Today 19.1

$\boxed{L21}$

Today 19.1

L21

Vibrations without damping

$\boxed{L21}$

<u>Today</u> 19.1

<u>Tuesday</u> 19.2, 19.3

Today 19.1

$\boxed{L21}$

Tuesday $\underbrace{19.2}$, 19.3

Free vibrations of rigid bodies

Today 19.1

$\boxed{\text{L21}}$

Tuesday 19.2, $\underbrace{19.3}_{\text{Energy Methods for vibrations}}$

Today 19.1

$\boxed{L21}$

Tuesday 19.2, 19.3

Thursday April 8th 19.4

$\boxed{\text{L21}}$

Today 19.1

Tuesday 19.2, 19.3

Thursday April 8th $\underbrace{19.4}_{\text{Forced vibrations}}$

L21

Today 19.1

Tuesday 19.2, 19.3

Thursday April 8th 19.4

Tuesday April 13th 19.5

L21

Today 19.1

Tuesday 19.2, 19.3

Thursday April 8th 19.4

Tuesday April 13th $\underbrace{19.5}_{\text{Damped Vibrations}}$

L21

Today 19.1

Tuesday 19.2, 19.3

Thursday April 8th 19.4

Tuesday April 13th 19.5

Thursday April 15th Review

Today 19.1

L21

Tuesday 19.2, 19.3

Thursday April 8th 19.4

Tuesday April 13th 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

L21

Today 19.1

Tuesday 19.2, 19.3

Thursday April 8th 19.4

Tuesday April 13th 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

Thursday April 22nd Day of reckoning

L21

Today 19.1

Tuesday 19.2, 19.3

Thursday April 8th 19.4

Tuesday April 13th 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

Thursday April 22nd Day of reckoning

will know grade for course if you decide not to take final exam

Today 19.1

L21

Tuesday 19.2, 19.3

Thursday April 8th 19.4

Tuesday April 13th 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

Thursday April 22nd Day of reckoning

Thursday April 29th Final exam
from 7:30 AM to
9:20 AM

Let there be some function of t: $f(t)$, such that $\frac{d^2 f}{dt^2} = -\omega^2 f$,

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$\frac{d}{dt} \sin(\omega t)$

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$$\frac{d}{dt}\sin(\omega t) = \omega\cos(\omega t)$$

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Look like we found two solutions

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Look like we found two solutions ☺

Now we can write $f_n = A\sin(\omega t) + B\cos(\omega t)$, Now we can write $f_h = A\sin(\omega t) + B\cos(\omega t)$, where A & B are constants that have to be determined by initial conditions

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For example, we could be told that $\omega = 1$ & $f_1(t=0) = 5 \Rightarrow$

$B = 5$ & $\dot{f}_1(t=0) = 3$

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so $A = 3/\omega = 3$

Now we can write $f = A\sin(\omega t) + B\cos(\omega t)$, where A & B are constants that have to be determined by initial conditions

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Now $f(t) = 3\sin(t) + 5\cos(t)$

Note: $A\sin(x) + B\cos(x) = C\sin(x+\phi)$, Note: $A\sin(x) + B\cos(x) = C\sin(x+\phi)$,

where $C = [A^2 + B^2]^{1/2}$ & $\phi = \tan^{-1}\left(\frac{-B}{A}\right)$

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This means we can trade the unknowns A & B in $f_h = A\sin(\omega t) + B\cos(\omega t)$

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This means we can trade the unknowns A & B in $f_h = A\sin(\omega t) + B\cos(\omega t)$ for the unknowns C & ϕ in

$$f_h = C\sin(\omega t + \phi)$$

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Anatomy of a wave:

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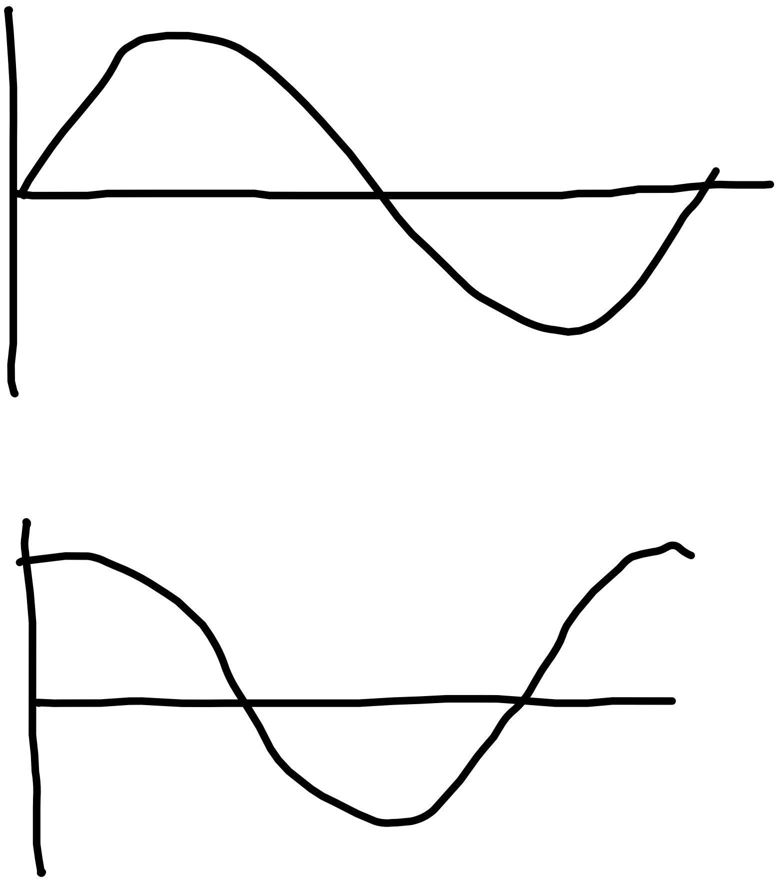
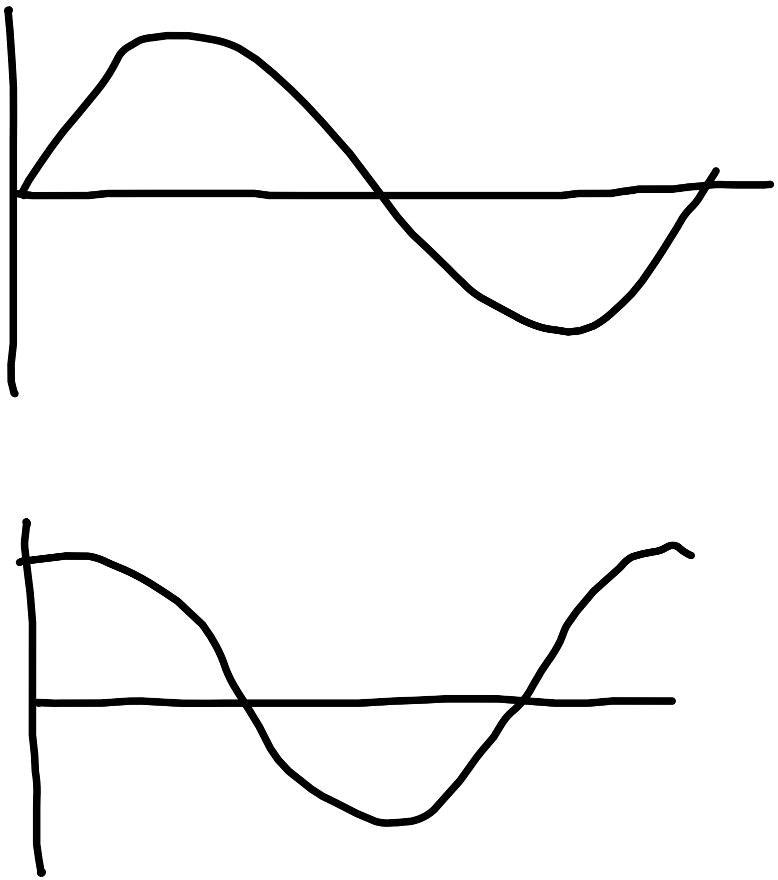
Let subscript "n" denote "natural" so that $\omega_n \equiv$ natural circular frequence



Anatomy of a wave:

Let subscript "n" denote "natural" so that $\omega_n \equiv$ natural circular frequence & $\tau_n \equiv$ natural period



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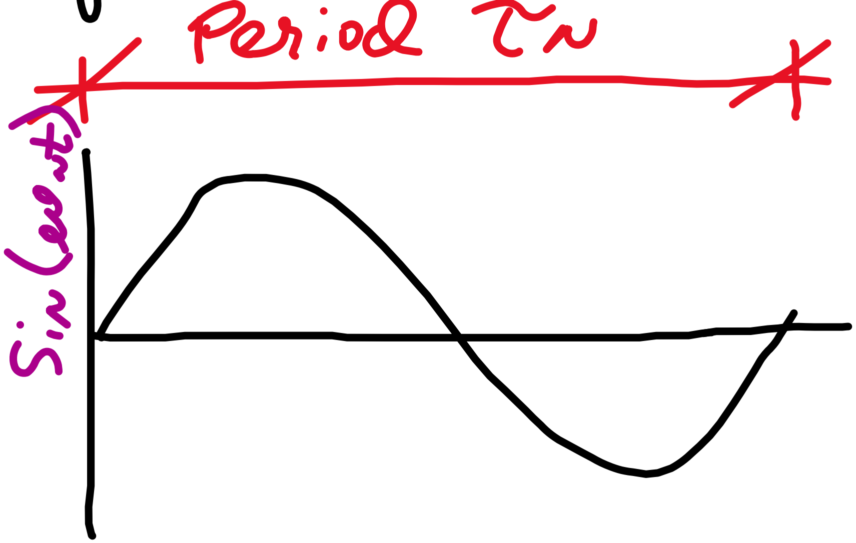

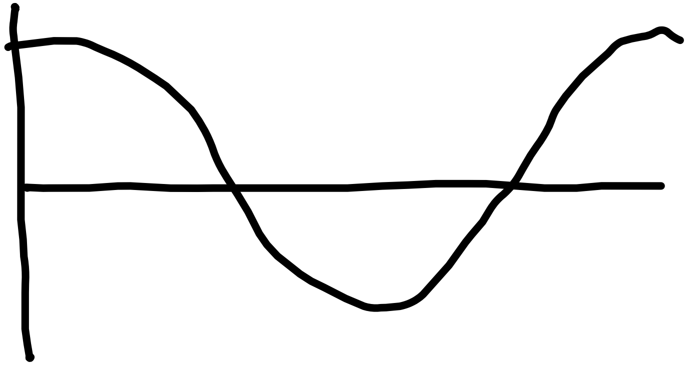

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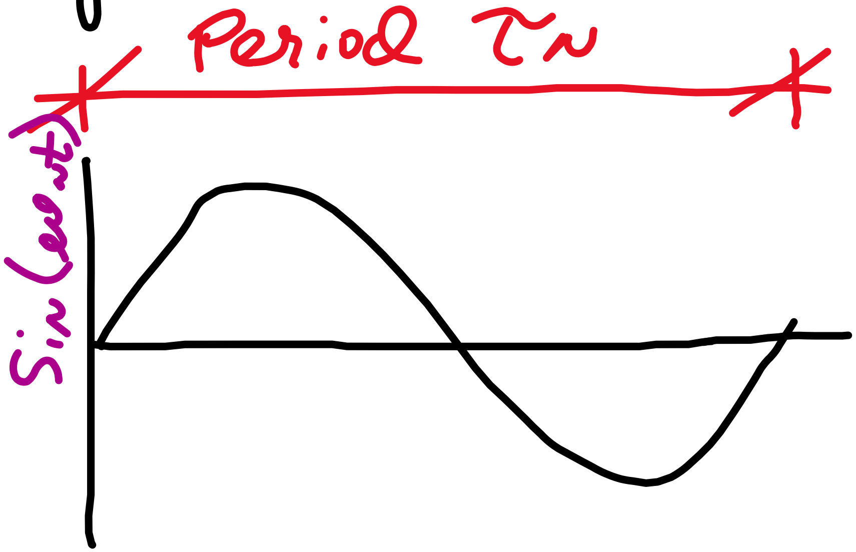

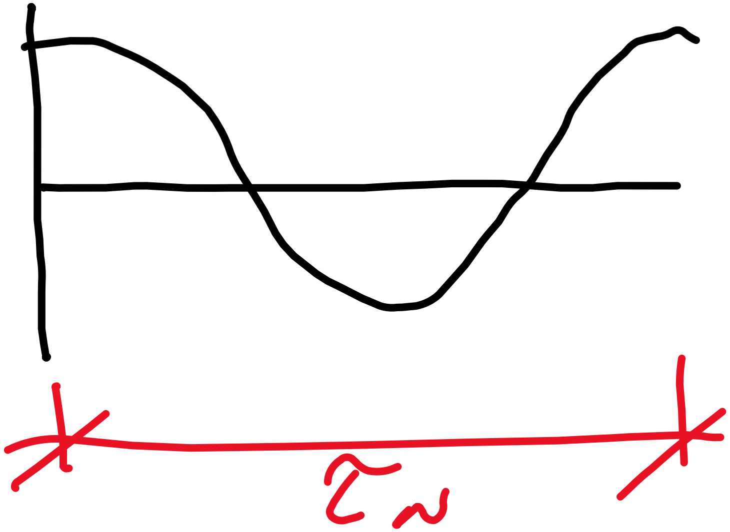

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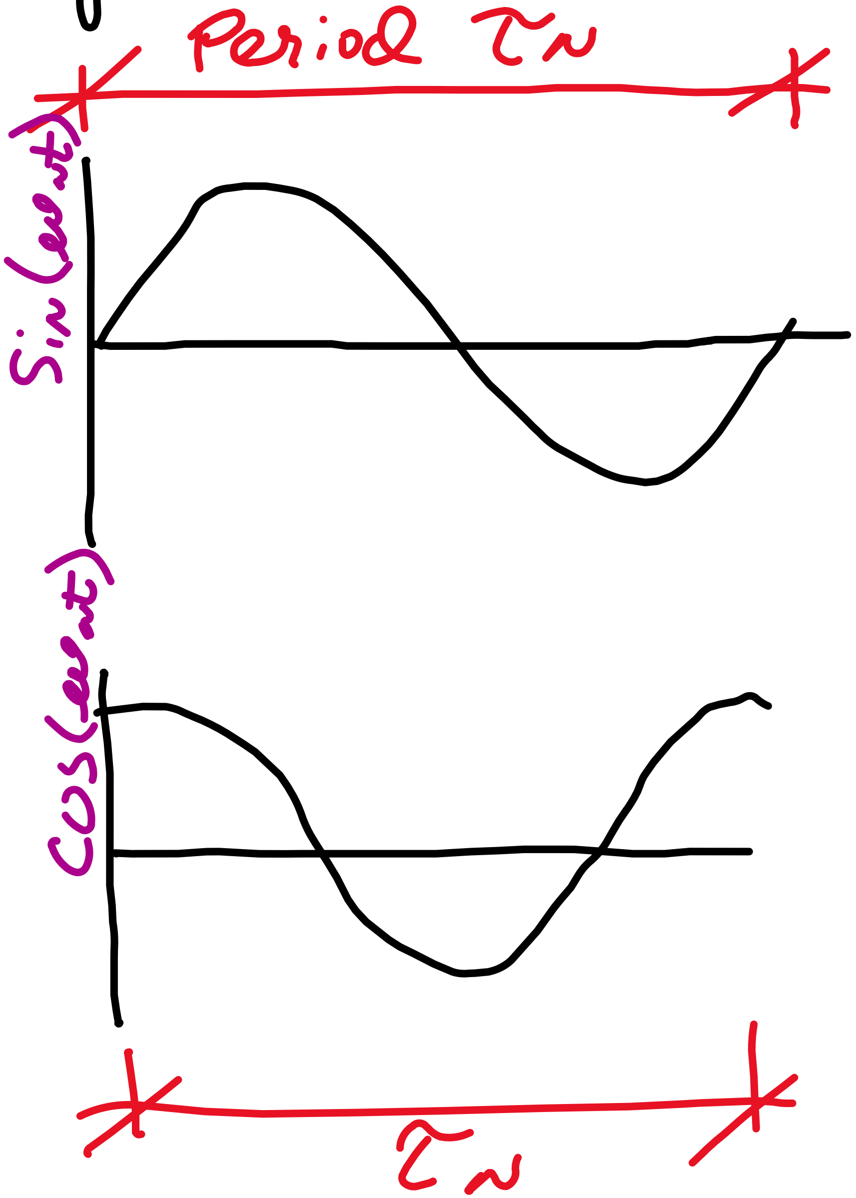

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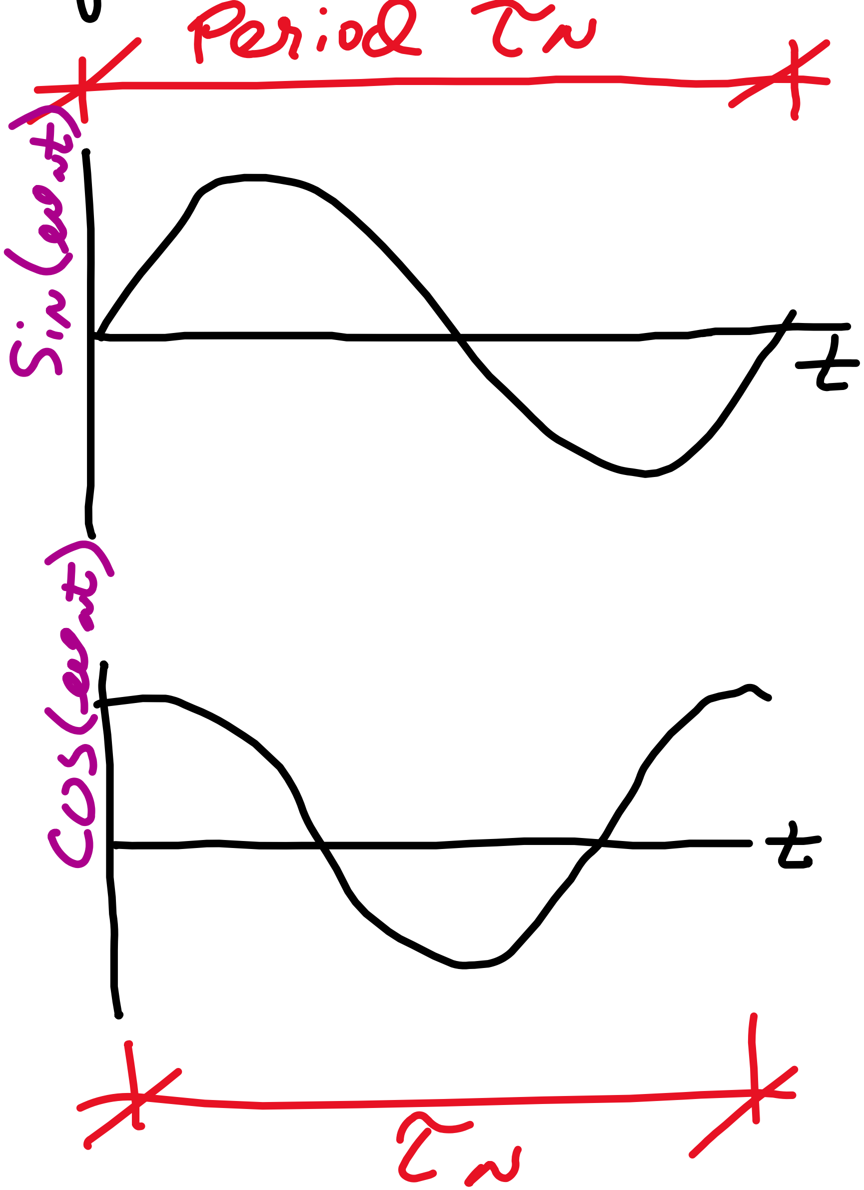

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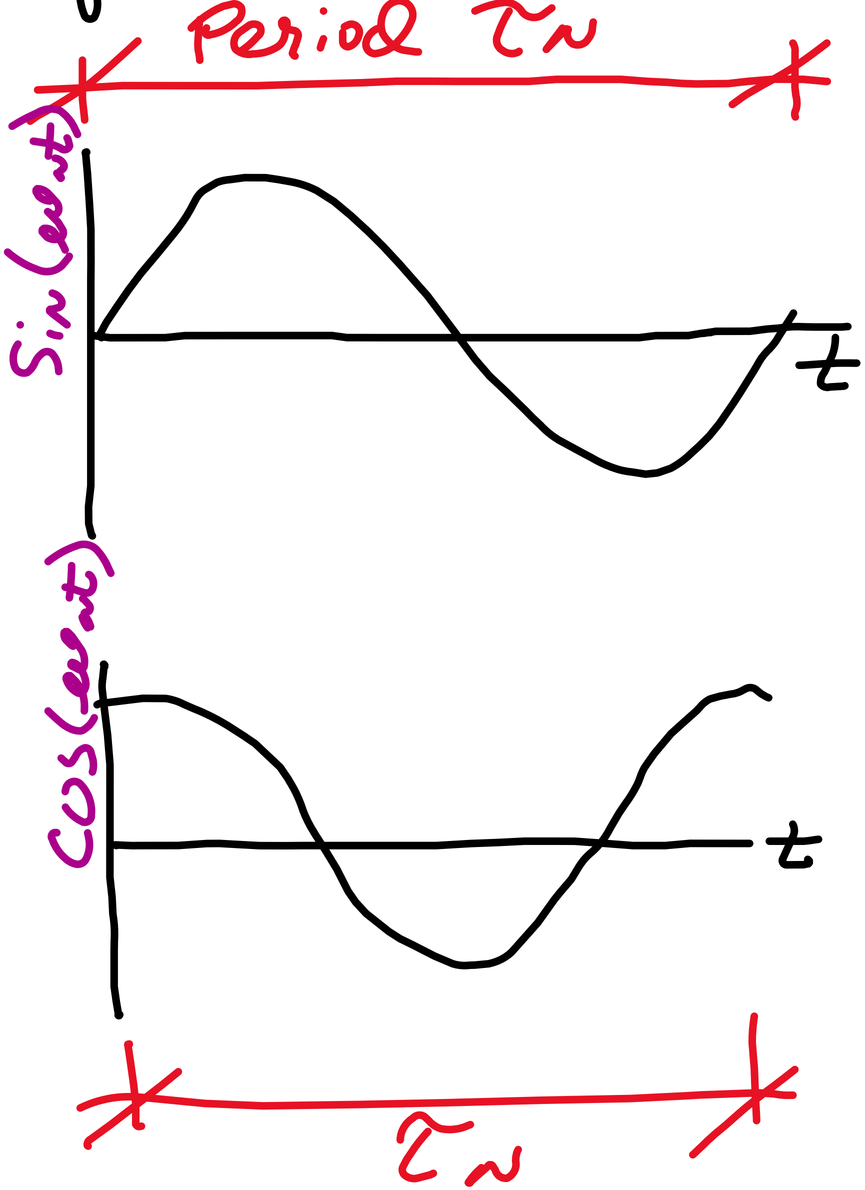
Full wave when

$\omega_n \tau_n = 2\pi$



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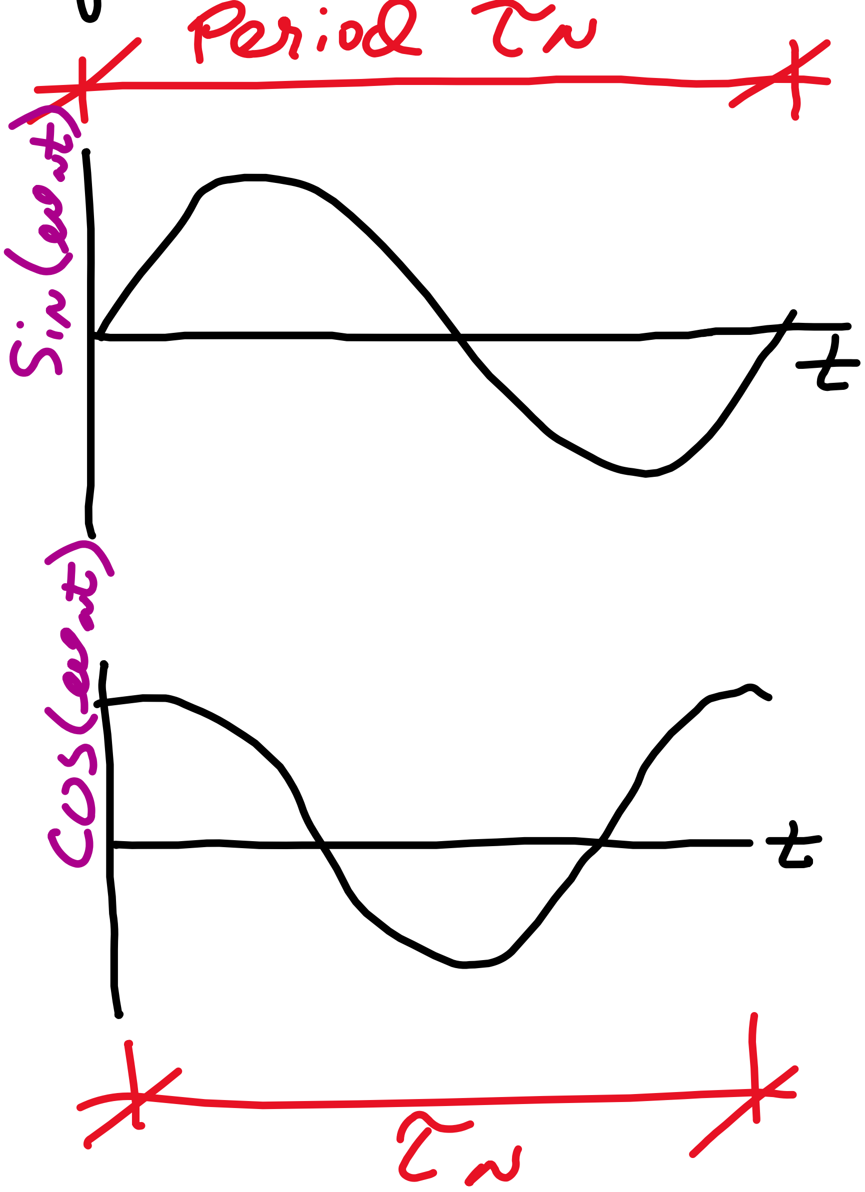

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This means that

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Example: mass hanging by a spring in equilibrium

<u>Example</u>: mass hanging by a spring in equilibrium [i.e. everything naturally at rest]

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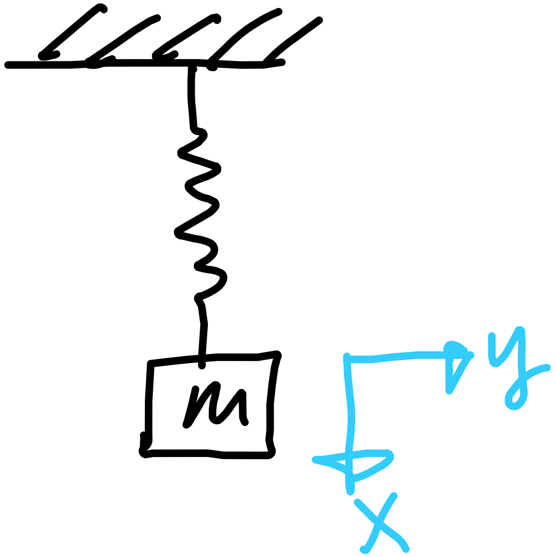

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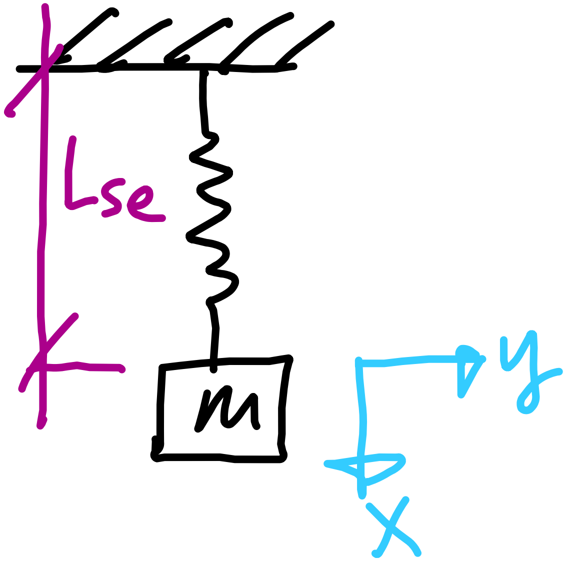
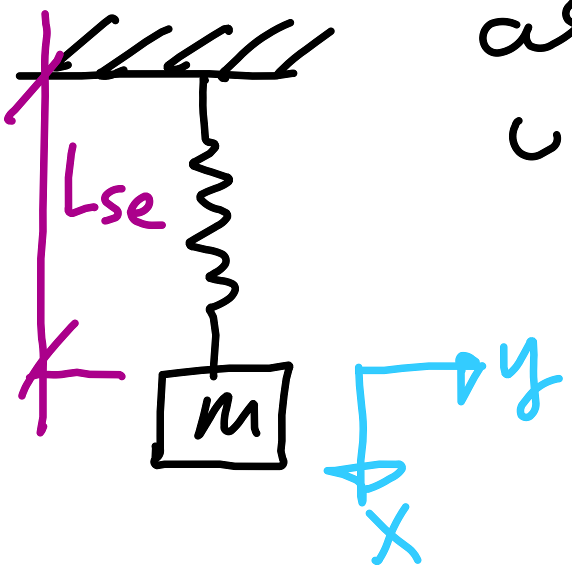


Example: mass hanging by a spring in equilibrium [i.e. everything naturally at rest]. **Note**: we are going to change our usual notation a bit:



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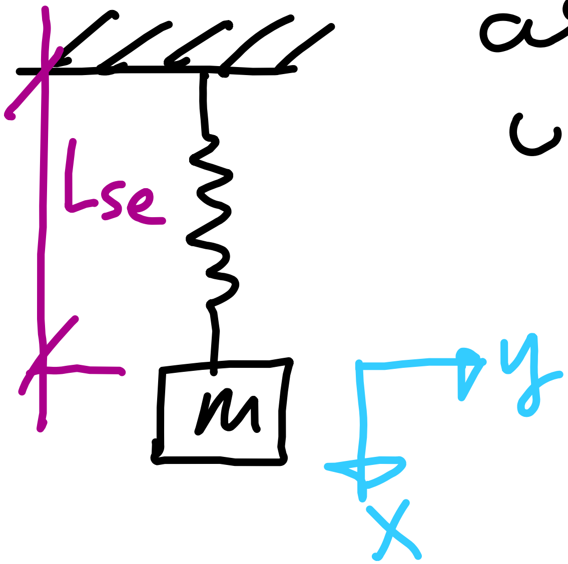

Let δ be displacement of spring from natural length L_o.

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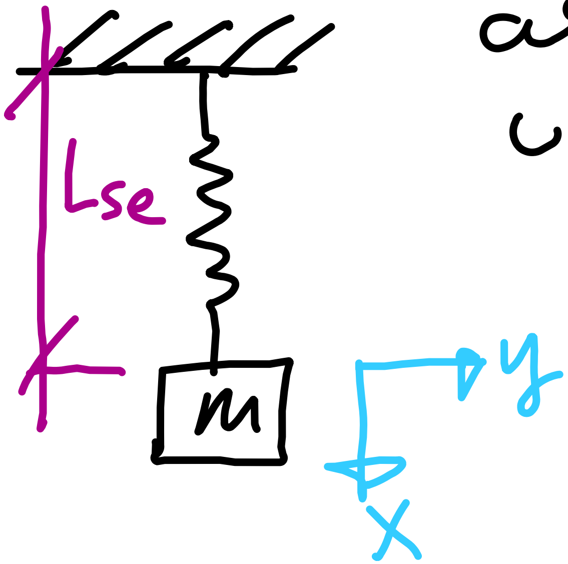

Let δ be displacement of spring from natural length L_0. This means

$L_{se} = L_0 + \delta_0$,

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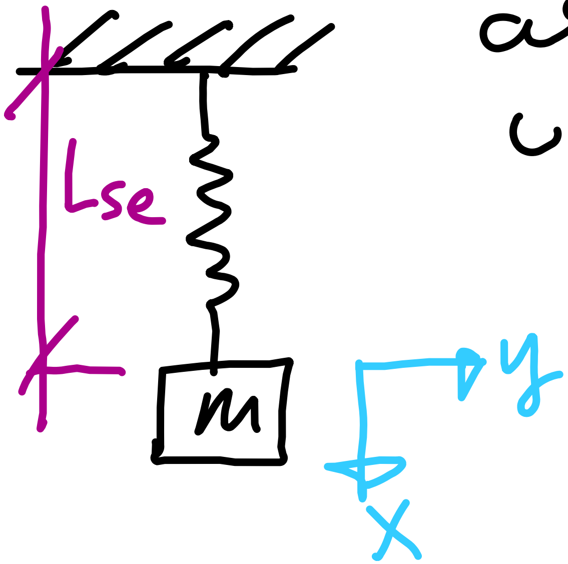

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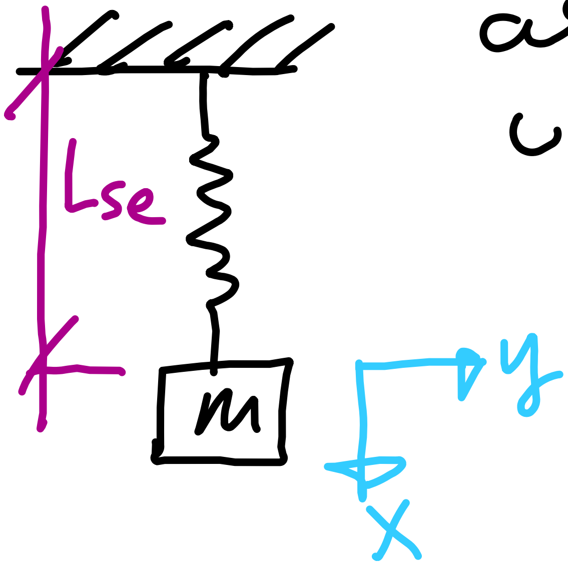
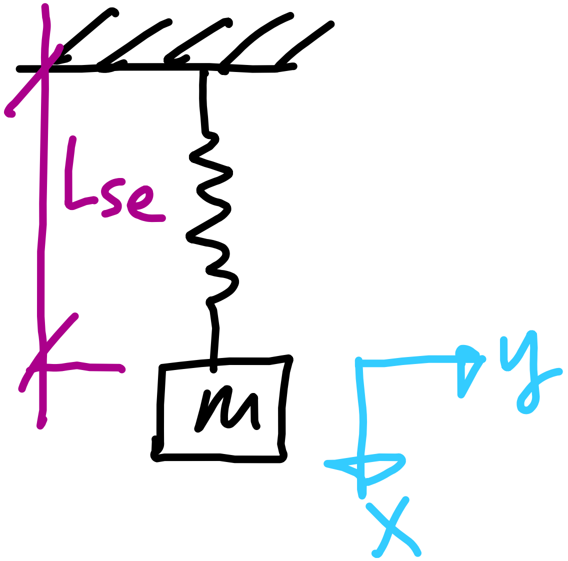
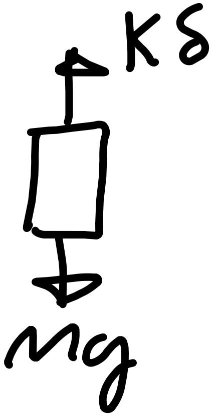


Let δ be displacement of spring from natural length L_0. This means $L_{se} = L_0 + \delta_0$, where $L_{se} \equiv$ length of spring when in equilibrium. We are going to set our coordinate system such that $x = 0$ at equilibrium

Example: mass hanging by a spring in equilibrium [i.e. everything naturally at rest]





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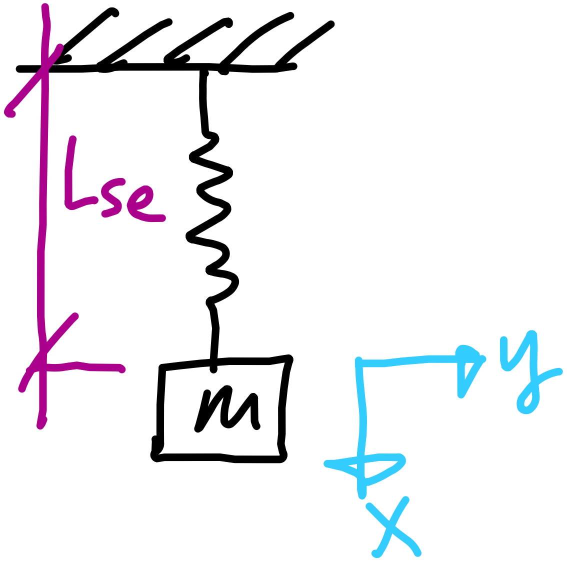

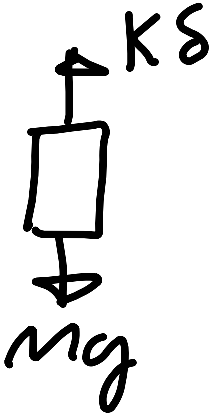

So $\sum F_x = \cancel{0} \Rightarrow$

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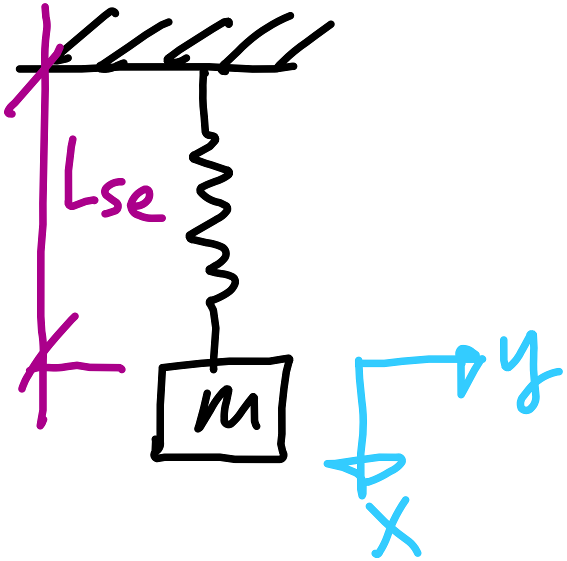

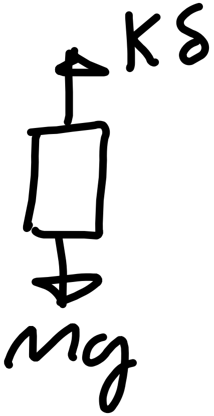

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$$-k\delta + mg = 0$$

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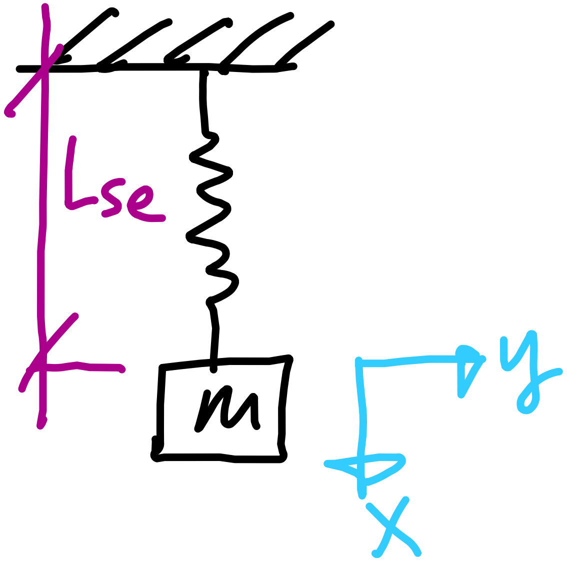

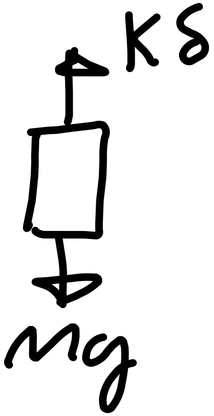

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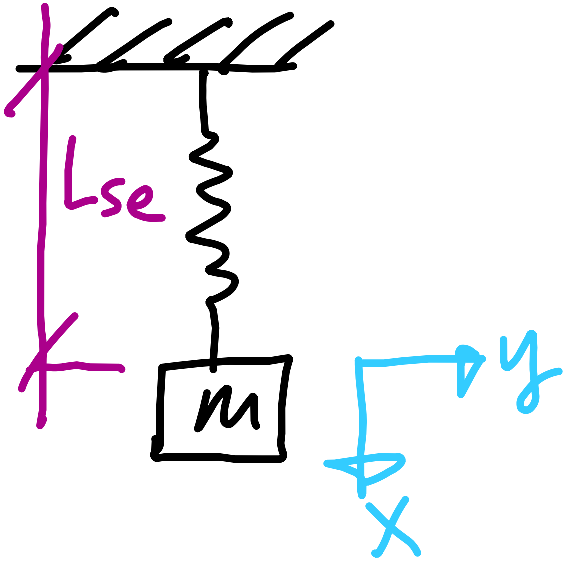

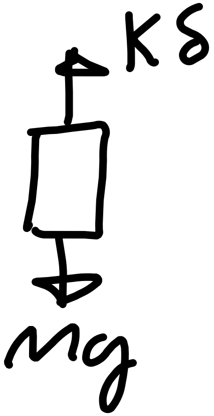

So $\sum F_x = 0 \Rightarrow$

$-k\delta + mg = 0 \Rightarrow k\delta = mg$

Now move away from equilibrium by displacing mass an amount x

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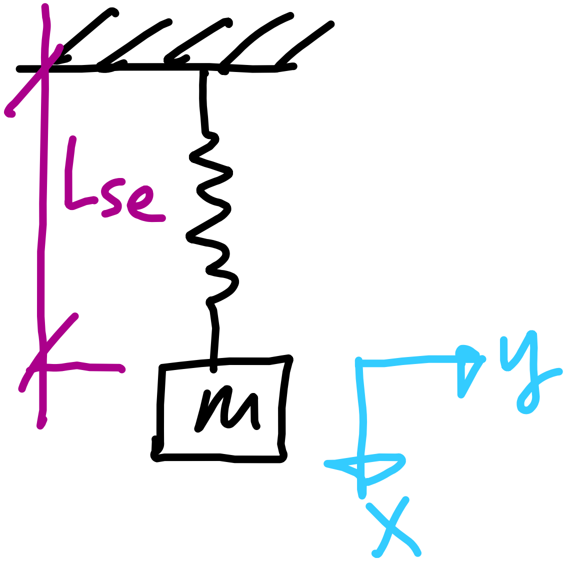
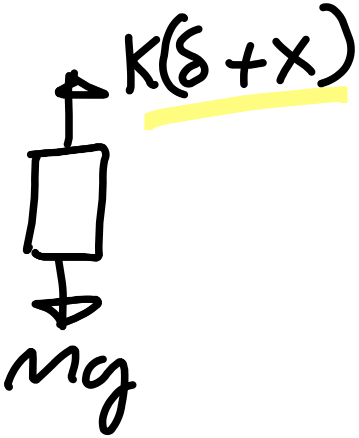
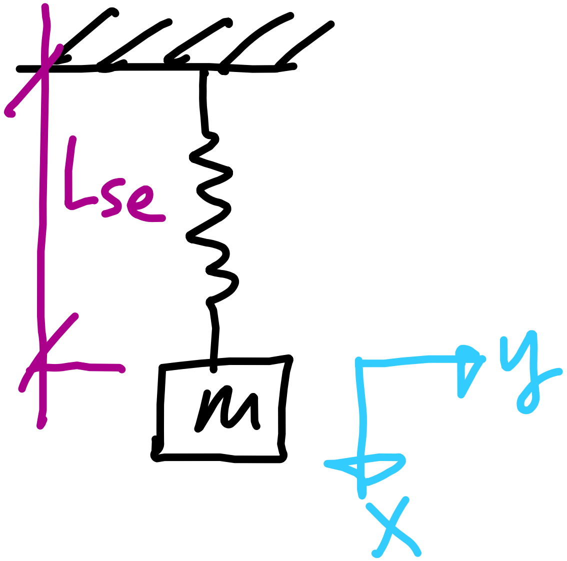
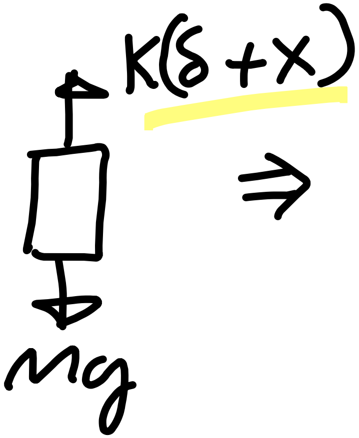




Example: mass hanging by a spring in equilibrium [i.e. everything naturally at rest]





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$\Rightarrow$

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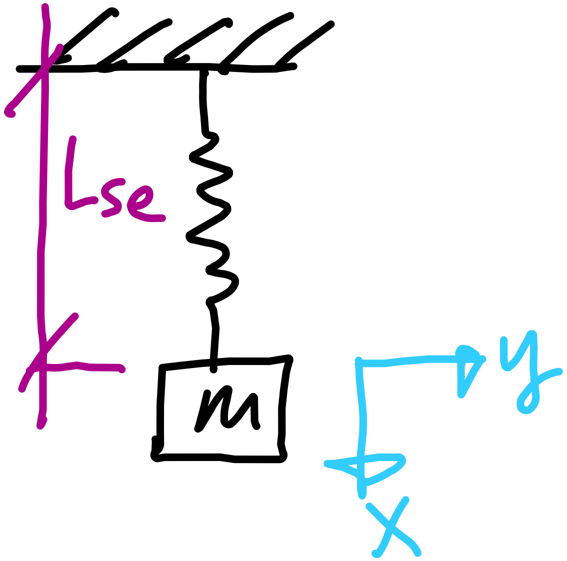

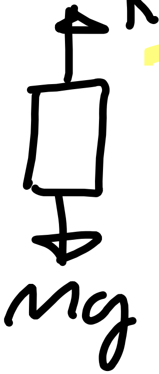

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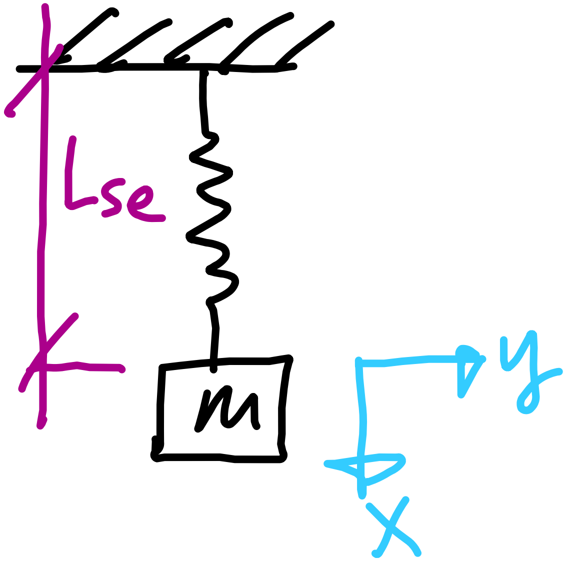

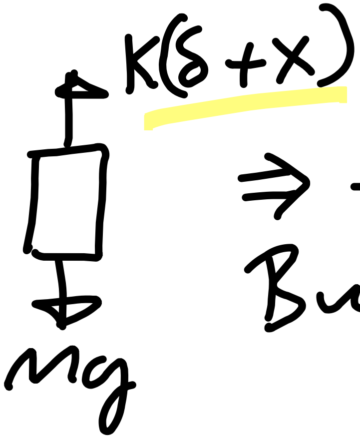

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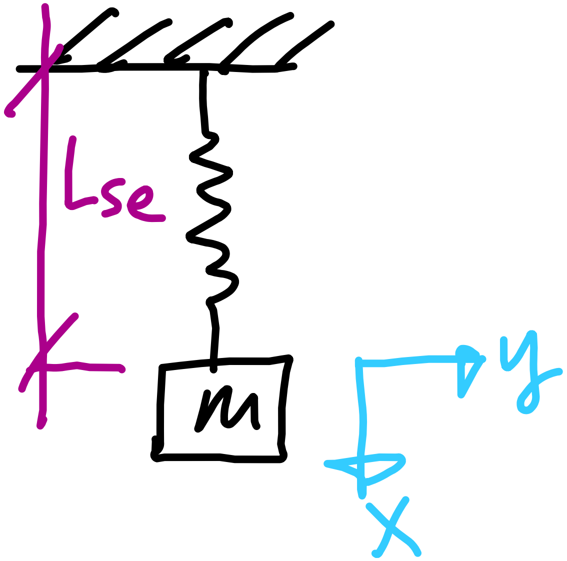

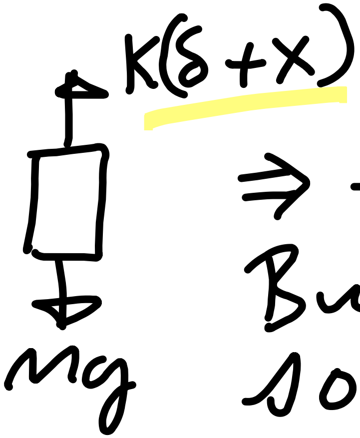

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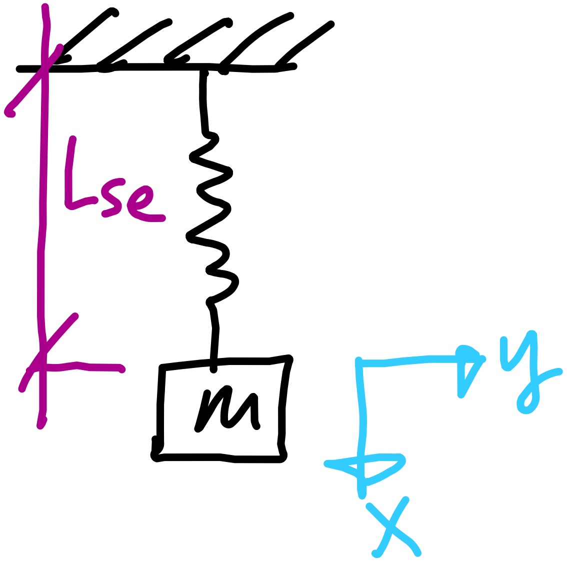

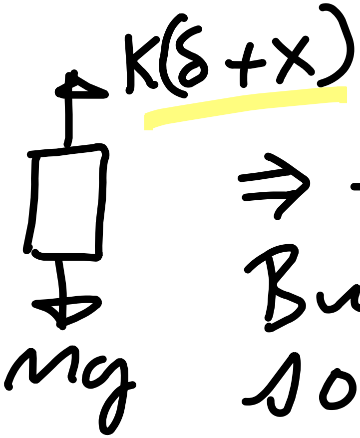

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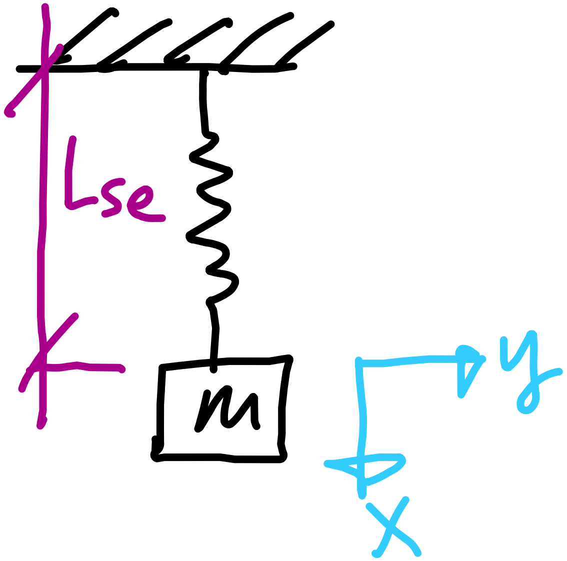

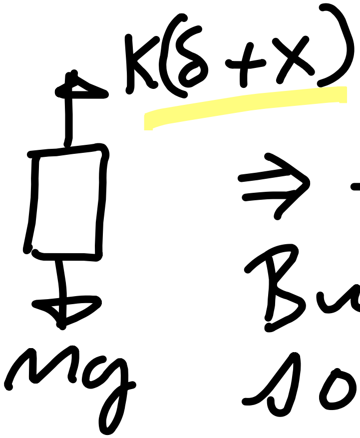

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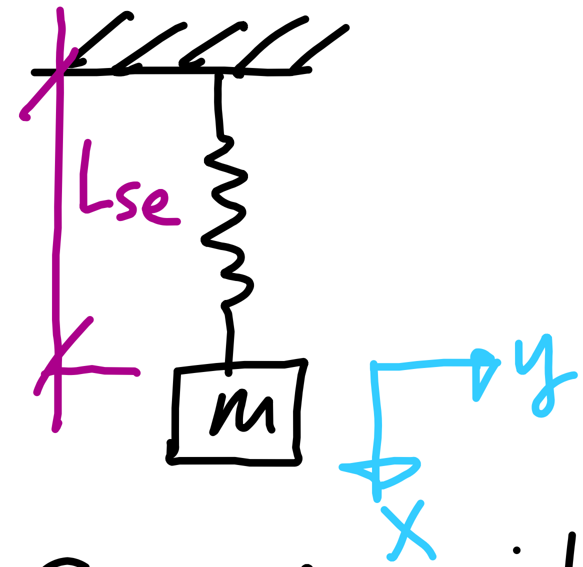

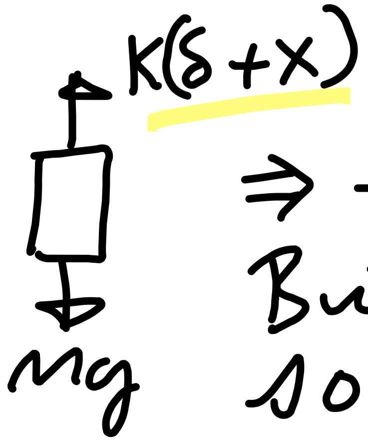

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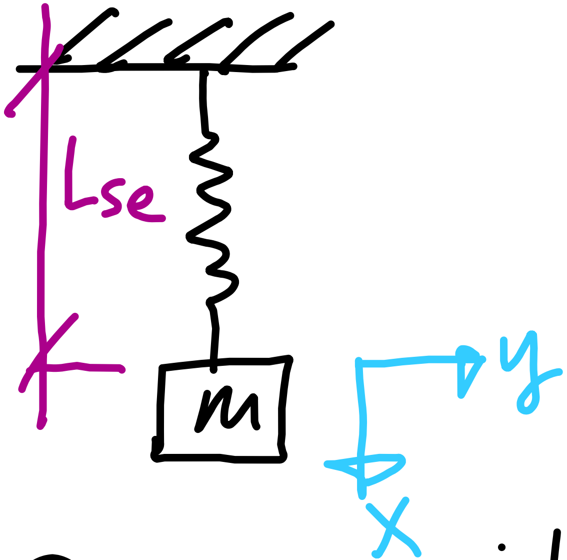

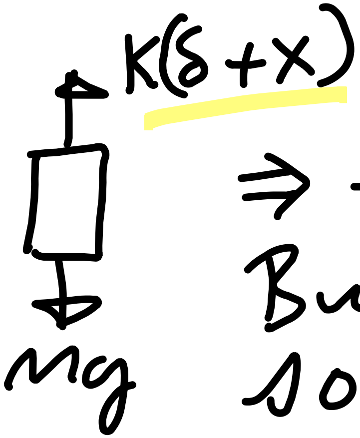

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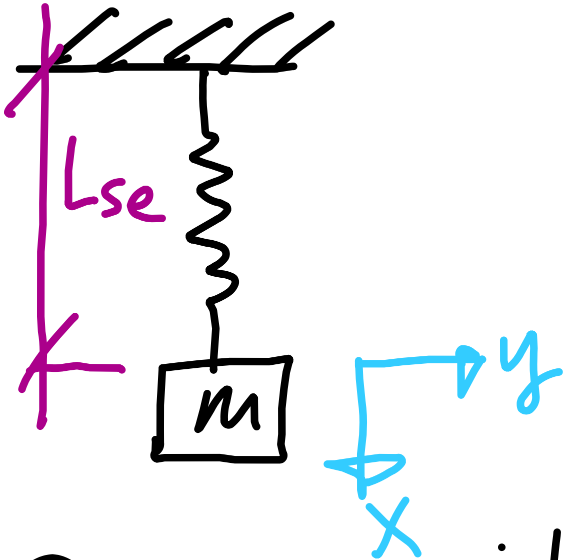

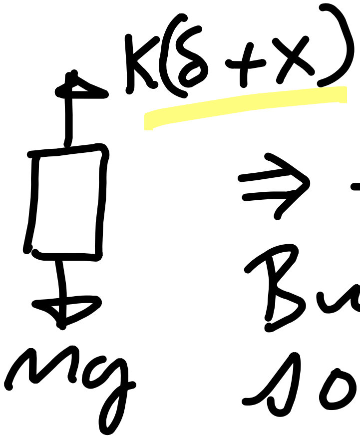

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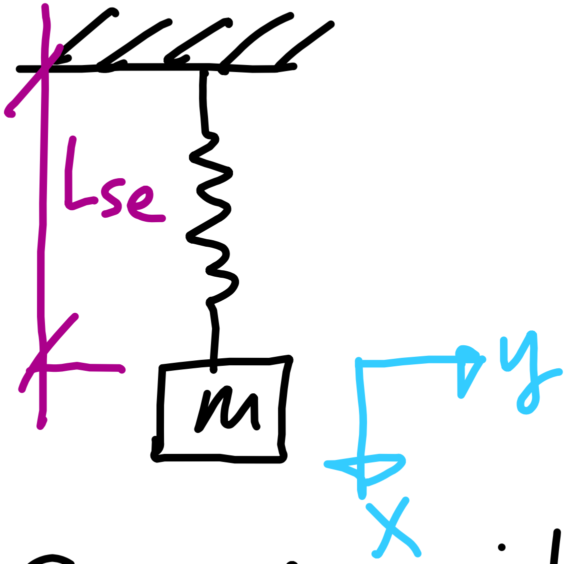

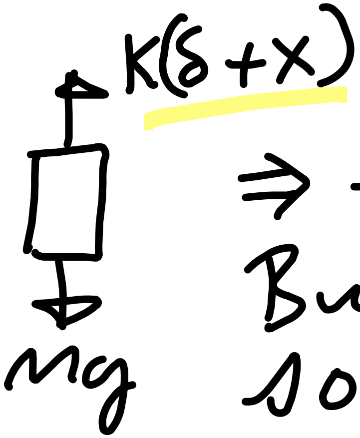

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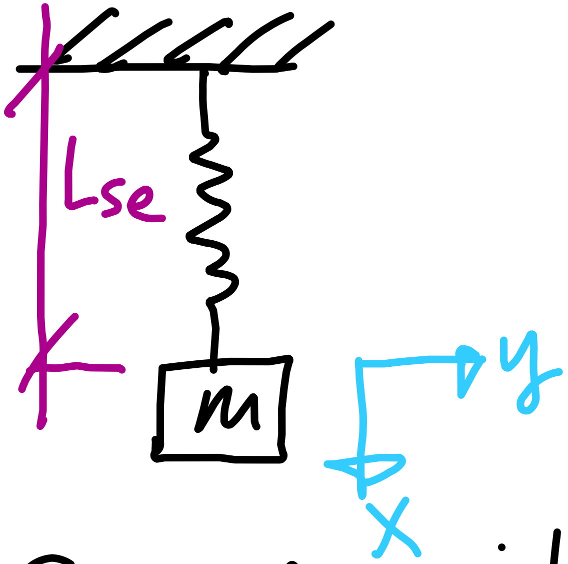

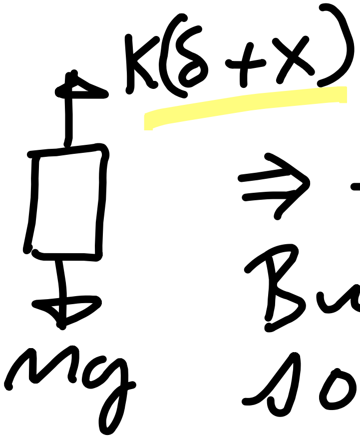

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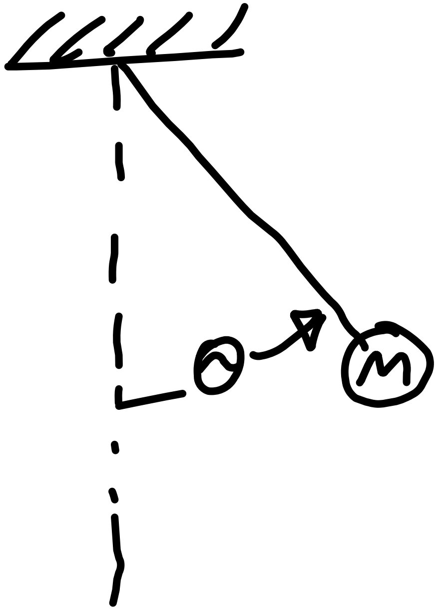
Example: Simple pendulum



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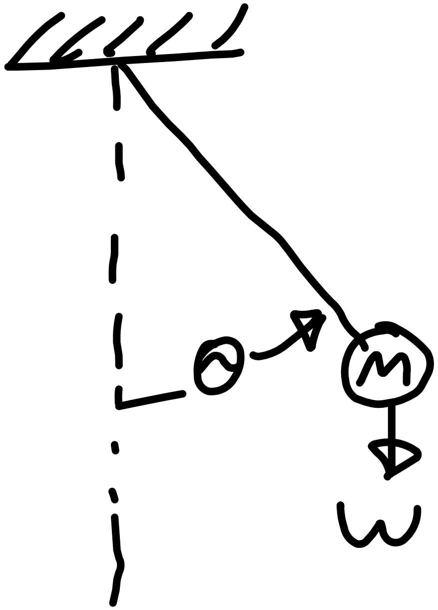

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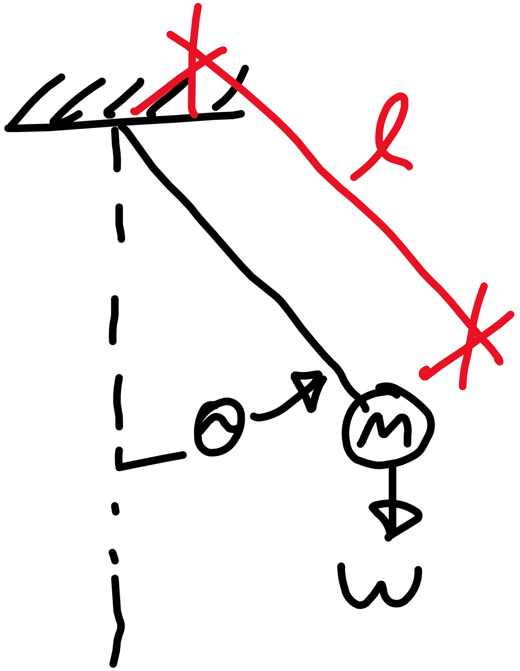

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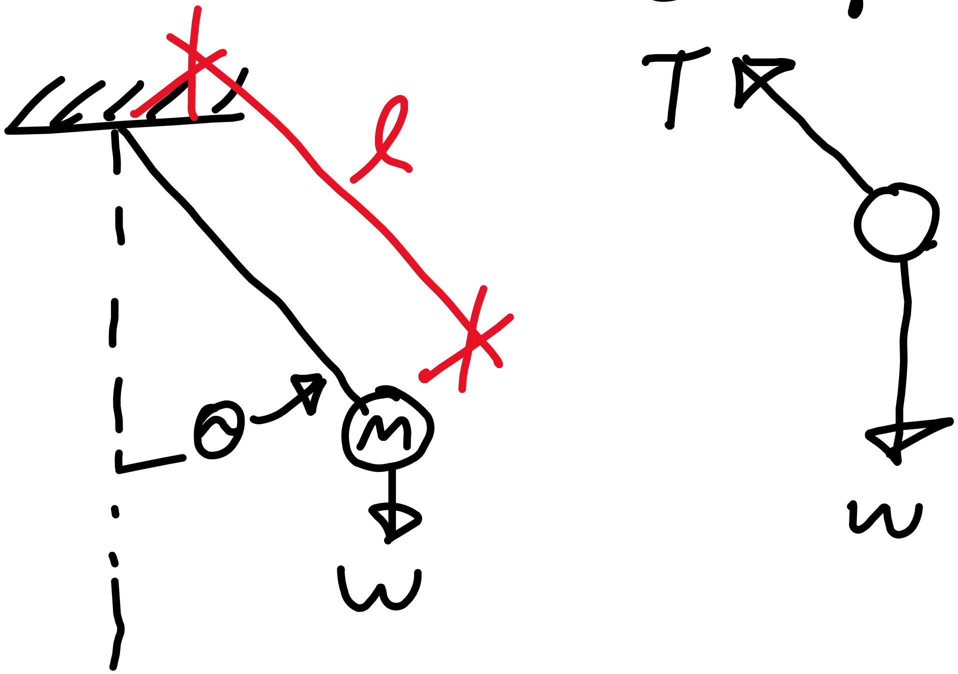

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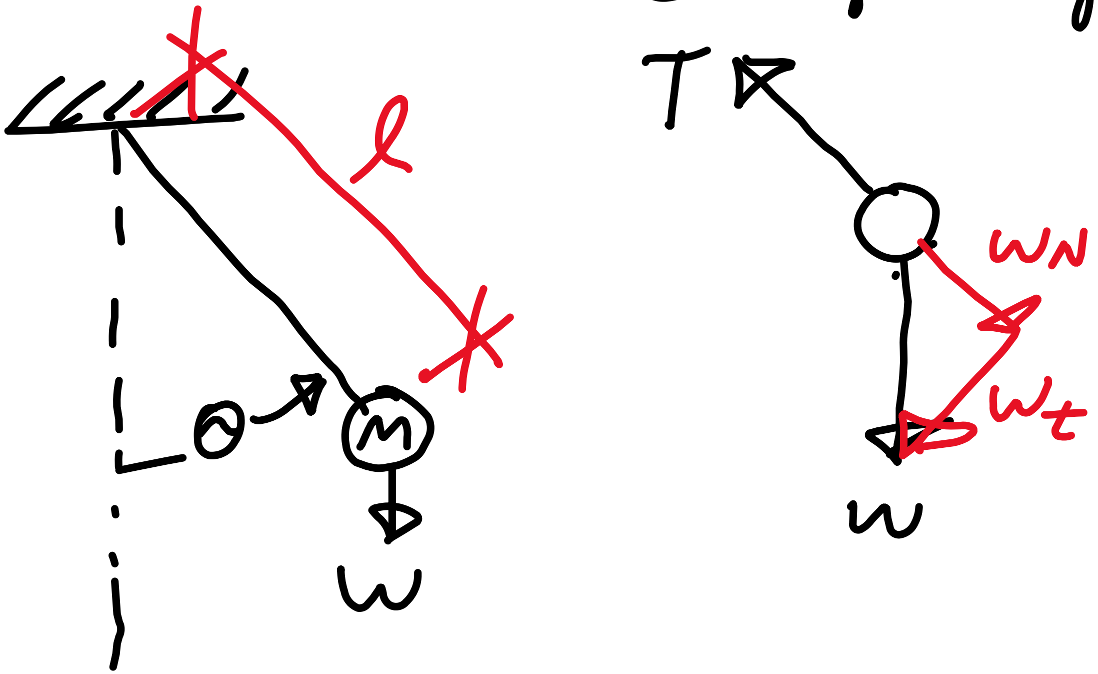

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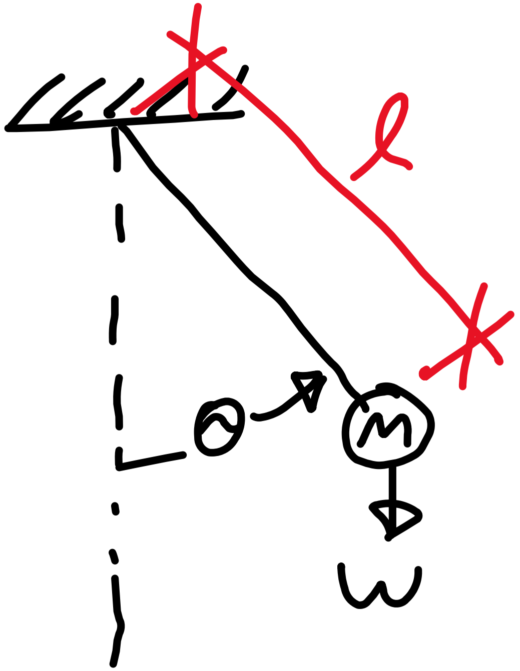
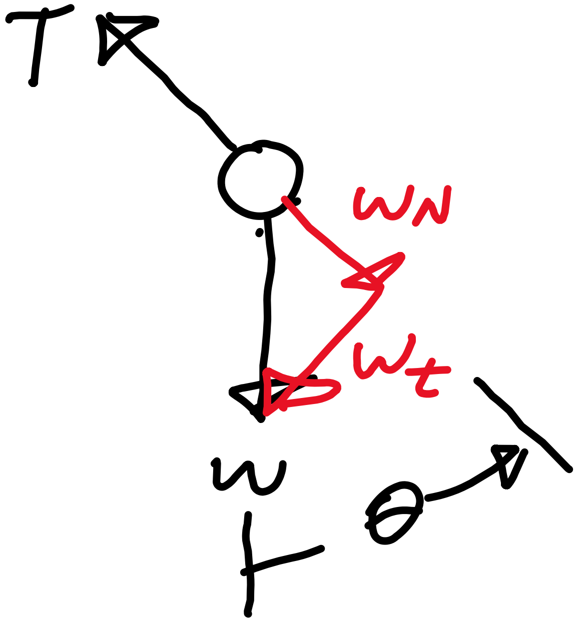
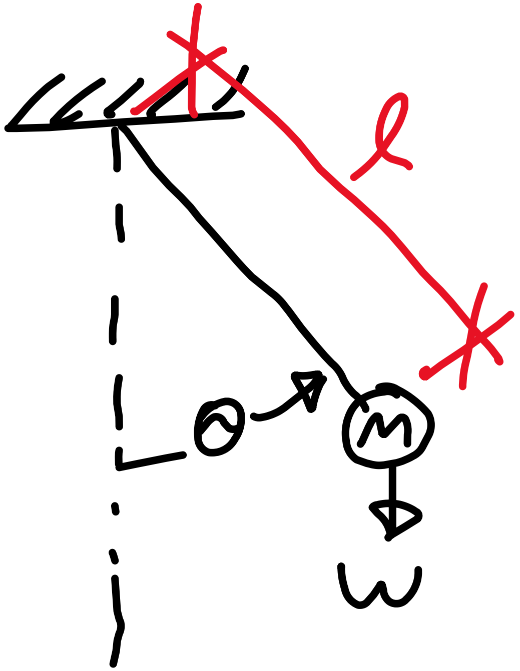
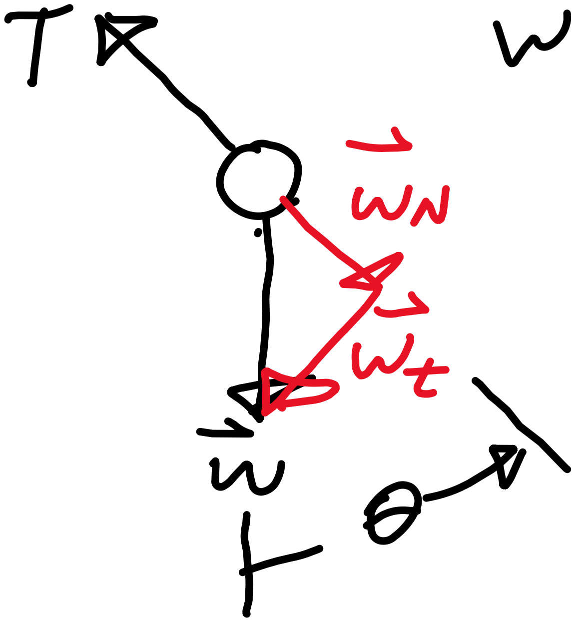




Example: Simple pendulum

$\vec{w} = \vec{w}_N + \vec{w}_t$





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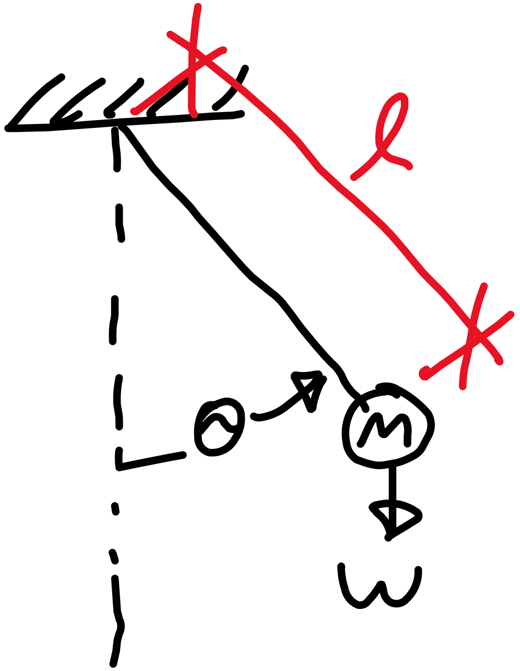
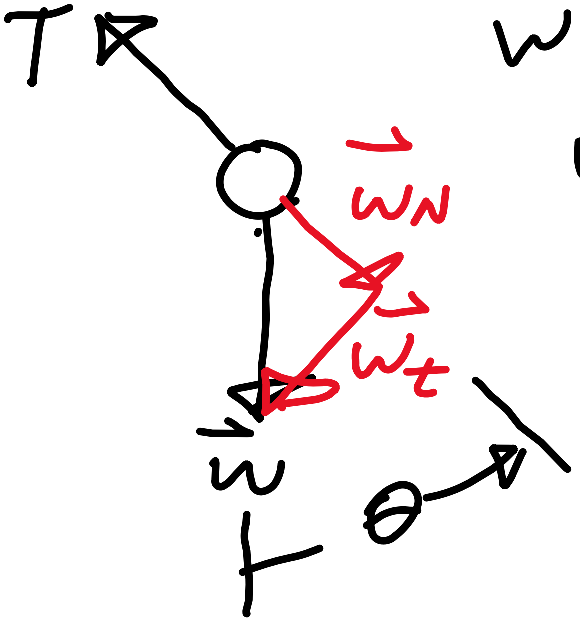
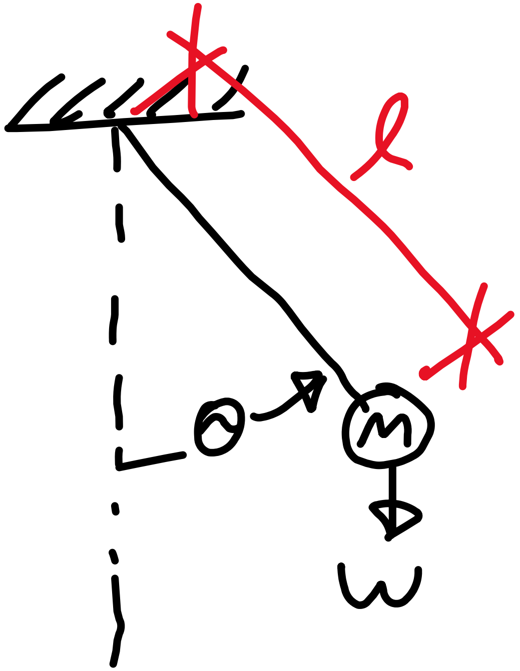


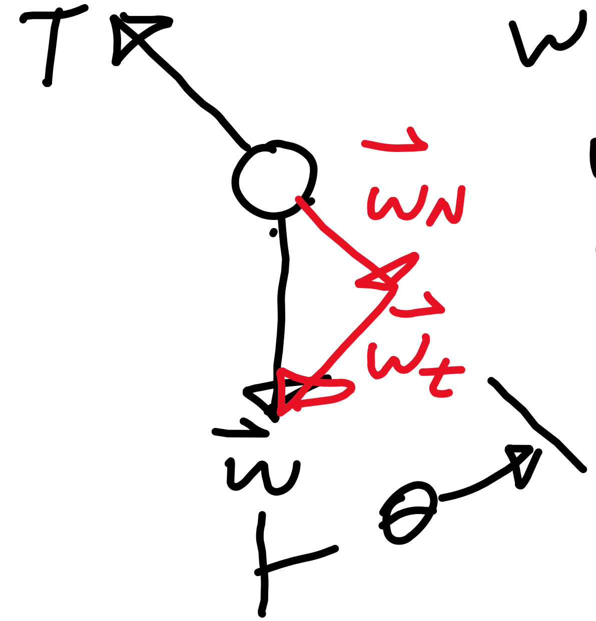


$\vec{W} = \vec{W}_N + \vec{W}_t$

$W_N = -W\cos\theta$

Example: Simple pendulum





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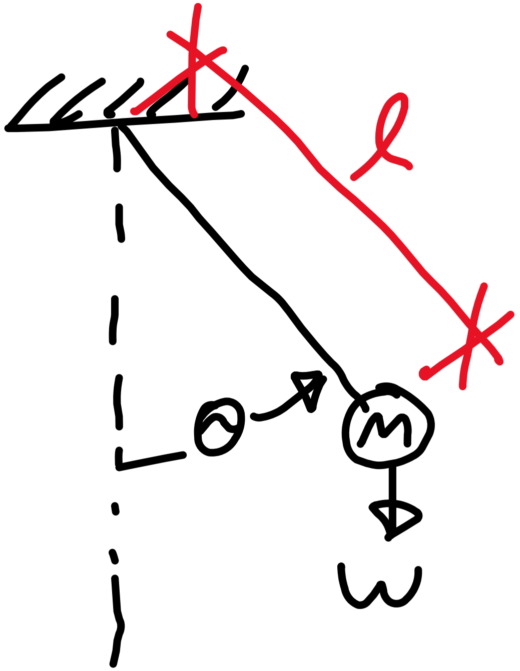

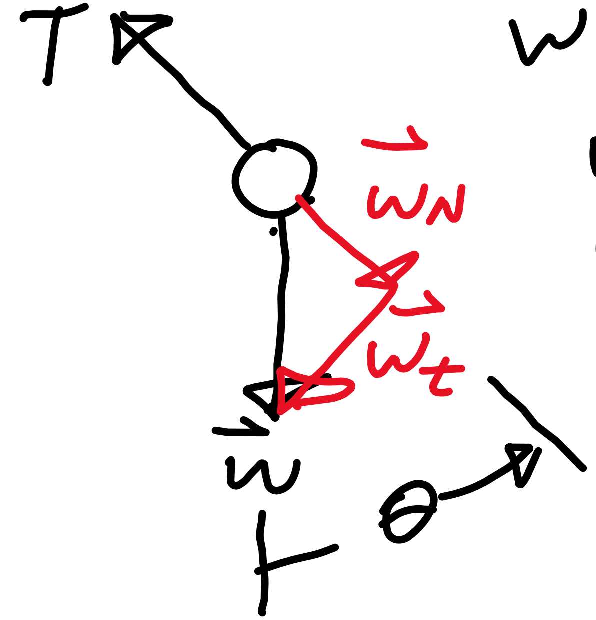

$$\vec{w} = \vec{w}_N + \vec{w}_t$$

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$$\Sigma F_N = 0 \Rightarrow$$

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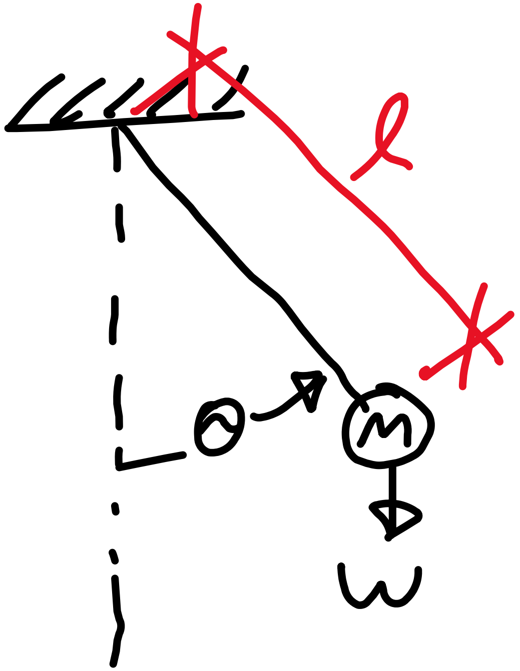

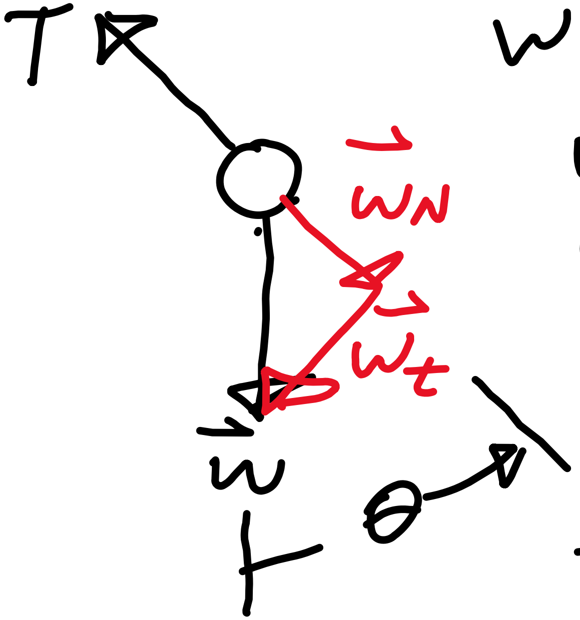

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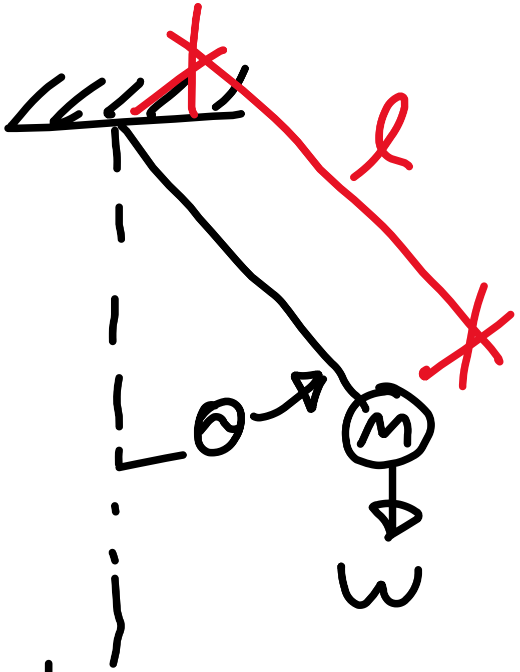

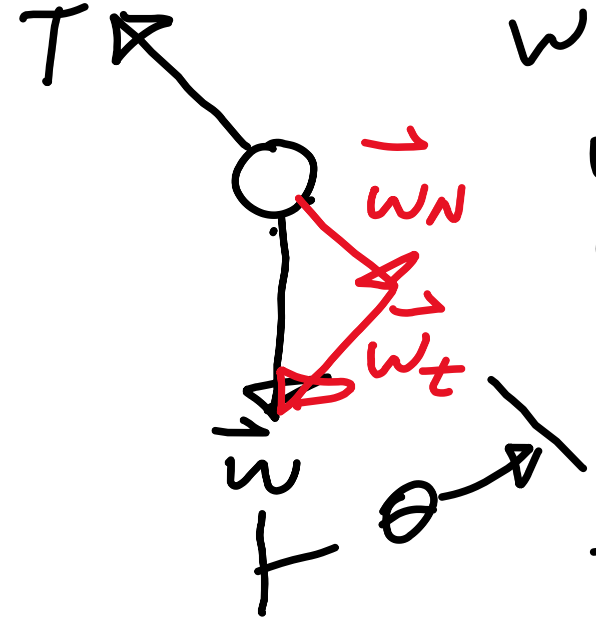

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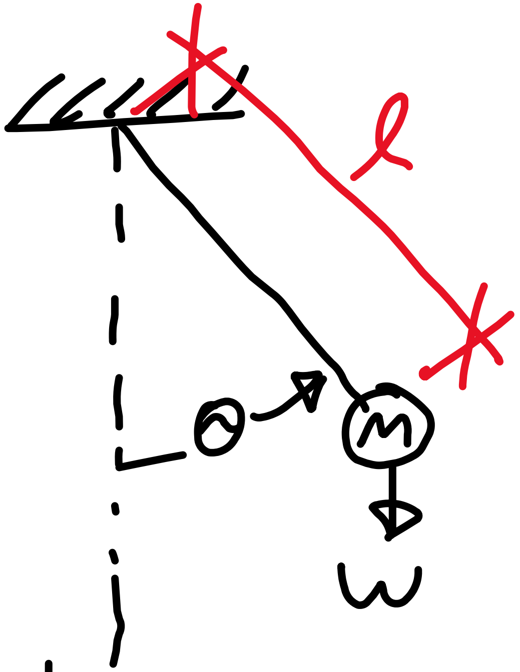

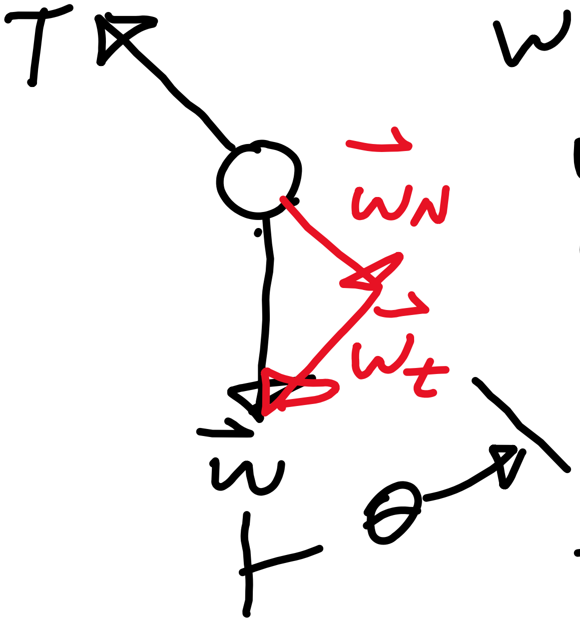

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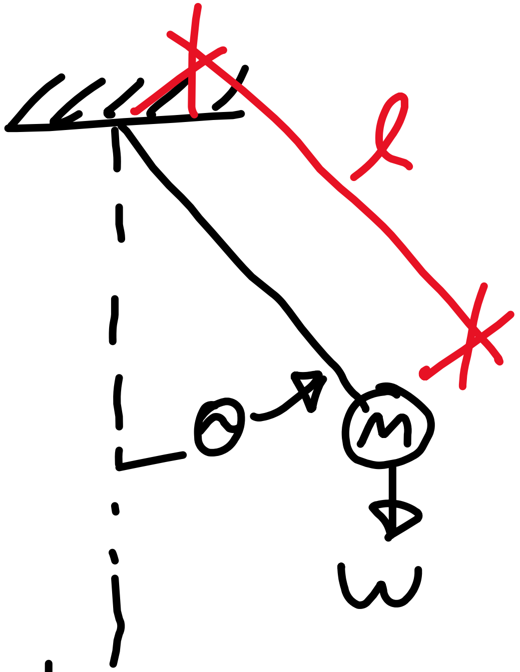

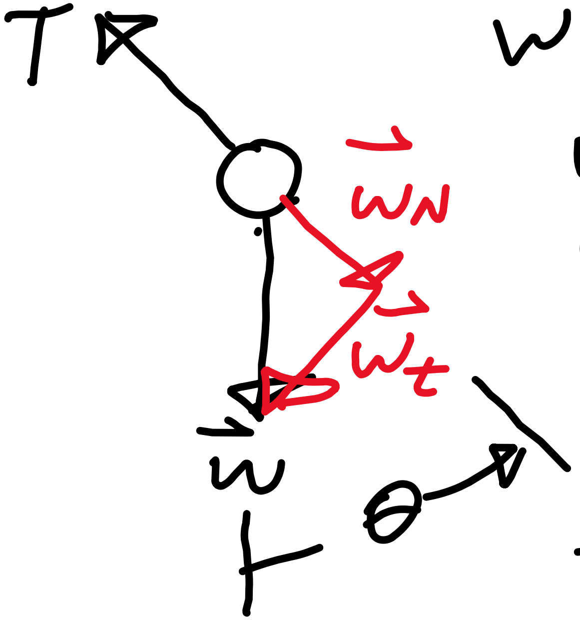

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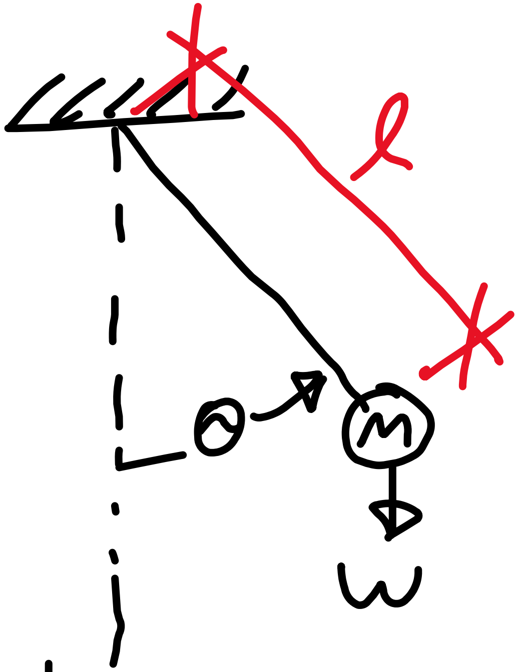

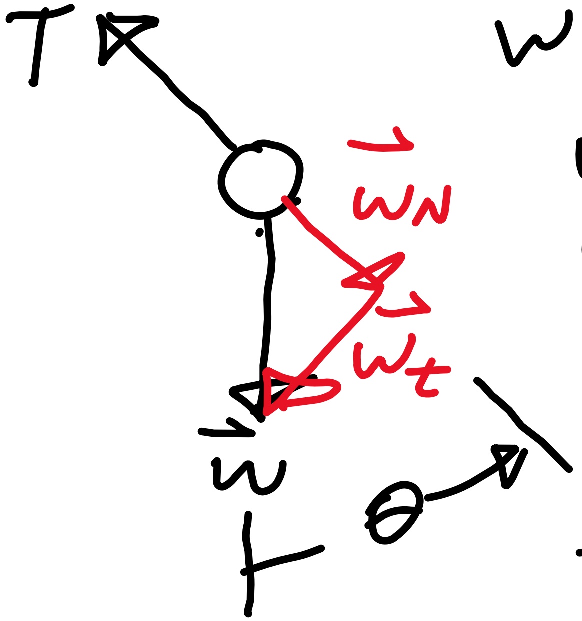

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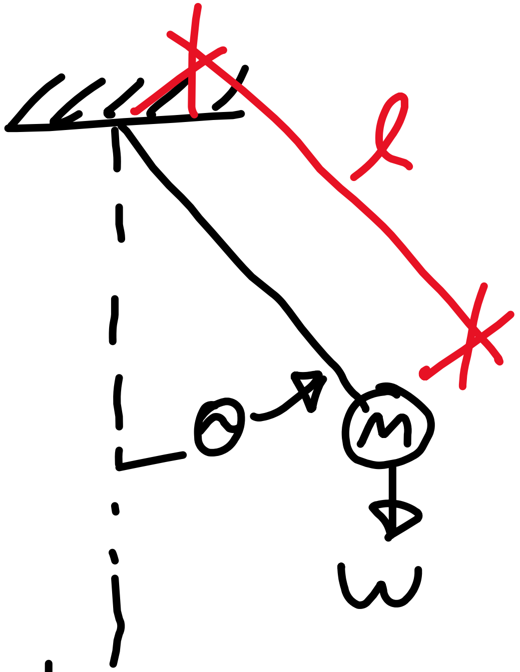

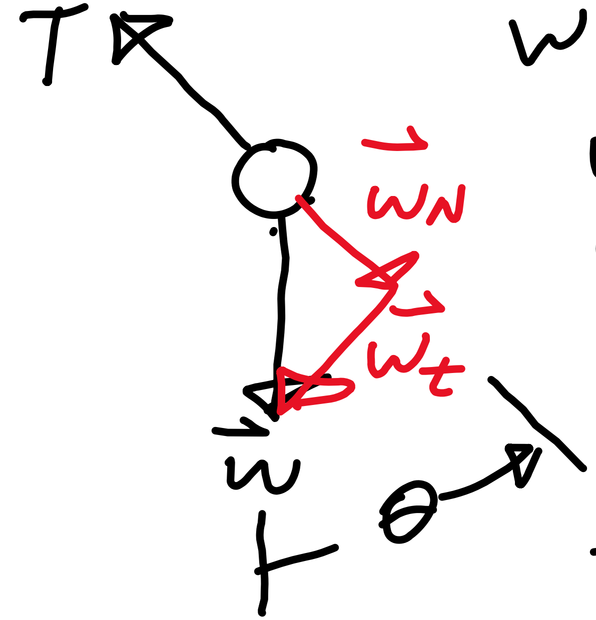

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$$\ddot{\theta} = -(\text{const})\theta$$

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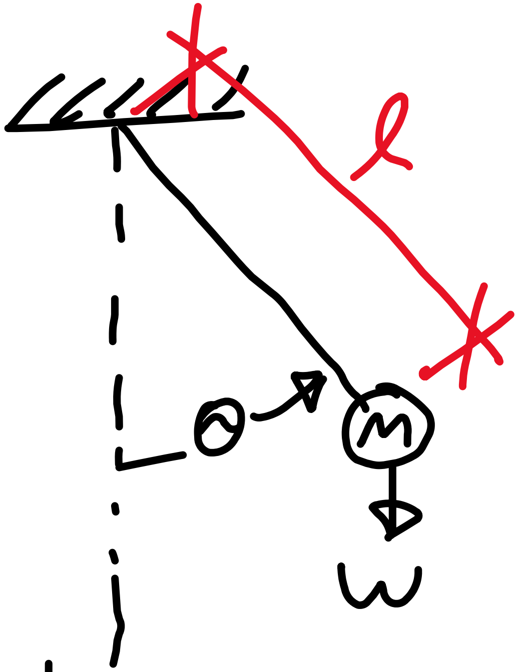

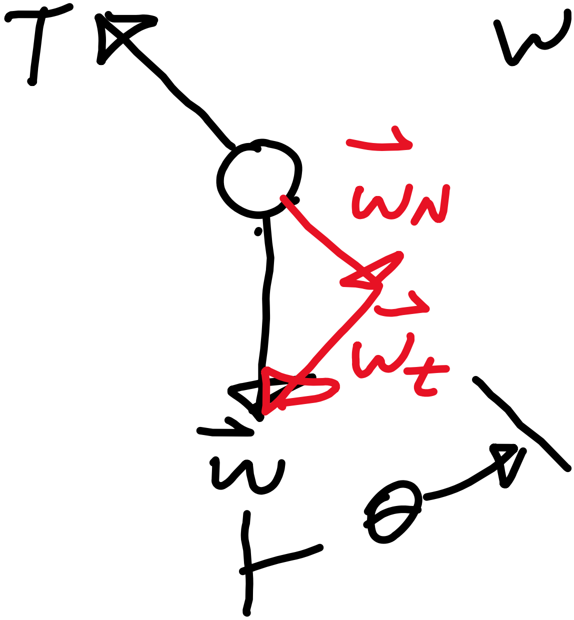

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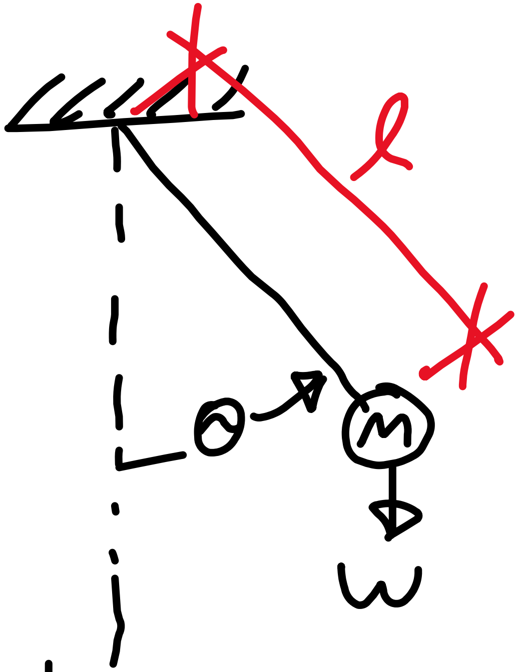

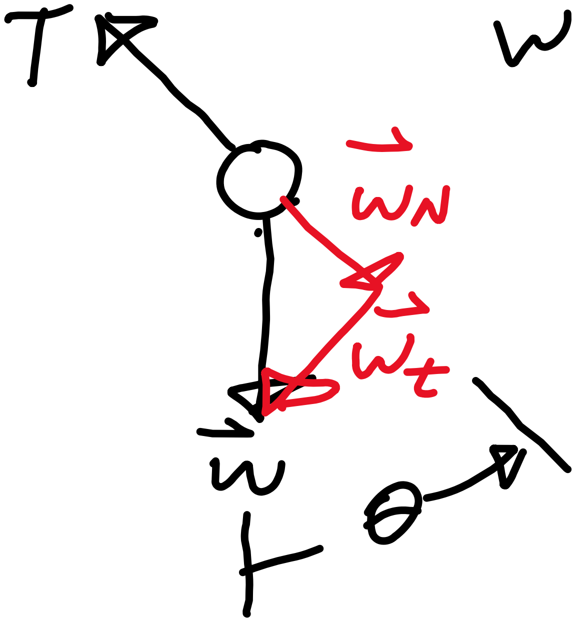

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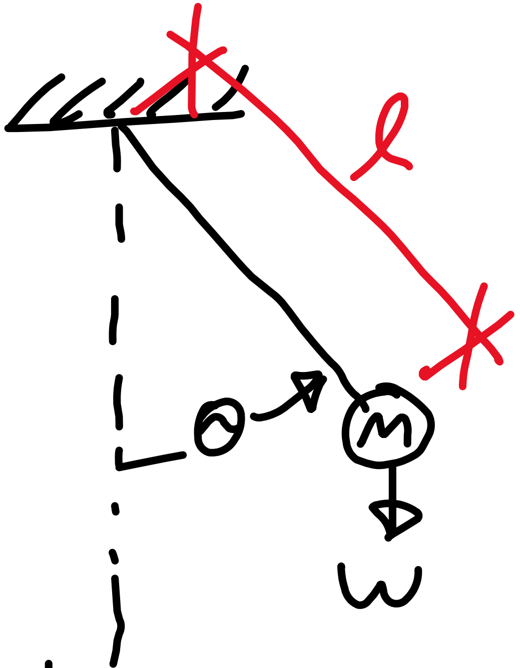
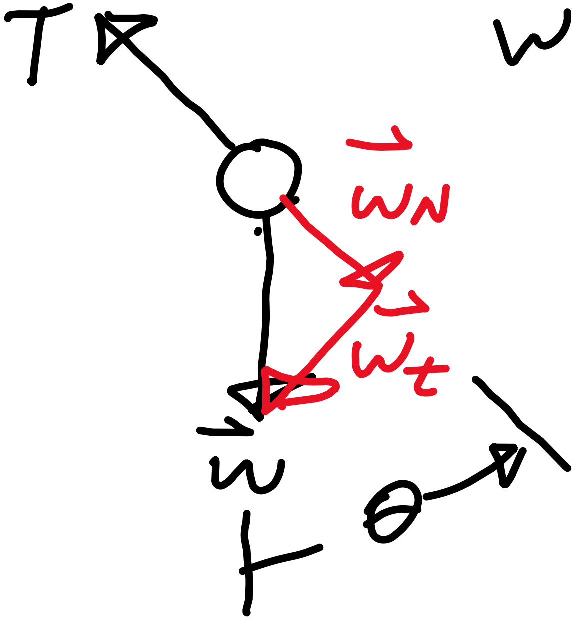
$\ddot{\theta} = -(\text{const})\sin\theta$ Need small angle approx

Example: Simple pendulum





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Note: $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$

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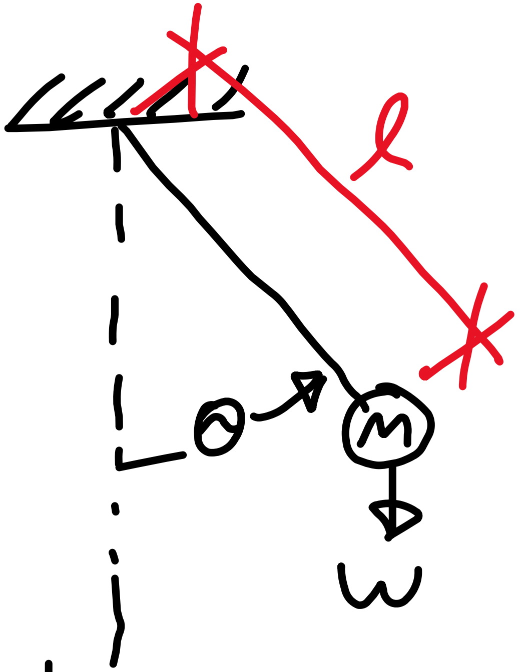

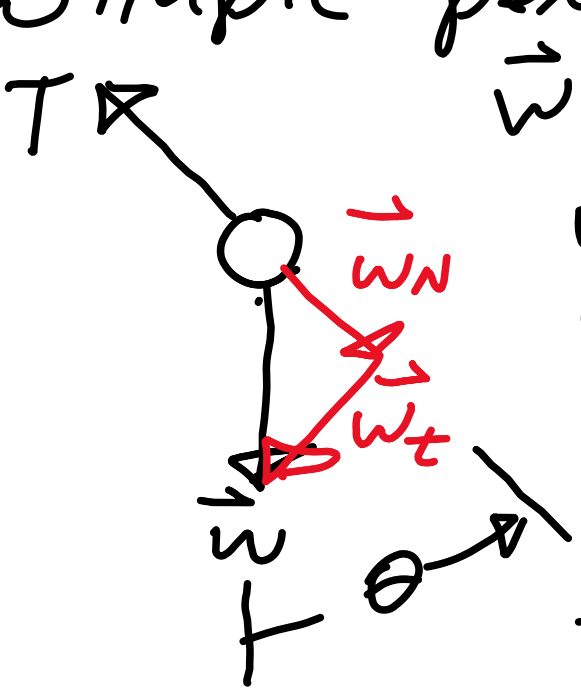

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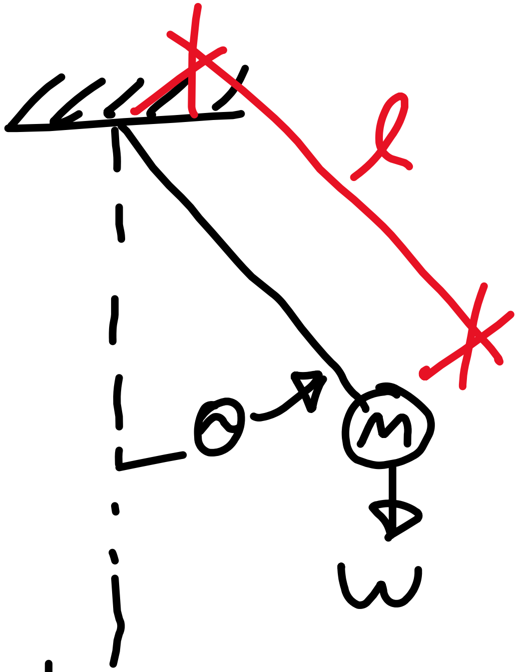

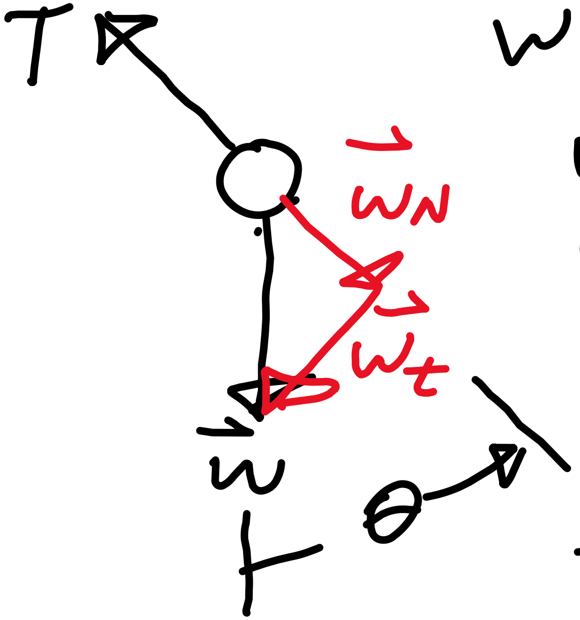

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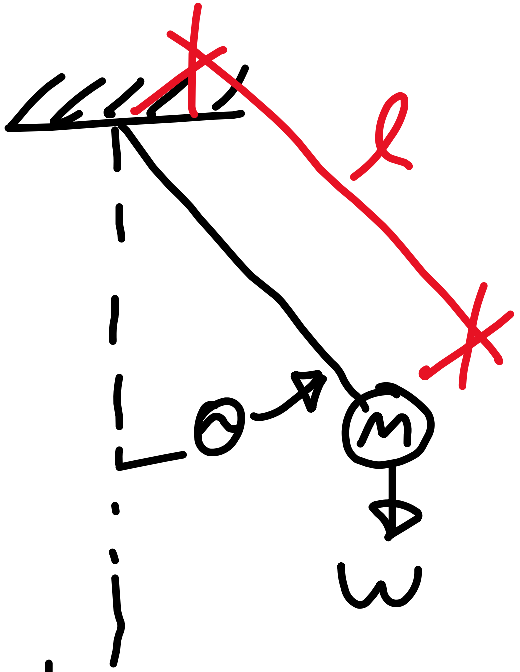

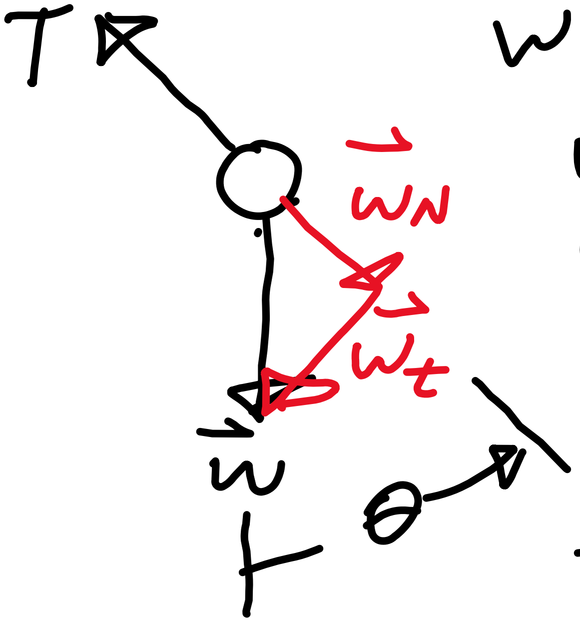

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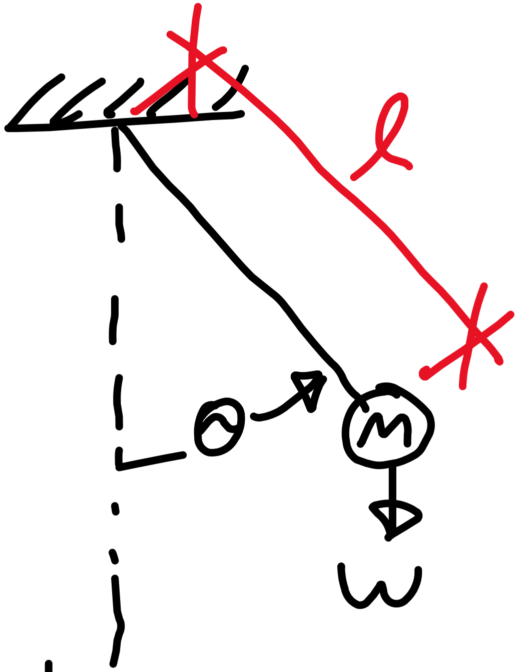

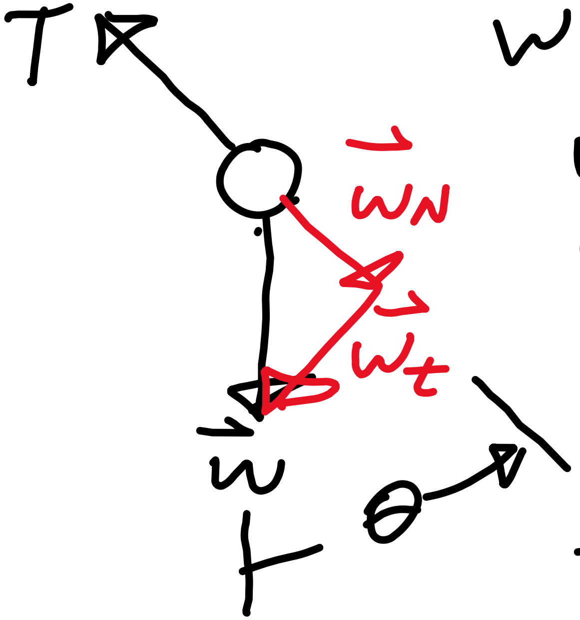

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So $\sin\theta \cong \theta$ for $\theta = 5°$ if we need $\sin\theta$ to within about 0.1%

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Notes on problems

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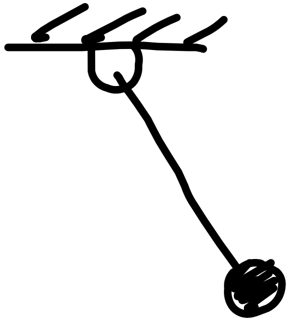

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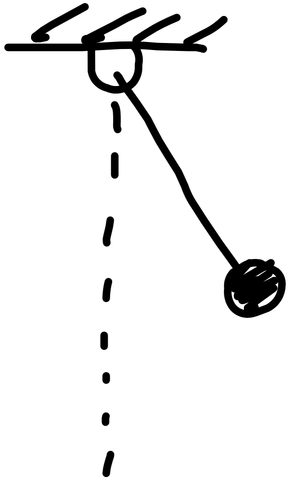

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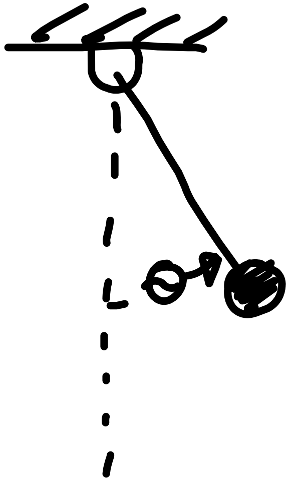

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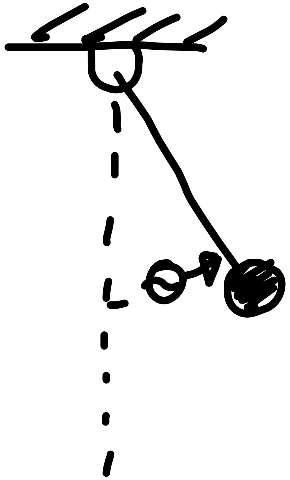
In part b you are asked to find the acceleration of the bob



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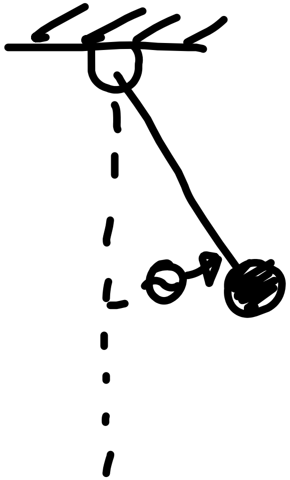

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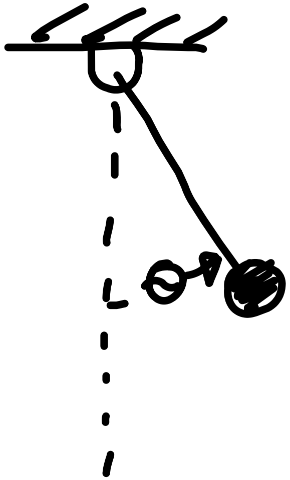

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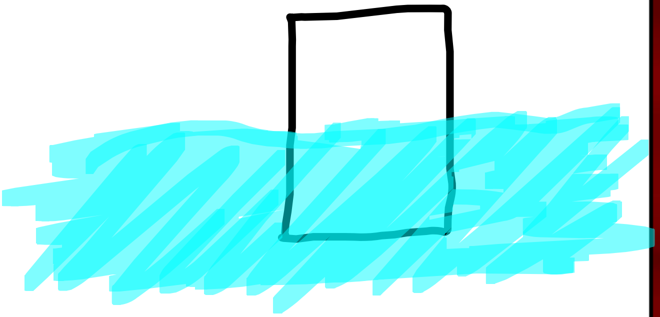
$$\text{i.e. } a = [a_n^2 + a_t^2]^{1/2}$$

Notes on problems

19.26:



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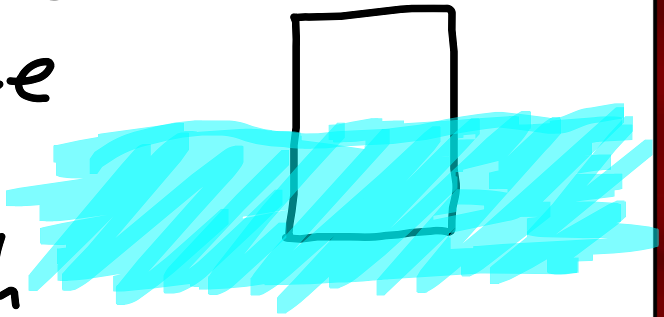
19.26: The key part is to use the hint stating that force on bottom of barrel can be modelled as spring with spring constant $k = \rho_{sw} g A$



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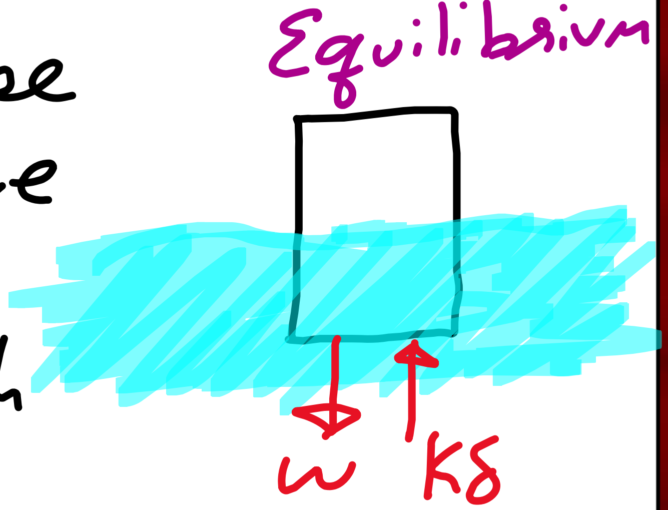

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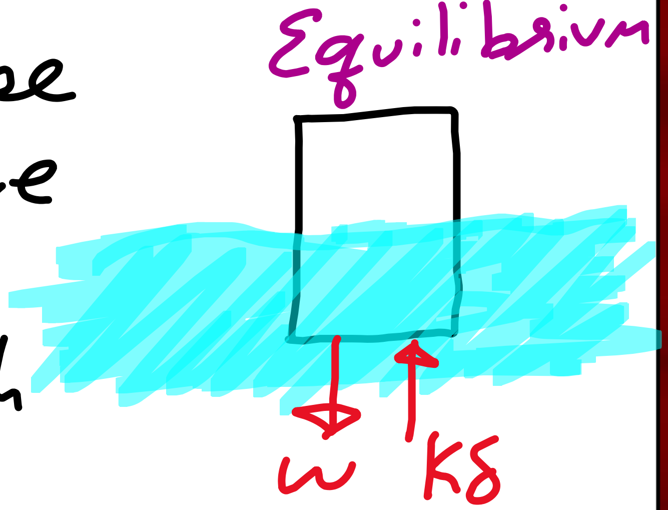

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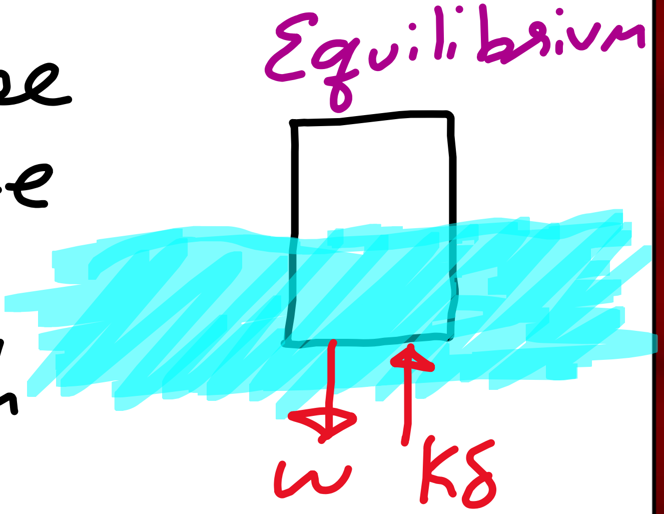

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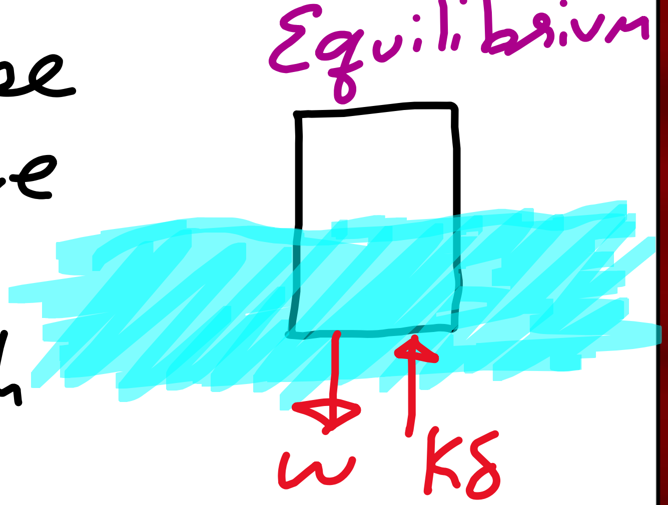

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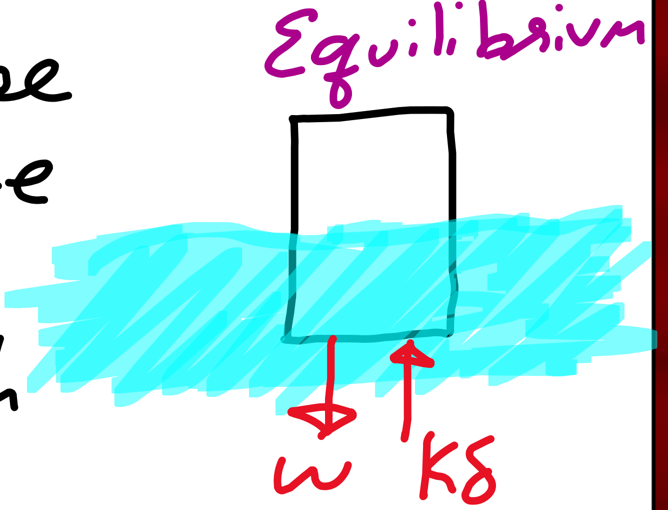

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Note: If you divide the two $[\tau_{NF}/\tau_{NE}]$ you can get nice expression relating M_{Full} to M_{empty}