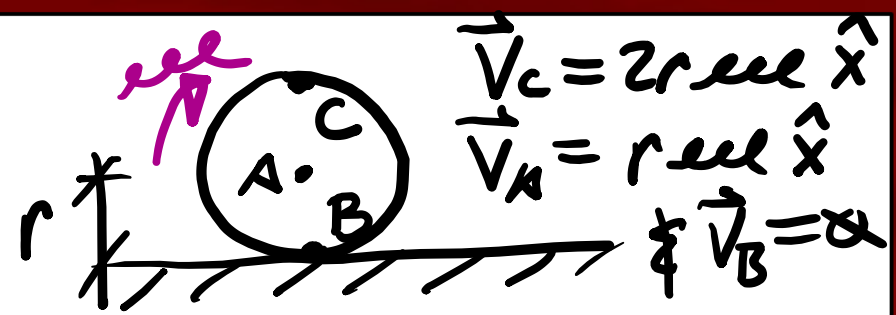
Wheel rolling with  
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$$\vec{v}_{C/B} = \vec{v}_C - \vec{v}_B = 2r\omega \hat{x}$$



Wheel rolling with  
no slipping

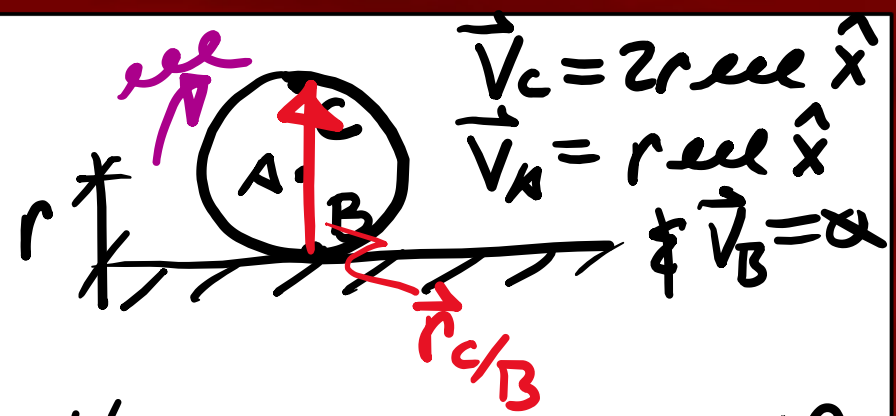


We can now write

$\vec{v}_{C/B} = \vec{v}_C - \vec{v}_B = 2r\omega \hat{x}$ . However, we could  
have just written this down using

$$v_{C/B} = r_{C/B} \omega$$

Wheel rolling with  
no slipping

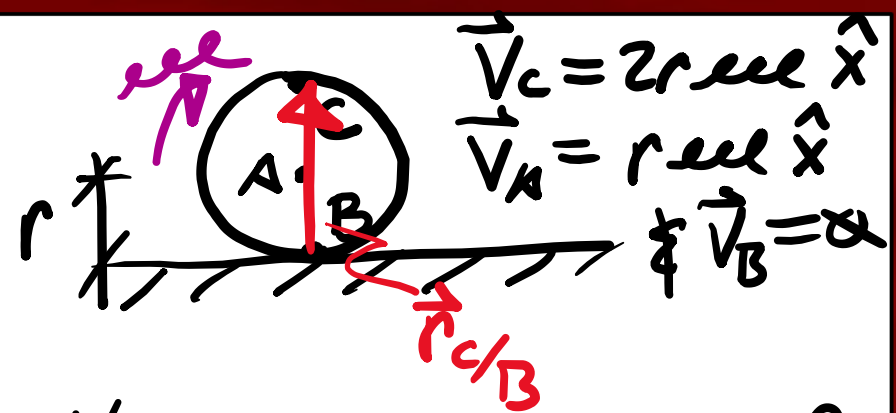


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Wheel rolling with  
no slipping

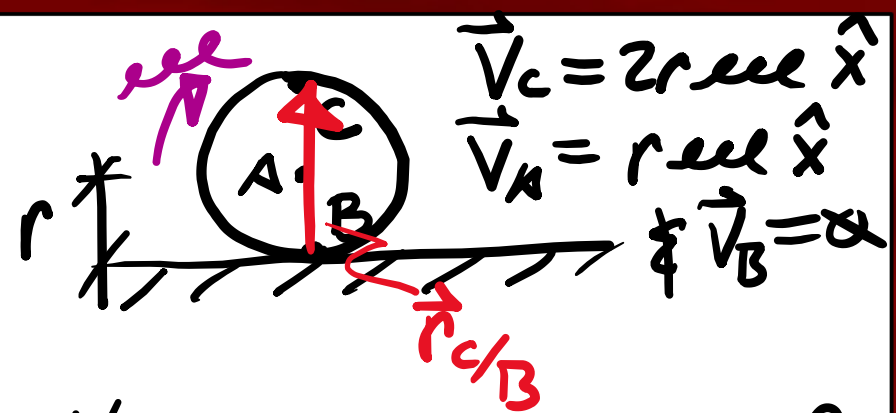


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have just written this down using

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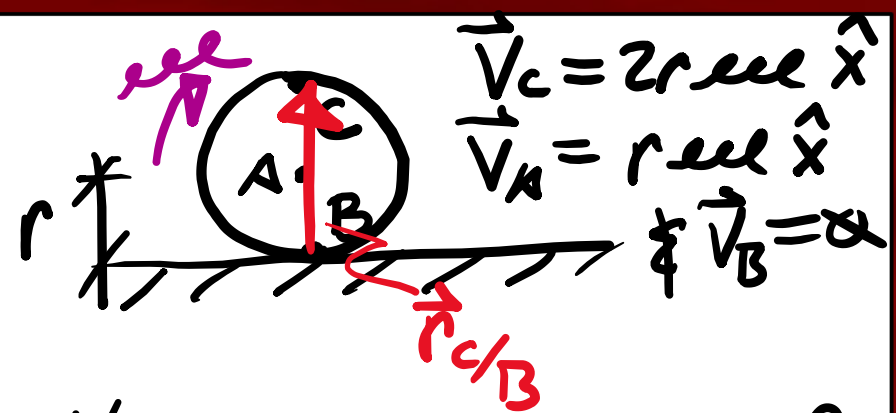
Wheel rolling with  
no slipping



We can now write

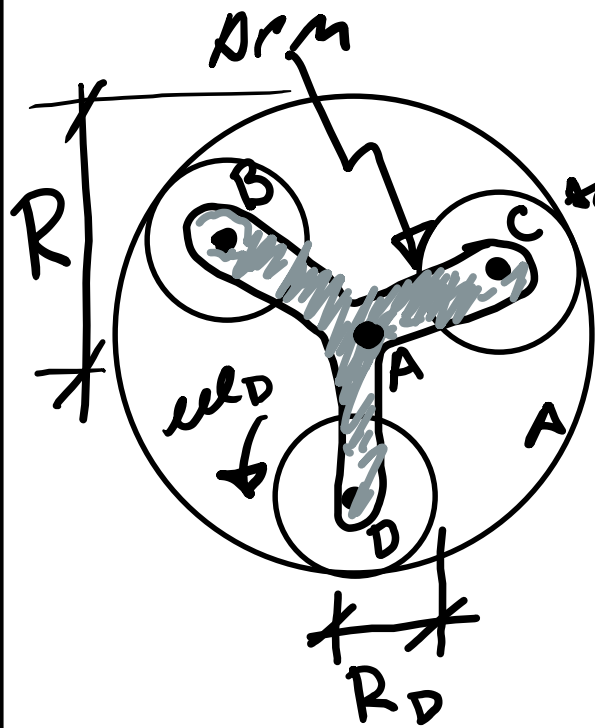
$\vec{v}_{C/B} = \vec{v}_C - \vec{v}_B = 2r\omega \hat{x}$ . However, we could  
have just written this down using  
 $v_{C/B} = r_{C/B} \omega = 2r\omega$ , since "The  
angular velocity is independent of the  
reference point"

Wheel rolling with  
no slipping



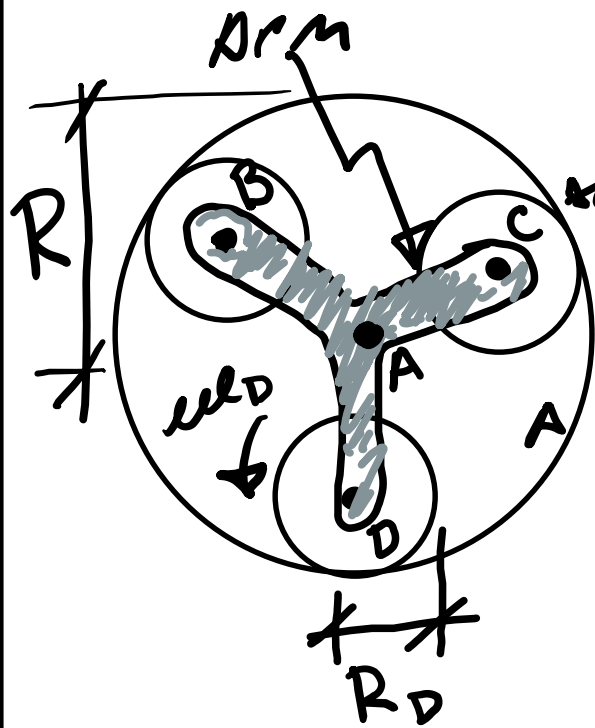
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 $v_{C/B} = r_{C/B} \omega = 2r\omega$ , since "The  
 angular velocity is independent of the  
 reference point"  $\& r_{C/B} = 2r$



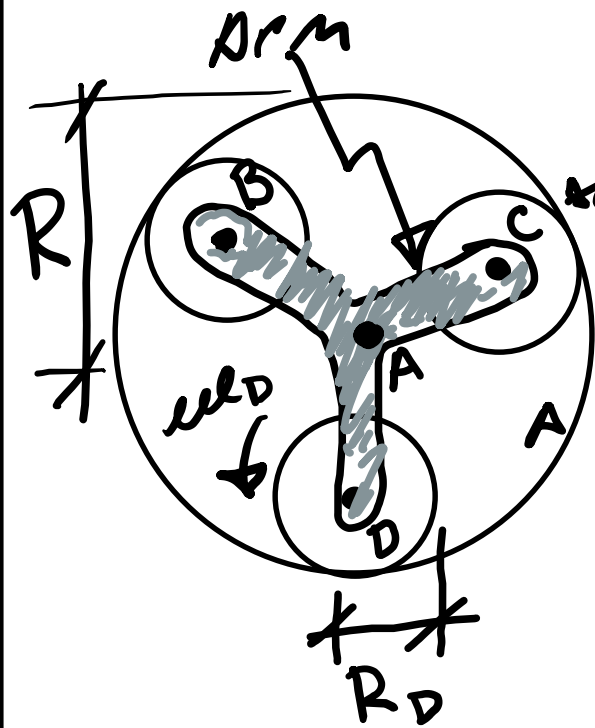
Stationary

Example: Find  $u_{arm}$



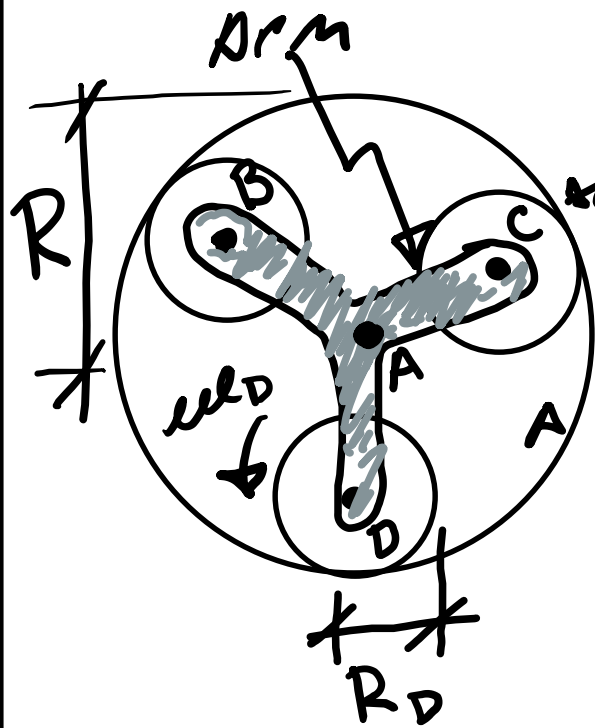
stationary Example: find  $\omega_{ARM}$

We want  $V_{D/A}$  & know that  $V_A = 0$



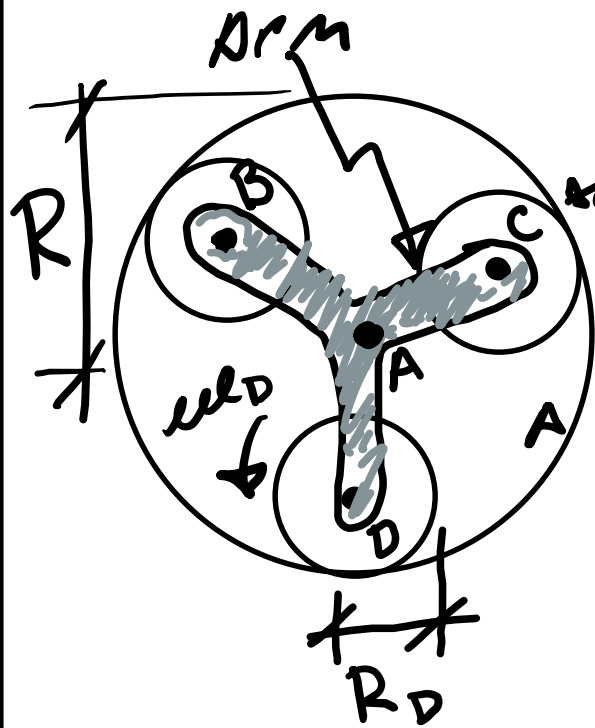
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We want  $v_{D/A}$  & know  
that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$



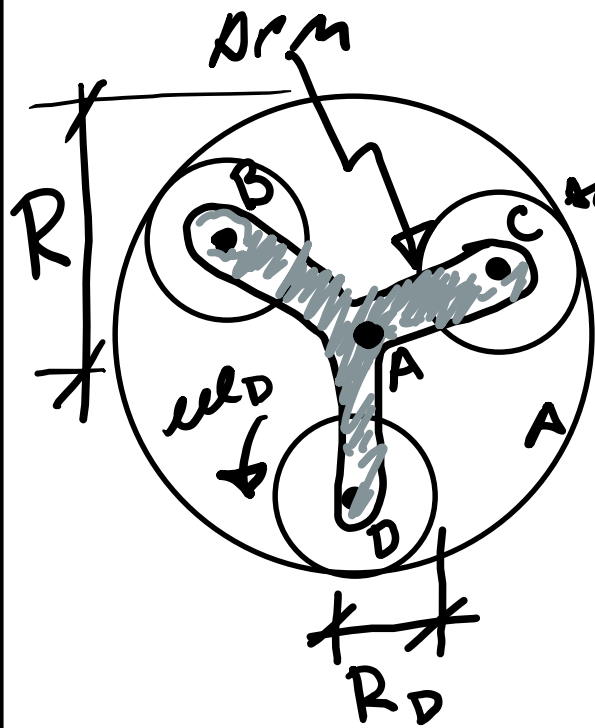
Example: Find  $\omega$

We want  $v_{D/A}$  & know that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$  (1)



Example: Find  $v_{D/A}$

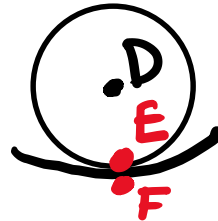
We want  $v_{D/A}$  & know that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$  (1)  
 Now connect the dots:

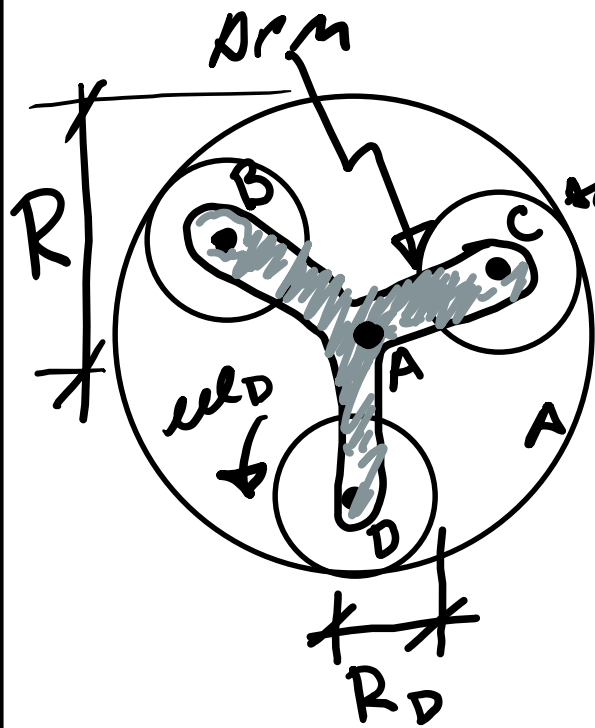


← stationary

Example: Find  $v_{D/A}$

We want  $v_{D/A}$  & know that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$  (1)  
 Now connect the dots





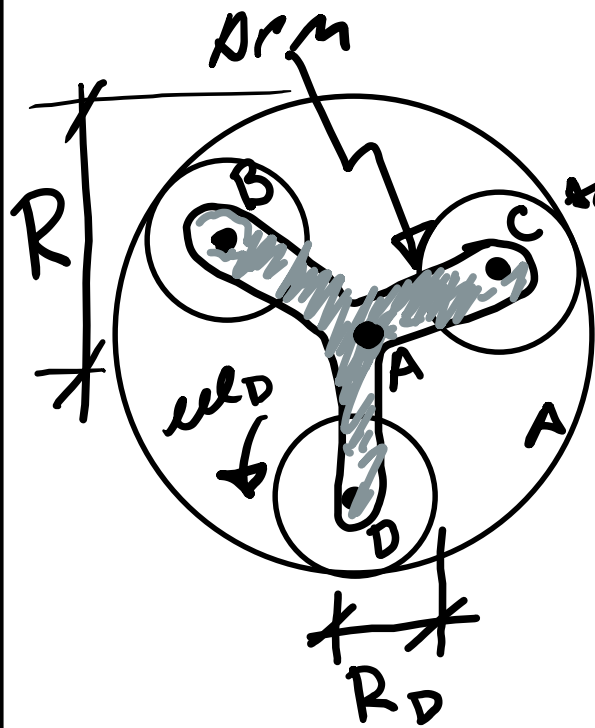
$\omega$  stationary

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 Now connect the dots

$$\vec{v}_D = \vec{v}_{D/E} + \vec{v}_E$$





$\omega$  stationary

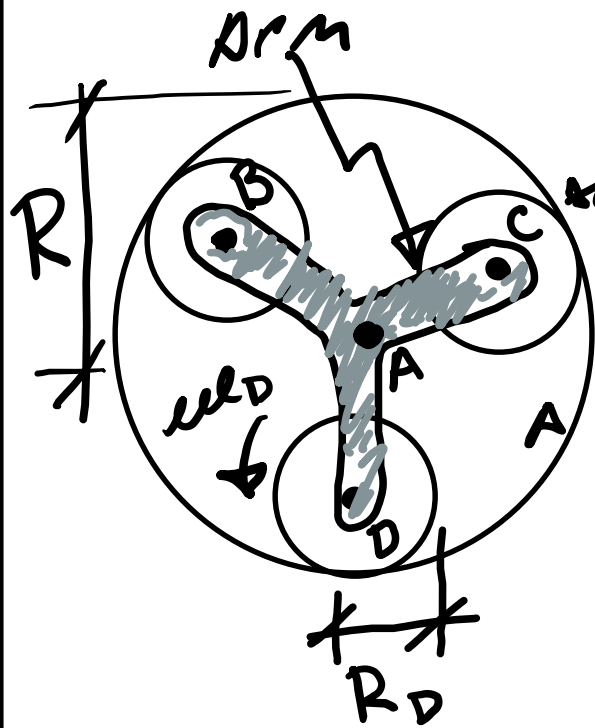
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 Now connect the dots



$$\vec{v}_D = \vec{v}_{D/E} + \vec{v}_E$$

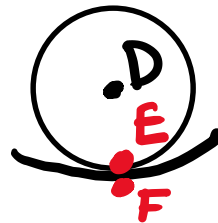
$$\& \vec{v}_E = \vec{v}_{E/F} + \vec{v}_F$$



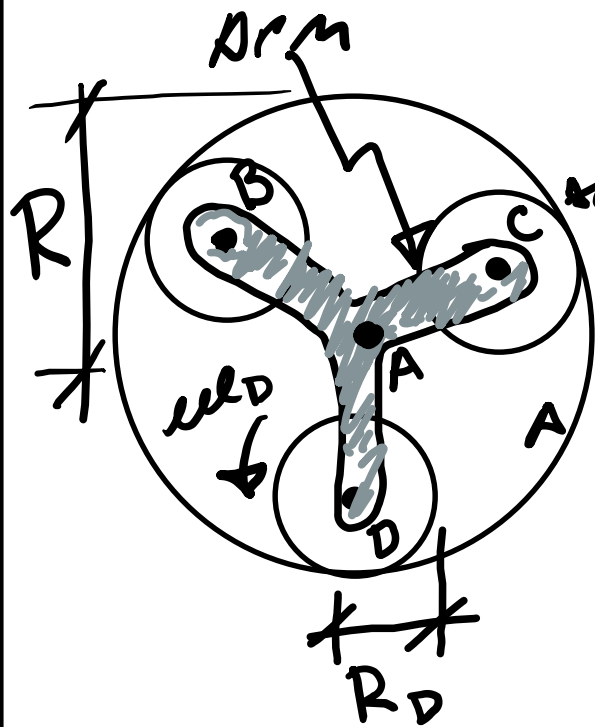
$\omega$  stationary

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 Now connect the dots

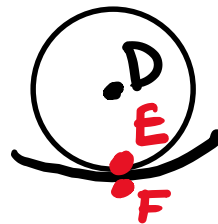


$$\left. \begin{aligned} \vec{v}_D &= \vec{v}_{D/E} + \vec{v}_E \\ \& \vec{v}_E &= \vec{v}_{E/F} + \vec{v}_F \end{aligned} \right\} \begin{array}{l} \text{But} \\ v_F = 0 \end{array}$$



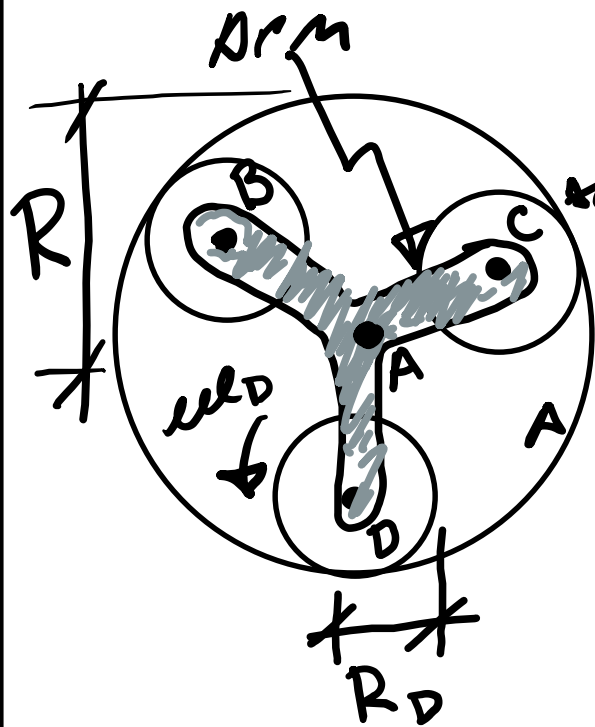
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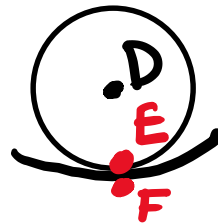
Roll  
no  
slip



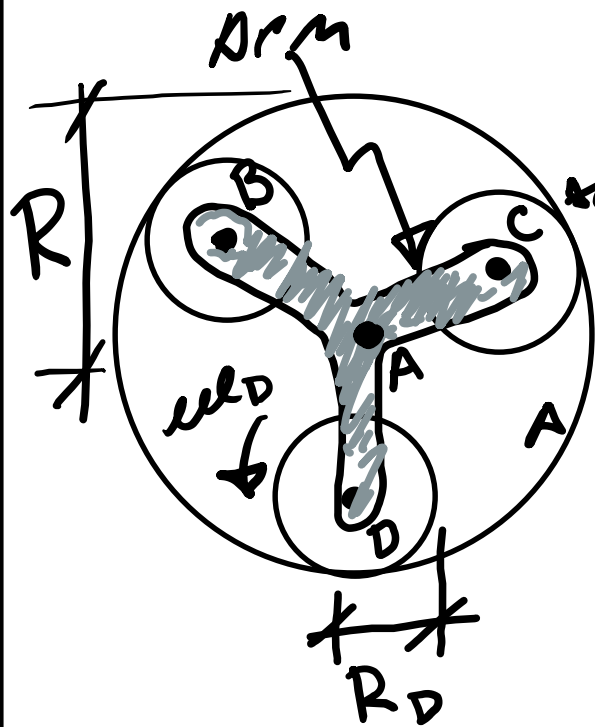
← stationary

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 Now connect the dots



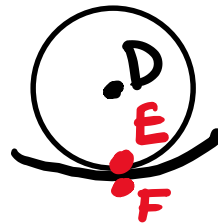
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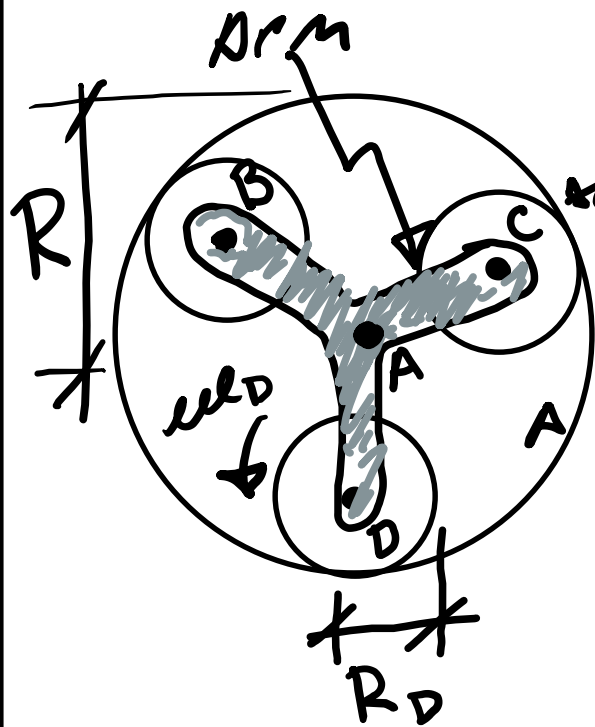
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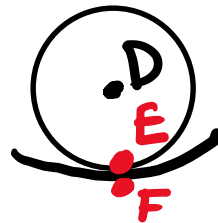


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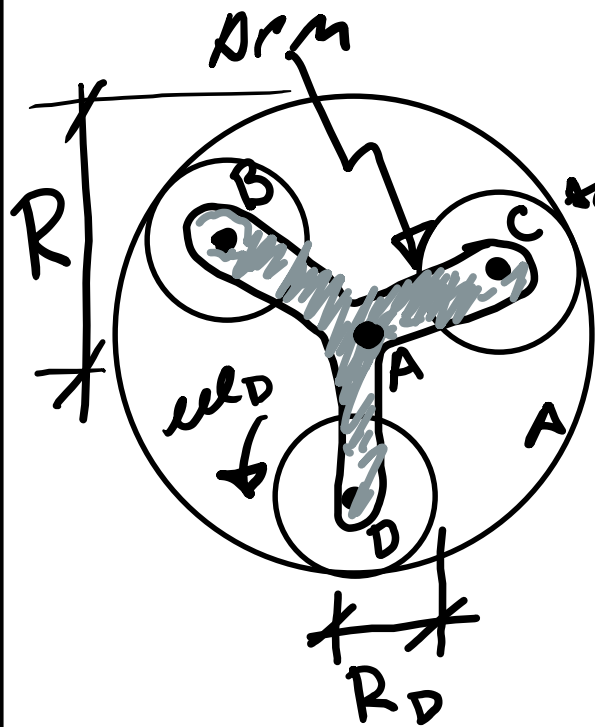
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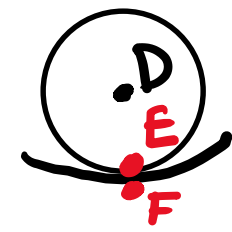
$$\text{So } \vec{v}_D = \vec{v}_{D/E}$$



← stationary

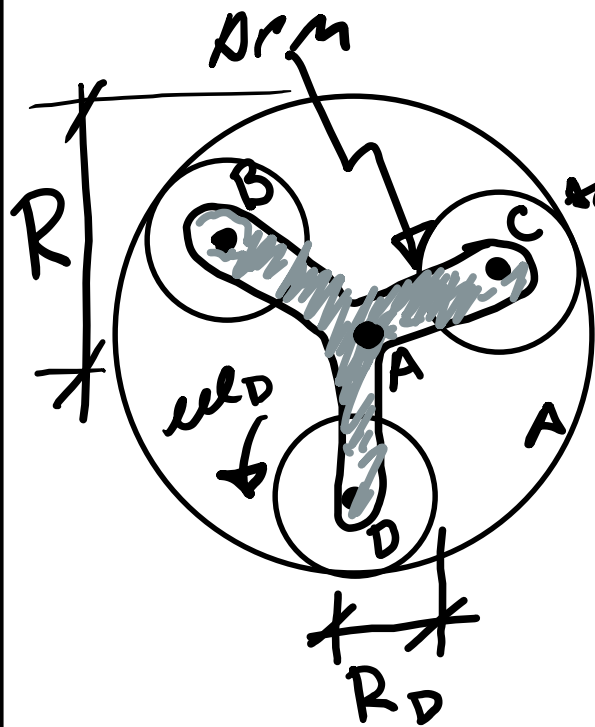
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We want  $v_{D/A}$  & know that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$  (1)  
 Now connect the dots



$\vec{v}_D = \vec{v}_{D/E} + \vec{v}_E$  } But  
 $\vec{v}_E = \vec{v}_{E/F} + \vec{v}_F$  }  $v_F = 0$   
 &  $v_{E/F}$

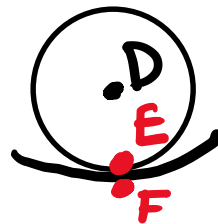
so  $\vec{v}_D = \vec{v}_{D/E}$  (2)



← stationary

Example: Find  $v_{D/A}$

We want  $v_{D/A}$  & know that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$  (1)  
 Now connect the dots



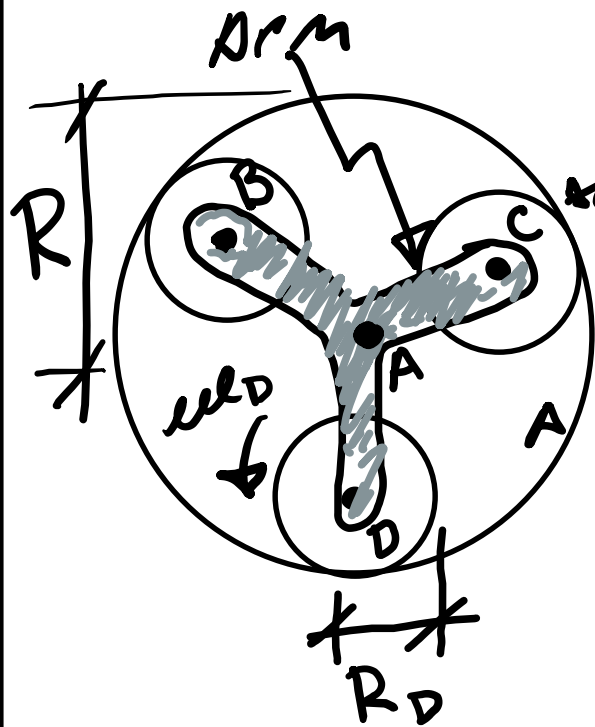
$$\vec{v}_D = \vec{v}_{D/E} + \vec{v}_E \quad \text{But}$$

$$\vec{v}_E = \vec{v}_{E/F} + \vec{v}_F \quad \text{& } v_F = 0 \text{ & } v_{E/F}$$

Eqns 1 & 2 give us

$$\text{So } \vec{v}_D = \vec{v}_{D/E} \quad (2)$$

$$\vec{v}_{D/E} = \vec{v}_{D/A}$$



stationary Example: find  $v_{D/A}$

We want  $v_{D/A}$  & know that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$  (1)  
 Now connect the dots



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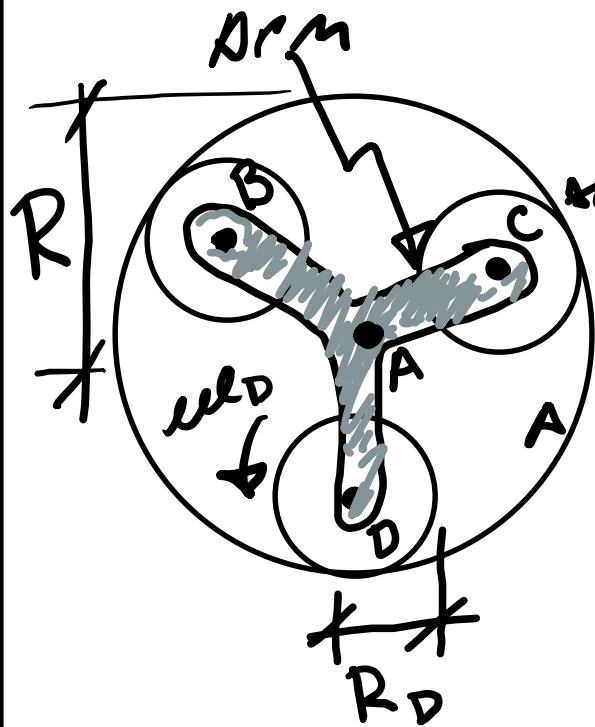
$$\& \vec{v}_E = \vec{v}_{E/F} + \vec{v}_F \quad \& v_F = 0$$

So  $\vec{v}_D = \vec{v}_{D/E}$  (2)

$\vec{v}_{D/E} = \vec{v}_{D/A}$

But  $v_{D/E} = R_D \omega$

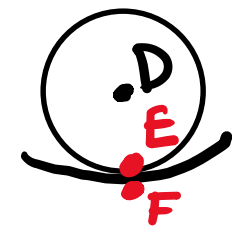
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← stationary

Example: Find  $v_{D/A}$

We want  $v_{D/A}$  & know that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$  (1)  
 Now connect the dots

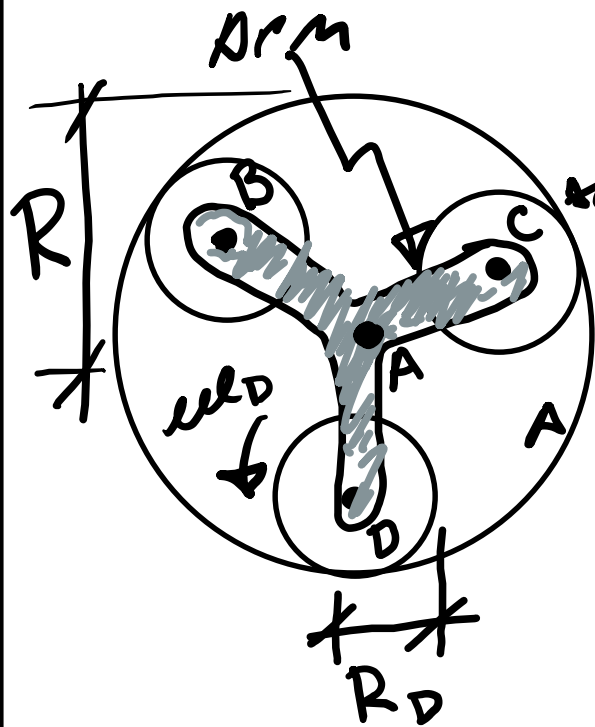


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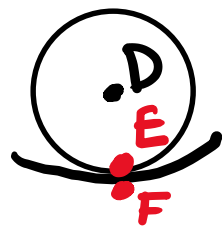
So  $\vec{v}_D = \vec{v}_{D/E}$  (2)      Eqs 1 & 2 give us

$\vec{v}_{D/E} = \vec{v}_{D/A}$       But  $v_{D/E} = R_D \omega_{D/E}$  &  $v_{D/A} = R_{D/A} \omega_{ARM}$



stationary Example: Find  $v_{D/A}$

We want  $v_{D/A}$  & know that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$  (1)  
 Now connect the dots



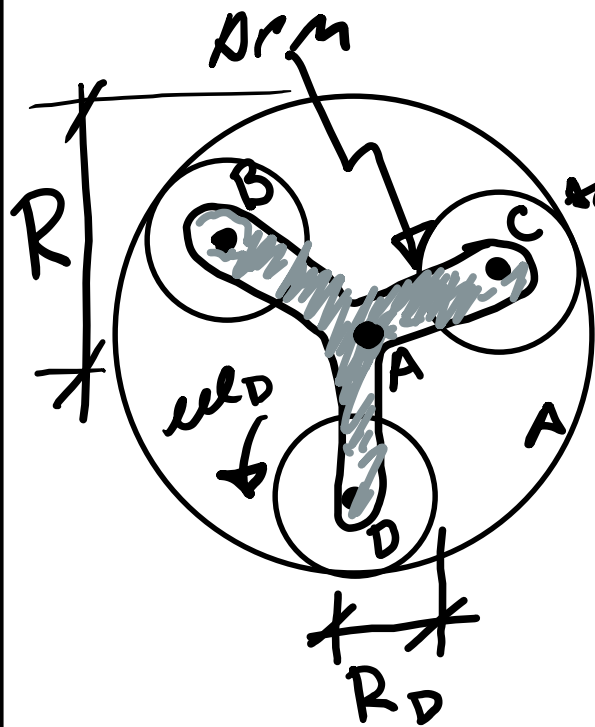
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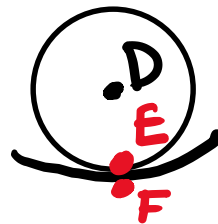
$$\vec{v}_{D/E} = \vec{v}_{D/A} \quad \text{But } v_{D/E} = R_D \omega_{D/E} \quad \& \quad v_{D/A} = R_{D/A} \omega_{D/A}$$

$$\text{So } \omega_{D/A} (R - R_D) = \omega_{D/E} R_D$$



stationary Example: Find  $\omega_{ARM}$

We want  $v_{D/A}$  & know that  $v_A = 0$  so  $\vec{v}_D = \vec{v}_{D/A}$  (1)  
 Now connect the dots



$$\left. \begin{aligned} \vec{v}_D &= \vec{v}_{D/E} + \vec{v}_E \\ \vec{v}_E &= \vec{v}_{E/F} + \vec{v}_F \end{aligned} \right\} \begin{array}{l} \text{But } v_E = 0 \\ \& v_F = 0 \\ \& v_{E/F} \end{array}$$

So  $\vec{v}_D = \vec{v}_{D/E}$  (2) Equns 1 & 2 give us

$$\vec{v}_{D/E} = \vec{v}_{D/A} \quad \text{But} \quad v_{D/E} = R_D \omega_{D/E} \quad \& \quad v_{D/A} = (R - R_D) \omega_{ARM}$$

$$\text{So } \omega_{ARM} (R - R_D) = \omega_{D/E} R_D \Rightarrow \omega_{ARM} = \frac{R_D \omega_{D/E}}{(R - R_D)}$$

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The method of connecting the dots should always work, however, if you feel confident in your understanding, you could perform the previous problem quickly. Could say  $\ell_{ARM} = \frac{V_D}{R - R_D}$ , but  $V_D = R_D \ell_D$  so

$$\ell_{ARM} = \frac{R_D \ell_D}{R - R_D}. \text{ The more}$$

complicated the problem, the more likely you will need to connect the dots methodically

# Kinetics :

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where  $\vec{M} = \vec{r} \times \vec{F}$  &  $\vec{H} = \vec{r} \times \vec{L}$

Kinetics :  $\Sigma \vec{F} = m\vec{a}$  &  $\Sigma \vec{M}_G = \dot{\vec{H}}_G$ ,  
where  $\vec{M} = \vec{r} \times \vec{F}$  &  $\vec{H} = \vec{r} \times \vec{L}$  &  $\vec{L} = m\vec{v}$

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For rigid body :

Kinetics :  $\Sigma \vec{F} = m\vec{a}$  &  $\Sigma \vec{M}_G = \dot{\vec{H}}_G$ ,

where  $\vec{M} = \vec{r} \times \vec{F}$  &  $\vec{H} = \vec{r} \times \vec{L}$  &  $\vec{L} = m\vec{v}$

For rigid body :  $\vec{H}_G = \bar{I} \vec{\omega}$

Kinetics :  $\Sigma \vec{F} = m\vec{a}$  &  $\Sigma \vec{M}_G = \dot{\vec{H}}_G$ ,

where  $\vec{M} = \vec{r} \times \vec{F}$  &  $\vec{H} = \vec{r} \times \vec{L}$  &  $\vec{L} = m\vec{v}$

For rigid body :  $\vec{H}_G = \bar{I} \vec{\omega} \Rightarrow \dot{\vec{H}}_G = \bar{I} \dot{\vec{\omega}}$

Kinetics :  $\Sigma \vec{F} = m\vec{a}$  &  $\Sigma \vec{M}_G = \dot{\vec{H}}_G$ ,

where  $\vec{M} = \vec{r} \times \vec{F}$  &  $\vec{H} = \vec{r} \times \vec{L}$  &  $\vec{L} = m\vec{v}$

For rigid body :  $\vec{H}_G = \bar{I} \vec{\omega} \Rightarrow \dot{\vec{H}}_G = \bar{I} \dot{\vec{\omega}}$

$\Rightarrow \Sigma \vec{M}_G = \bar{I} \dot{\vec{\omega}}$

Kinetics :  $\Sigma \vec{F} = m\vec{a}$  &  $\Sigma \vec{M}_G = \dot{\vec{H}}_G$ ,

where  $\vec{M} = \vec{r} \times \vec{F}$  &  $\vec{H} = \vec{r} \times \vec{L}$  &  $\vec{L} = m\vec{v}$

For rigid body :  $\vec{H}_G = \bar{I} \vec{\omega} \Rightarrow \dot{\vec{H}}_G = \bar{I} \dot{\vec{\omega}}$

$\Rightarrow \Sigma \vec{M}_G = \bar{I} \dot{\vec{\omega}}$  for rotation about

fixed point C:

Kinetics :  $\Sigma \vec{F} = m\vec{a}$  &  $\Sigma \vec{M}_G = \dot{\vec{H}}_G$ ,

where  $\vec{M} = \vec{r} \times \vec{F}$  &  $\vec{H} = \vec{r} \times \vec{L}$  &  $\vec{L} = m\vec{v}$

For rigid body :  $\vec{H}_G = \bar{I} \vec{\omega} \Rightarrow \dot{\vec{H}}_G = \bar{I} \dot{\vec{\omega}}$

$\Rightarrow \Sigma \vec{M}_G = \bar{I} \dot{\vec{\omega}}$  For rotation about

Fixed point C :  $\Sigma \vec{M}_C = I_C \dot{\vec{\omega}}$ ,

Kinetics :  $\Sigma \vec{F} = m\vec{a}$  &  $\Sigma \vec{M}_G = \dot{\vec{H}}_G$ ,

where  $\vec{M} = \vec{r} \times \vec{F}$  &  $\vec{H} = \vec{r} \times \vec{L}$  &  $\vec{L} = m\vec{v}$

For rigid body :  $\vec{H}_G = \bar{I} \vec{\omega} \Rightarrow \dot{\vec{H}}_G = \bar{I} \dot{\vec{\omega}}$

$\Rightarrow \Sigma \vec{M}_G = \bar{I} \dot{\vec{\omega}}$  for rotation about

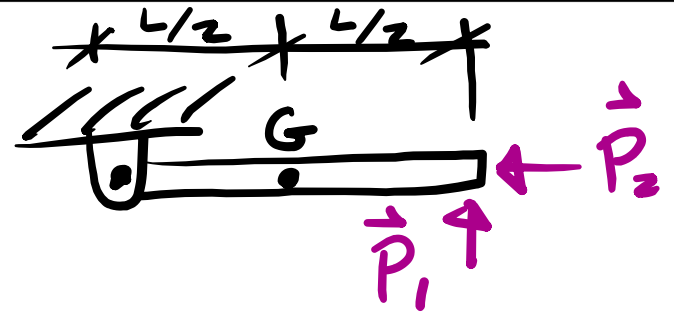
fixed point C :  $\Sigma \vec{M}_C = I_C \dot{\vec{\omega}}$ , where

$$I_C = \bar{I} + M r_{G/C}^2$$

Example problem:

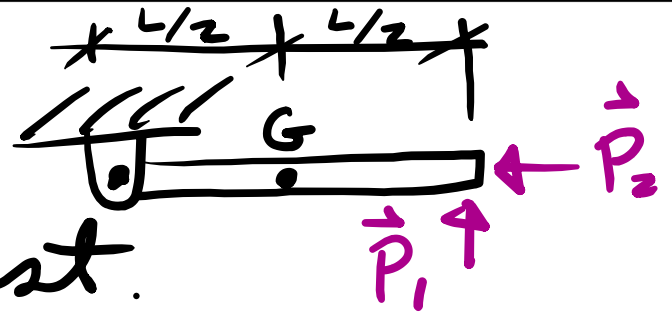


# Example problem:

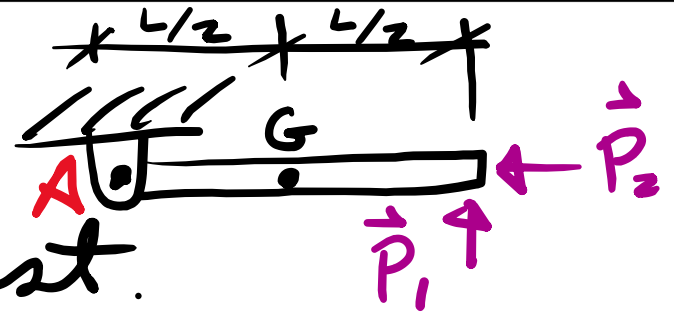


# Example problem:

Arm is initially at rest.

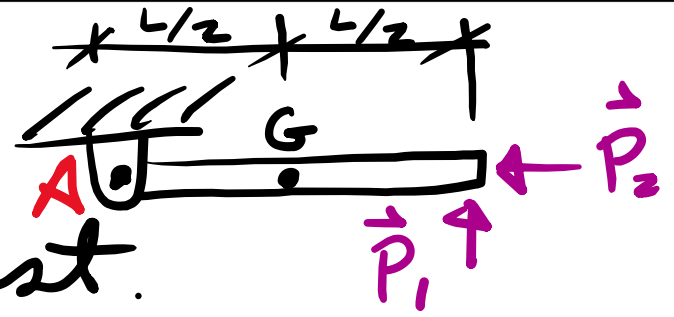


# Example problem:



Arm is initially at rest.  
Find reaction force at  $A$

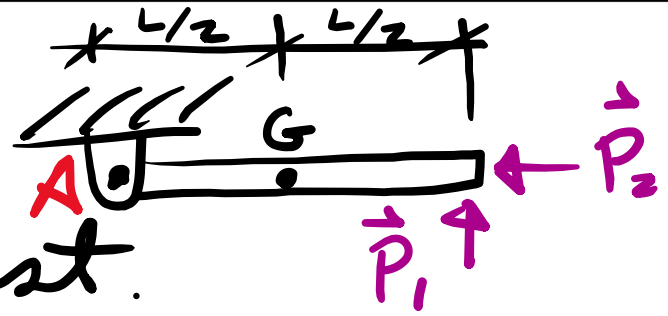
# Example problem:



Arm is initially at rest.

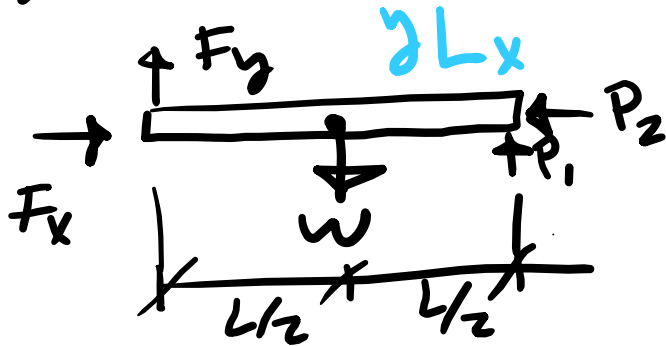
Find reaction force at  $A$  & arm

# Example problem:

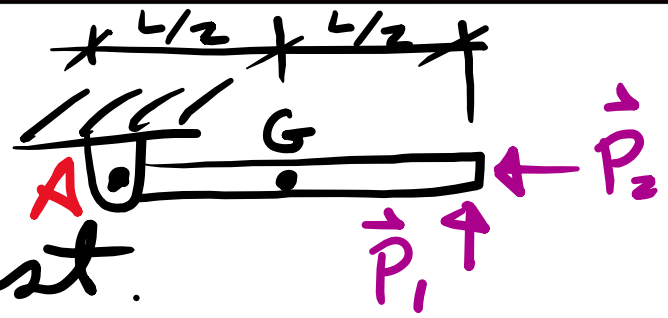


Arm is initially at rest.

Find reaction force at A & arm

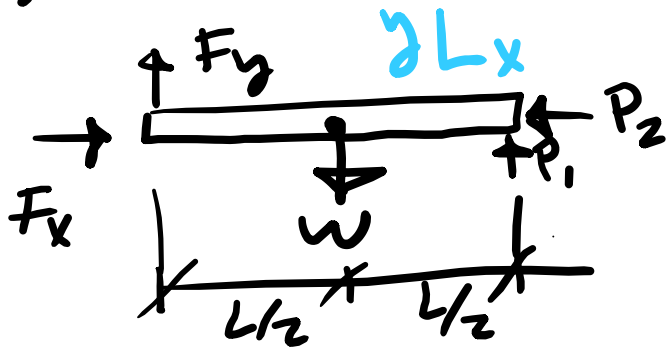


# Example problem:



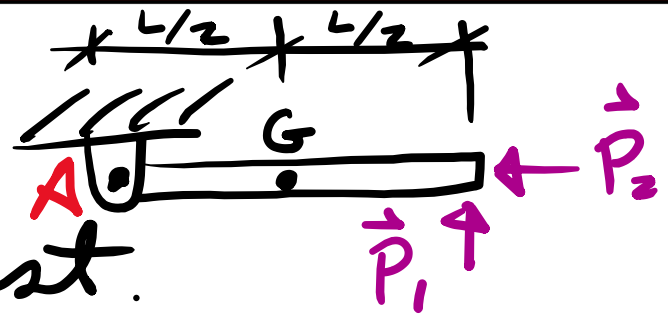
Arm is initially at rest.

Find reaction force at A & arm



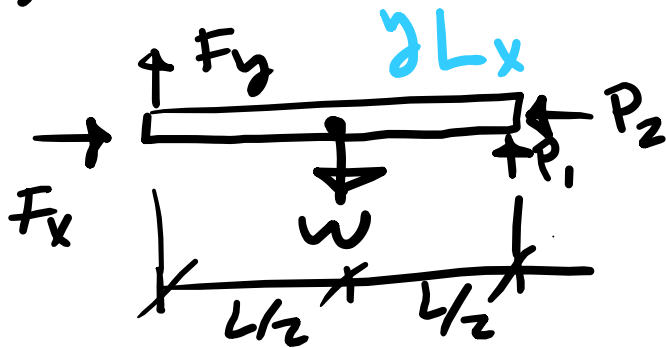
$$\Sigma F_x = m\bar{a}_x$$

# Example problem:



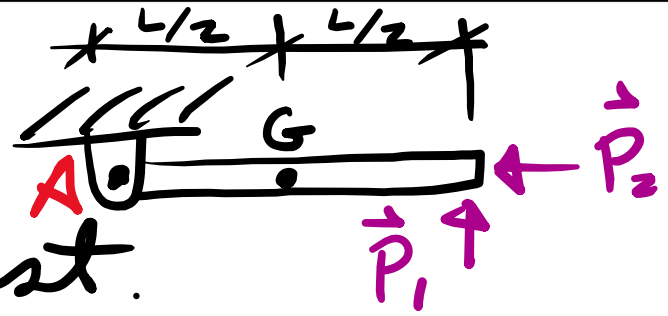
Arm is initially at rest.

Find reaction force at A & arm



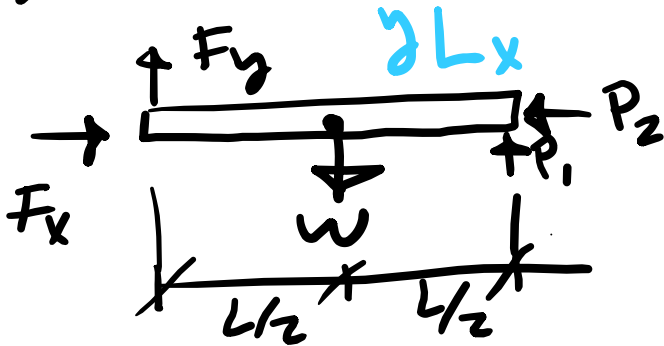
$$\Sigma F_x = m \ddot{x}$$

# Example problem:



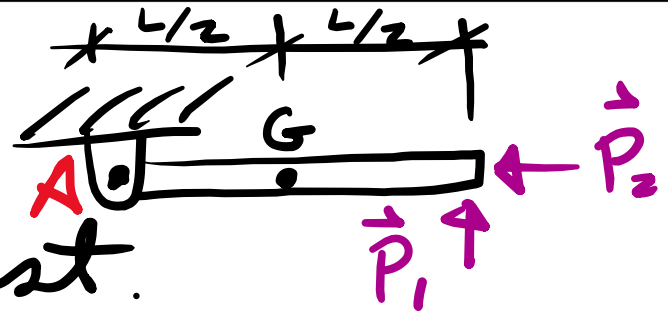
Arm is initially at rest.

Find reaction force at A &  $\alpha_{ARM}$



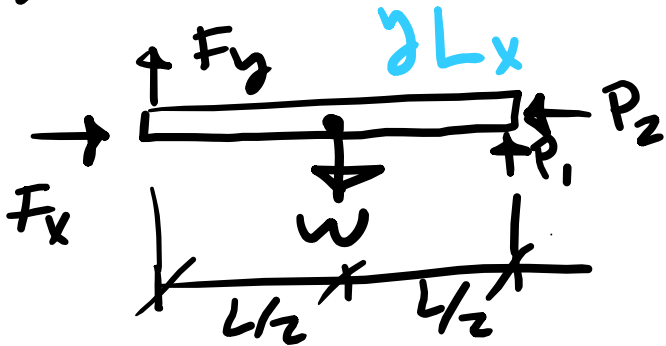
$$\Sigma F_x = m \cancel{a_x} \Rightarrow F_x = P_2$$

# Example problem:



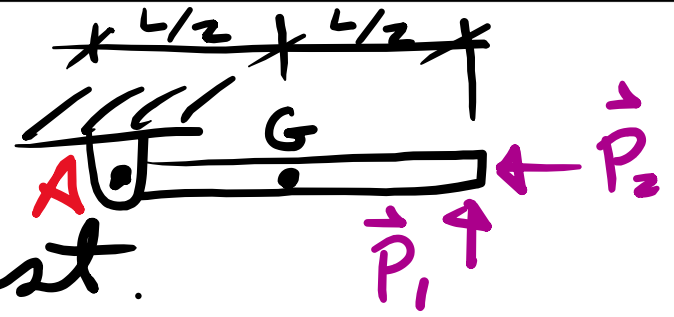
Arm is initially at rest.

Find reaction force at A &  $\alpha_{ARM}$



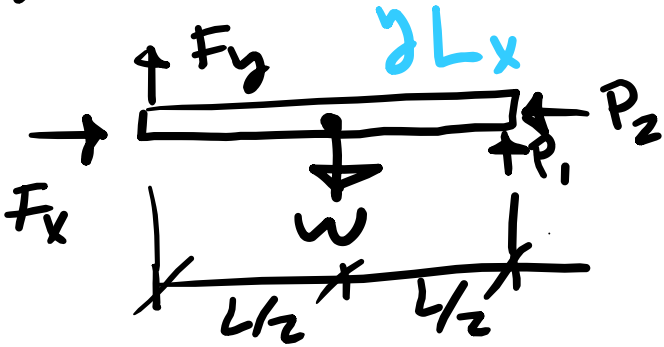
$$\Sigma F_x = m \cancel{a_x} \Rightarrow F_x = P_2 \quad (1)$$

# Example problem:



Arm is initially at rest.

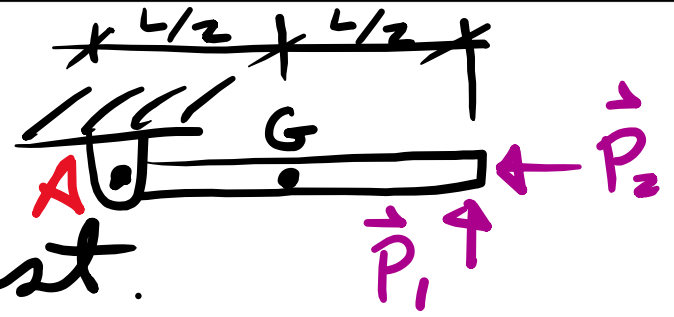
Find reaction force at A &  $\alpha_{ARM}$



$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

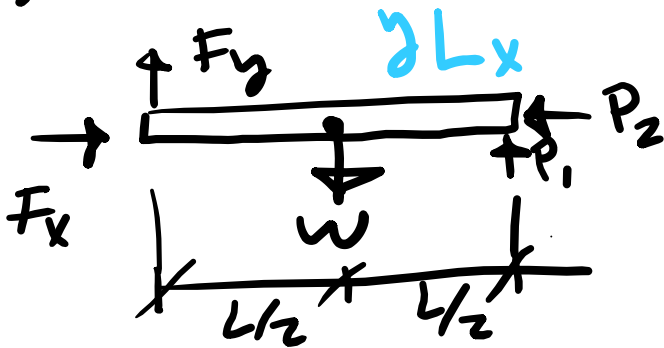
$$\Sigma F_y = m \bar{a}_y$$

# Example problem:



Arm is initially at rest.

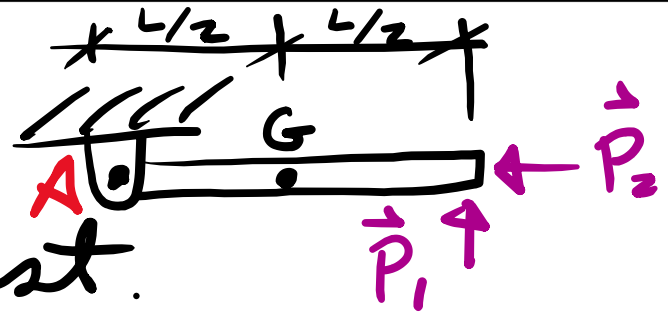
Find reaction force at A &  $\alpha_{ARM}$



$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

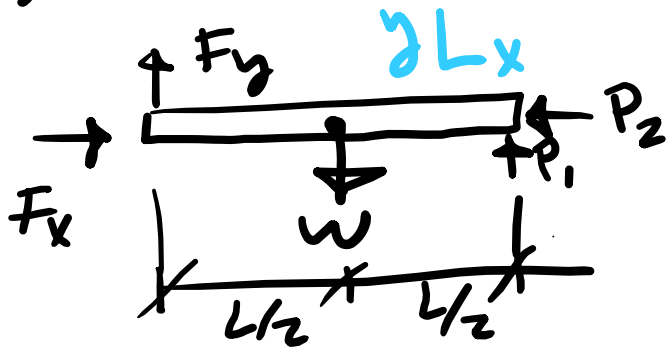
$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - w = m \bar{a}_y$$

# Example problem:



Arm is initially at rest.

Find reaction force at A &  $\alpha$  of arm

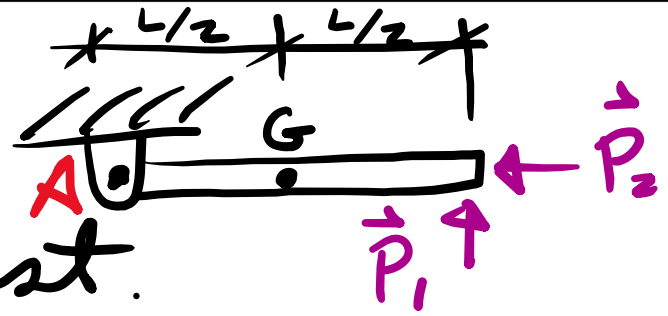


$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - w = m \bar{a}_y$$

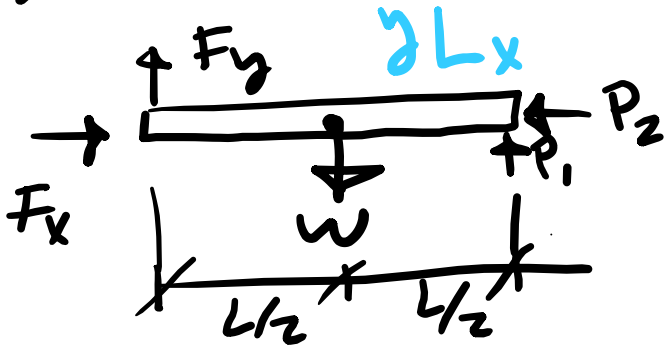
$$\text{But } \bar{a}_y = \alpha \frac{L}{2}$$

# Example problem:



Arm is initially at rest.

Find reaction force at A &  $\alpha$  of arm



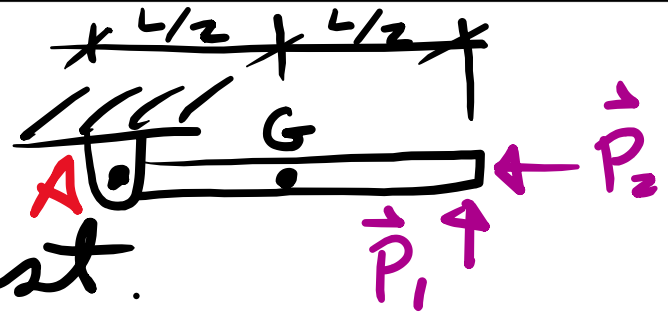
$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - W = m \bar{a}_y$$

But  $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

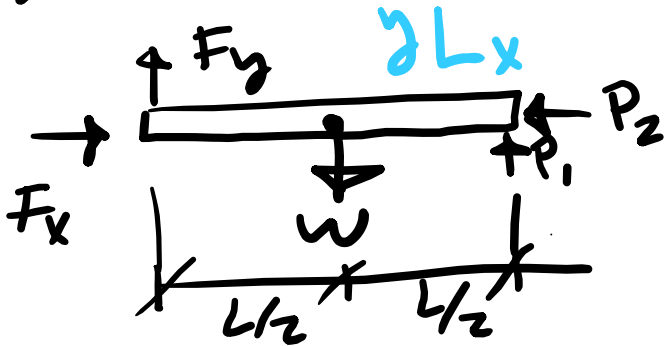
$$F_y + P_1 - W = m \left( \frac{L}{2} \right) \alpha$$

# Example problem:



Arm is initially at rest.

Find reaction force at A &  $\alpha$  of arm



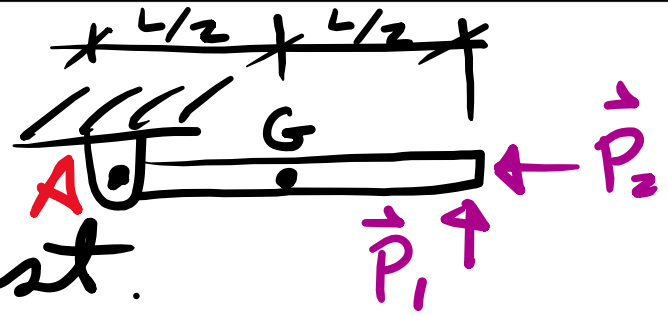
$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - W = m \bar{a}_y$$

But  $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

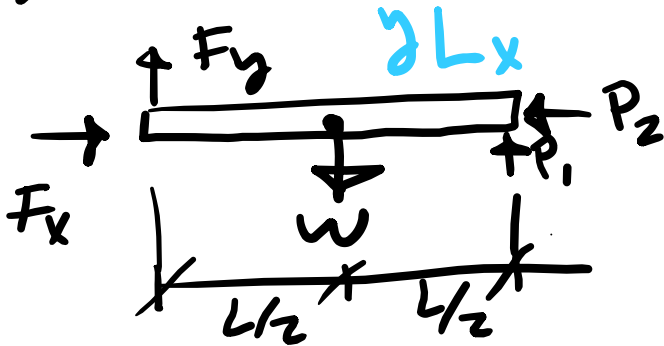
$$F_y + P_1 - W = m \left( \frac{L}{2} \right) \alpha \quad (2)$$

# Example problem:



Arm is initially at rest.

Find reaction force at A &  $\alpha$  of arm



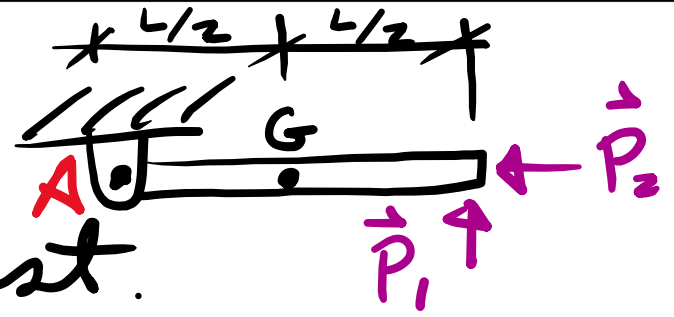
$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - W = m \bar{a}_y$$

But  $\bar{a}_y = \alpha \frac{L}{2}$  so

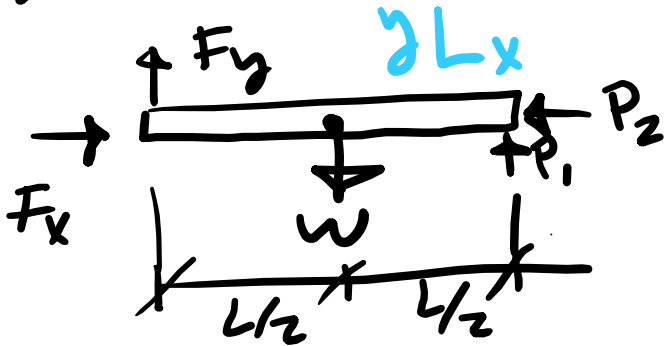
$$F_y + P_1 - W = m \left(\frac{L}{2}\right) \alpha \quad (2) \quad \& \Sigma M_A = I_A \alpha$$

# Example problem:



Arm is initially at rest.

Find reaction force at A &  $\alpha$  of arm



$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

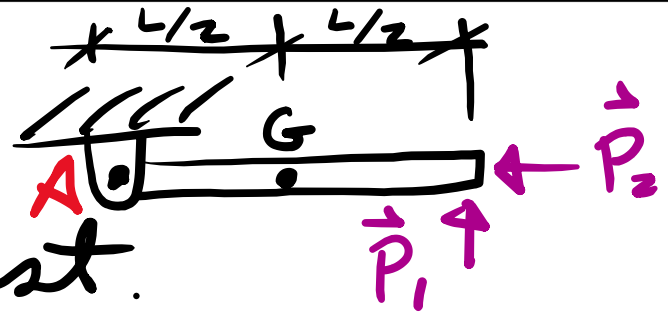
$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - W = m \bar{a}_y$$

But  $\bar{a}_y = \alpha \frac{L}{2}$  so

$$F_y + P_1 - W = m \left( \frac{L}{2} \right) \alpha \quad (2) \quad \Sigma M_A = I_A \alpha \Rightarrow$$

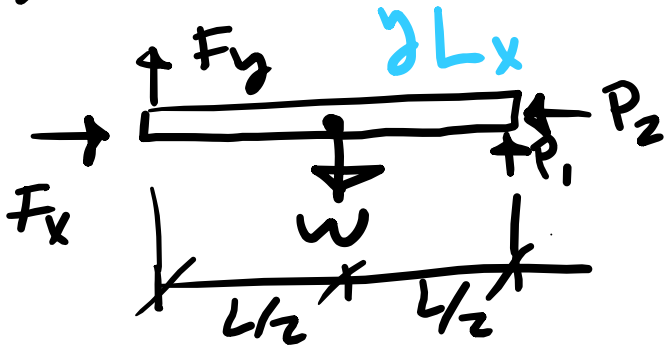
$$-W \frac{L}{2} + P_1 L = I_A \alpha$$

# Example problem:



Arm is initially at rest.

Find reaction force at A &  $\alpha$  of arm



$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

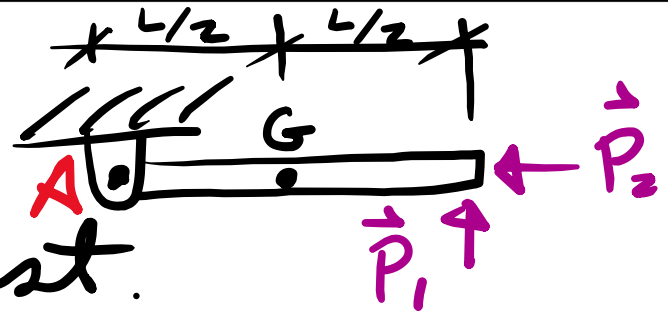
$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - W = m \bar{a}_y$$

But  $\bar{a}_y = \alpha \frac{L}{2}$  so

$$F_y + P_1 - W = m \left( \frac{L}{2} \right) \alpha \quad (2) \quad \Sigma M_A = I_A \alpha \Rightarrow$$

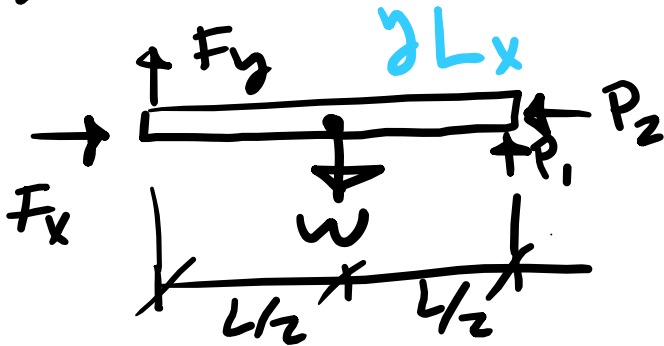
$$-W \frac{L}{2} + P_1 L = I_A \alpha \Rightarrow \alpha = \frac{P_1 L - W \frac{L}{2}}{I_A}$$

# Example problem:



Arm is initially at rest.

Find reaction force at A &  $\alpha$  of arm



$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - W = m \bar{a}_y$$

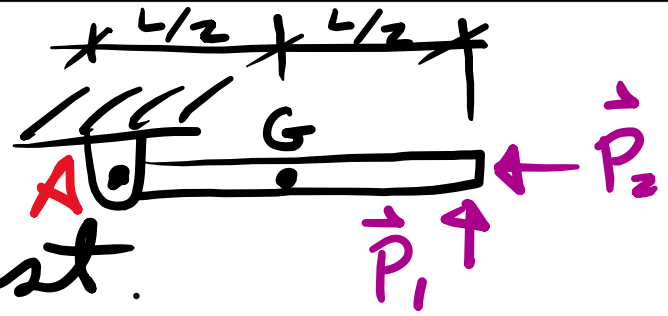
But  $\bar{a}_y = \alpha \frac{L}{2}$  so

$$F_y + P_1 - W = m \left( \frac{L}{2} \right) \alpha \quad (2) \quad \Sigma M_A = I_A \alpha \Rightarrow$$

$$-W \frac{L}{2} + P_1 L = I_A \alpha \Rightarrow \alpha = \frac{P_1 L - W \frac{L}{2}}{I_A} \text{ But}$$

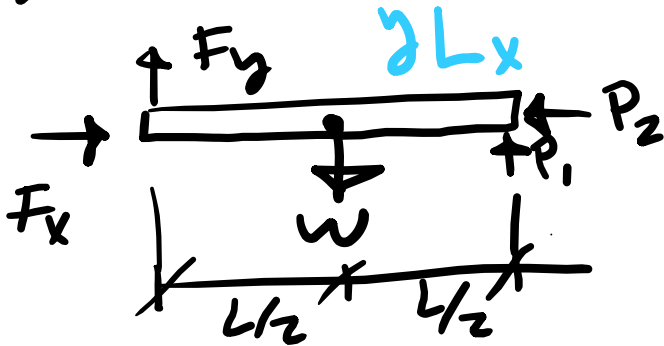
$$I_A = \frac{M L^2}{12} + M \frac{L^2}{4}$$

# Example problem:



A rod is initially at rest.

Find reaction force at A &  $\alpha$  of rod



$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - W = m \bar{a}_y$$

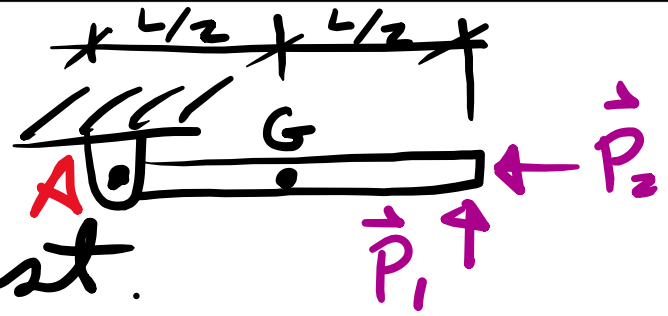
But  $\bar{a}_y = \alpha \frac{L}{2}$  so

$$F_y + P_1 - W = m \left( \frac{L}{2} \right) \alpha \quad (2) \quad \Sigma M_A = I_A \alpha \Rightarrow$$

$$-W \frac{L}{2} + P_1 L = I_A \alpha \Rightarrow \alpha = \frac{P_1 L - W \frac{L}{2}}{I_A} \text{ But}$$

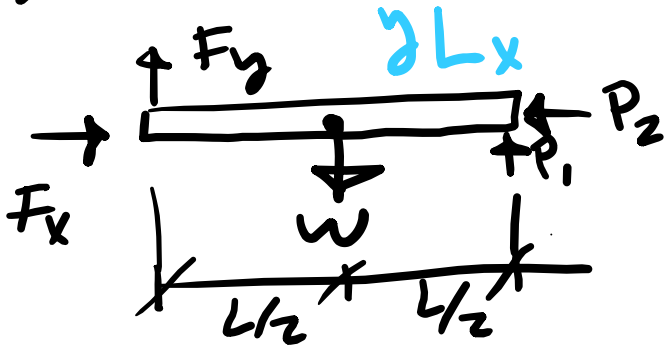
$$I_A = \frac{M L^2}{12} + M \frac{L^2}{4} = \frac{M L^2}{3}$$

# Example problem:



A rod is initially at rest.

Find reaction force at A &  $\alpha$  of rod



$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - W = m \bar{a}_y$$

But  $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

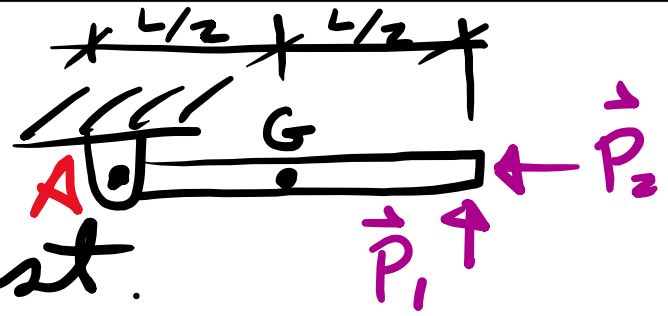
$$F_y + P_1 - W = m \left( \frac{L}{2} \right) \alpha \quad (2) \quad \Sigma M_A = I_A \alpha \Rightarrow$$

$$-W \frac{L}{2} + P_1 L = I_A \alpha \Rightarrow \alpha = \frac{P_1 L - W \frac{L}{2}}{I_A} \text{ But}$$

$$I_A = \frac{ML^2}{12} + M \frac{L^2}{4} = \frac{ML^2}{3}$$

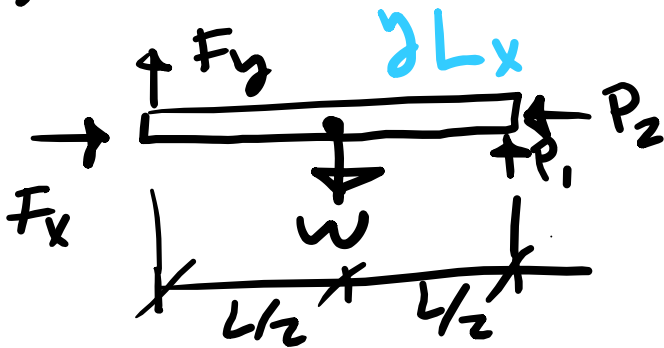
$$\alpha = \left[ \frac{P_1 - W/2}{ML/3} \right] \alpha$$

# Example problem:



A rod is initially at rest.

Find reaction force at A &  $\alpha$  of rod



$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - w = m \bar{a}_y$$

But  $\bar{a}_y = \alpha \frac{L}{2} \neq 0$

$$F_y + P_1 - w = m \left(\frac{L}{2}\right) \alpha \quad (2) \quad \Sigma M_A = I_A \alpha \Rightarrow$$

$$-w \frac{L}{2} + P_1 L = I_A \alpha \Rightarrow \alpha = \frac{P_1 L - w \frac{L}{2}}{I_A} \text{ But}$$

$$I_A = \frac{ML^2}{12} + M \frac{L^2}{4} = \frac{ML^2}{3}$$

Equation 2 now

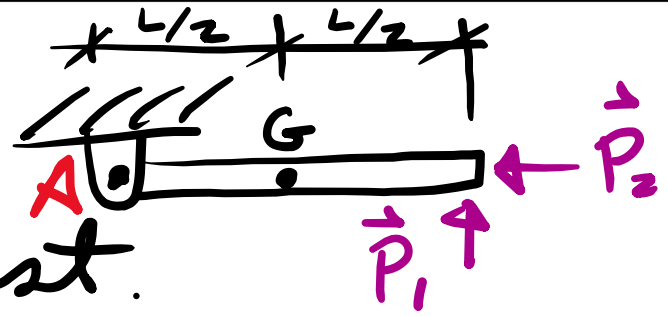
$$\alpha = \left[ \frac{P_1 - w/2}{ML/3} \right] \alpha$$



gives

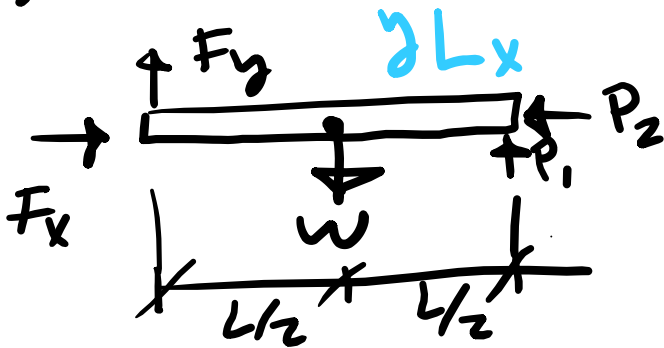
$$F_y = w - P_1 + \left(\frac{3}{2}\right) [P_1 - w/2]$$

# Example problem:



A rod is initially at rest.

Find reaction force at A &  $\alpha$  of rod



$$\Sigma F_x = m \bar{a}_x \Rightarrow F_x = P_2 \quad (1)$$

$$\Sigma F_y = m \bar{a}_y \Rightarrow F_y + P_1 - W = m \bar{a}_y$$

But  $\bar{a}_y = \alpha \frac{L}{2}$  so

$$F_y + P_1 - W = m \left(\frac{L}{2}\right) \alpha \quad (2) \quad \Sigma M_A = I_A \alpha \Rightarrow$$

$$-W \frac{L}{2} + P_1 L = I_A \alpha \Rightarrow \alpha = \frac{P_1 L - W \frac{L}{2}}{I_A} \text{ But}$$

$$I_A = \frac{ML^2}{12} + M \frac{L^2}{4} = \frac{ML^2}{3}$$

Equation 2 now

$$\alpha = \left[ \frac{P_1 - W/2}{ML/3} \right] \alpha$$



gives

$$F_y = W - P + \left(\frac{3}{2}\right) [P_1 - W/2]$$

$$\Rightarrow F_y = \frac{W}{4} + \frac{P_1}{2}$$

# Work

Work

$$U_{1 \rightarrow 2} = \Delta T,$$

Work  $U_{1 \rightarrow 2} = \Delta T$ , for moment couple

$$U_{1 \rightarrow 2} = \int_{\theta_1}^{\theta_2} M d\theta$$

Work  $U_{1 \rightarrow 2} = \Delta T$ , for moment couple

$$U_{1 \rightarrow 2} = \int_{\theta_1}^{\theta_2} M d\theta \quad \& \quad T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2$$

Work  $U_{1 \rightarrow 2} = \Delta T$ , for moment couple

$$U_{1 \rightarrow 2} = \int_{\theta_1}^{\theta_2} M d\theta \quad \& \quad T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2.$$

About a fixed point C:

## Work

$U_{1 \rightarrow 2} = \Delta T$ , for moment couple

$$U_{1 \rightarrow 2} = \int_{\theta_1}^{\theta_2} M d\theta \quad \& \quad T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2.$$

About a fixed point C:  $T = \frac{1}{2} I_C \omega^2$

## Work

$U_{1 \rightarrow 2} = \Delta T$ , for moment couple

$$U_{1 \rightarrow 2} = \int_{\theta_1}^{\theta_2} M d\theta \quad \& \quad T = \frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2.$$

About a fixed point C:  $T = \frac{1}{2} I_C \omega^2$

Example for T:

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Example for T: Disk rolling &

not slipping

## Work

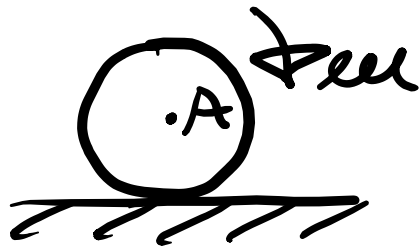
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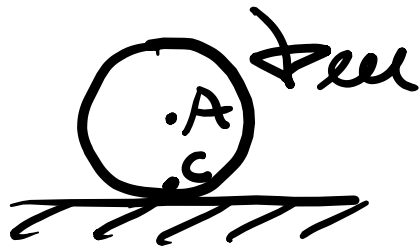
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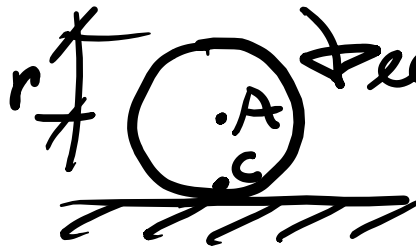
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Here  $\bar{v} = r \bar{\omega}$

## Work

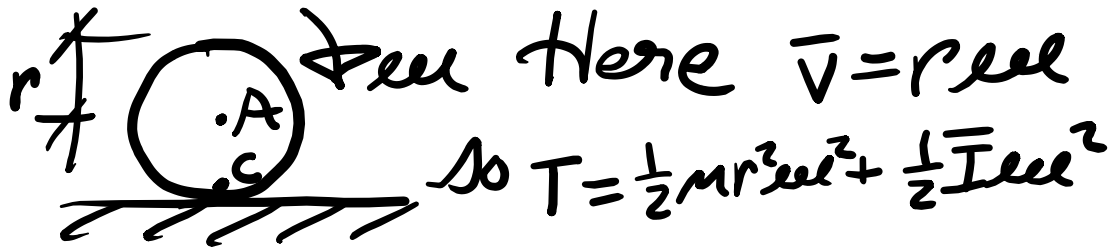
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## Work

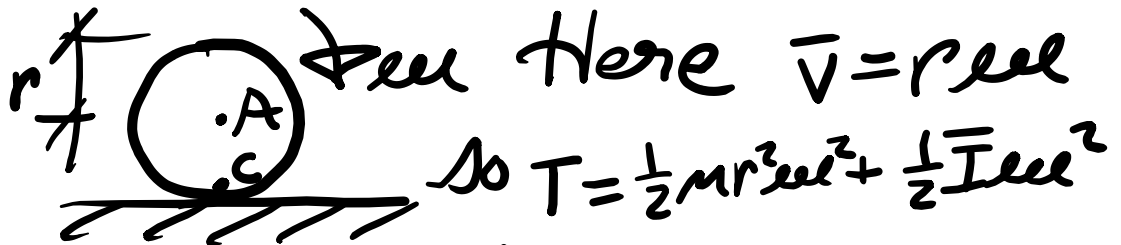
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If we looked at the problem as a rotation about point C:

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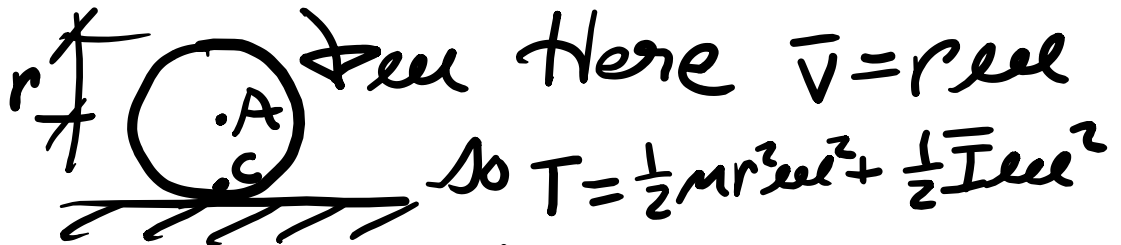
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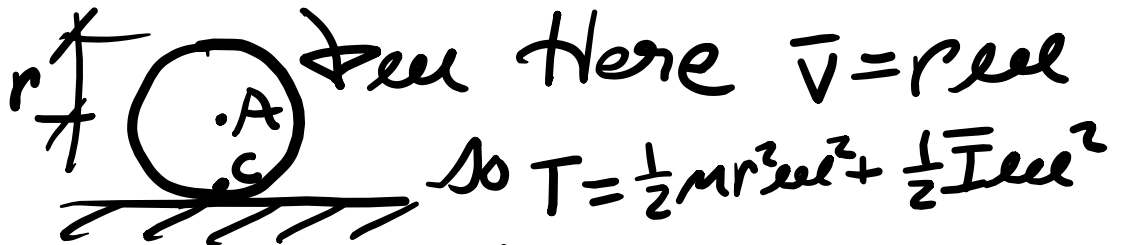
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If we looked at the problem as a rotation about point C:  $T = \frac{1}{2} I_C \omega^2$ , where

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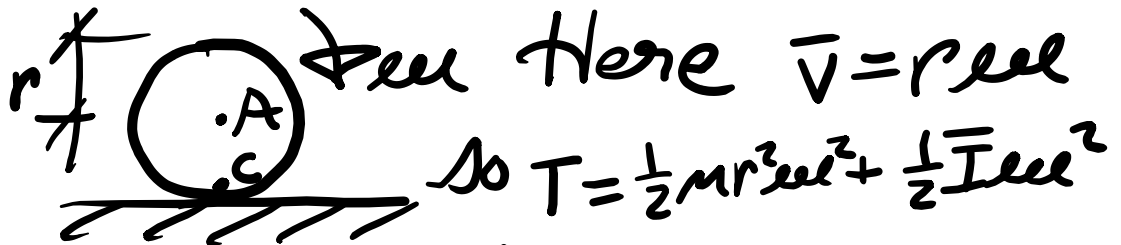
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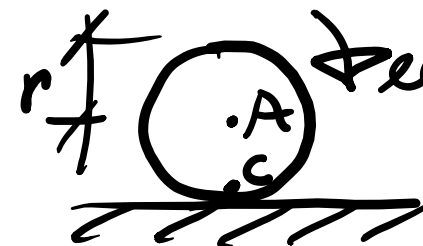
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Example for T: Disk rolling &

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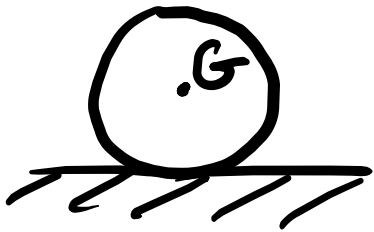
[Same as before]

Example: Disk rolling w/out slipping while brake is applied.

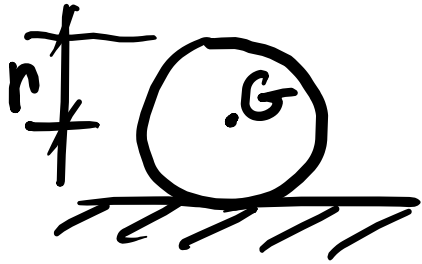
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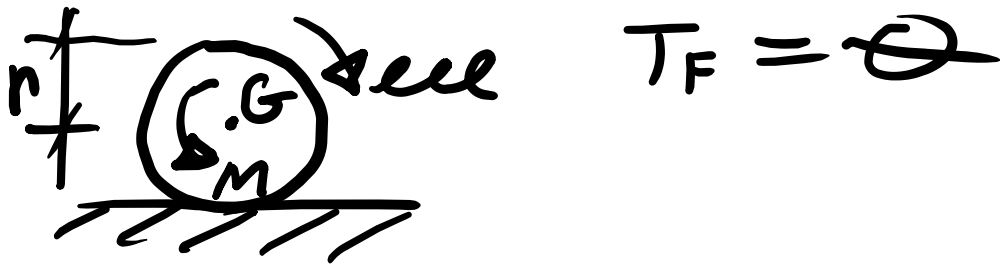
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$T_I - M\frac{d}{r} = 0 \Rightarrow T_I = M\frac{d}{r} \Rightarrow d = \frac{T_I r}{M}$

# Energy conservation

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$$T_I + V_I + U_{I \rightarrow F}^{NC} = T_F + V_F$$

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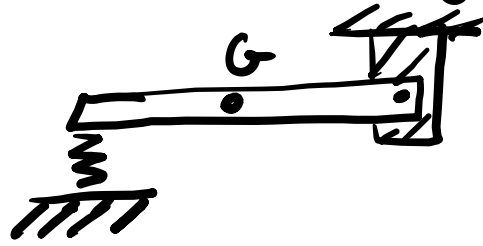
If only conservative forces, then  $U_{I \rightarrow F}^{NC} = 0$

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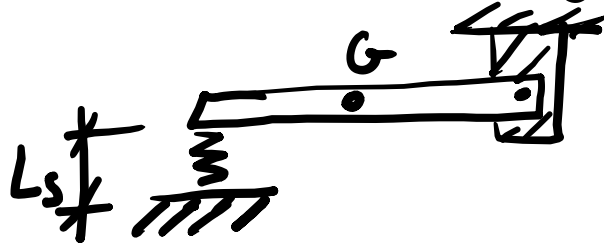


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## Example

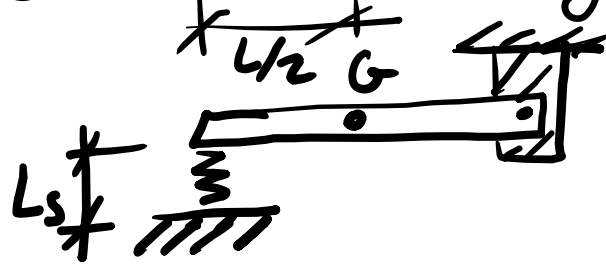


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## Example

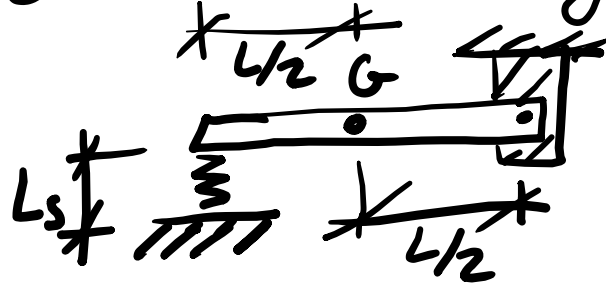


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## Example

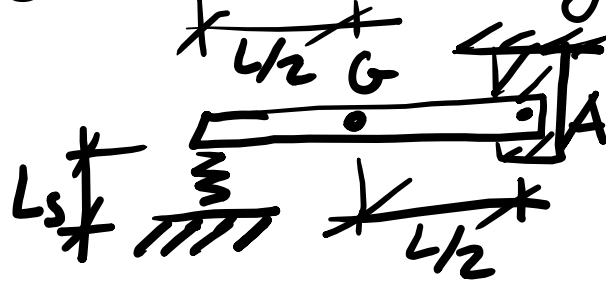


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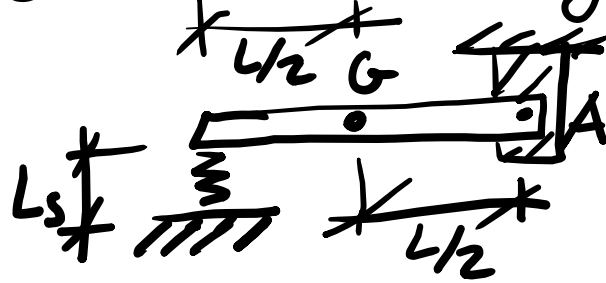


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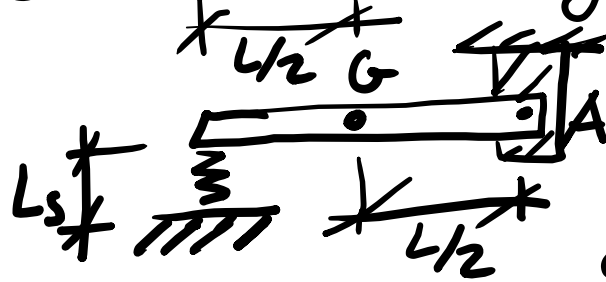
Mass of arm =  $M$

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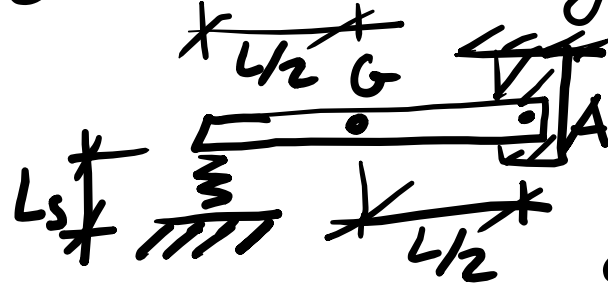
Mass of arm =  $M$   
Natural length of spring =  $L_0$

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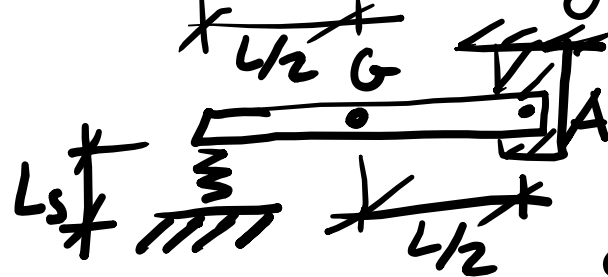
Arm pivots about point  $A$ .

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## Example



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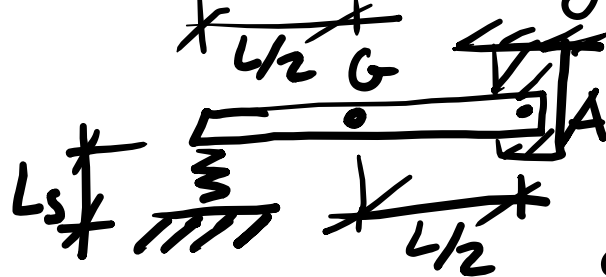
Arm pivots about point  $A$ . Initially at rest, the arm compressed a spring with spring constant  $k$ .

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$$T_I + V_I + U_{I \rightarrow F}^{NC} = T_F + V_F$$

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## Example



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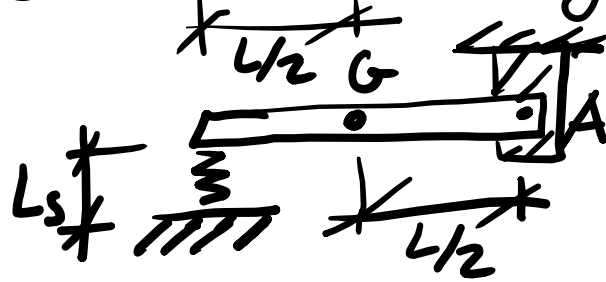
Arm pivots about point  $A$ . Initially at rest, the arm compressed a spring with spring constant  $k$ . What is the angular velocity when the arm is vertical (rotates  $90^\circ$  upwards)?

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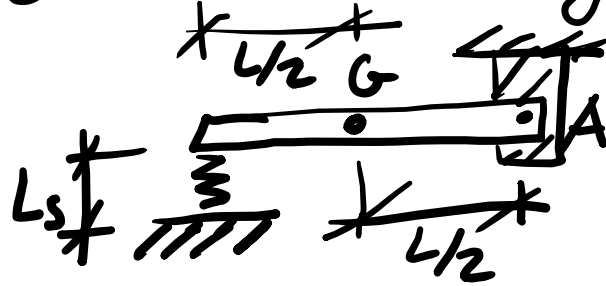
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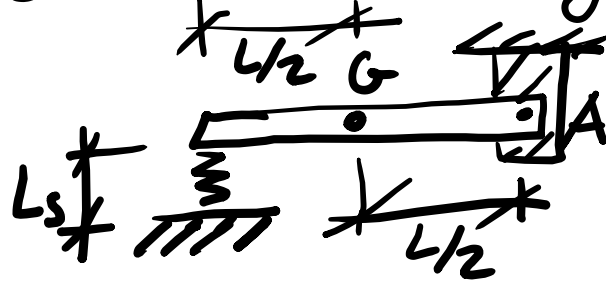
$$T_I = 0, V_I = \frac{1}{2} k x_I^2$$

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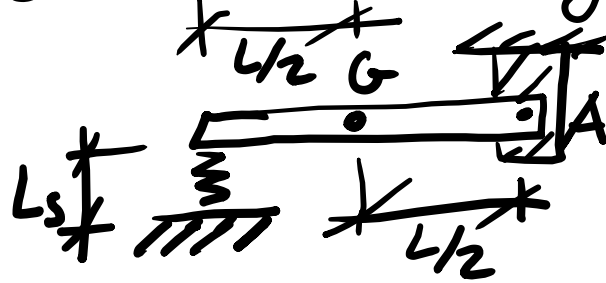
$$T_I = 0, V_I = \frac{1}{2} k x_I^2$$
$$V_F = mgh$$

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$$T_I = 0, V_I = \frac{1}{2} k x_I^2$$
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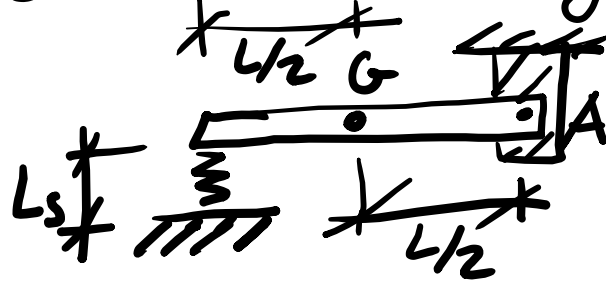
$$h = L/2$$

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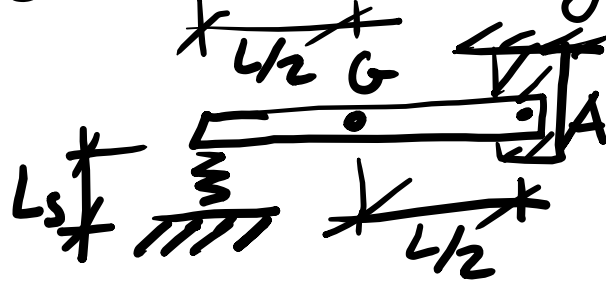
$$h = \frac{L}{2} \quad \text{Now} \quad T_I + V_I = T_F + V_F$$

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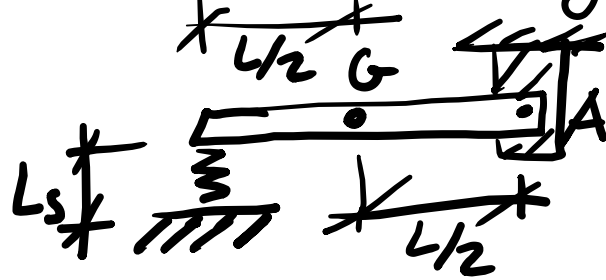
Now  ~~$T_I + V_I = T_F + V_F$~~

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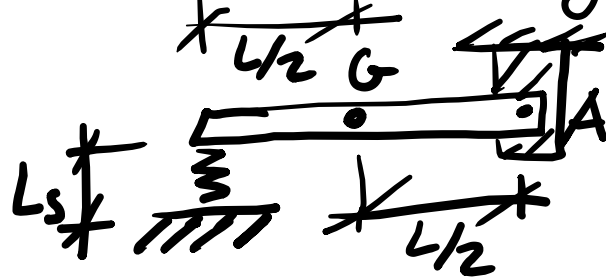
$$\frac{1}{2} k x_I^2 = \frac{1}{2} I_A \omega_F^2 + mg \frac{L}{2}$$

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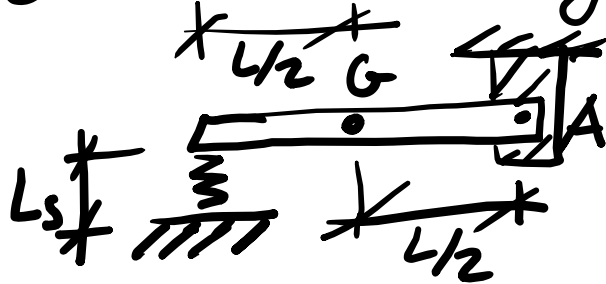
$$\frac{1}{2} k x_I^2 = \underbrace{\frac{1}{2} I_A \omega_F^2}_{T_F} + mg \frac{L}{2}$$

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$$T_I + V_I + U_{I \rightarrow F}^{NC} = T_F + V_F$$

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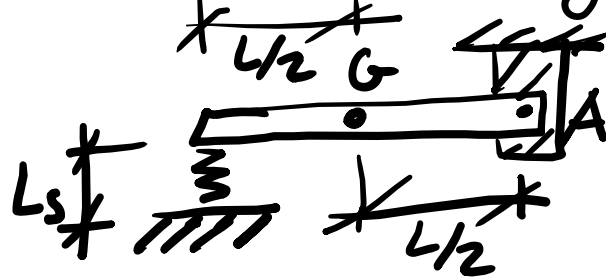
$$\frac{1}{2} k x_I^2 = \frac{1}{2} I_A \omega_F^2 + mg \frac{L}{2} \Rightarrow \frac{1}{2} I_A \omega_F^2 = \frac{1}{2} k x_I^2 - mg \frac{L}{2}$$

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## Example



$$T_I = 0, V_I = \frac{1}{2} k x_I^2$$

$$V_F = mgh, \text{ where}$$

$h = L/2$  Now  ~~$T_I + V_I = T_F + V_F$~~   $\Rightarrow$

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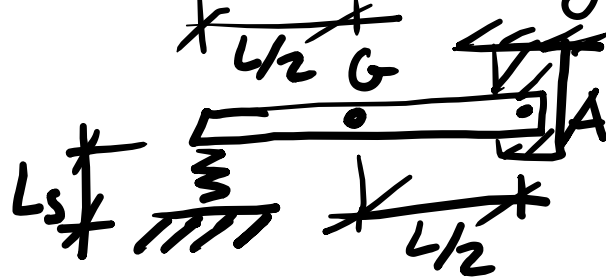
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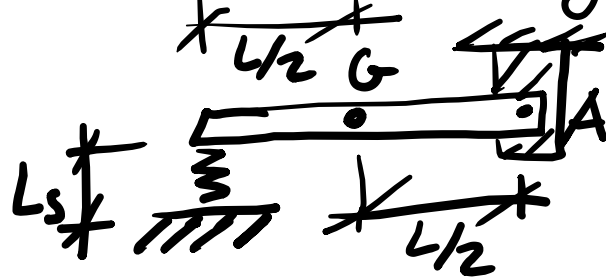
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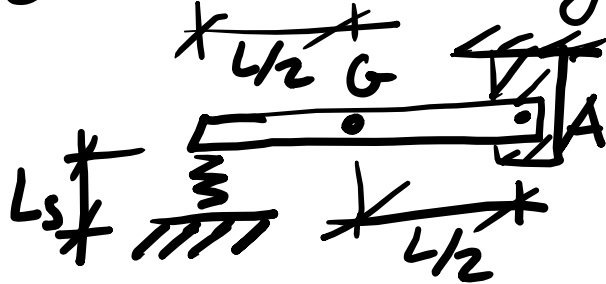
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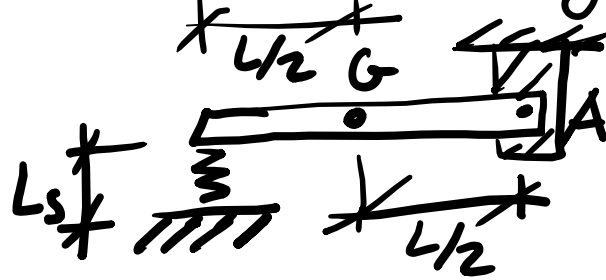
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$$\omega_F = \sqrt{\frac{3[k(L_s - L_0)^2 - mgL]}{ML^2}}$$

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Note:  $M$   
is a brake

so  $M < 0$

if  $\omega_I > 0$



