Today 19.2, 19.3

$\boxed{L22}$

Today 19.2, 19.3

L22

Free Vibrations of rigid bodies

Today 19.2, $\underbrace{19.3}$

L22

Energy Methods for vibrations

L22

Today 19.2, 19.3

Thursday 19.4

$\boxed{L22}$

Today 19.2, 19.3

Thursday $\underbrace{19.4}_{\text{Forced vibrations}}$

ASU

$\boxed{L22}$

Today 19.2, 19.3

Thursday 19.4

Tuesday April 13th 19.5

$\boxed{L22}$

Today 19.2, 19.3

Thursday 19.4

Tuesday April 13^{th} $\underbrace{19.5}_{}$

Damped vibrations

L22

Today 19.2, 19.3

Thursday 19.4

Tuesday April 13th 19.5

Thursday April 15th Review

L22

Today 19.2, 19.3

Thursday 19.4

Tuesday April 13th 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

L22

Today 19.2, 19.3

Thursday 19.4

Tuesday April 13th 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

Thursday April 22nd Day of reckoning

L22

Today 19.2, 19.3

Thursday 19.4

Tuesday April 13^{th} 19.5

Thursday April 15^{th} Review

Tuesday April 20^{th} Exam 4

Thursday April 22^{nd} Day of reckoning

will know grade for course if you decide NOT to take final exam

L22

Today 19.2, 19.3

Thursday 19.4

Tuesday April 13th 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

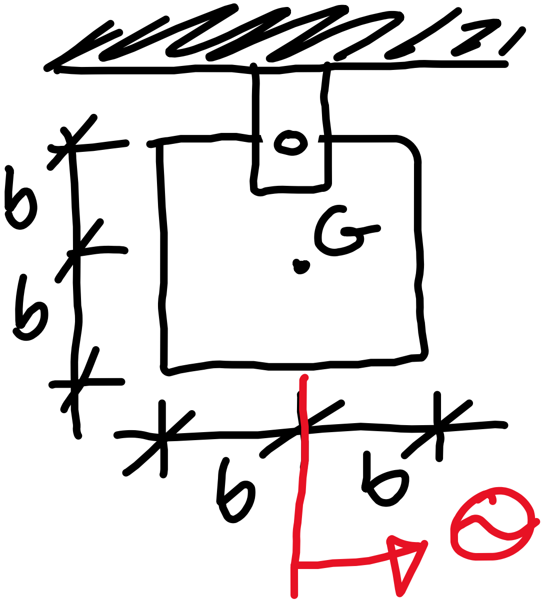
Thursday April 22nd Day of reckoning

Thursday April 29th Final exam

from 7:30 AM to 9:20 AM

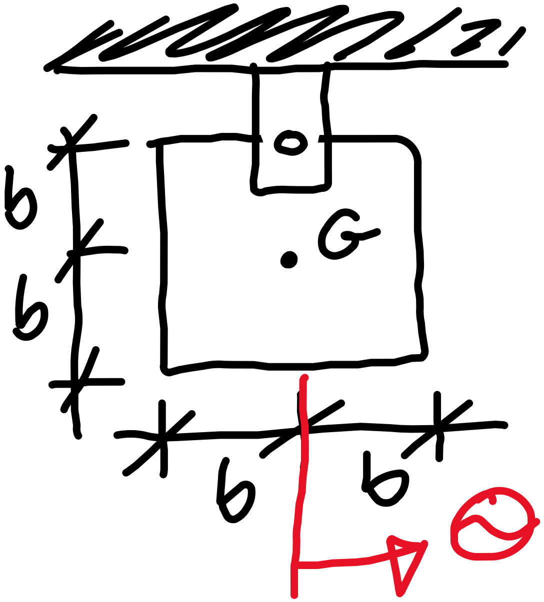
ASU

Rigid body example



Rigid body example

Displace small amount θ & find period



Rigid body example

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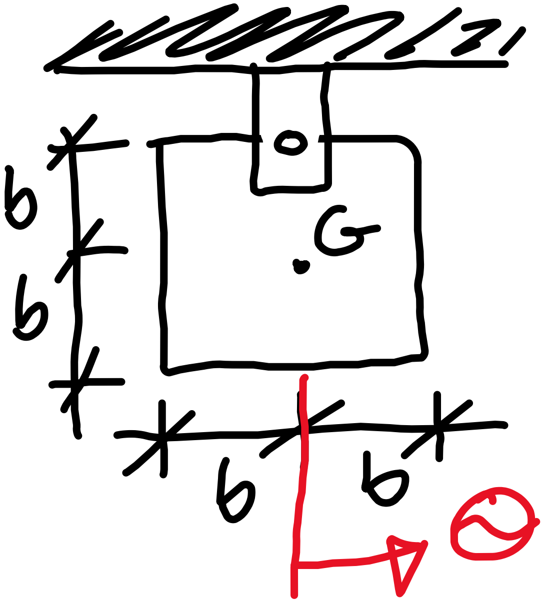
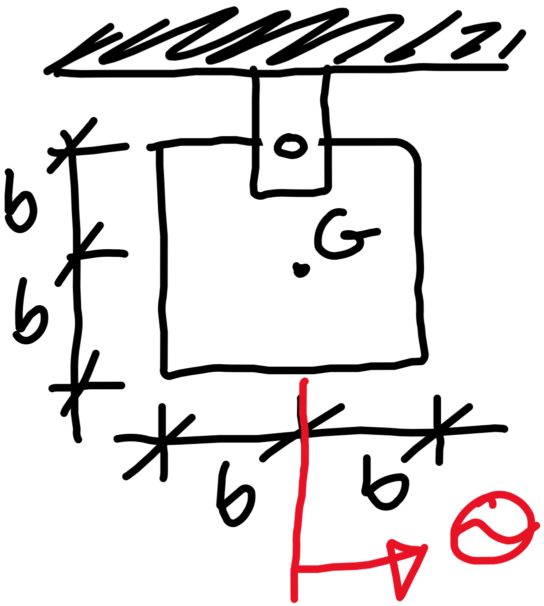


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right]$$

Rigid body example

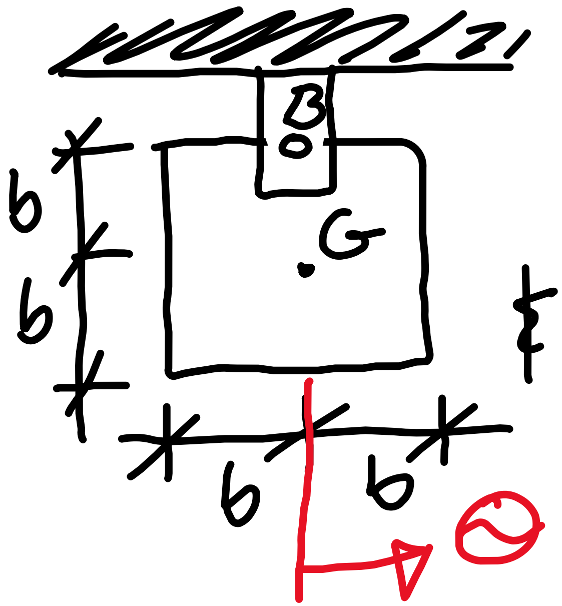
Displace small amount θ & find period



$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

Rigid body example

Displace small amount θ & find period

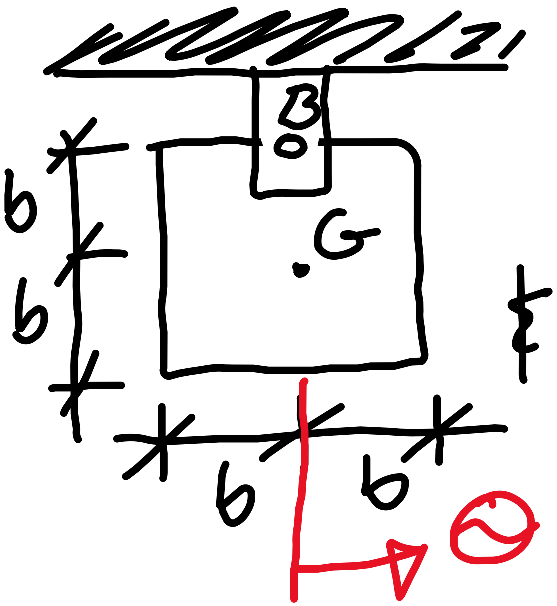


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \quad I_B = \bar{I} + m r_{G/B}^2$$

Rigid body example

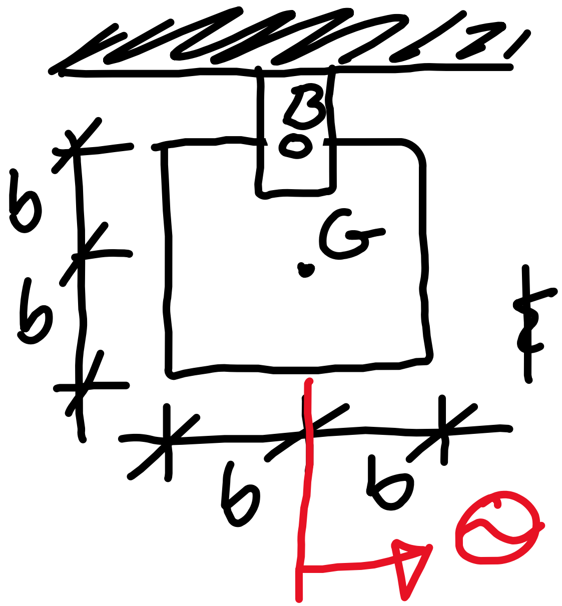
Displace small amount θ & find period



$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2 + (2b)^2\right] = \frac{8}{12} m b^2$$

$$\& \; I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12} m b^2 +$$

Rigid body example



Displace small amount θ & find period

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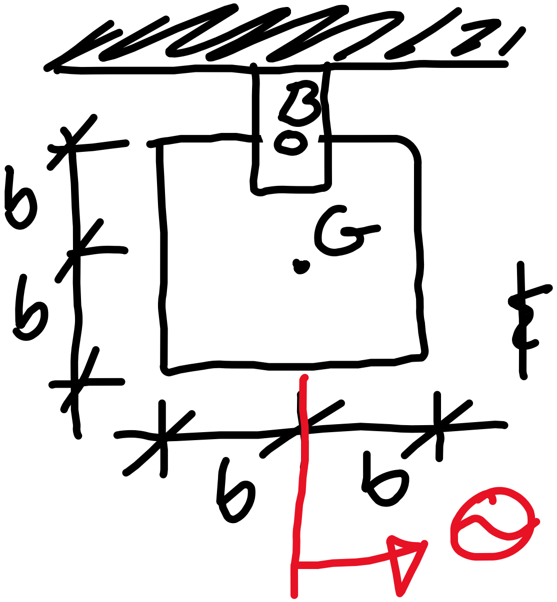
Rigid body example

Displace small amount θ & find period



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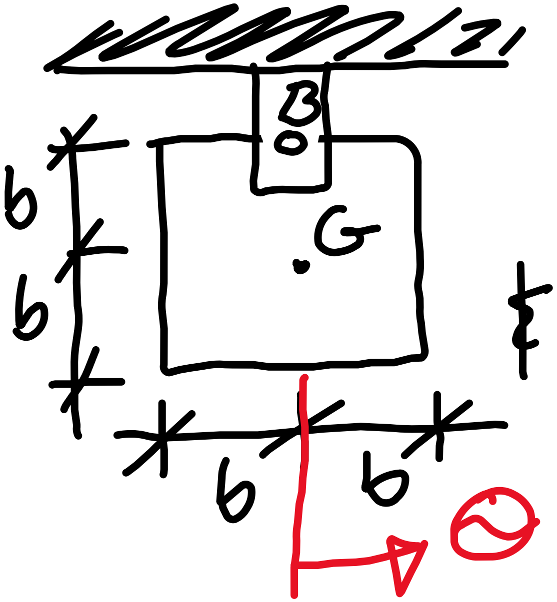
$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2$$

Rigid body example

Displace small amount θ & find period



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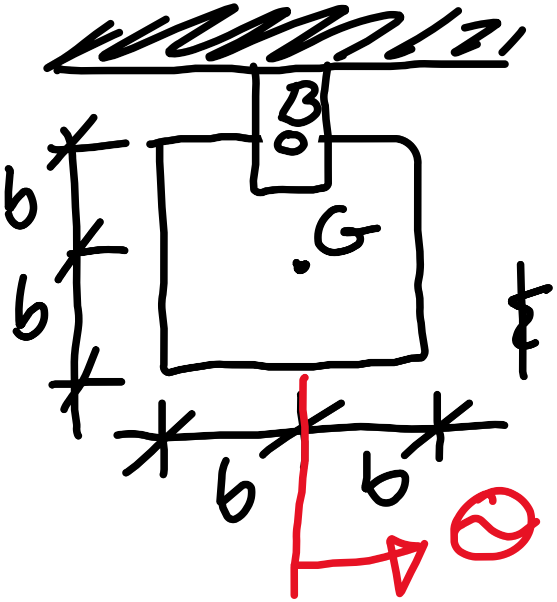
$$= \left(\frac{8}{12}+\frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2$$

Rigid body example



Displace small amount θ & find period

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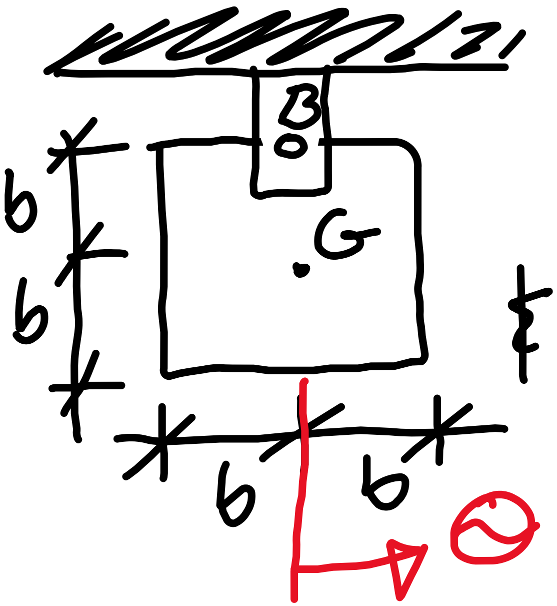
$$= \left(\frac{8}{12} + \frac{12}{12}\right) m b^2 = \frac{20}{12} m b^2 = \frac{5}{3} m b^2$$

Rigid body example

Displace small amount θ & find period



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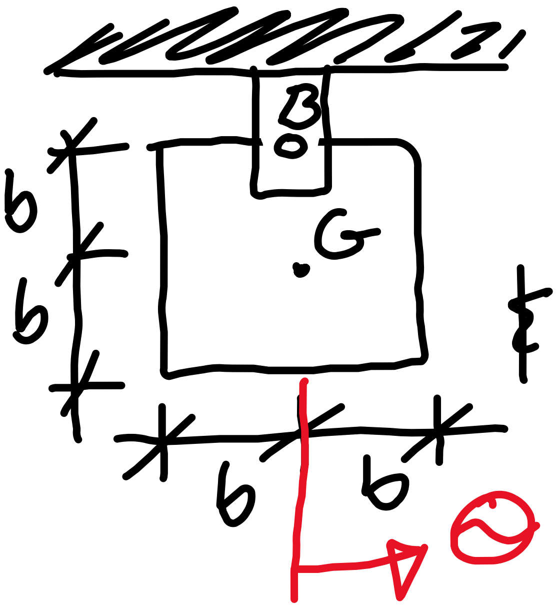
At equilibrium

Rigid body example

Displace small amount θ & find period



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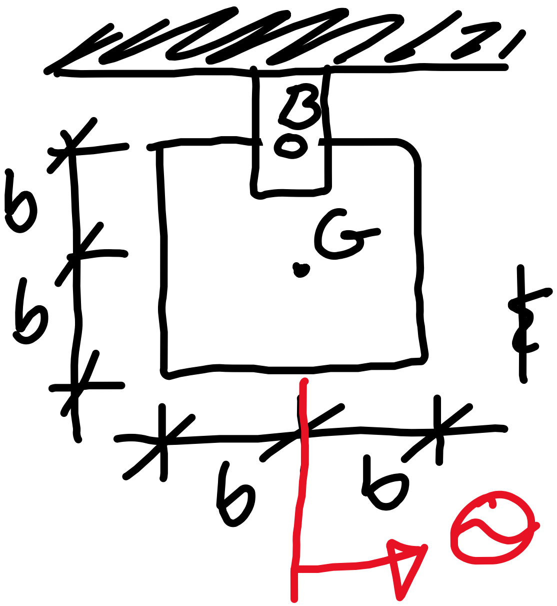
$$= \left(\frac{8}{12} + \frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2 = \frac{5}{3}mb^2$$

At equilibrium $\quad \circlearrowleft \sum \vec{M}_B = I_B \vec{\alpha}$

Rigid body example

Displace small amount θ & find period



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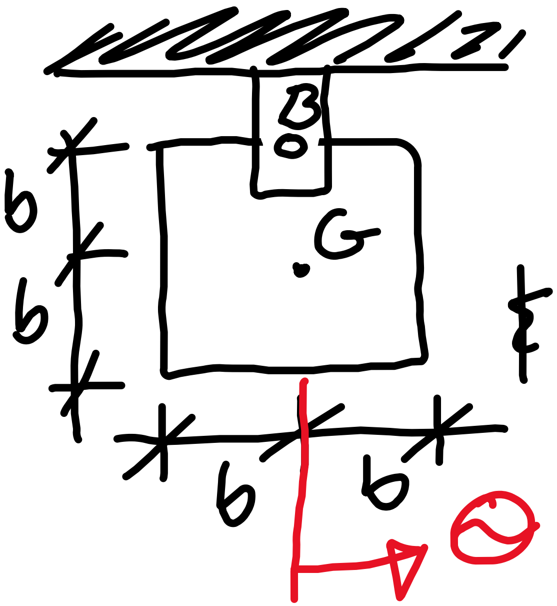
At equilibrium $\quad \circlearrowleft \sum \vec{M}_B = I_B \vec{\alpha} = \cancel{0}$

Rigid body example



Displace small amount θ & find period

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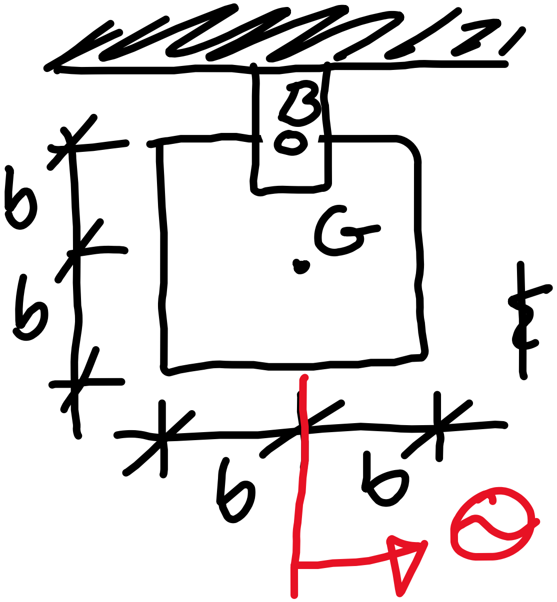
At equilibrium $\circlearrowleft \sum \vec{M}_B = I_B \vec{\alpha} = \cancel{0}$

Nothing interesting going on at equilibrium

Rigid body example

Displace small amount θ & find period



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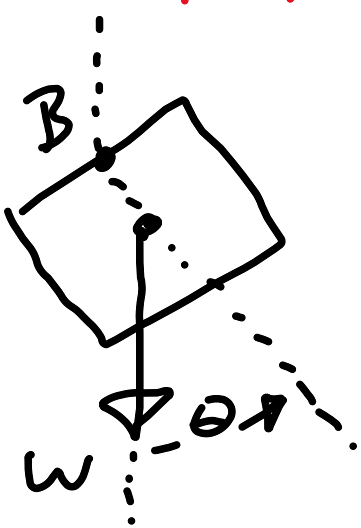
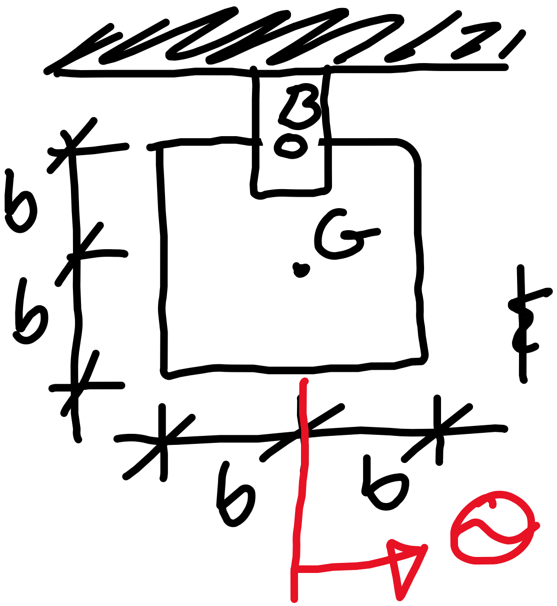
Displace amount θ:



Rigid body example

Displace small amount θ & find period



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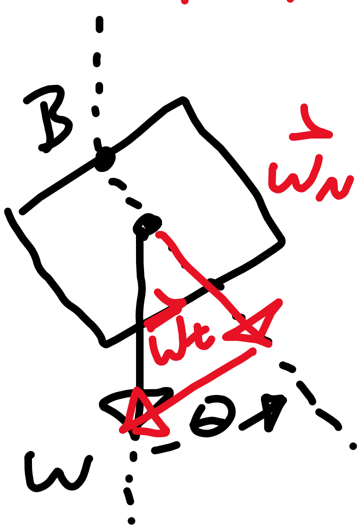
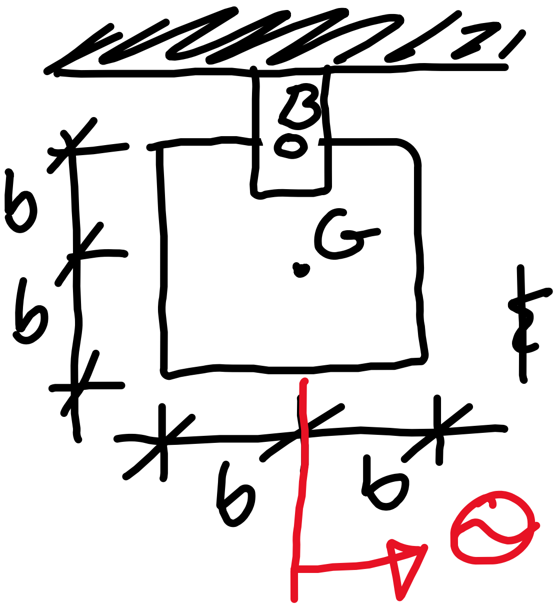


Rigid body example

Displace small amount θ & find period



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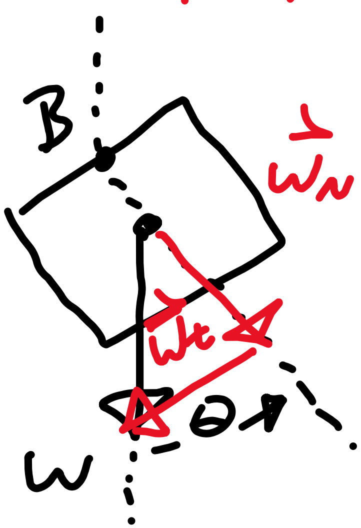
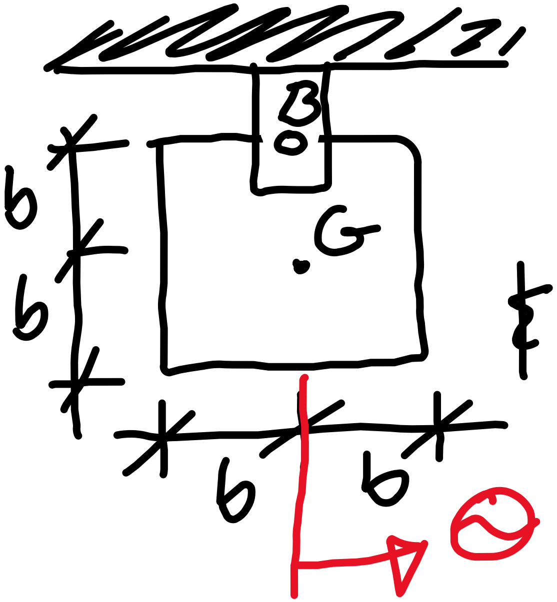
Displace amount θ:

$$w_n = -w\cos\theta$$



Rigid body example



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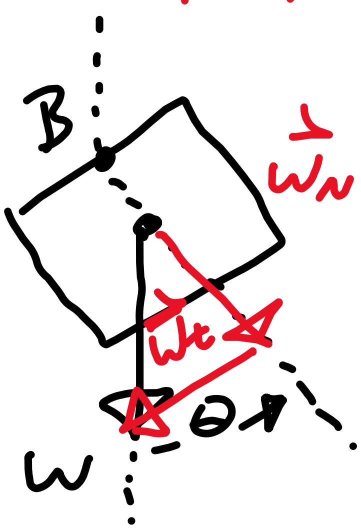
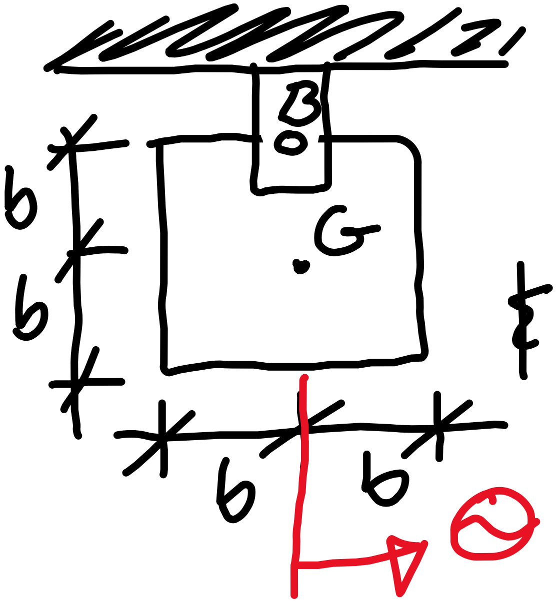
Displace amount θ:

$$W_N = -W\cos\theta \;\; \& \;\; W_t = -W\sin\theta$$



Rigid body example

Displace small amount θ & find period



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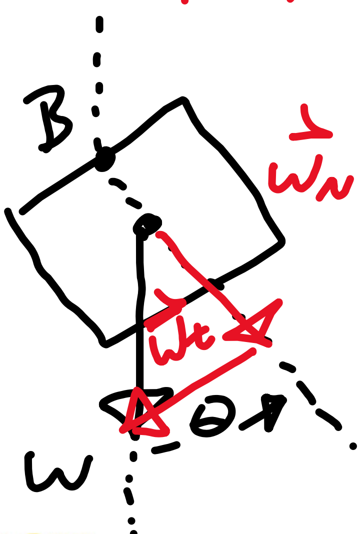
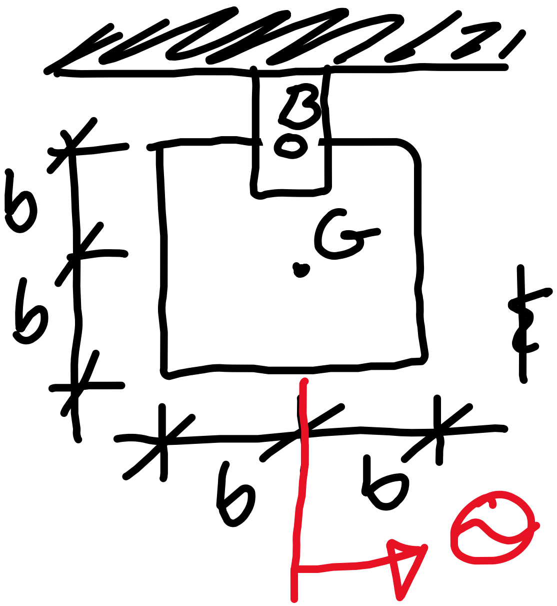
[just like for simple pendulum]



Rigid body example

Displace small amount θ & find period



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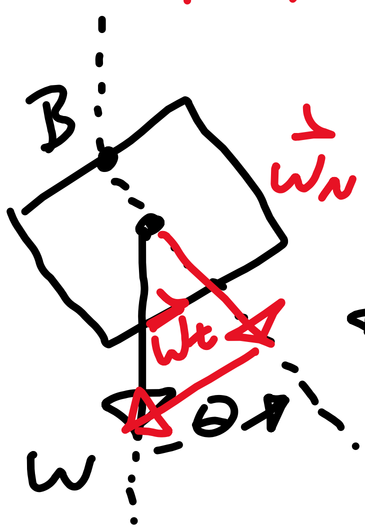


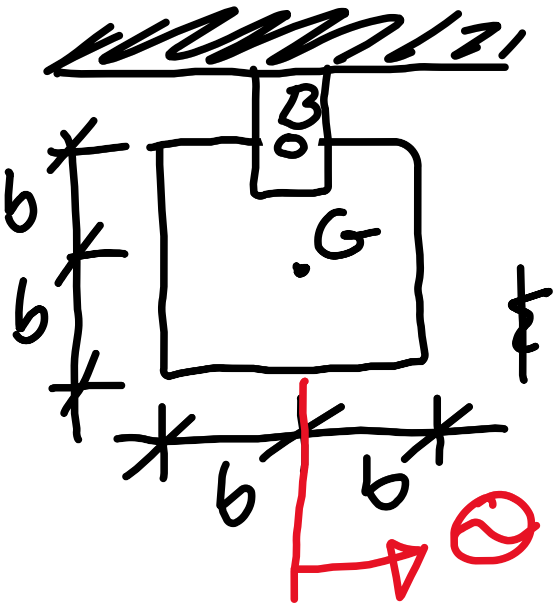
$w_N = -W\cos\theta$ & $w_t = -W\sin\theta$

[just like for simple pendulum]

$$\circlearrowleft \sum M_B = I_B \alpha$$

Rigid body example

Displace small amount θ & find period



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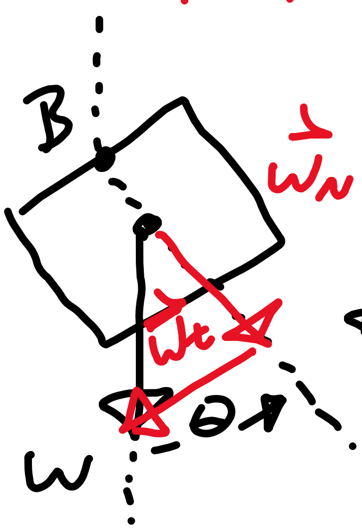

$w_N = -w\cos\theta$ & $w_t = -w\sin\theta$

[just like for simple pendulum]

$$\circlearrowleft \sum M_B = I_B\alpha \Rightarrow -bmg\sin\theta = I_B\ddot{\theta}$$

Rigid body example

Displace small amount θ & find period

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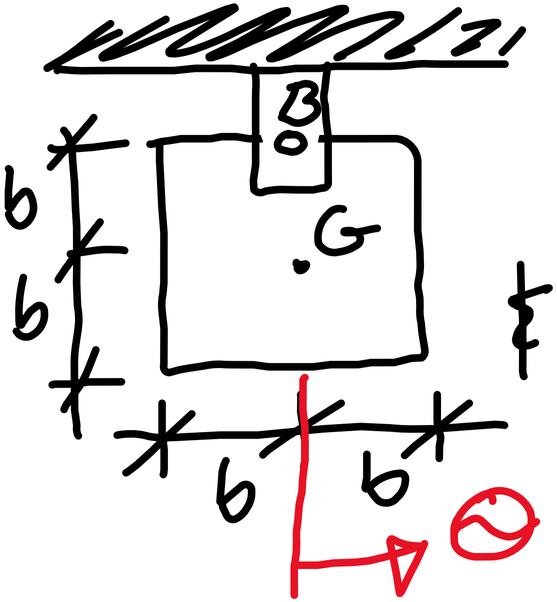

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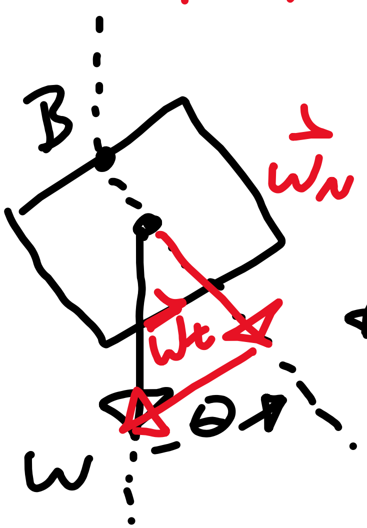
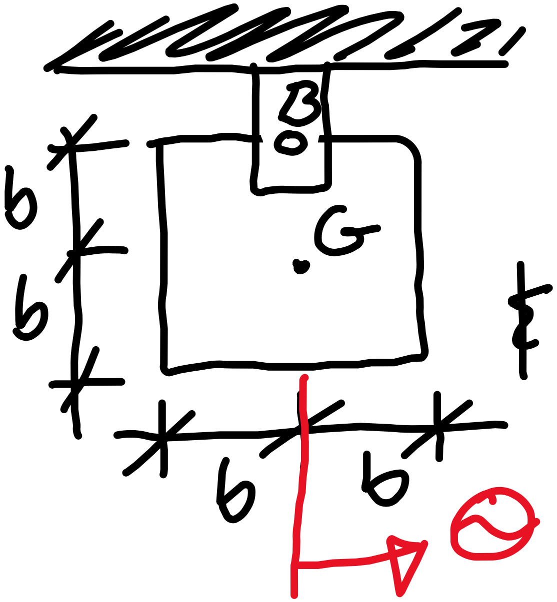


$$\circlearrowleft \sum M_B = I_B\alpha \Rightarrow -bmg\sin\theta = I_B\ddot{\theta}$$

Take θ small such that $\sin\theta \cong \theta$

Rigid body example

Displace small amount θ & find period



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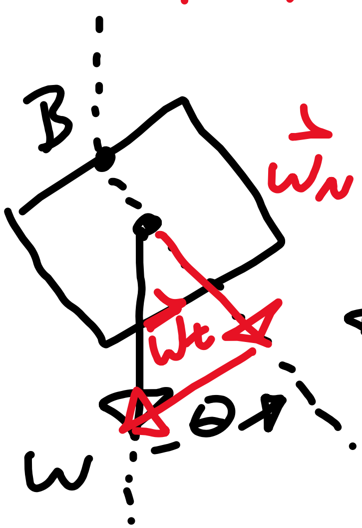
Displace amount θ:



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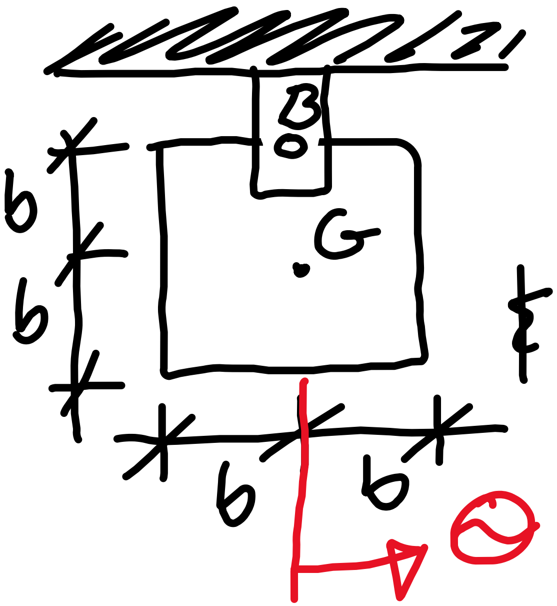
Rigid body example

Displace small amount θ & find period



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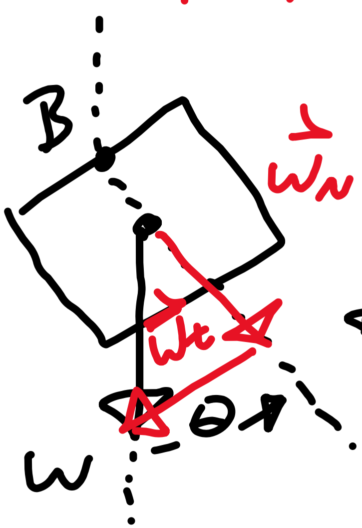
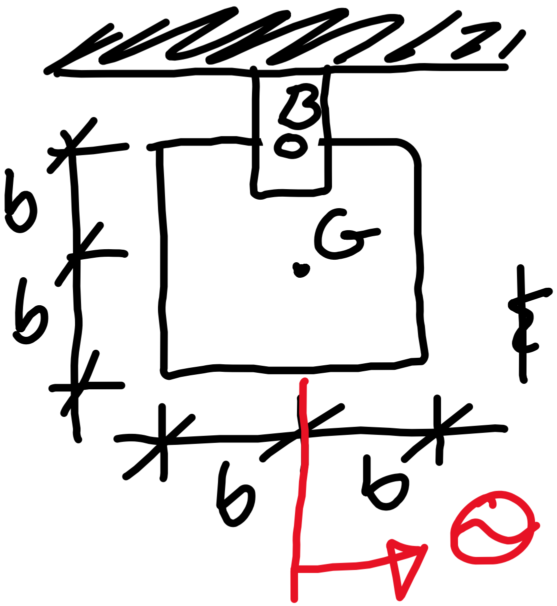


$$\circlearrowleft \sum M_B = I_B \alpha \Rightarrow -bmg\sin\theta = I_B \ddot{\theta}$$

$$\Rightarrow -bmg\theta \cong I_B \ddot{\theta} \Rightarrow \ddot{\theta} = -\omega^2\theta,$$

Rigid body example

Displace small amount θ & find period



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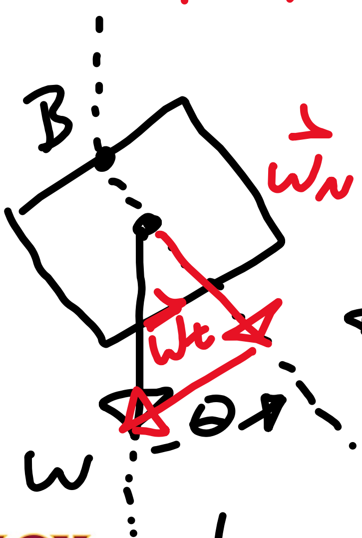

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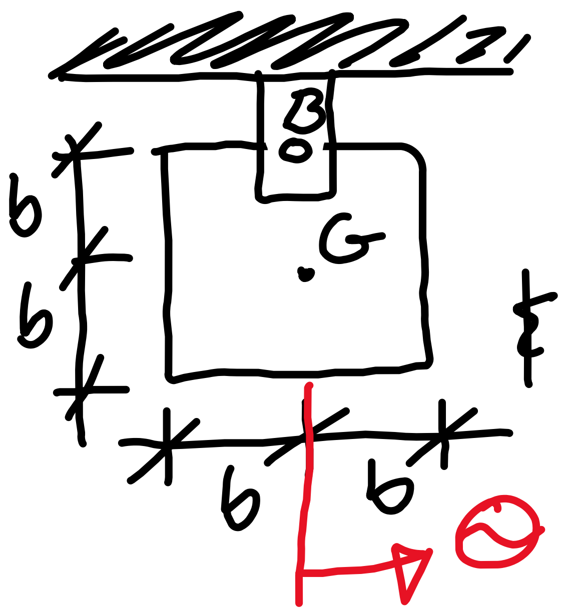
$$\circlearrowleft \sum M_B = I_B\alpha \Rightarrow -bmg\sin\theta = I_B\ddot{\theta}$$

$$\Rightarrow -bmg\theta \cong I_B\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega^2\theta,$$

where $\omega = \sqrt{\dfrac{bmg}{I_B}}$

Rigid body example

Displace small amount θ & find period

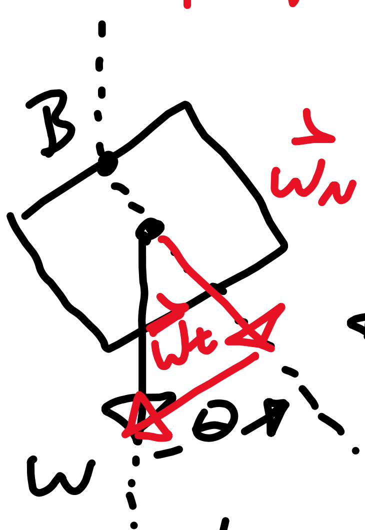


$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2+(2b)^2\right] = \frac{8}{12}mb^2$$

$$\text{\&}\ I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12}mb^2 + mb^2$$

$$= \left(\frac{8}{12}+\frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2 = \frac{5}{3}mb^2$$

Displace amount θ:



$w_n = -w\cos\theta$ & $w_t = -w\sin\theta$

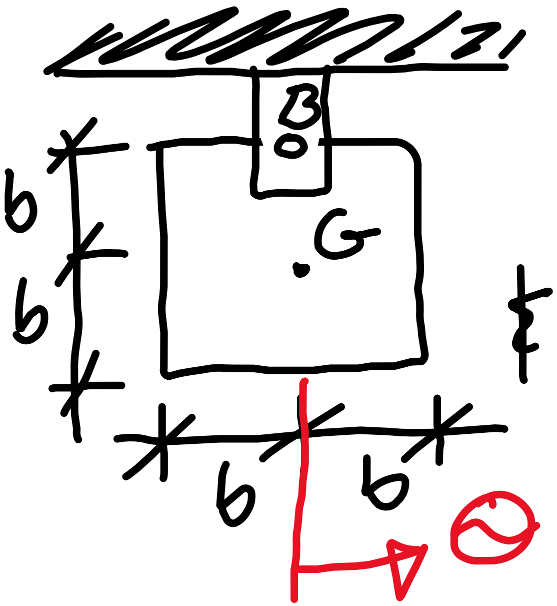
[just like for simple pendulum]

$$\circlearrowleft \sum M_B = I_B\alpha \Rightarrow -bmg\sin\theta = I_B\ddot{\theta}$$

$$\Rightarrow -bmg\,\theta \cong I_B\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega^2\theta,$$

where $\omega = \sqrt{\dfrac{bmg}{I_B}} = \sqrt{\dfrac{bmg}{5mb^2/3}}$

Rigid body example



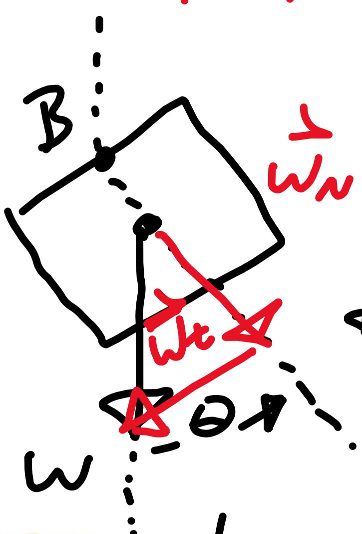
Displace small amount θ & find period

$$\bar{I} = \left(\frac{m}{12}\right)\left[(2b)^2+(2b)^2\right] = \frac{8}{12}mb^2$$

$$\& \ I_B = \bar{I} + m r_{G/B}^2 = \frac{8}{12}mb^2 + mb^2$$

$$= \left(\frac{8}{12}+\frac{12}{12}\right)mb^2 = \frac{20}{12}mb^2 = \frac{5}{3}mb^2$$

Displace amount θ:



$$w_n = -w\cos\theta \ \ \& \ \ w_t = -w\sin\theta$$

[just like for simple pendulum]

$$\circlearrowleft \sum M_B = I_B\alpha \Rightarrow -bmg\sin\theta = I_B\ddot{\theta}$$

$$\Rightarrow -bmg\theta \cong I_B\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega^2\theta,$$

where $\omega = \sqrt{\frac{bmg}{I_B}} = \sqrt{\frac{bmg}{5mb^2/3}} = \sqrt{\frac{3g}{5b}}$

From previous slide

$$\omega = \sqrt{\frac{3g}{5b}}$$

From previous slide

$$\omega = \sqrt{\frac{3g}{5b}}$$ & since $\omega\tau = 2\pi$

From previous slide

$$\omega = \sqrt{\frac{3g}{5b}}$$ & since $\omega\tau = 2\pi$

then $$\tau = \frac{2\pi}{\omega}$$

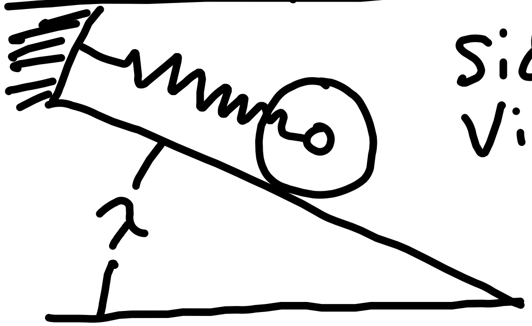
From previous slide

$$\omega = \sqrt{\frac{3g}{5b}}$$ & since $\omega\tau = 2\pi$

then $\tau = \frac{2\pi}{\omega} \Rightarrow$

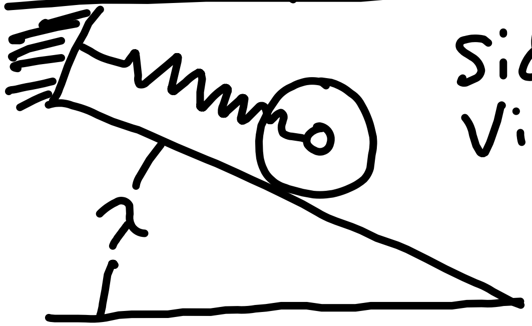
$$\boxed{\tau = 2\pi\sqrt{\frac{5b}{3g}}}$$

Example



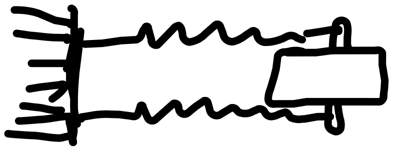
side
view

Example

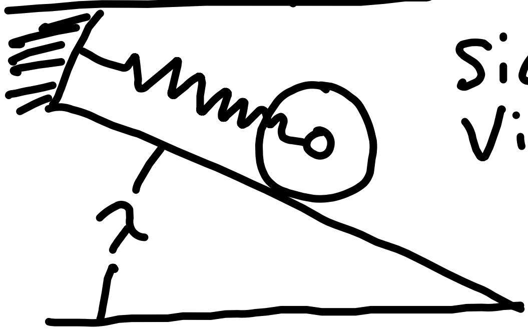


Side View

Top View

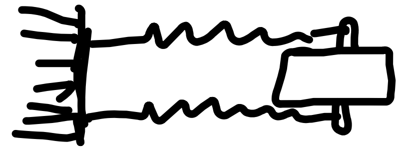


Example



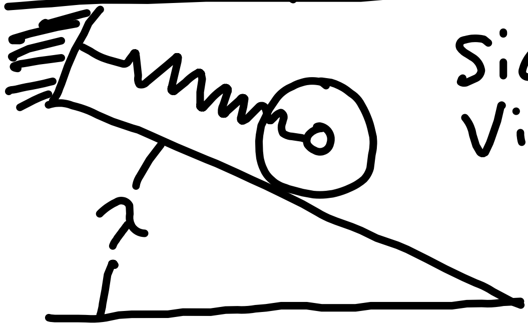
Side View

Top View



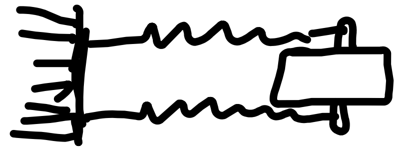
push disk down hill an amount d.

Example



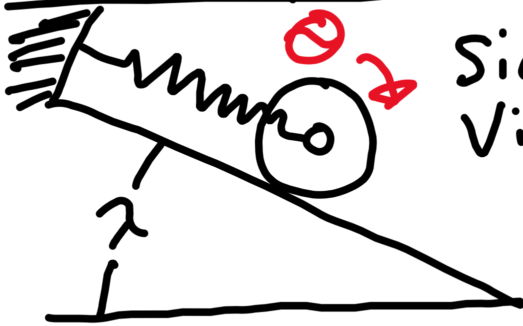
Side View

Top View



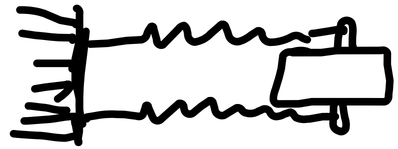
push disk down hill an amount d, find period

Example



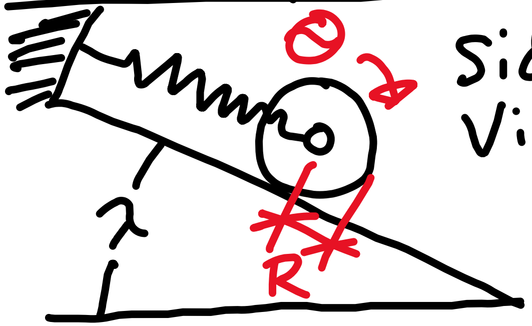
Side View

Top View



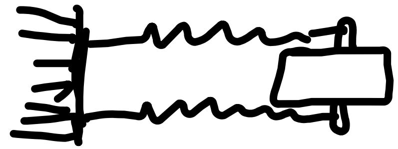
push disk down hill an amount d, find period

Example



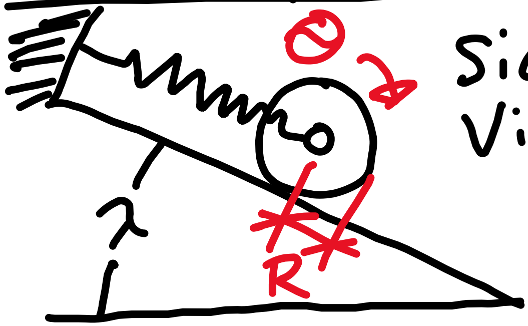
Side View

Top View



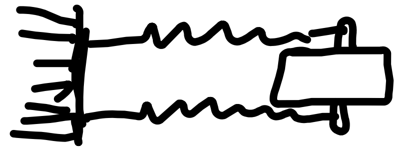
push disk down hill an amount d, find period

Example



Side View

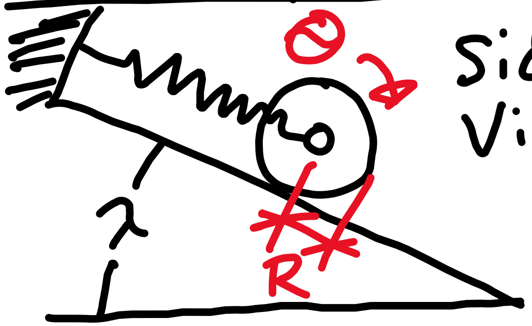
Top View



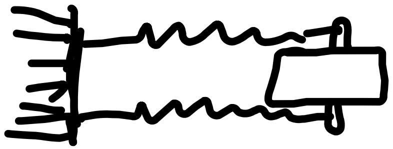
push disk down hill an amount d, find period

Equilibrium:

Example



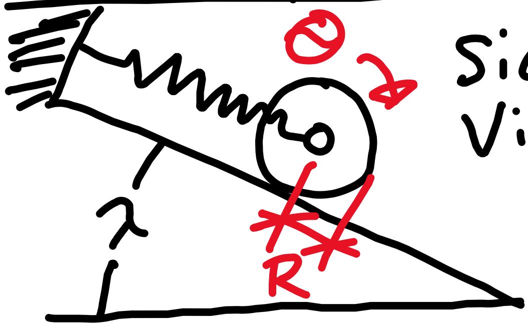
Top View



push disk down hill an amount d, find period

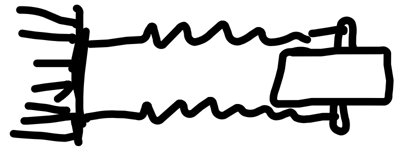
Equilibrium:

Example



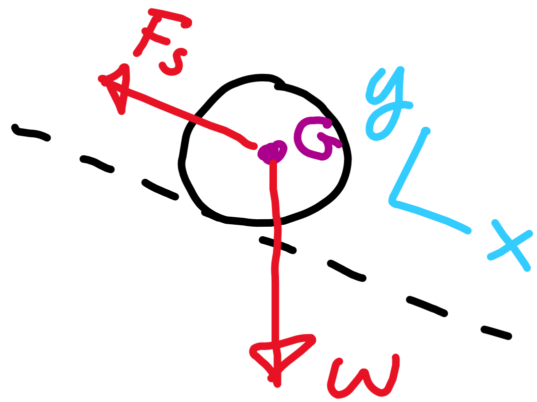
Side View

Top View

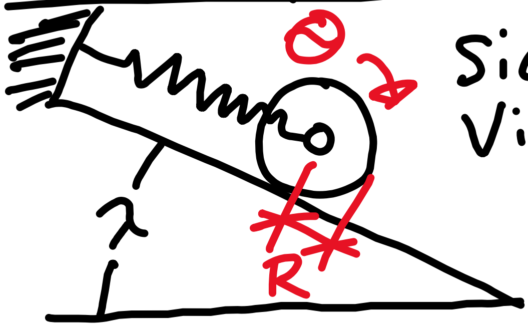


push disk down hill an amount d, find period

Equilibrium:

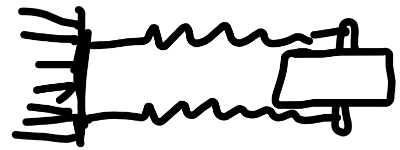


Example



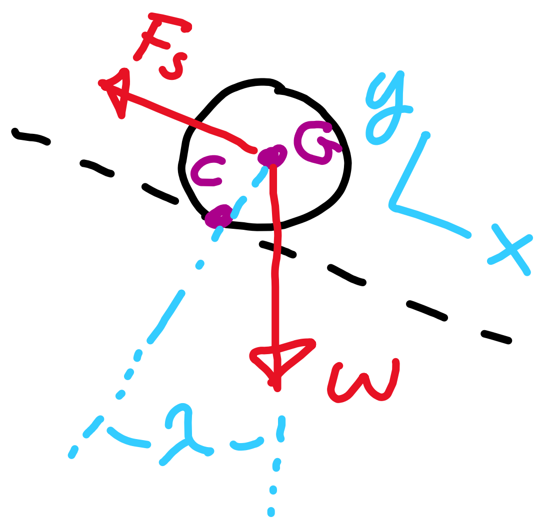
Side View

Top View

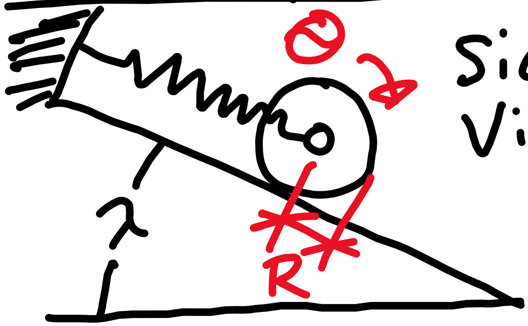


push disk down hill an amount d, find period

Equilibrium:

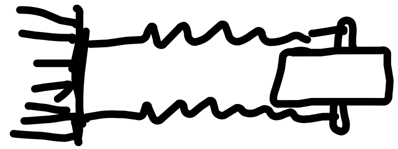


Example



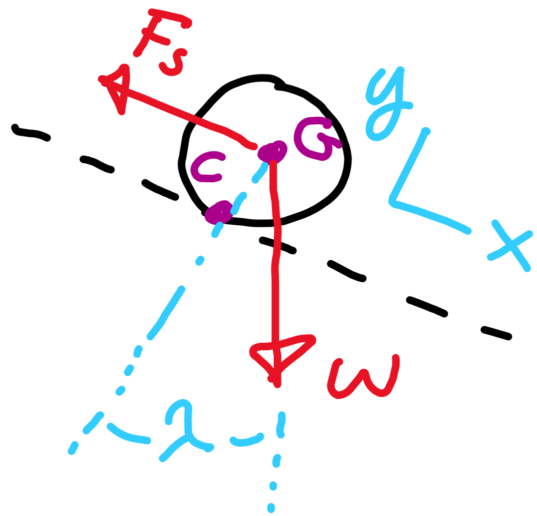
Side View

Top View



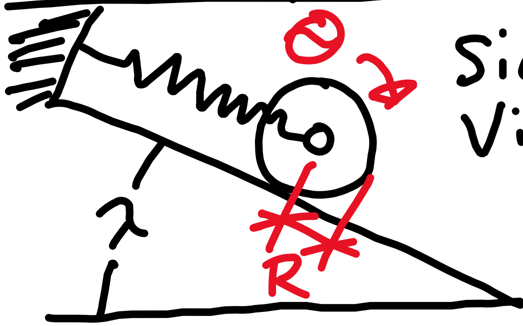
push disk down hill an amount d, find period

Equilibrium:



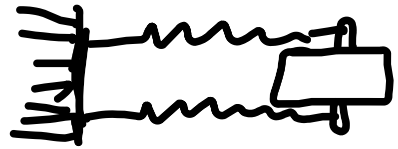
$$\vec{w} = \hat{x} W \sin\lambda + \hat{y} W \cos\lambda$$

Example



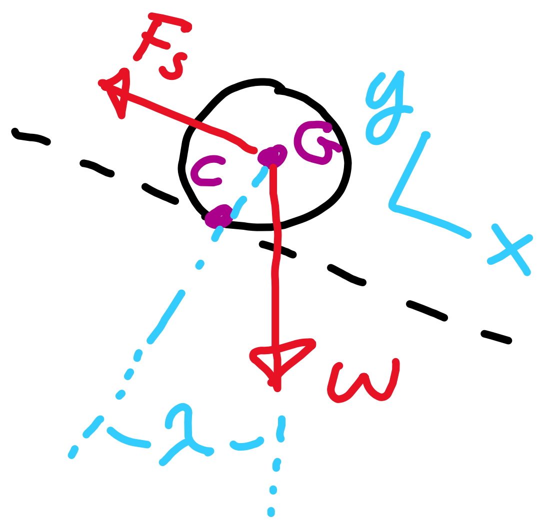
Side View

Top View



push disk down hill an amount d, find period

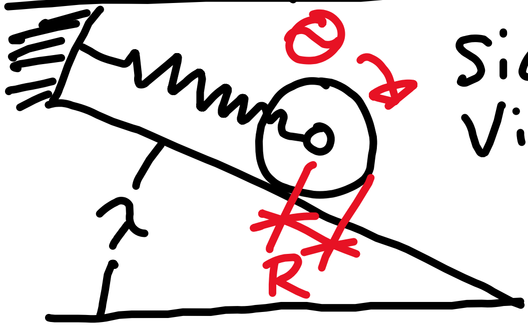
Equilibrium:



$\vec{w} = \hat{x} w \sin\lambda + \hat{y} w \cos\lambda$

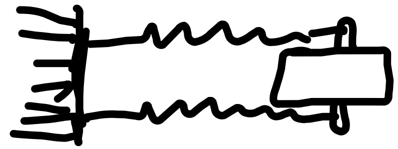
$\sum \vec{M}_c = \cancel{0}$

Example



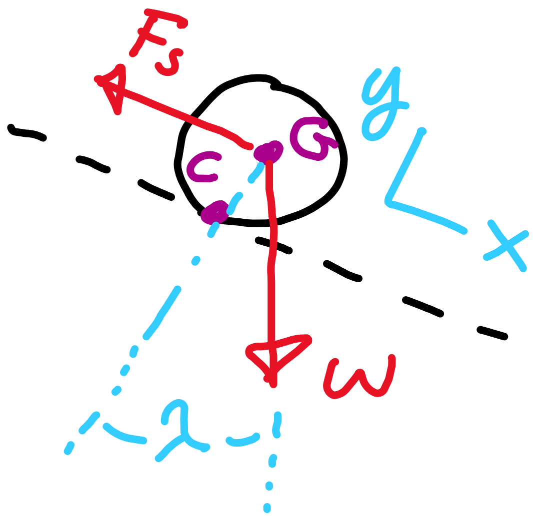
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:

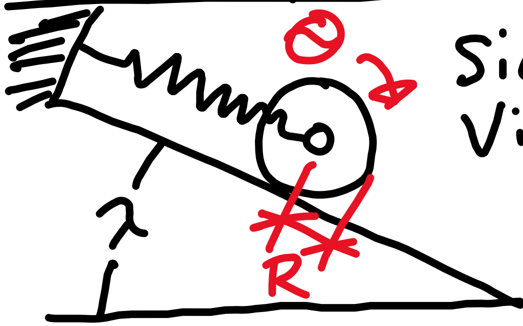


$\circlearrowright \sum \vec{M}_c = 0 \Rightarrow$

$\vec{w} = \hat{x} w \sin\lambda + \hat{y} w \cos\lambda$

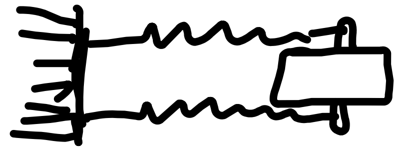
$R w \sin\lambda - R F_s = 0$

Example



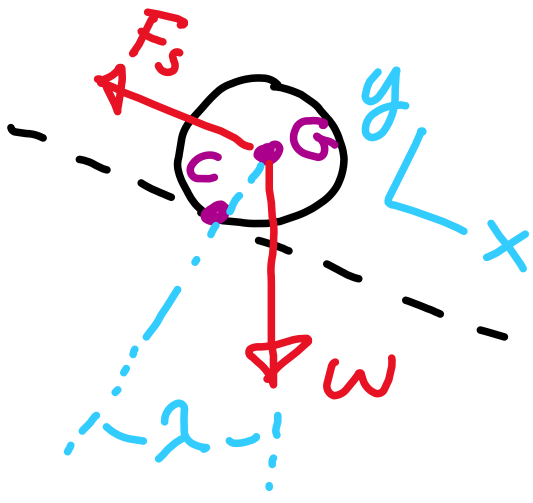
Side View

Top View



push disk down hill an amount δ, find period

Equilibrium:



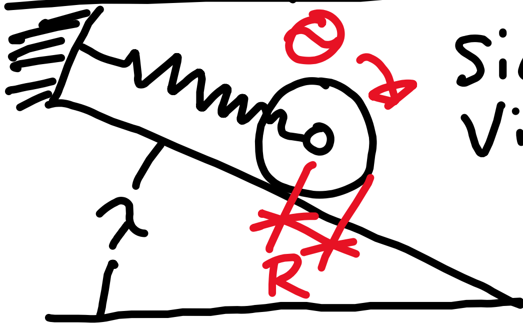
$\vec{W} = \hat{x} W \sin\lambda + \hat{y} W \cos\lambda$

$\sum \vec{M}_c = 0 \Rightarrow$

$R W \sin\lambda - R F_s = 0$

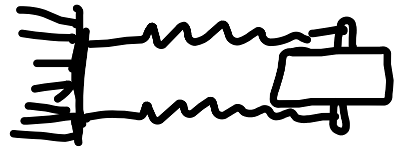
$\Rightarrow R W \sin\lambda = R k \delta$

Example



Side View

Top View

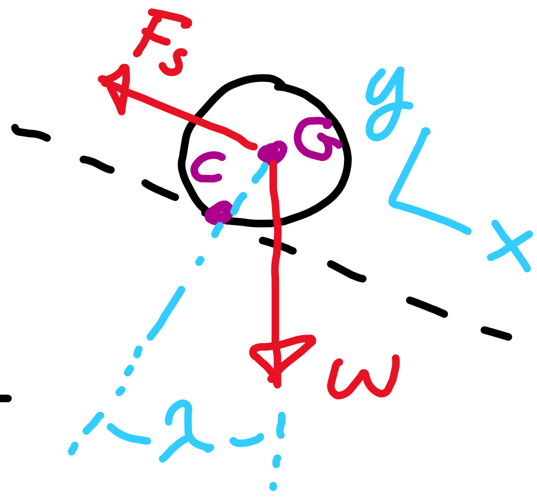


push disk down hill an amount δ, find period

Equilibrium:

$\sum \vec{M}_c = 0 \Rightarrow$

$\Rightarrow RW\sin\lambda = Rk\delta$

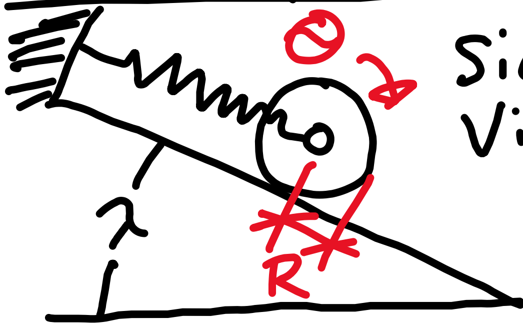


$\vec{W} = \hat{x}W\sin\lambda + \hat{y}W\cos\lambda$

$RW\sin\lambda - RF_s = 0$

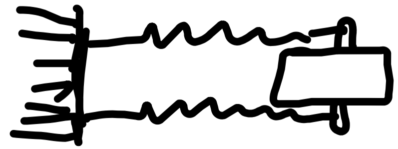
push down amount δ:

Example



Side View

Top View

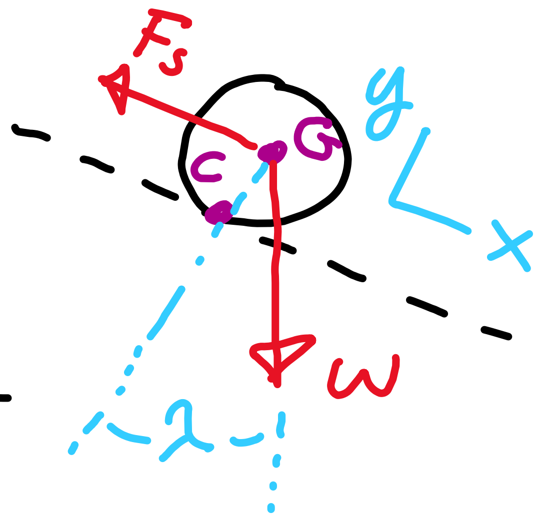


push disk down hill an amount δ, find period

Equilibrium:

$\circlearrowright \sum \vec{M}_c = 0 \Rightarrow$

$\Rightarrow Rw\sin\lambda = Rk\delta$



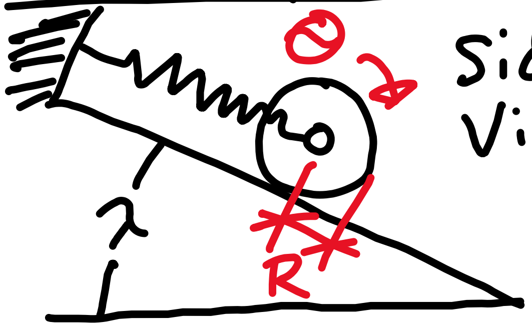
$\vec{w} = \hat{x}\, w\sin\lambda + \hat{y}\, w\cos\lambda$

$Rw\sin\lambda - RF_s = 0$

push down amount δ:

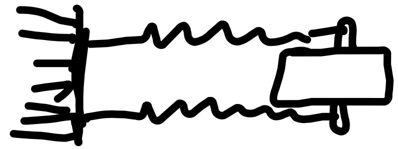
$\circlearrowright \sum \vec{M}_c = I_c\ddot{\theta}$

Example



Side View

Top View

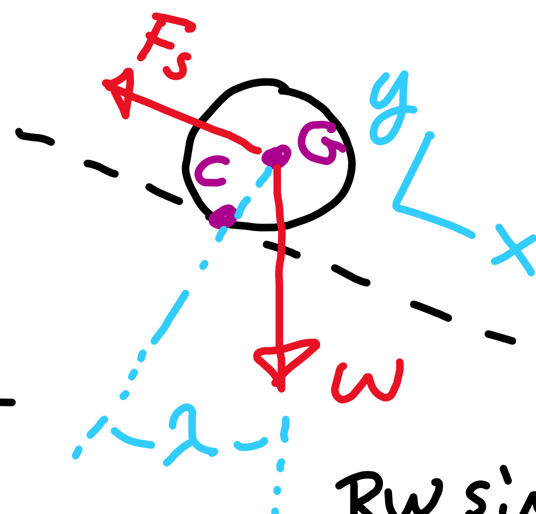


push disk down hill an amount d, find period

Equilibrium:

$\sum \vec{M}_c = 0 \Rightarrow$

$\Rightarrow Rw\sin\lambda = Rk\delta$



$\vec{w} = \hat{x}\, w\sin\lambda + \hat{y}\, w\cos\lambda$

$Rw\sin\lambda - RF_s = 0$

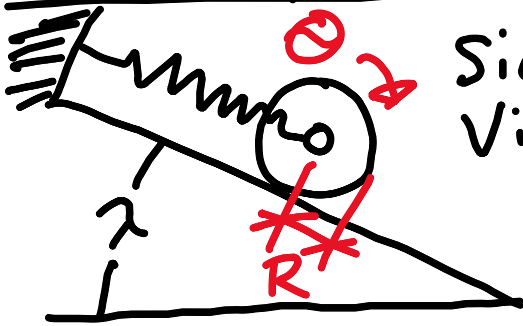
push down amount d:

$\sum \vec{M}_c = I_c\ddot{\theta} \Rightarrow$

But $Rw\sin\lambda = Rk\delta$

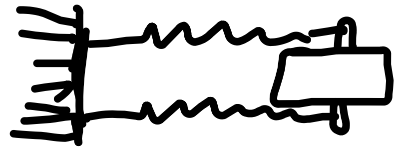
$Rw\sin\lambda - Rk(\delta + d) = I_c\ddot{\theta}$

Example



Side View

Top View



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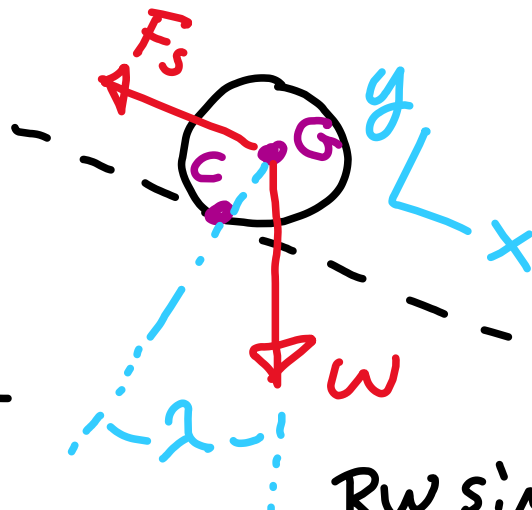
Equilibrium:



$\circlearrowright \sum \vec{M}_c = 0 \Rightarrow$

$\Rightarrow Rw\sin\lambda = RK\delta$

$\vec{w} = \hat{x}\, w\sin\lambda + \hat{y}\, w\cos\lambda$

$RW\sin\lambda - RF_s = 0$

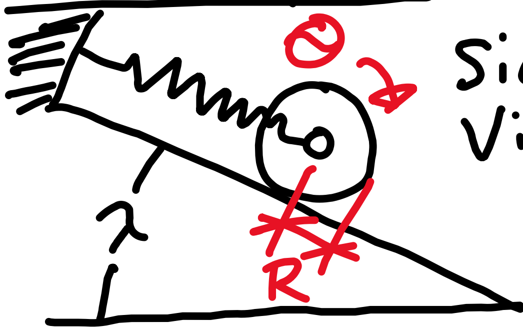
push down amount d:

$\circlearrowright \sum \vec{M}_c = I_c\ddot{\theta} \Rightarrow$

$Rw\sin\lambda - RK(\delta + d) = I_c\ddot{\theta}$

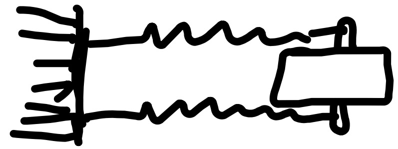
But $Rw\sin\lambda = RK\delta$ so $-RKd = I_c\ddot{\theta}$

Example



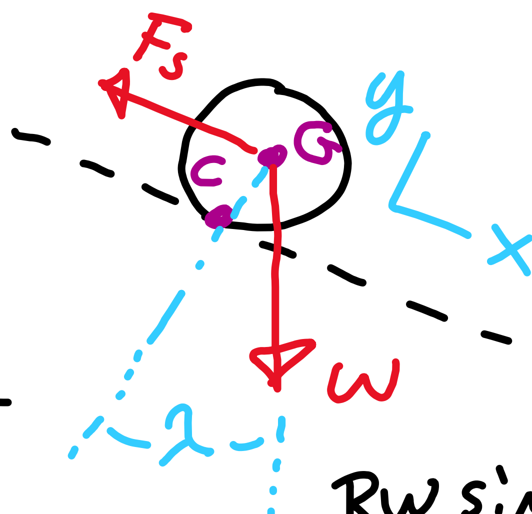
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



$\vec{w} = \hat{x} w \sin\lambda + \hat{y} w \cos\lambda$

$\curvearrowright \Sigma \vec{M}_c = 0 \Rightarrow$

$R W \sin\lambda - R F_s = 0$

$\Rightarrow R w \sin\lambda = R K \delta$

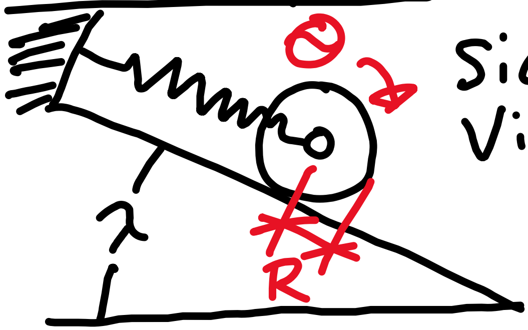
push down amount d:

$\curvearrowright \Sigma \vec{M}_c = I_c \ddot{\theta} \Rightarrow$

$R w \sin\lambda - RK(\delta + d) = I_c \ddot{\theta}$

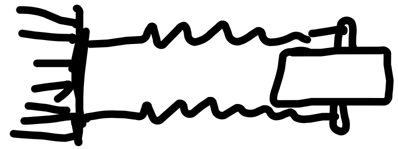
But $R w \sin\lambda = R K \delta$ so $- R K d = I_c \ddot{\theta}$ & since $d = R\theta$

Example



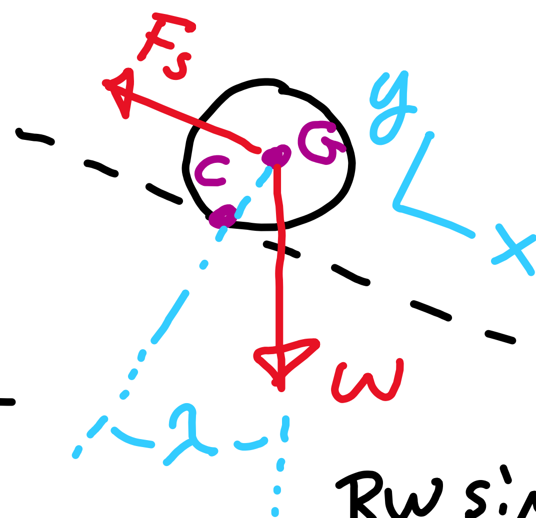
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



$\curvearrowright \sum \vec{M}_c = 0 \Rightarrow$

$\Rightarrow Rw\sin\lambda = RK\delta$

$\vec{w} = \hat{x}\, w\sin\lambda + \hat{y}\, w\cos\lambda$

$RW\sin\lambda - RF_s = 0$

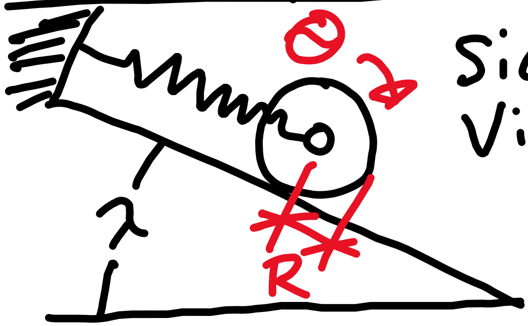
push down amount d:

$\curvearrowright \sum \vec{M}_c = I_c\ddot{\theta} \Rightarrow$

$Rw\sin\lambda - RK(\delta + d) = I_c\ddot{\theta}$

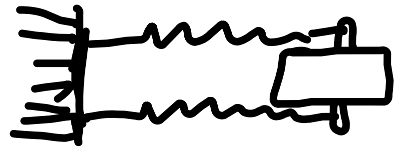
But $Rw\sin\lambda = RK\delta$ so $-RKd = I_c\ddot{\theta}$ & since $d = R\theta$ then $-R^2K\theta = I_c\ddot{\theta}$

Example



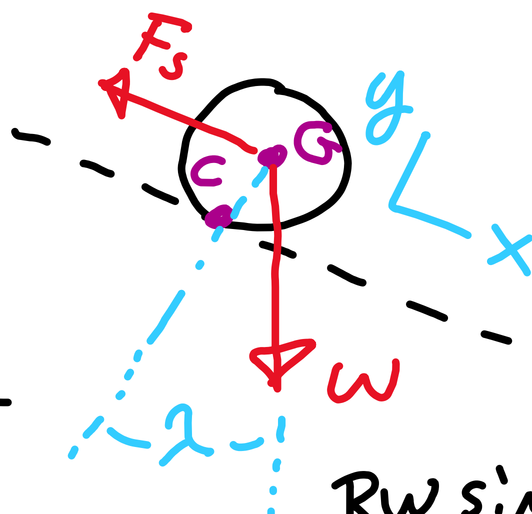
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



$\curvearrowleft \sum \vec{M}_c = 0 \Rightarrow$

$\Rightarrow Rw\sin\lambda = RK\delta$

$\vec{w} = \hat{x}\, w\sin\lambda + \hat{y}\, w\cos\lambda$

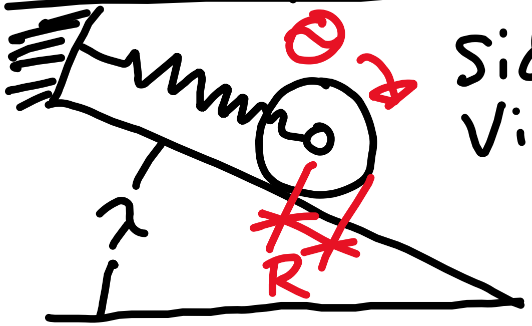
$RW\sin\lambda - RF_s = 0$

push down amount d:

$\curvearrowleft \sum \vec{M}_c = I_c\ddot{\theta} \Rightarrow \quad Rw\sin\lambda - RK(\delta + d) = I_c\ddot{\theta}$

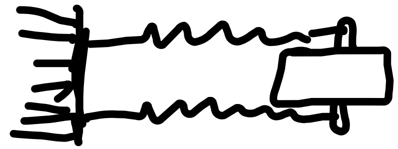
But $Rw\sin\lambda = RK\delta$ so $-RKd = I_c\ddot{\theta}$ & since $d = R\theta$ then $-R^2K\theta = I_c\ddot{\theta}$, where $I_c = \bar{I} + mR^2$

Example



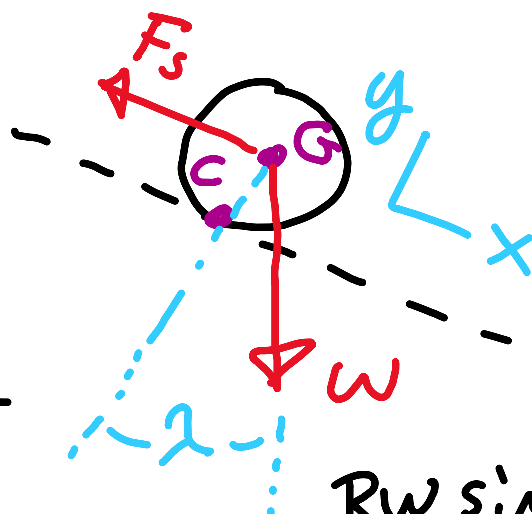
Side View

Top View



push disk down hill an amount d, find period

Equilibrium:



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push down amount d:

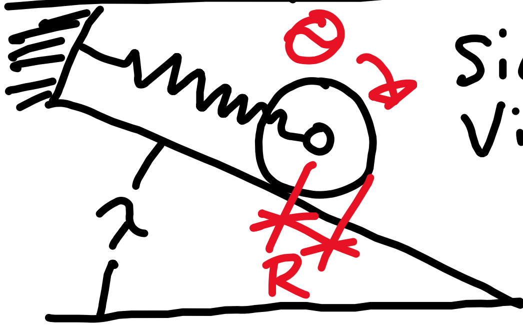
$\circlearrowright \sum \vec{M}_c = I_c\ddot{\theta} \Rightarrow$

$Rw\sin\lambda - RK(\delta + d) = I_c\ddot{\theta}$

But $Rw\sin\lambda = RK\delta$ so $-RKd = I_c\ddot{\theta}$ & since $d = R\theta$ then $-R^2K\theta = I_c\ddot{\theta}$, where $I_c = \bar{I} + mR^2$

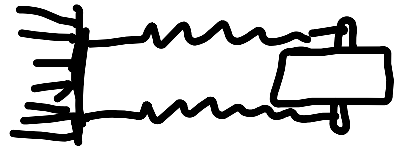
$\Rightarrow I_c = \frac{1}{2}mR^2 + mR^2$

Example



Side View

Top View

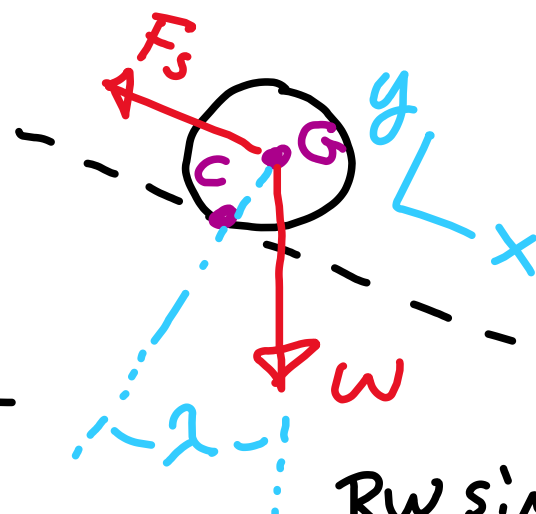


push disk down hill an amount d, find period

Equilibrium:

$\circlearrowleft \sum \vec{M}_c = 0 \Rightarrow$

$\Rightarrow Rw\sin\lambda = Rk\delta$



$\vec{w} = \hat{x}\, w\sin\lambda + \hat{y}\, w\cos\lambda$

$RW\sin\lambda - RF_s = 0$

push down amount d:

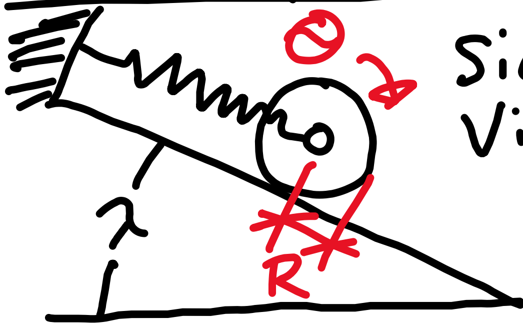
$\circlearrowleft \sum \vec{M}_c = I_c\ddot{\theta} \Rightarrow$

$Rw\sin\lambda - Rk(\delta + d) = I_c\ddot{\theta}$

But $Rw\sin\lambda = Rk\delta$ so $-Rkd = I_c\ddot{\theta}$ & since $d = R\theta$ then $-R^2k\theta = I_c\ddot{\theta}$, where $I_c = \bar{I} + mR^2$

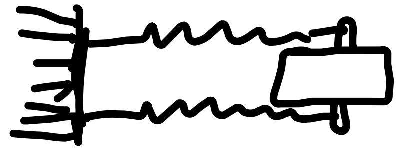
$\Rightarrow I_c = \frac{1}{2}mR^2 + mR^2$

Example



Side View

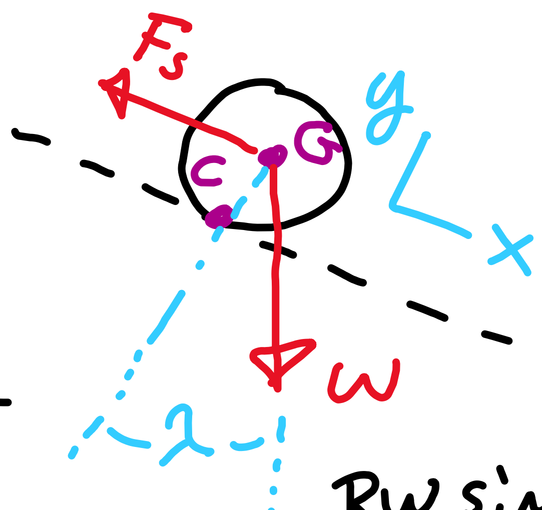
Top View



push disk down hill an amount d, find period

Equilibrium:

$\circlearrowright \sum \vec{M}_c = 0 \Rightarrow$



$\vec{w} = \hat{x} w \sin\lambda + \hat{y} w \cos\lambda$

$R W \sin\lambda - R F_s = 0$

$\Rightarrow R w \sin\lambda = R K \delta$

push down amount d:

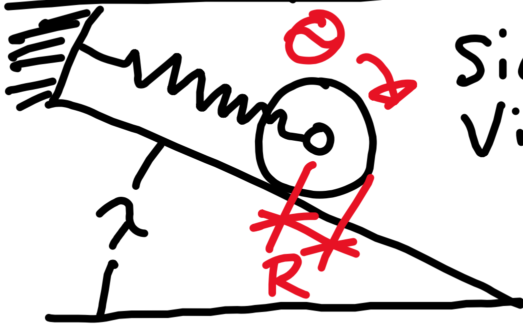
$\circlearrowright \sum \vec{M}_c = I_c \ddot{\theta} \Rightarrow$

$R w \sin\lambda - R K(\delta + d) = I_c \ddot{\theta}$

But $R w \sin\lambda = R K \delta$ so $-R K d = I_c \ddot{\theta}$ & since $d = R\theta$ then $-R^2 K \theta = I_c \ddot{\theta}$, where $I_c = \bar{I} + m R^2$

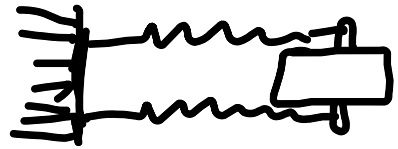
$\Rightarrow I_c = \frac{1}{2} m R^2 + m R^2 = \frac{3}{2} m R^2$

Example



Side View

Top View



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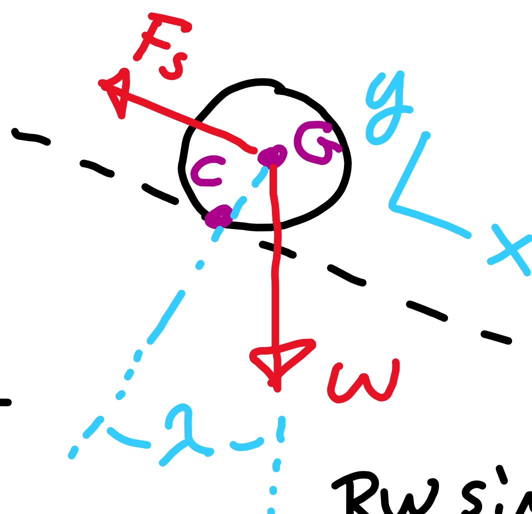

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$RW \sin\lambda - RF_s = 0$

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push down amount d:

$\circlearrowright \sum \vec{M}_c = I_c \ddot{\theta} \Rightarrow$

$RW\sin\lambda - RK(\delta + d) = I_c \ddot{\theta}$

But $RW\sin\lambda = RK\delta$ so $-RKd = I_c\ddot{\theta}$ & since $d = R\theta$ then $-R^2K\theta = I_c\ddot{\theta}$, where $I_c = \bar{I} + mR^2$

$\Rightarrow I_c = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$ so

$R^2K\theta = \frac{3}{2}MR^2\ddot{\theta}$

From previous slide

$$-R^2 K \theta = \frac{3}{2} M R^2 \ddot{\theta}$$

From previous slide

$$-R^2 K \Theta = \frac{3}{2} M R^2 \ddot{\Theta} \implies -\frac{2}{3}\left(\frac{K}{M}\right)\Theta = \ddot{\Theta}$$

From previous slide

$$-R^2K\Theta = \frac{3}{2}MR^2\ddot{\Theta} \Rightarrow -\frac{2}{3}\left(\frac{K}{M}\right)\Theta = \ddot{\Theta}$$

or $\quad \ddot{\Theta} = -\omega^2\Theta,$

From previous slide

$$-R^2K\Theta = \frac{3}{2}MR^2\ddot{\Theta} \implies -\frac{2}{3}\left(\frac{K}{m}\right)\Theta = \ddot{\Theta}$$

or $\ddot{\Theta} = -\omega^2\Theta$, where $\omega = \sqrt{\frac{2K}{3m}}$

From previous slide

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From previous slide

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or $\ddot{\Theta} = -\omega^2 \Theta$, where $\omega = \sqrt{\frac{2K}{3m}}$

& since $\omega \tau = 2\pi$

$$\tau = \frac{2\pi}{\omega}$$

From previous slide

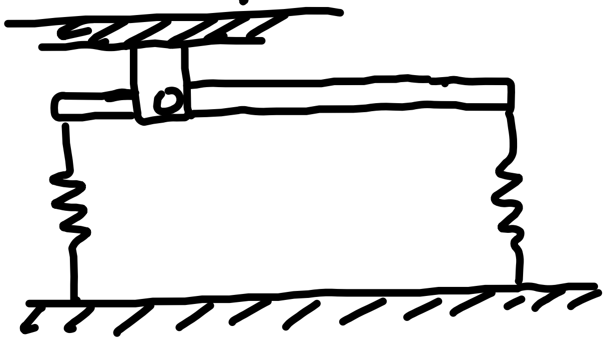
$$-R^2 K\Theta = \frac{3}{2} M R^2 \ddot{\Theta} \Rightarrow -\frac{2}{3}\left(\frac{K}{M}\right)\Theta = \ddot{\Theta}$$

or $\ddot{\Theta} = -\omega^2 \Theta$, where $\omega = \sqrt{\frac{2K}{3M}}$

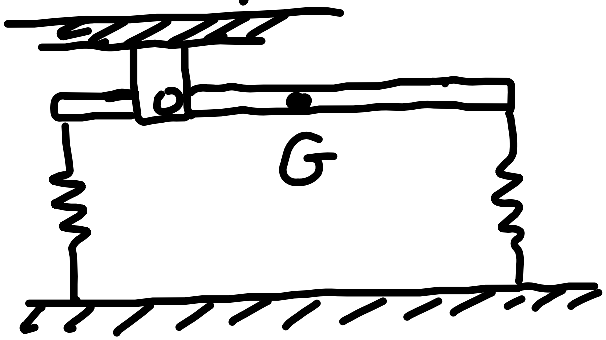
& since $\omega\tau = 2\pi$

$$\boxed{\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3M}{2K}}}$$

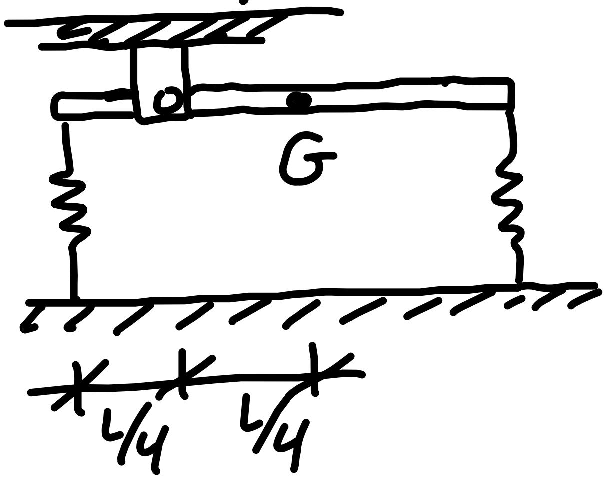
Example



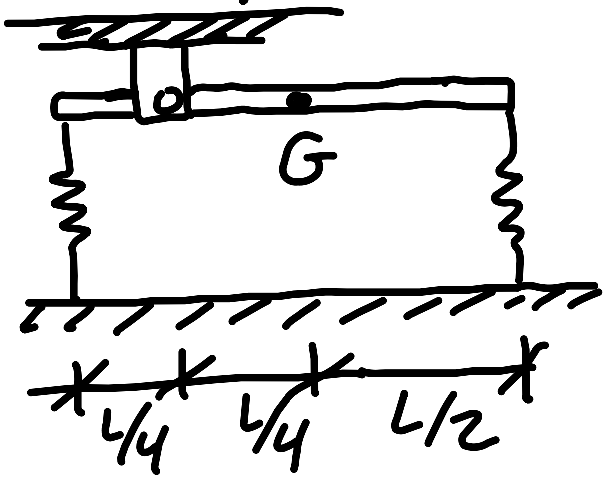
Example



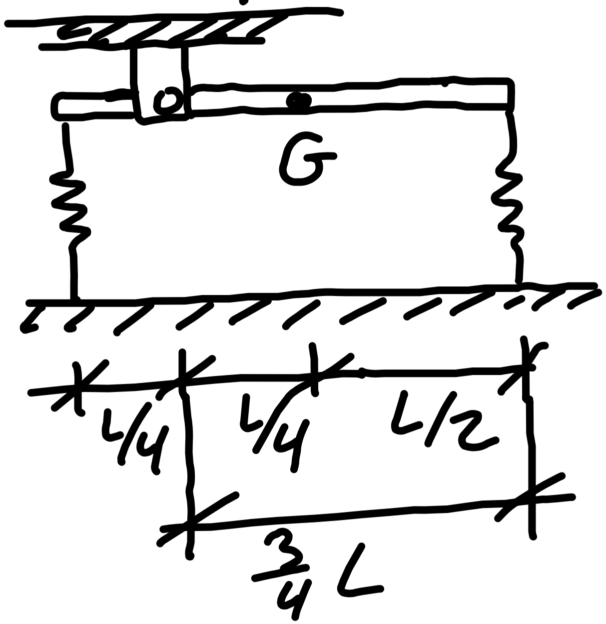
Example



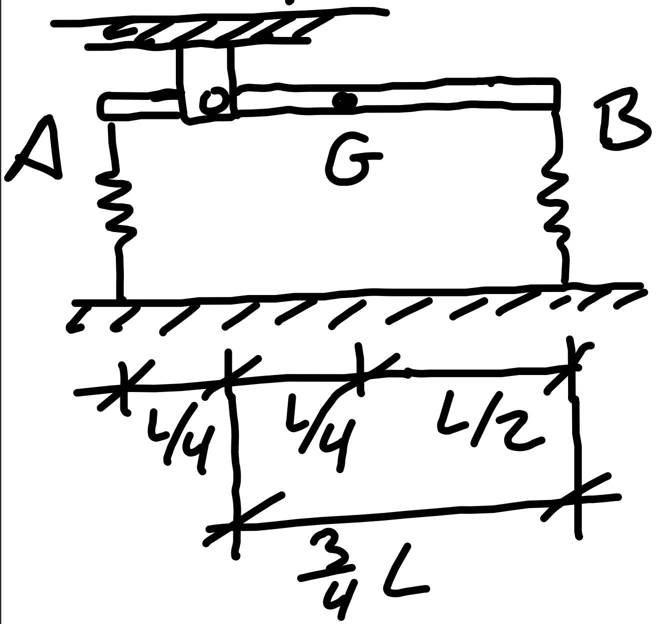
Example



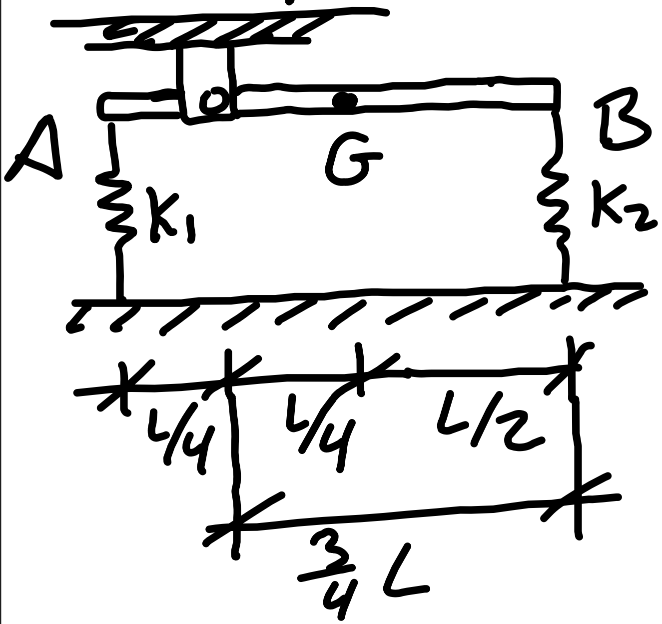
Example



Example

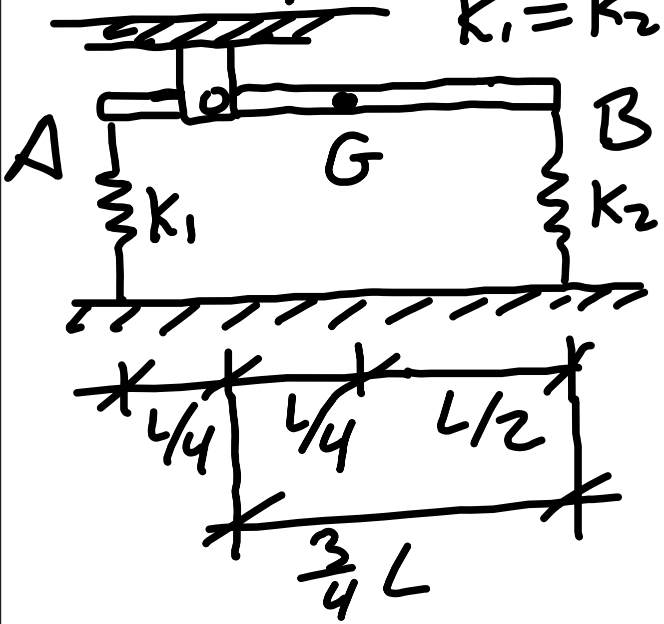


Example

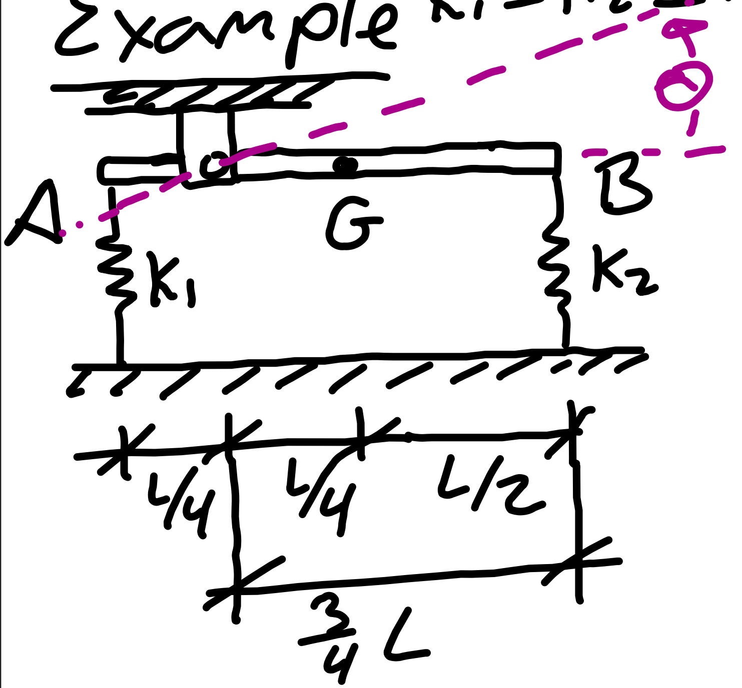


Example

$k_1 = k_2 = k$

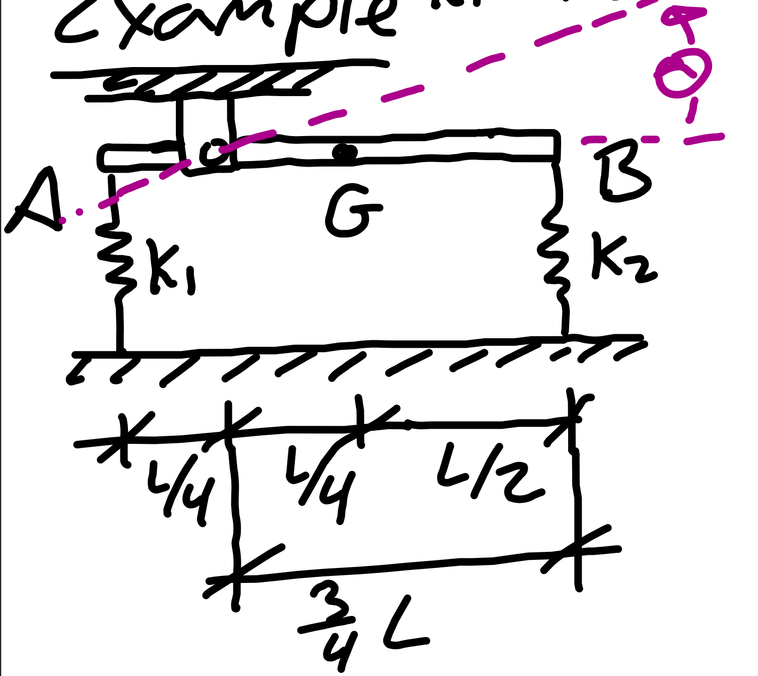


Example $k_1 = k_2 = k$



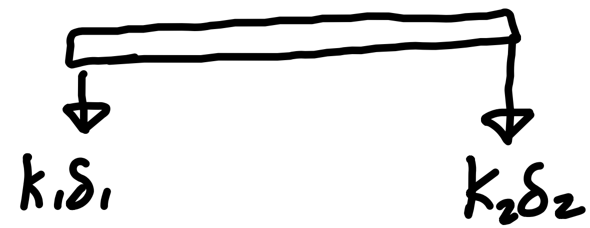
Push side B up a small distance d & find ω_n

Example $k_1 = k_2 = k$



Push side B up a small distance δ & Find ω_n:

Equilibrium



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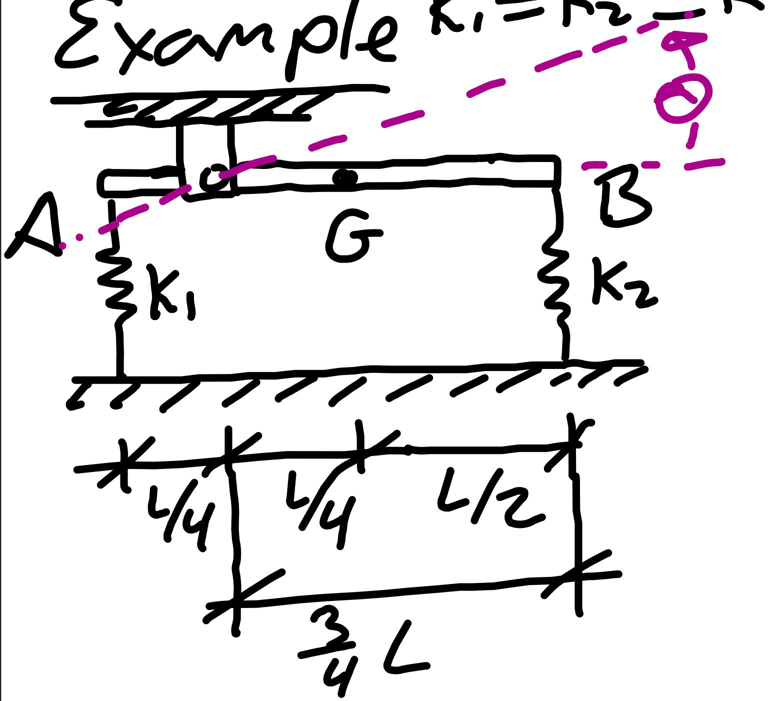

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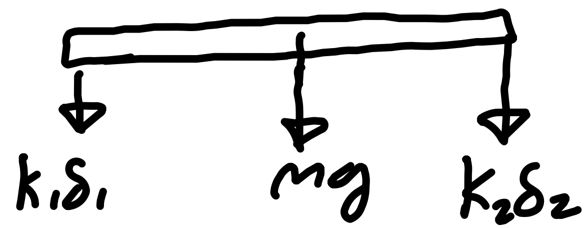

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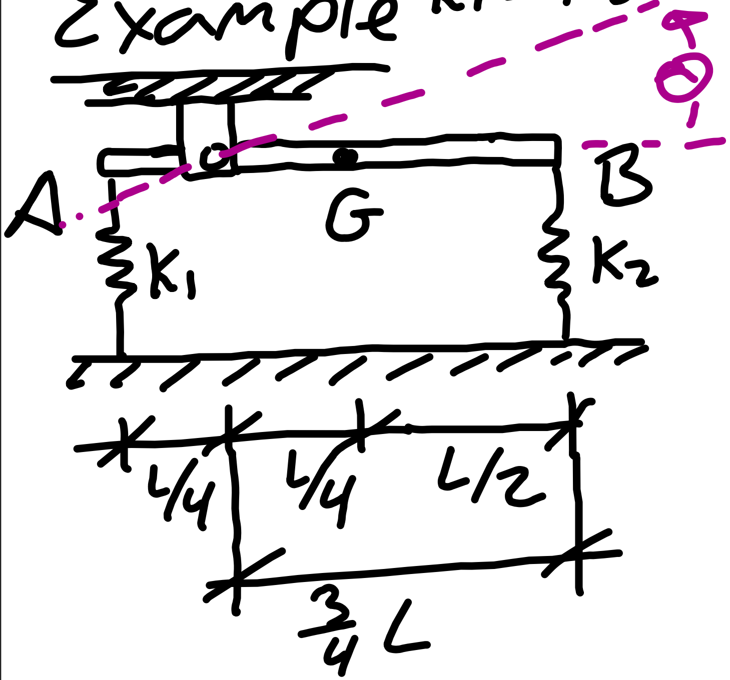

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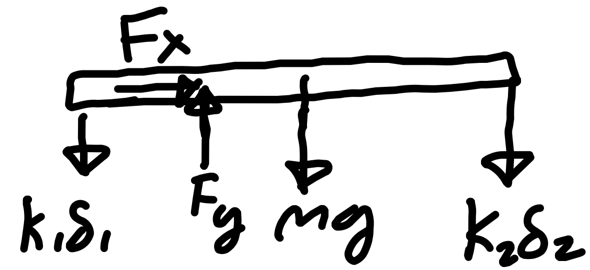

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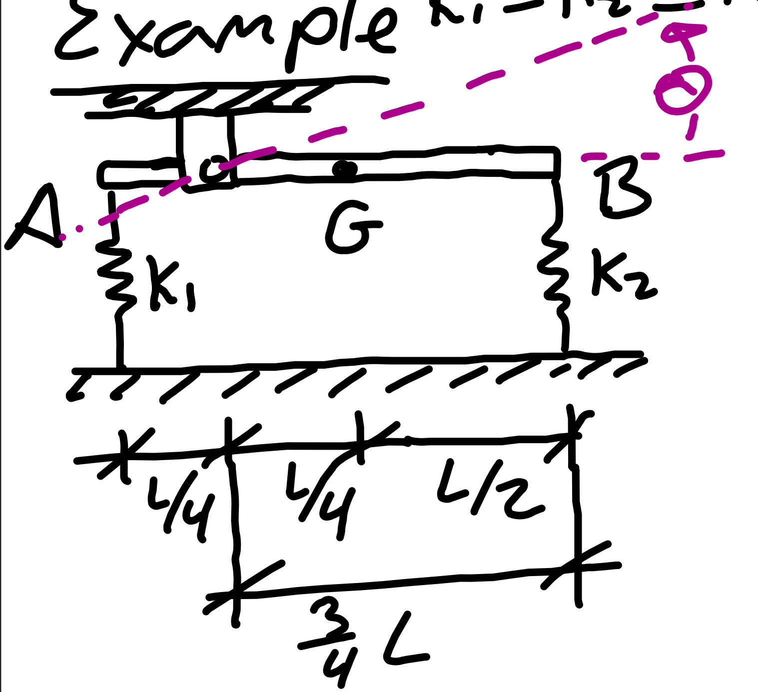

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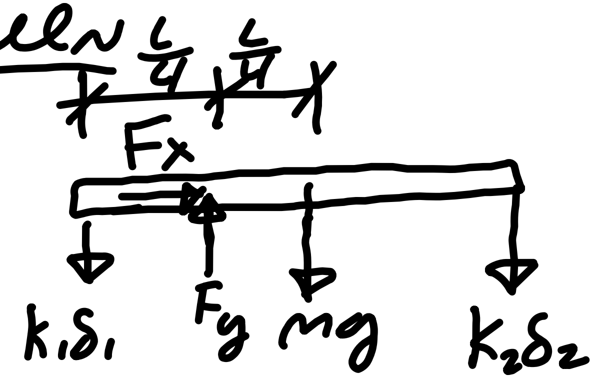

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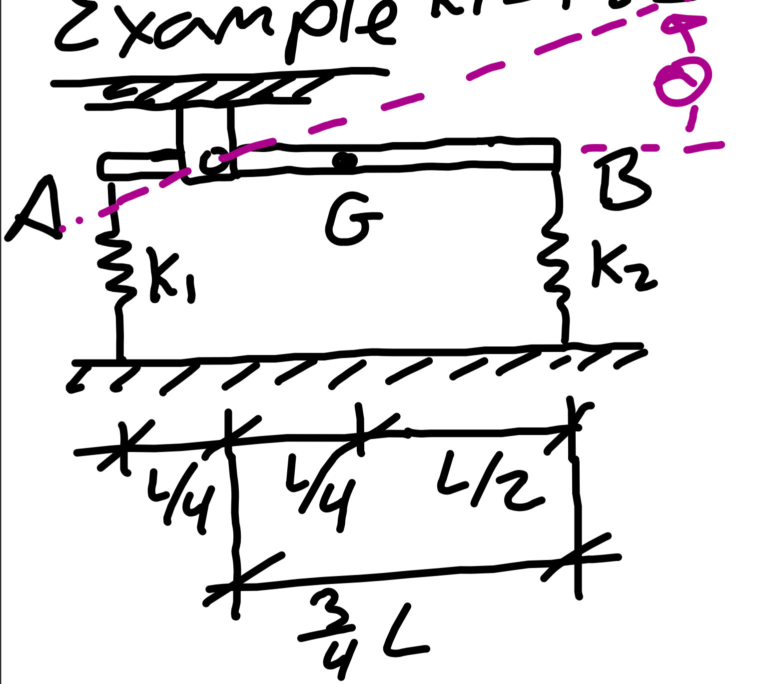
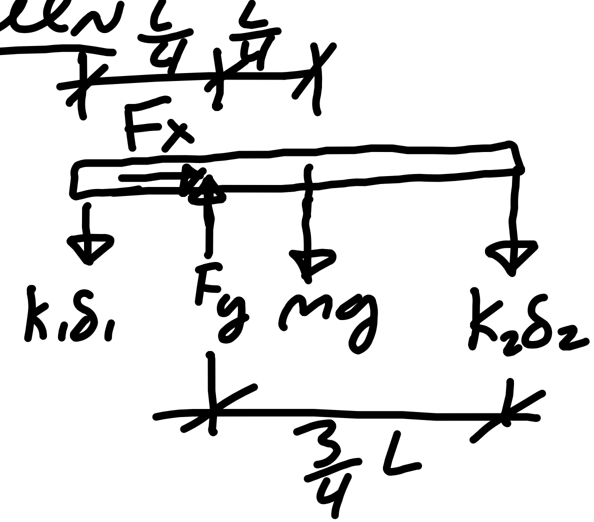


Push side B up a small distance δ & Find eqn

Equilibrium



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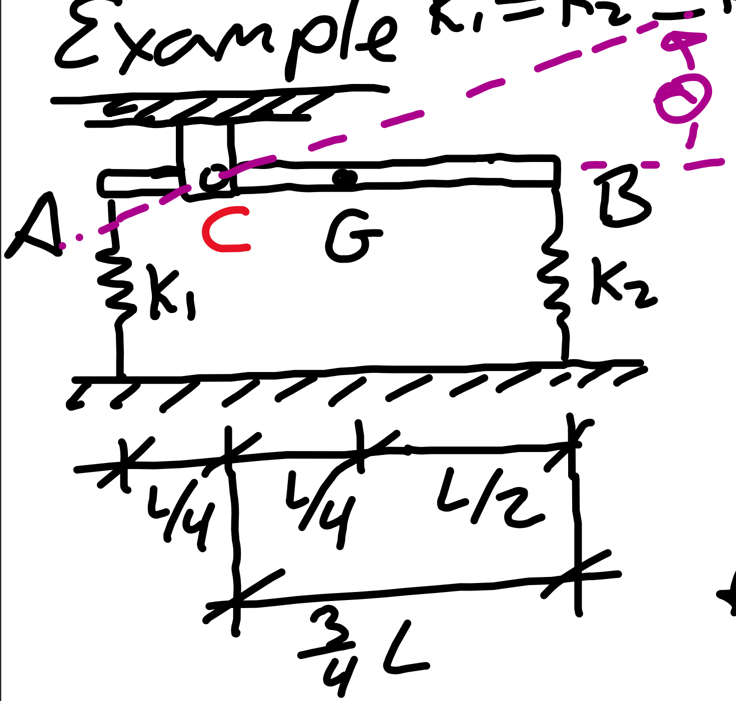
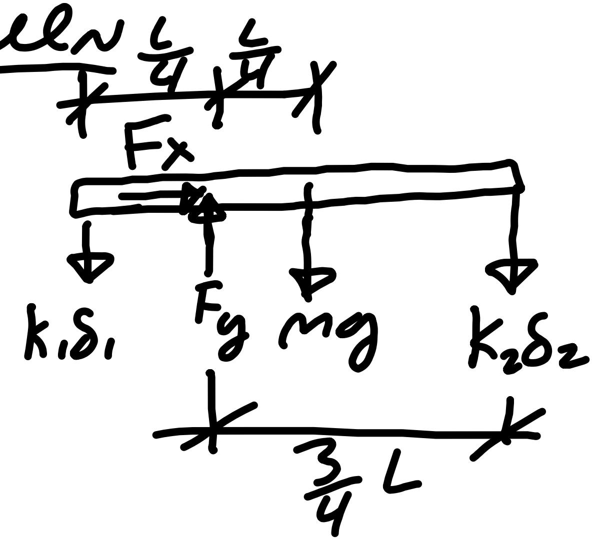


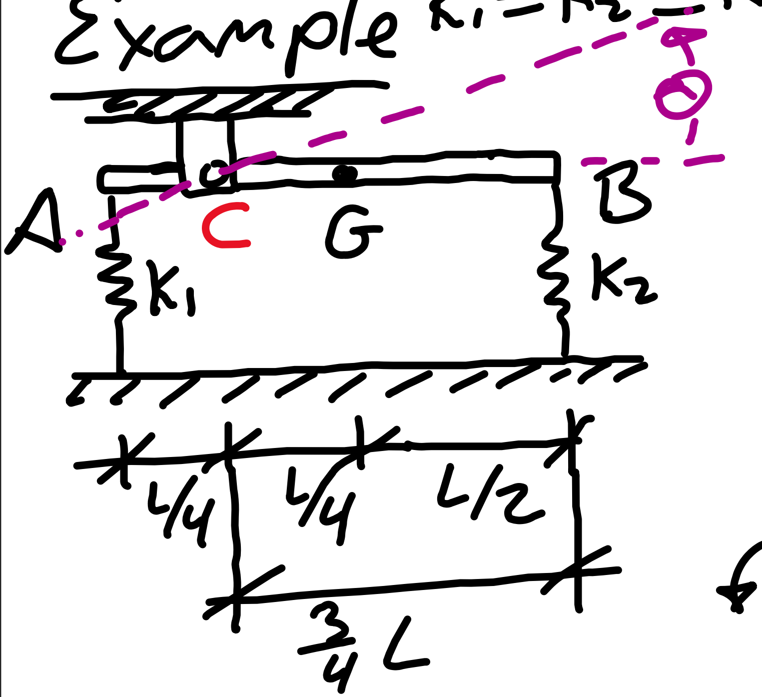
Push side B up a small distance δ & Find eqn



Equilibrium

$$\circlearrowleft \Sigma M_C = I_C \alpha$$

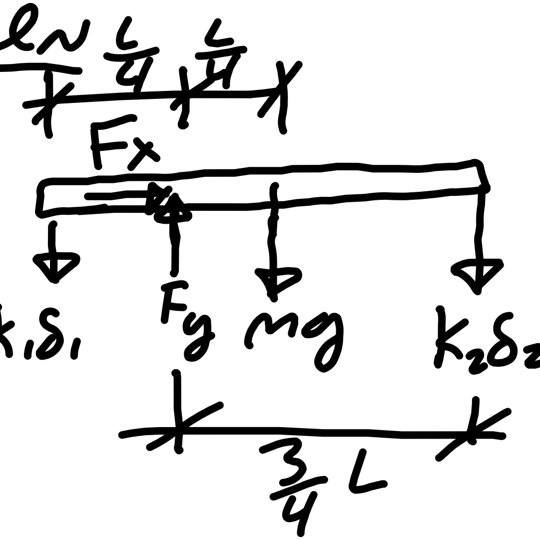
Example $k_1 = k_2 = k$



Push side B up a small distance δ & Find ω_n

Equilibrium

$\circlearrowleft \sum M_C = I_C \alpha$ θ



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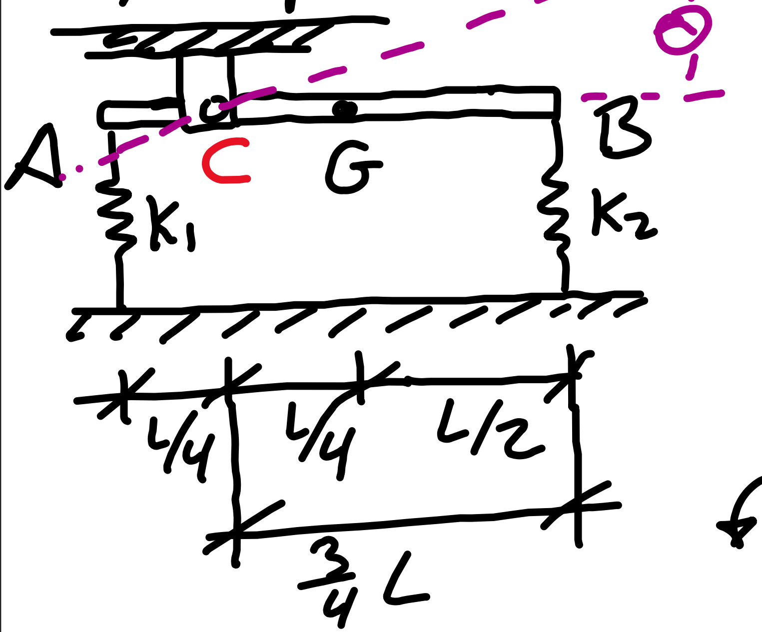
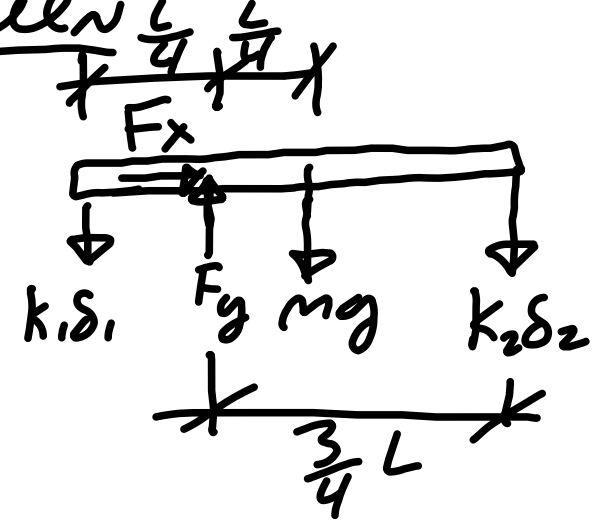


Push side B up a small distance δ & Find eqn

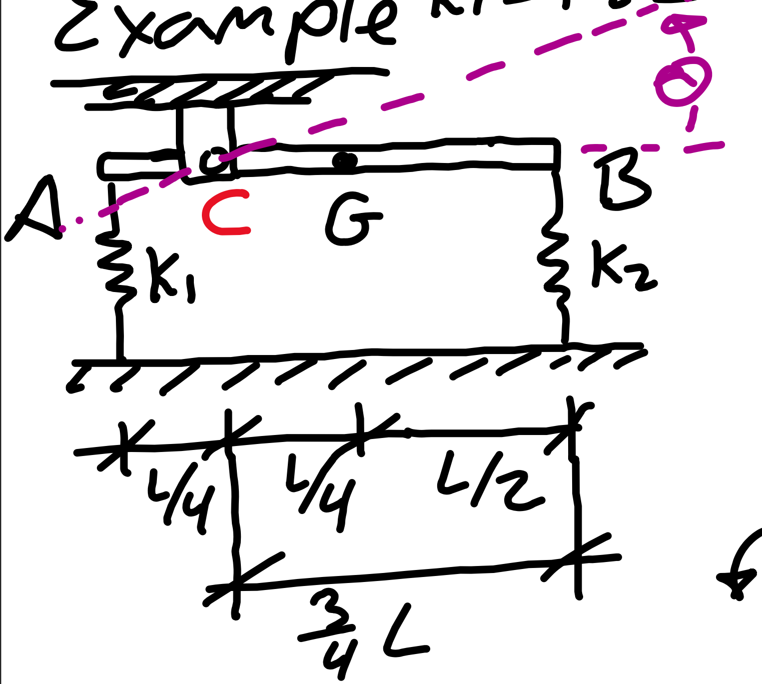


Equilibrium

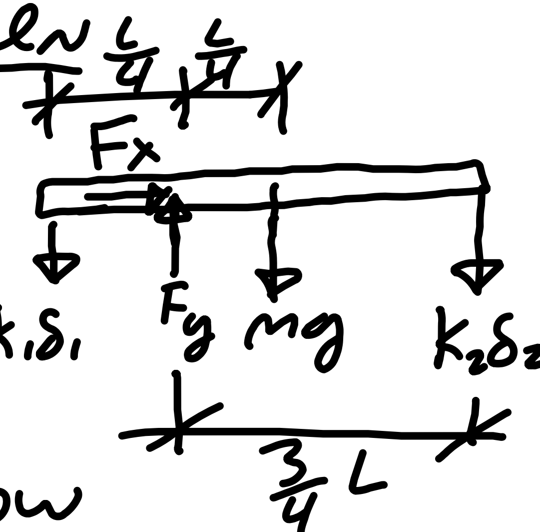
$\circlearrowleft \sum M_C = I_C \alpha$ θ

so $k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0$

Example $k_1 = k_2 = k$



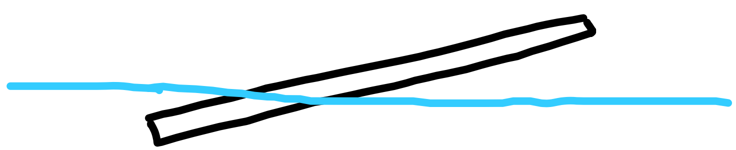
Push side B up a small distance d & Find eqn



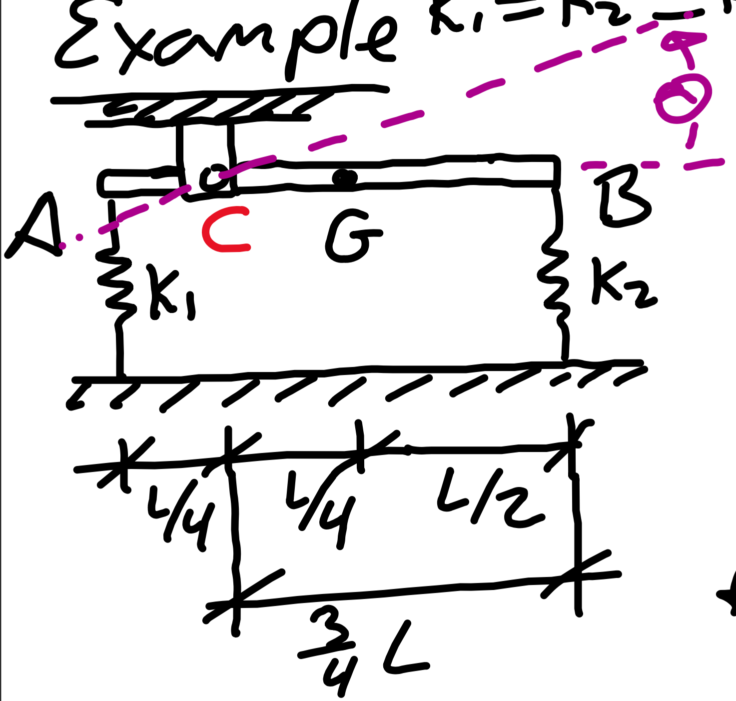
Equilibrium

$\circlearrowleft \Sigma M_C = \cancel{I_C \alpha}$ 0

So $k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\,\delta_2 \frac{3}{4}L = 0$ Now push up side B a distance d



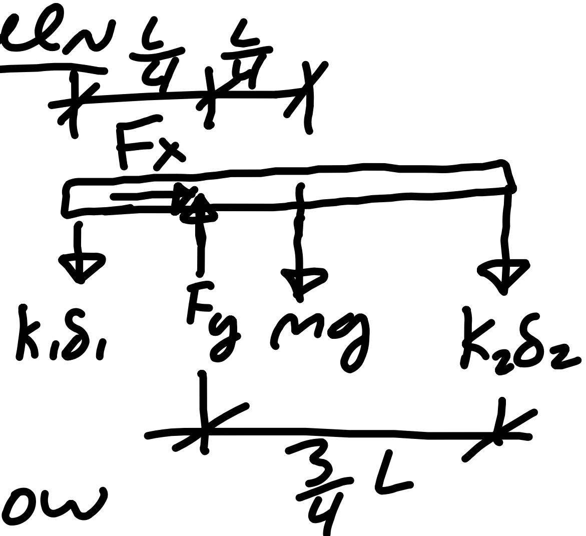
Example $k_1 = k_2 = k$



Push side B up a small distance d & find eq'n

Equilibrium

$\circlearrowleft \sum M_C = \cancel{I_C \alpha}$ θ



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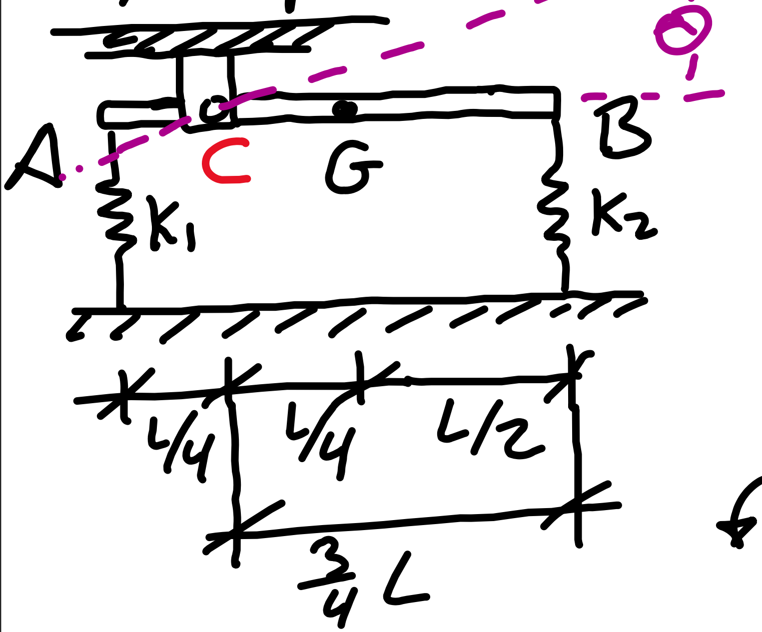
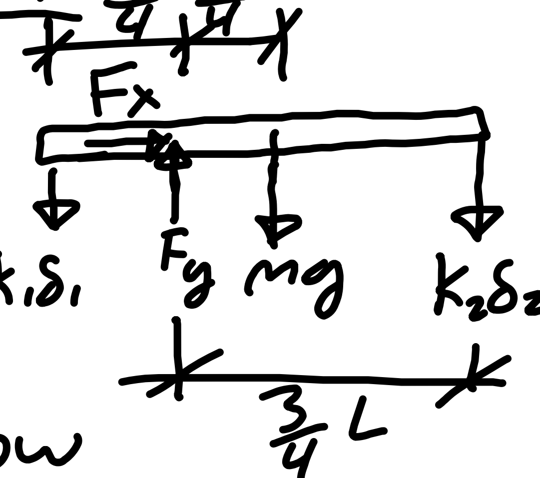
Example $k_1 = k_2 = k$



Push side B up a small distance d & Find ω_n

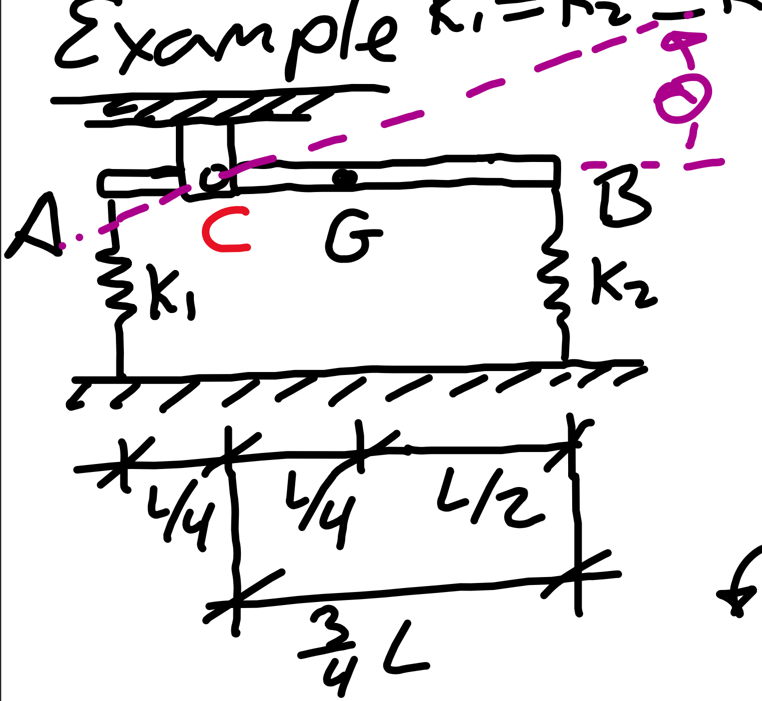
Equilibrium

$\circlearrowleft \sum M_c = \cancel{I_c\alpha}\ 0$

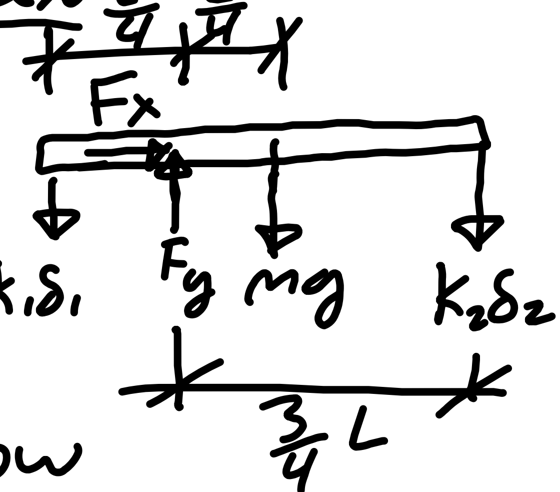


So $k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = 0$ Now push up side B a distance d

Example $k_1 = k_2 = k$



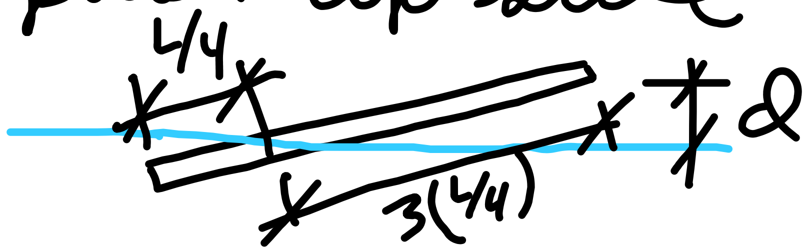
Push side B up a small distance d & Find eqn



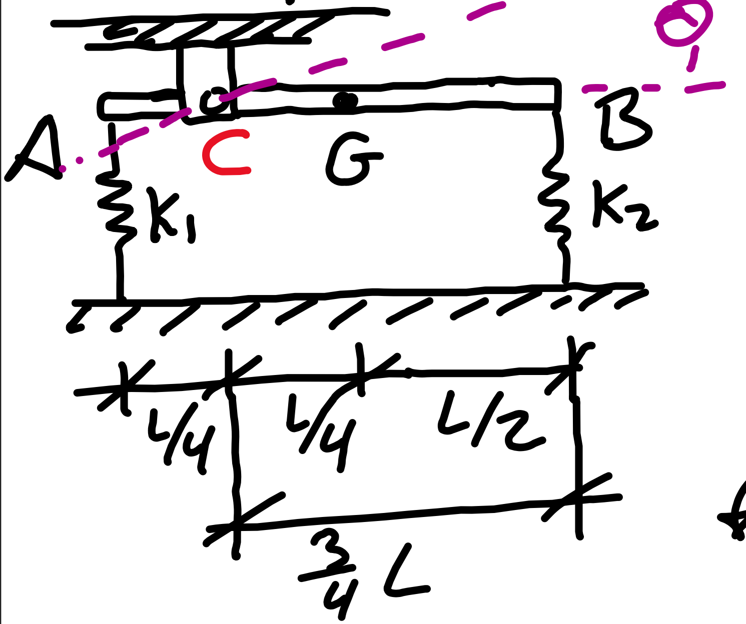
Equilibrium

$\circlearrowleft \sum M_C = I_C\alpha$ 0

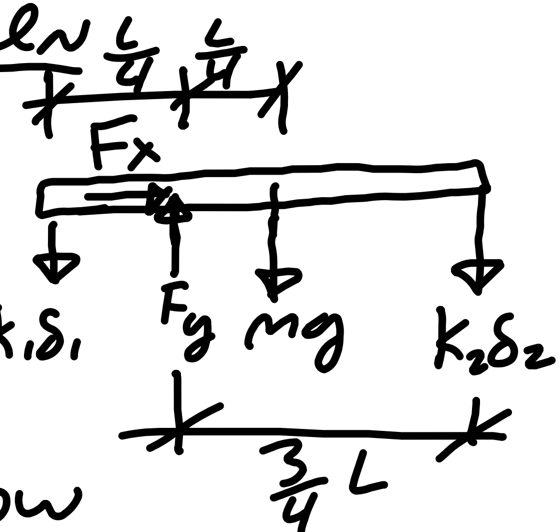
So $k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0$ Now push up side B a distance d



Example $k_1 = k_2 = k$



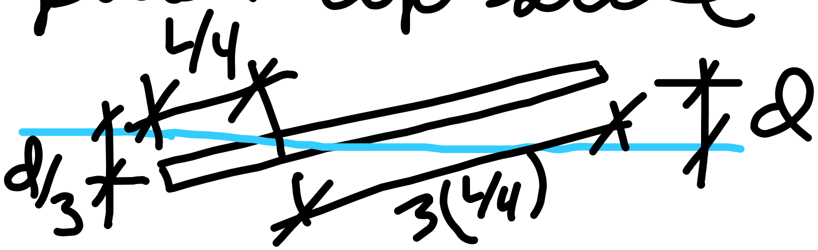
Push side B up a small distance d & Find ω_n



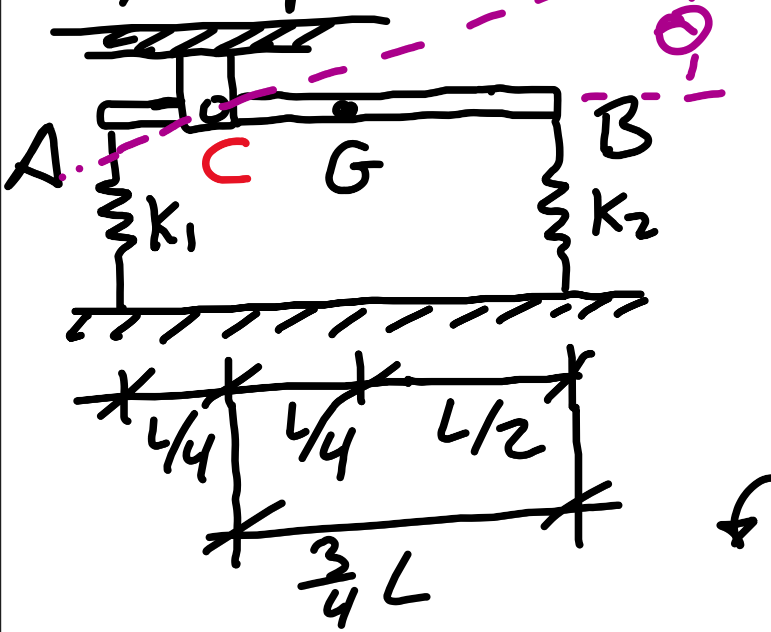
Equilibrium

$\circlearrowleft \sum M_C = \cancel{I_C\alpha}\ 0$

So $k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = 0$ Now push up side B a distance d



Example $k_1 = k_2 = k$



Push side B up a small distance d & Find ω_n

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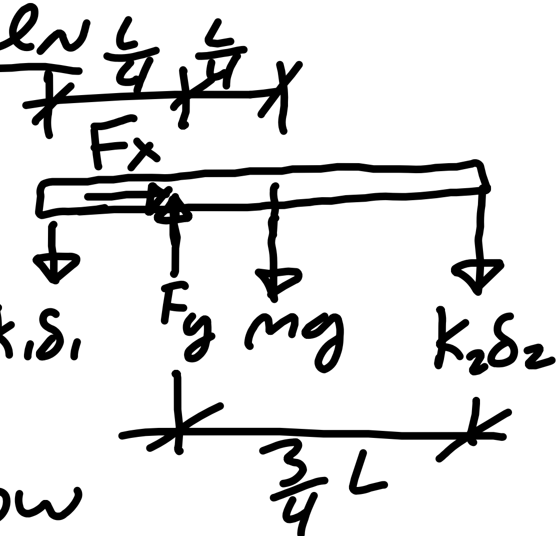

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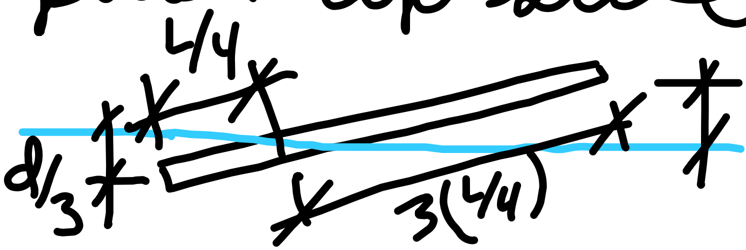

d So $\circlearrowleft \sum M_C = I_C \alpha$

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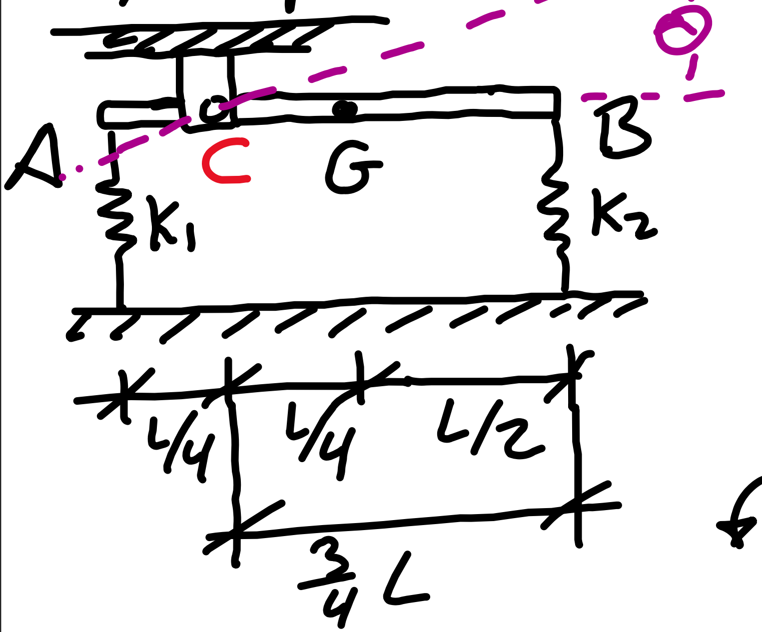

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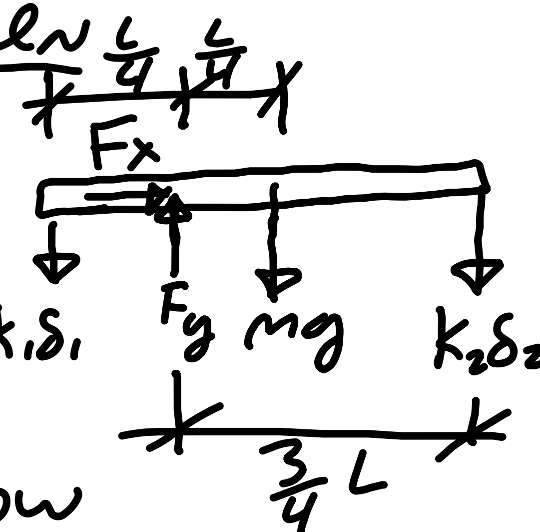

Equilibrium

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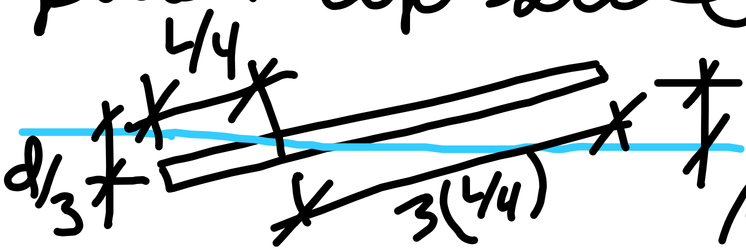
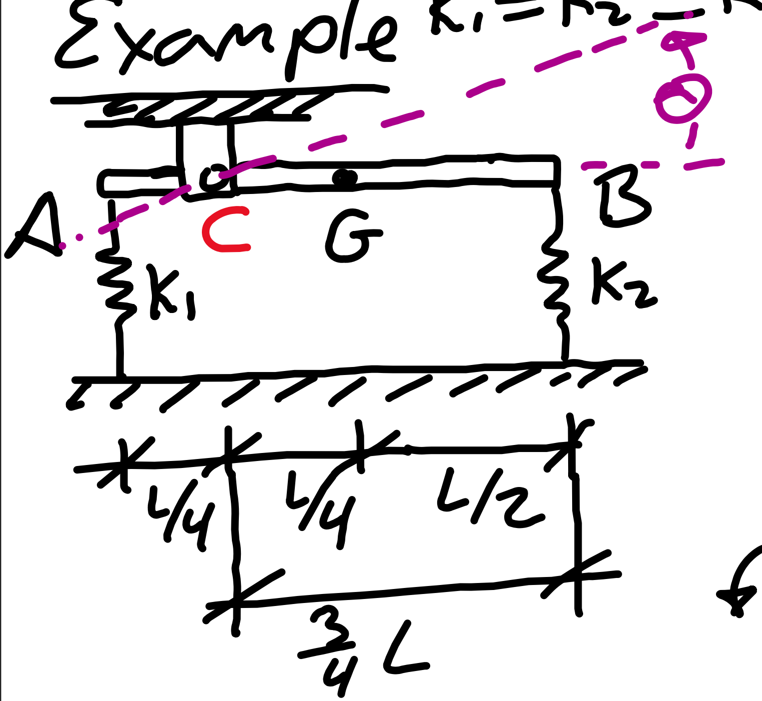


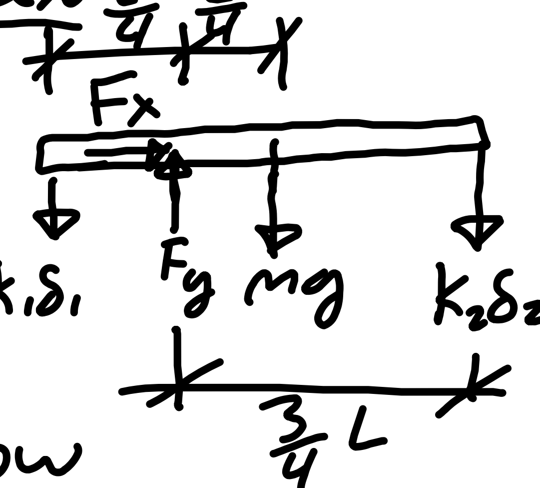
so $\circlearrowleft M_c = I_c\alpha \Rightarrow$

$M_c = k(\delta - d/3)\frac{L}{4} -$

Example $k_1 = k_2 = k$



Push side B up a small distance d & Find eqn



Equilibrium

$\circlearrowleft \sum M_c = I_c\alpha$ 0

So $k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = 0$ Now

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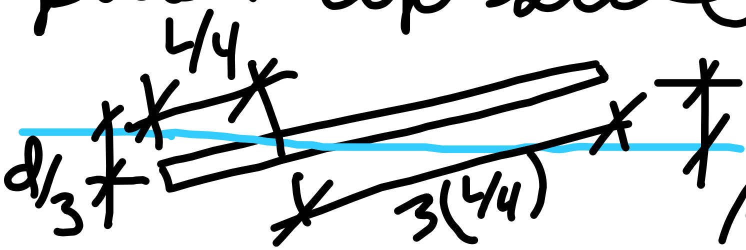

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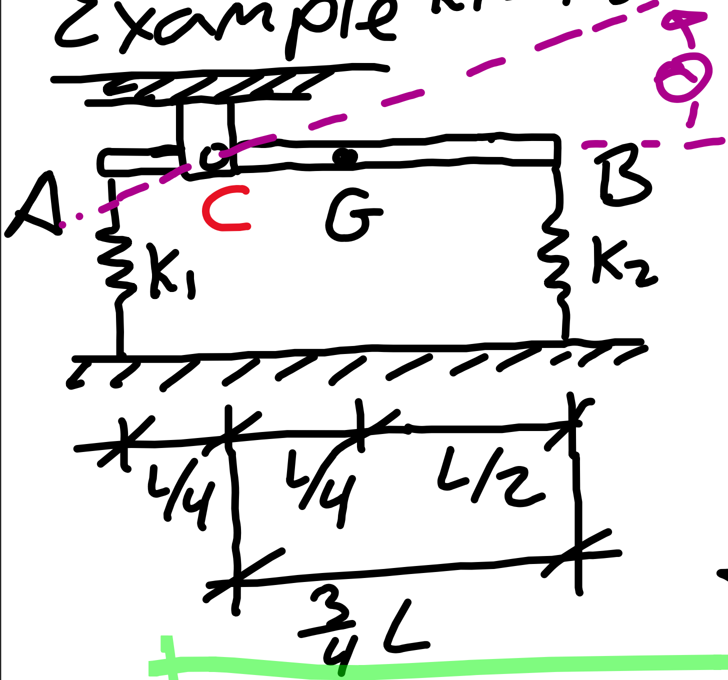
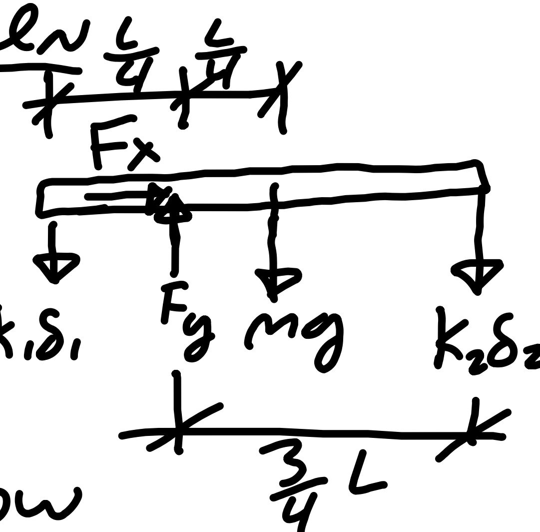


Push side B up a small distance d & Find eqn

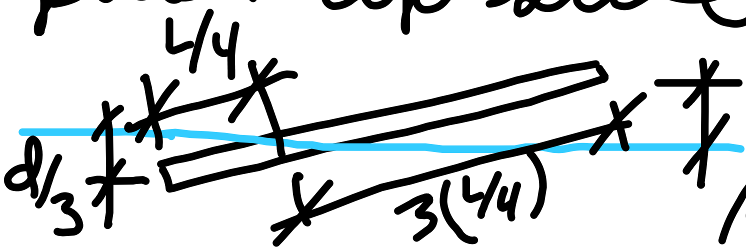
Equilibrium



$\circlearrowleft \sum M_C = I_C\alpha$ θ

so $\boxed{k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = \theta}$ Now

push up side B a distance d



d so $\circlearrowleft M_C = I_C\alpha \Rightarrow$

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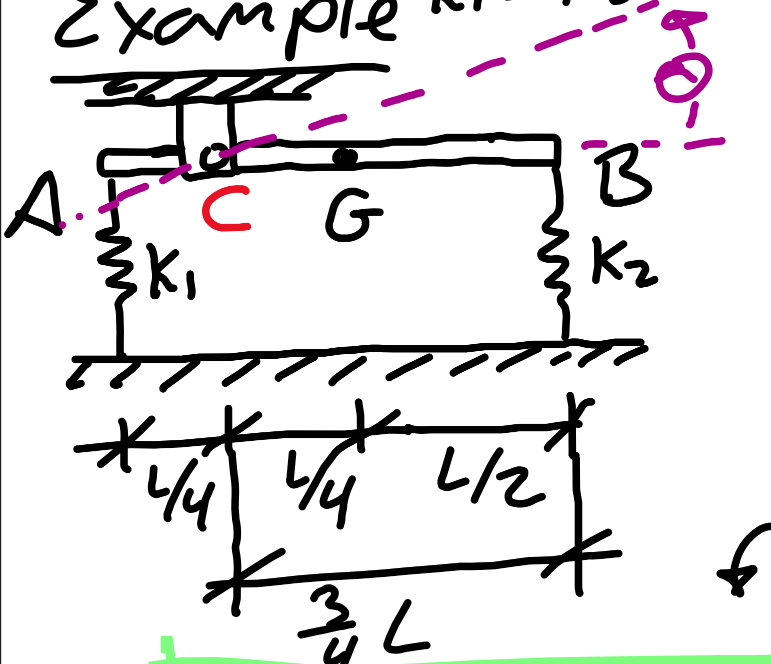
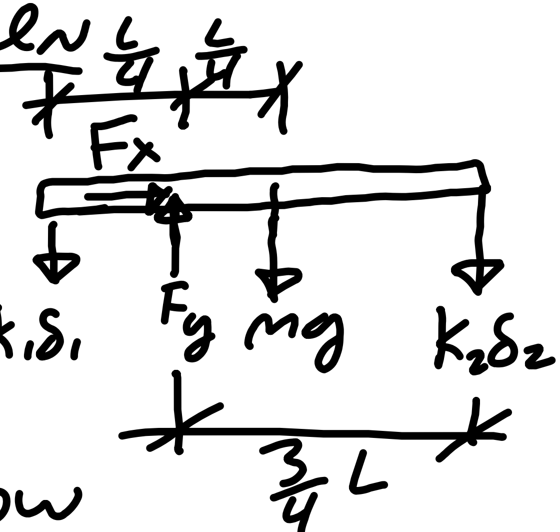


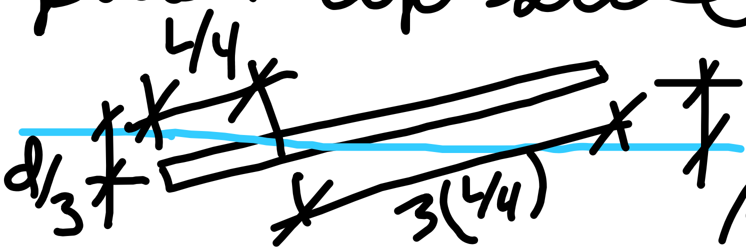
Push side B up a small distance δ & find eqn



Equilibrium

$\circlearrowleft \sum M_C = I_C\alpha$ θ

So $\boxed{k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = 0}$ Now push up side B a distance δ

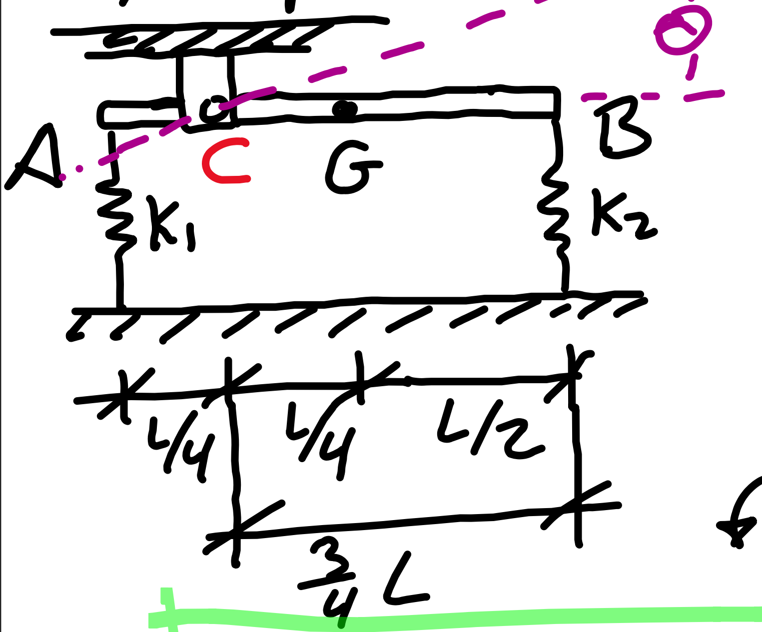


So $\circlearrowleft M_C = I_C\alpha \Rightarrow$

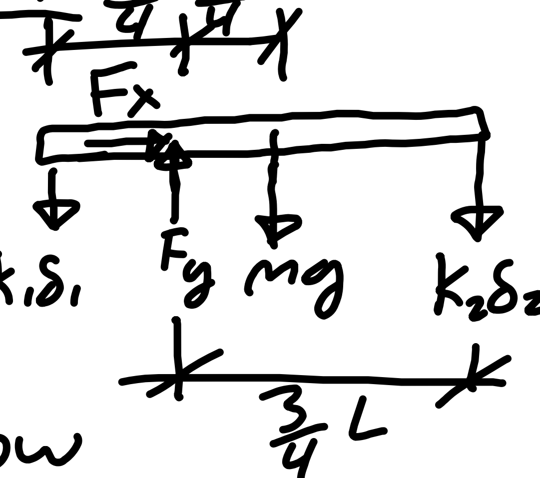
$$M_C = k(\delta_1 - \delta/3)\frac{L}{4} - mg\frac{L}{4} - k(\delta_2 + \delta)\frac{3L}{4}$$

$$\Rightarrow -k\frac{\delta L}{12} - k\frac{3\delta L}{4} = I_C\ddot{\theta}$$

Example $k_1 = k_2 = k$



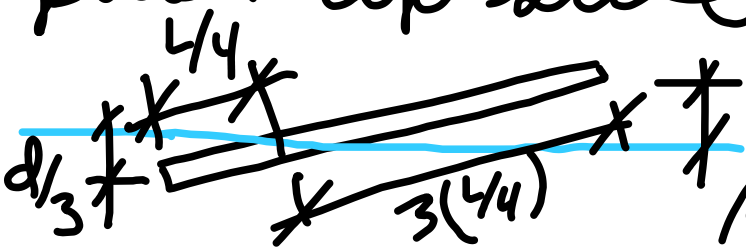
Push side B up a small distance d & Find ω_n



Equilibrium

$$\circlearrowleft \sum M_c = I_c\alpha \quad \theta$$

so $\boxed{k\delta_1\frac{L}{4} - mg\frac{L}{4} - k\delta_2\frac{3}{4}L = 0}$ Now push up side B a distance d

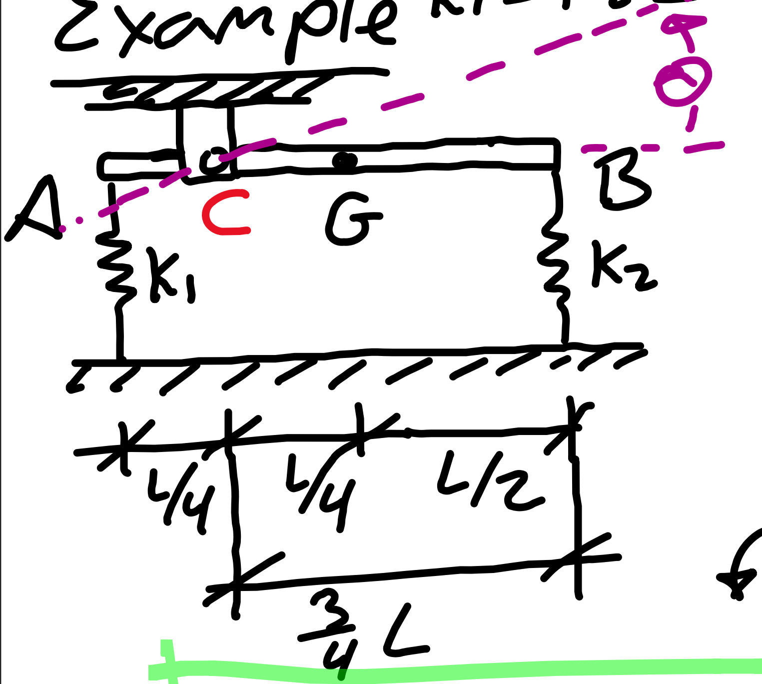


so $\circlearrowleft M_c = I_c\alpha \Rightarrow$

$$M_c = k(\delta_1 - d/3)\frac{L}{4} - mg\frac{L}{4} - k(\delta_2 + d)\frac{3L}{4}$$

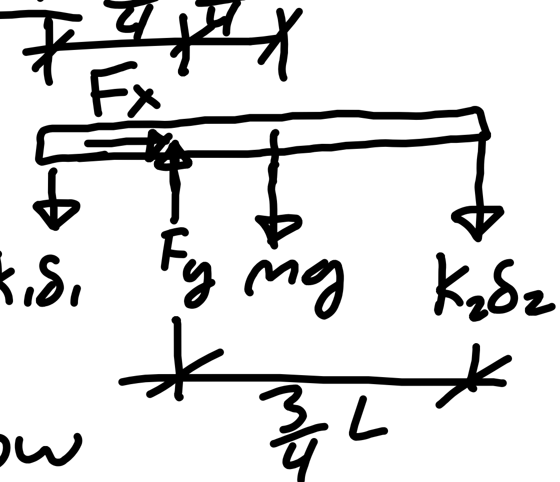
$$\Rightarrow -k\frac{dL}{12} - k\frac{3dL}{4} = I_c\ddot{\theta} \Rightarrow -\frac{kdL}{12}(1+9) = I_c\ddot{\theta}$$

Example $k_1 = k_2 = k$



Push side B up a small distance δ & Find eqn

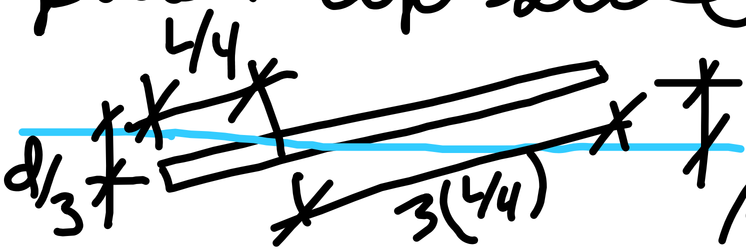
Equilibrium



$\circlearrowleft \sum M_C = I_C \alpha$ θ

so $\boxed{k\delta_1 \frac{L}{4} - mg\frac{L}{4} - k\delta_2 \frac{3}{4}L = 0}$

Now push up side B a distance δ



so $\circlearrowleft M_C = I_C\alpha \Rightarrow$

$M_C = k(\delta_1 - \delta/3)\frac{L}{4} - mg\frac{L}{4} - k(\delta_2 + \delta)\frac{3L}{4}$

$\Rightarrow -k\frac{\delta L}{12} - k\frac{3\delta L}{4} = I_C\ddot{\theta} \Rightarrow -\frac{k\delta L}{12}(1+9) = I_C\ddot{\theta}$

From previous slide

From previous slide

$$-\frac{k\,l\,L}{12}(1+9) = I_c\ddot{\theta}$$

From previous slide

$$-\frac{k d L}{12}(1+9) = I_c \ddot{\theta} \quad \text{But} \quad d = L\sin\theta$$

From previous slide

$-\frac{k d L}{12}(1+9) = I_c \ddot{\theta}$ But $d = L\sin\theta \cong L\theta$

From previous slide

$-\frac{kdL}{12}(1+9) = I_c\ddot{\theta}$ But $d = L\sin\theta \cong L\theta$

So $-KL^2\left(\frac{5}{6}\right) = I_c\ddot{\theta}$

From previous slide

$-\frac{k d L}{12}(1+9) = I_c \ddot{\theta}$ But $d = L\sin\theta \cong L\theta$

So $-kL^2\left(\frac{5}{6}\right) = I_c \ddot{\theta}$ or

$\ddot{\theta} = -\omega^2 \theta$

From previous slide

$$-\frac{k d L}{12}(1+9) = I_c\ddot{\theta} \quad \text{But} \quad d = L\sin\theta \cong L\theta$$

So $\quad -KL^2\left(\frac{5}{6}\right) = I_c\ddot{\theta} \quad$ or

$$\ddot{\theta} = -\omega^2\theta, \text{ where } \boxed{\omega = L\sqrt{\frac{5K}{6I_c}}}$$

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<u>or</u>

$$\ddot{\Theta} = -\omega^2 \Theta$$

New game

New game

Since $a = v\frac{dv}{dx}$

New game

Since $a = v\frac{\partial v}{\partial x}$ & $a = \ddot{x}$,

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$$

$$v\,dv = -\omega^2 x\,dx$$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$$

$$\int v\,dv = -\omega^2 \int x\,dx$$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$

$\int v\,dv = -\omega^2 \int x\,dx \Rightarrow \frac{1}{2}v^2 + C_1 = -\omega^2 \frac{1}{2}x^2 + \omega^2 C_2$

New game

Since $a = v\frac{dv}{dx}$ & $a = \ddot{x}$, then

$$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$$

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or $\quad v^2 + \omega^2 x^2 = \text{const.}$

New game

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$$\ddot{x} = -\omega^2 x \Rightarrow v\frac{dv}{dx} = -\omega^2 x \Rightarrow$$

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or $\quad v^2 + \omega^2 x^2 = \text{const.}$

or $\quad \dot{x}^2 + \omega^2 x^2 = \text{const.}$

Old game :

Old game:

Get problem into the form

$$\ddot{x} = -\omega^2 x$$

Old game:

Get problem into the form

$\ddot{x} = -\omega^2 x$ using forces

& moments

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New game:

Get problem into the form

$$\dot{x}^2 + \omega^2 x^2 = \text{constant}$$

Old game:

Get problem into the form

$$\ddot{x} = -\omega^2 x$$ using forces & moments

New game:

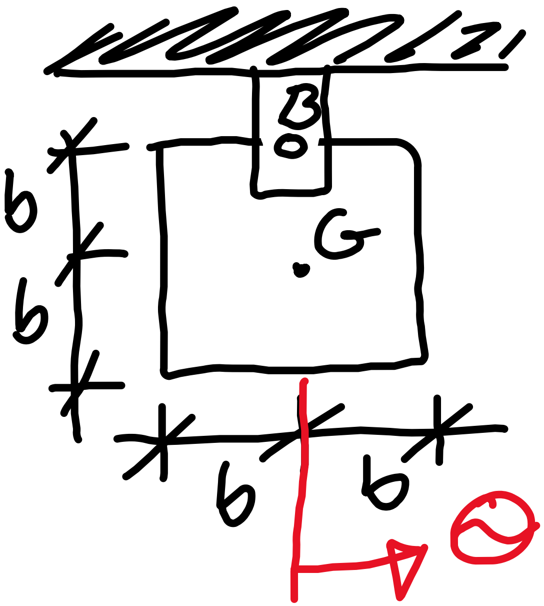
Get problem into the form

$$\dot{x}^2 + \omega^2 x^2 = \text{constant}$$

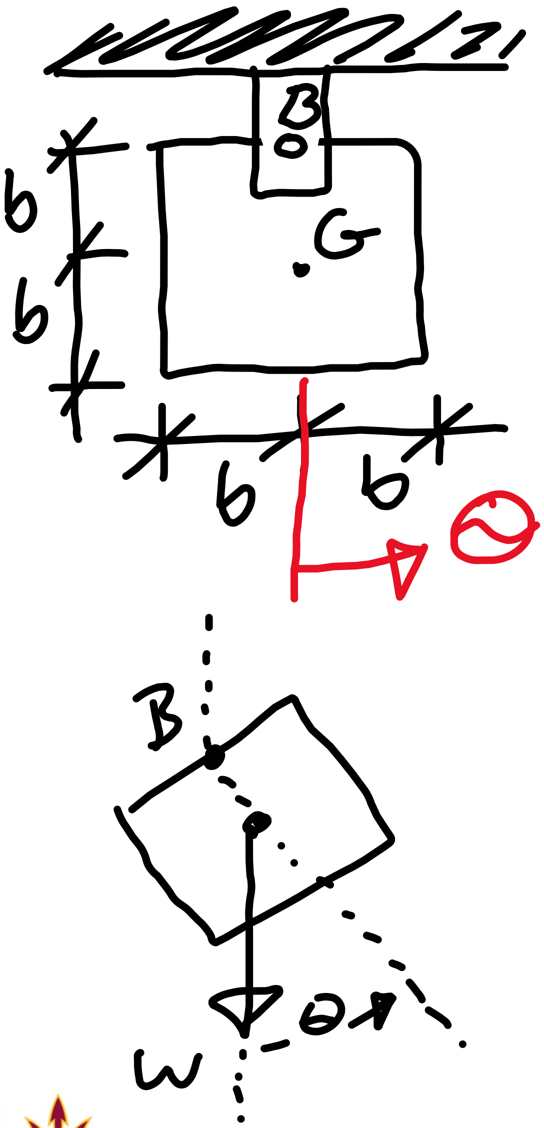
using energy conservation

$$T + V = \text{constant}.$$

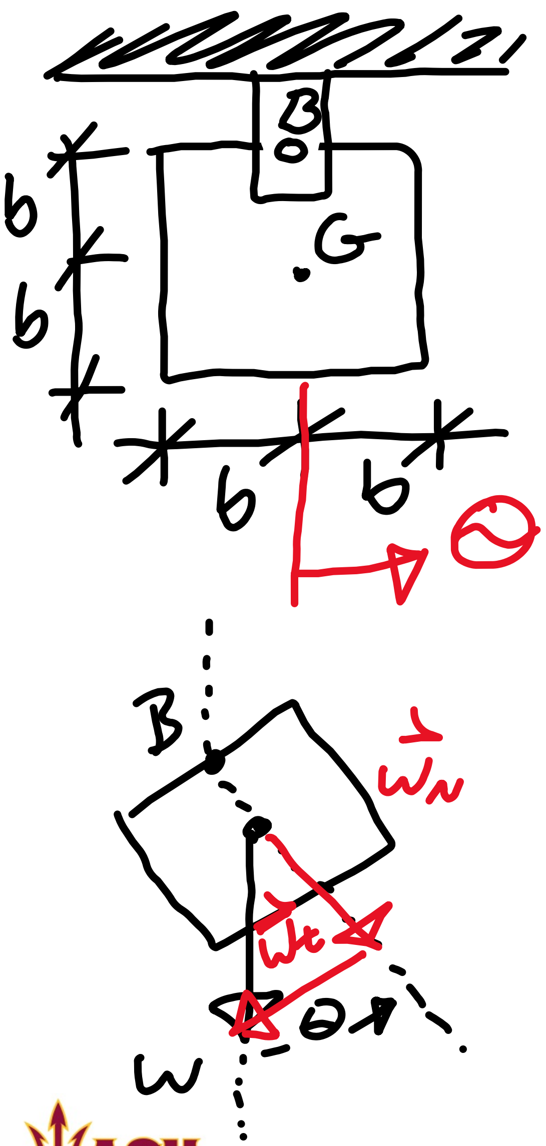
Old style



Old style

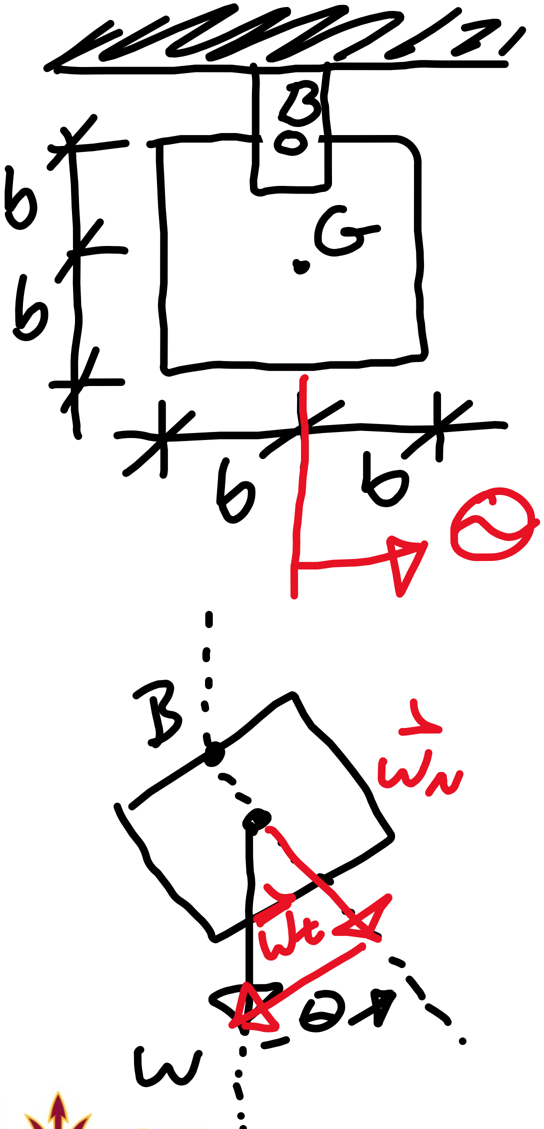


Old style



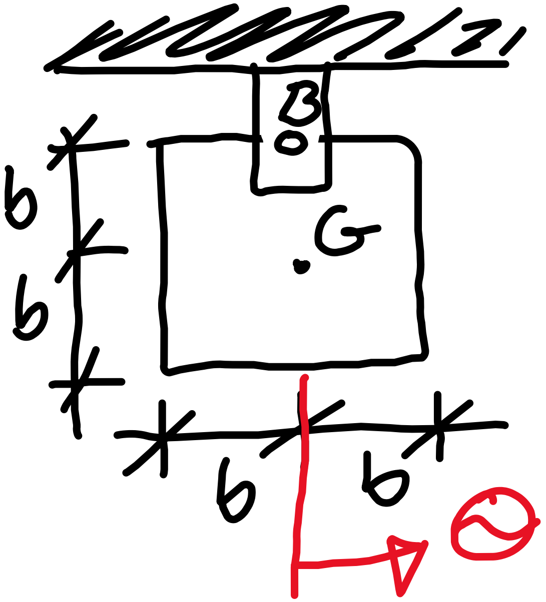
Old style

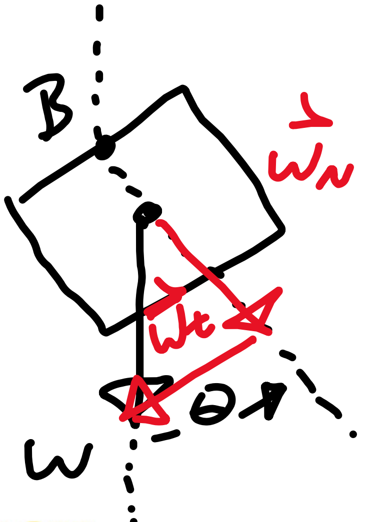
$$w_N = -W\cos\theta$$



Old style

$w_n = -W\cos\theta \;\&\; w_t = -W\sin\theta$

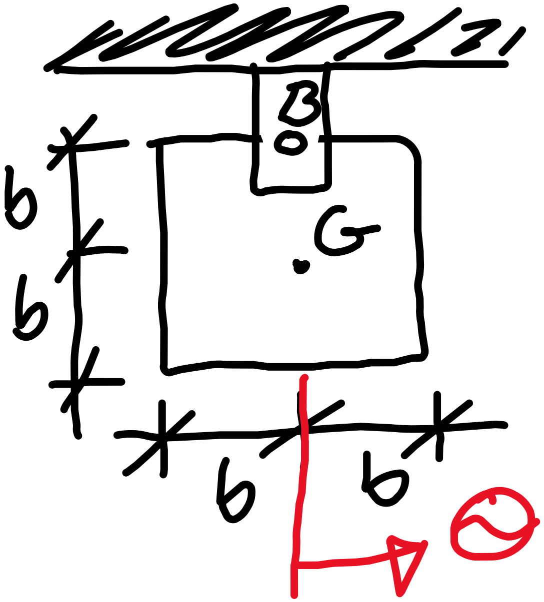


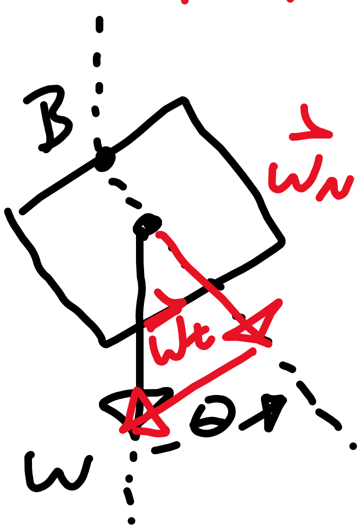


Old style

$$w_N = -W\cos\theta \;\&\; w_t = -W\sin\theta$$

$$\circlearrowleft \sum M_B = I_B\theta$$

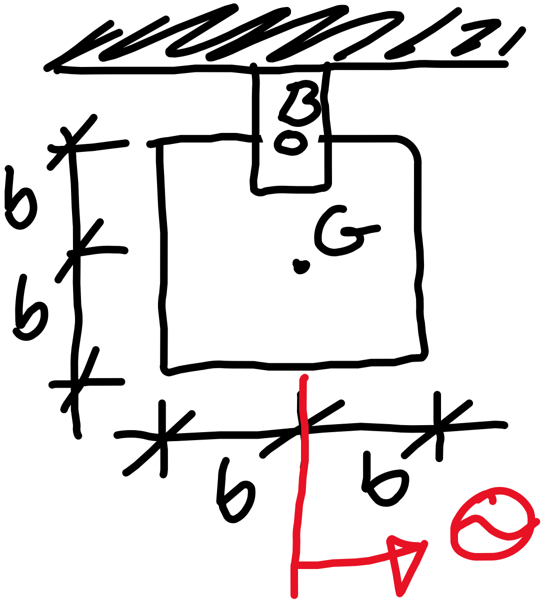


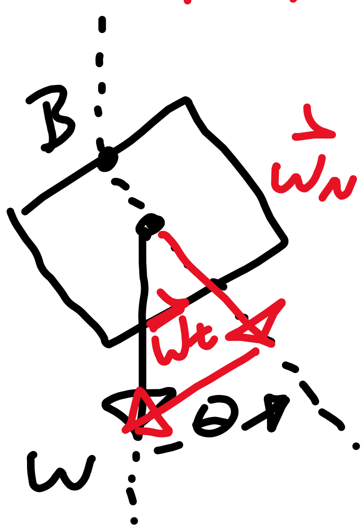


Old style

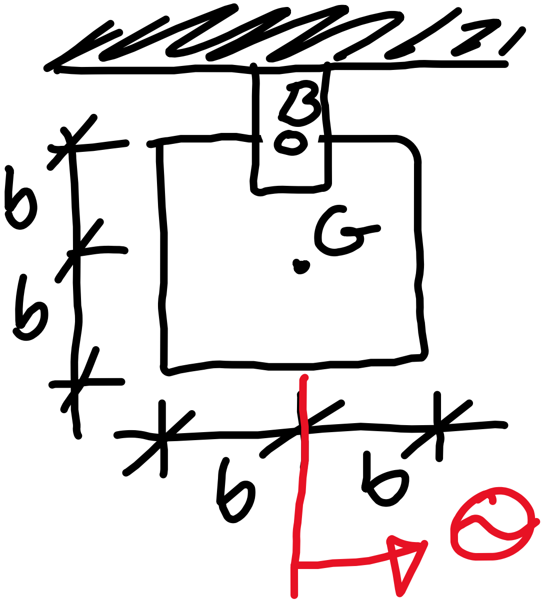
$w_N = -W\cos\theta$ & $w_t = -W\sin\theta$

$\circlearrowleft \sum M_B = I_B\,\theta \Rightarrow b\,w_t = I_B\,\theta$





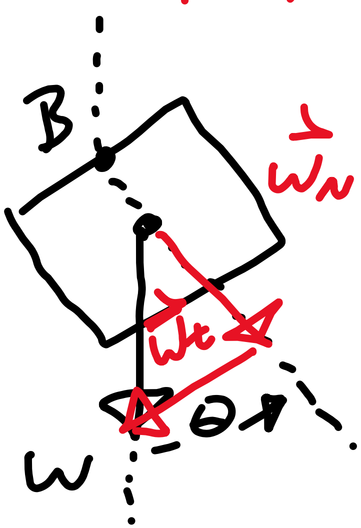
Old style



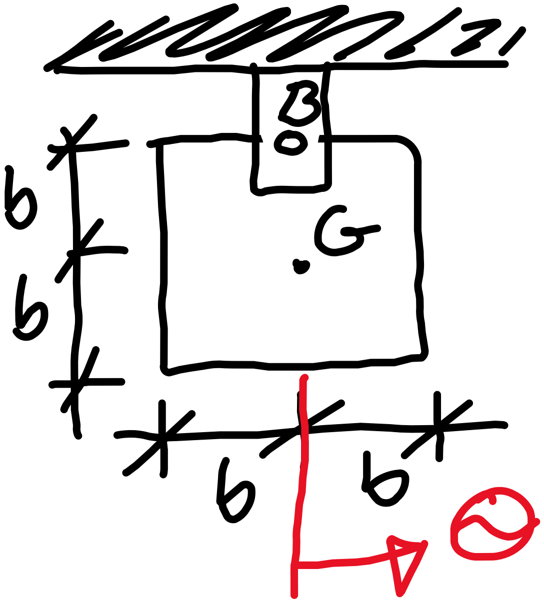
$$w_N = -W\cos\theta \;\&\; w_t = -w\sin\theta$$

$$\circlearrowleft \sum M_B = I_B\ddot{\theta} \Rightarrow b w_t = I_B\ddot{\theta}$$

$$\Rightarrow -bw\sin\theta = (\bar{I} + mb^2)\ddot{\theta}$$



Old style



$$w_N = -W\cos\theta \;\; \& \;\; w_t = -w\sin\theta$$

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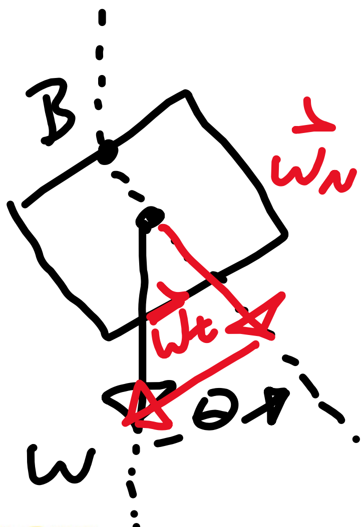
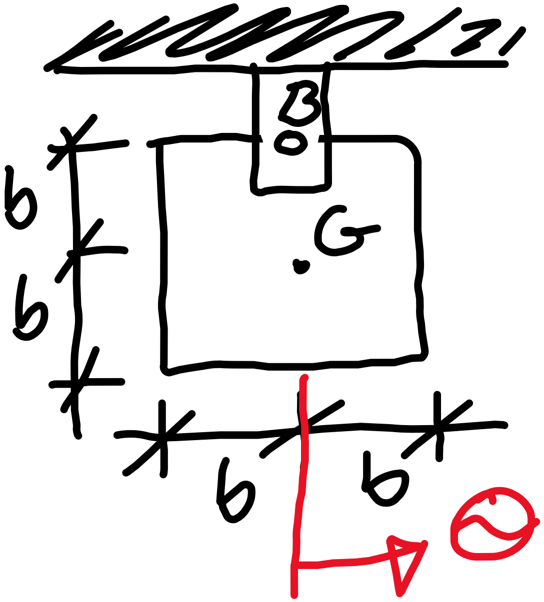
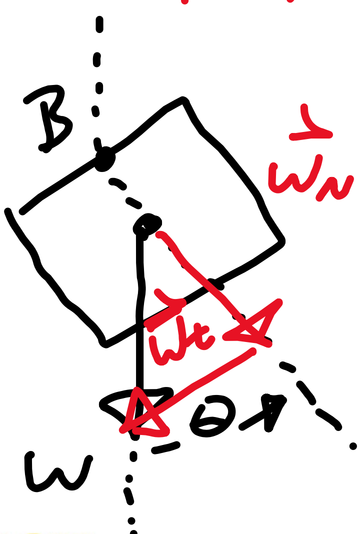
$$\Rightarrow -bw\theta \cong (\bar{I} + mb^2)\ddot{\theta}$$



Old style





$$w_N = -W\cos\theta \;\&\; w_t = -w\sin\theta$$

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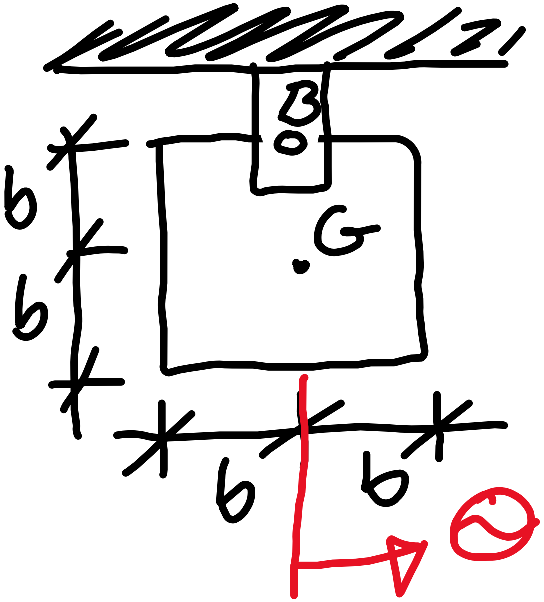
$$\Rightarrow \ddot{\theta} = -\omega^2\theta$$

Old style

$W_N = -W\cos\theta$ & $W_t = -W\sin\theta$



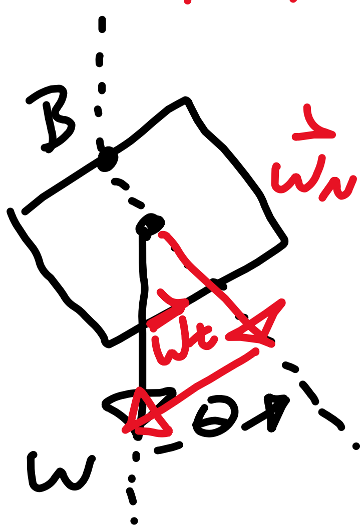
$\circlearrowleft \sum M_B = I_B\ddot{\theta} \Rightarrow bW_t = I_B\ddot{\theta}$

$\Rightarrow -bW\sin\theta = (\bar{I} + mb^2)\ddot{\theta}$

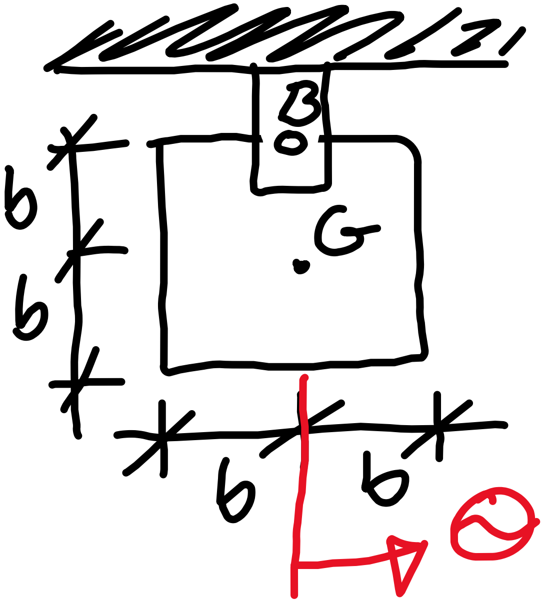
$\Rightarrow -bW\theta \cong (\bar{I} + mb^2)\ddot{\theta}$

$\Rightarrow \ddot{\theta} = -eee^2\theta$, where

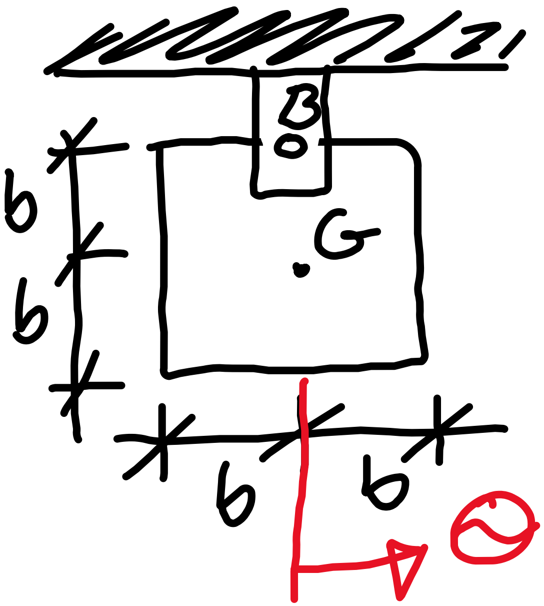
$$eee = \sqrt{\frac{bW}{\bar{I} + mb^2}}$$

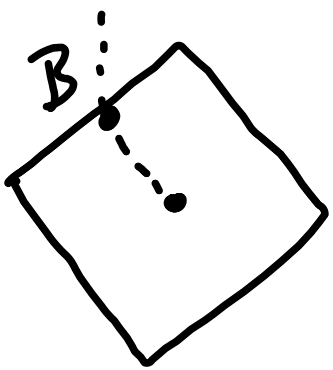


New style



New style





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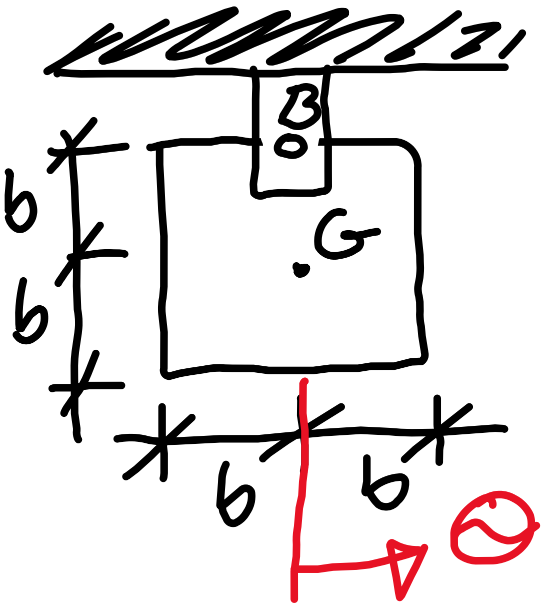
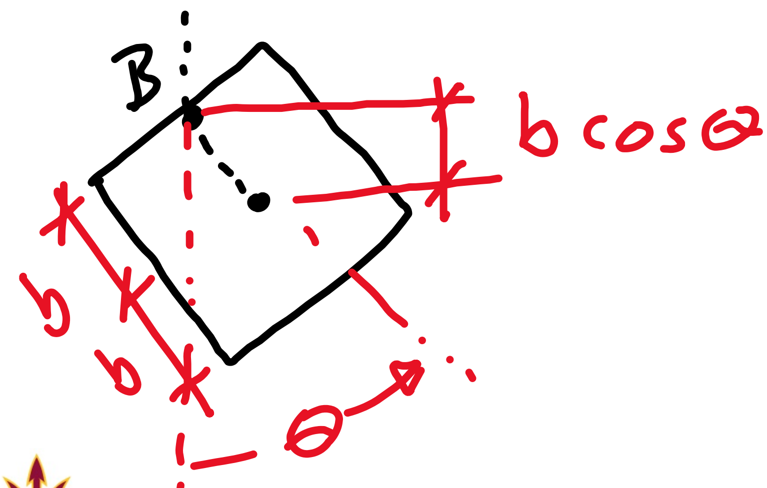
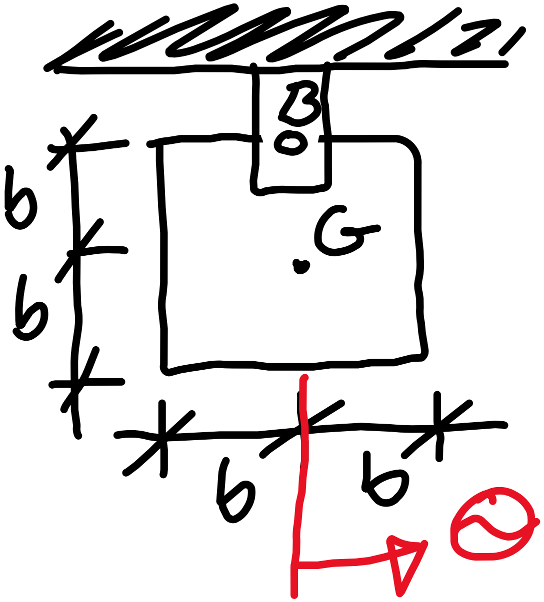
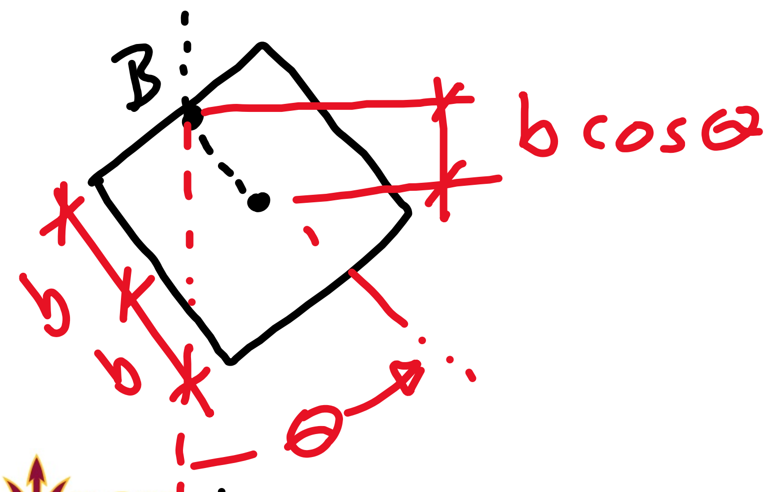




New style

$T + V = \text{const.}$





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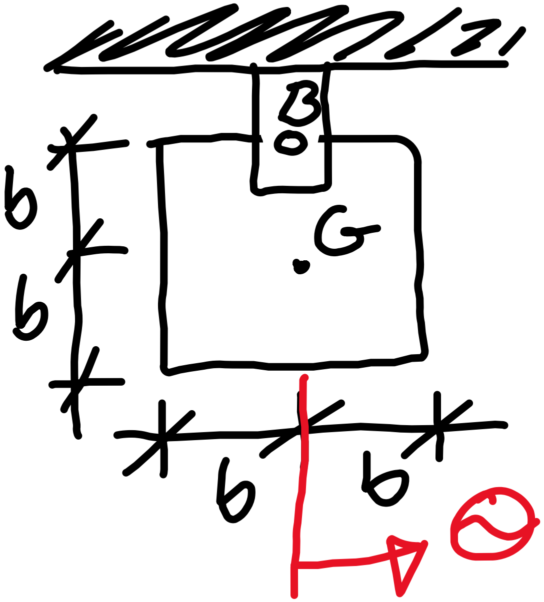
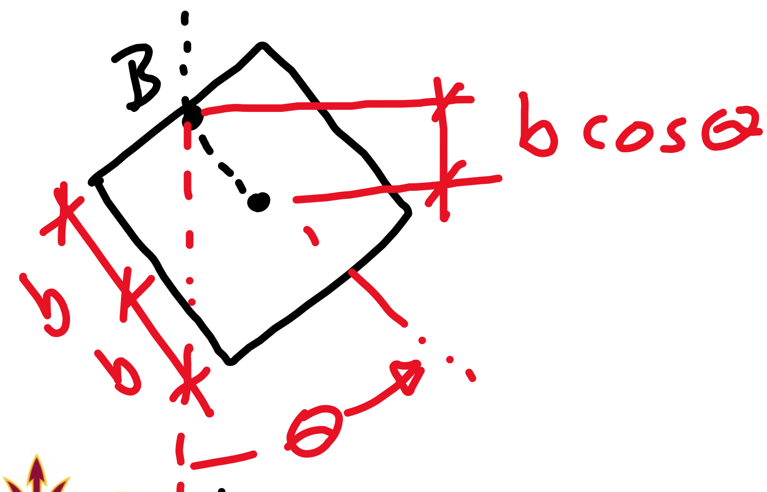


$T + V = \text{const.},$ where

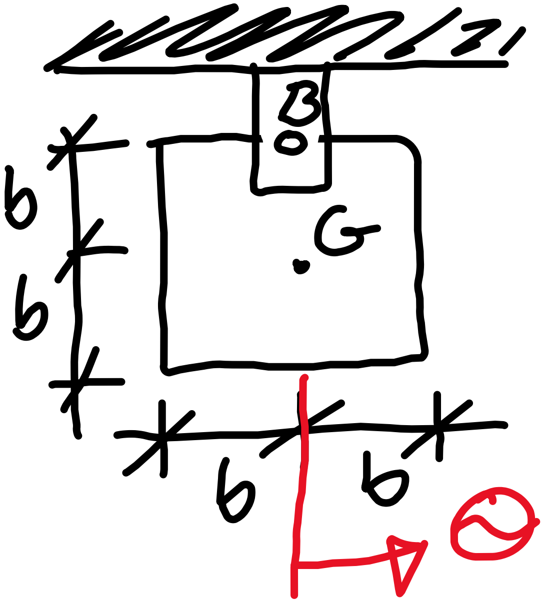
$T = \frac{1}{2} I_B \dot{\theta}^2$

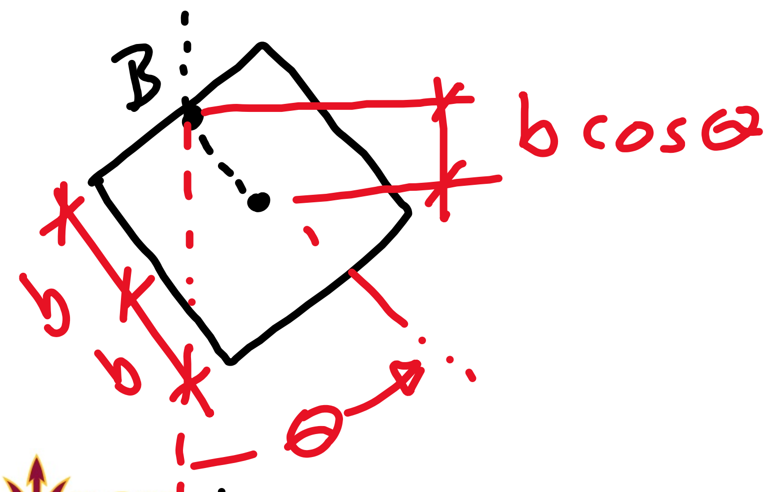


New style

$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \omega^2$ & $V = mgh$



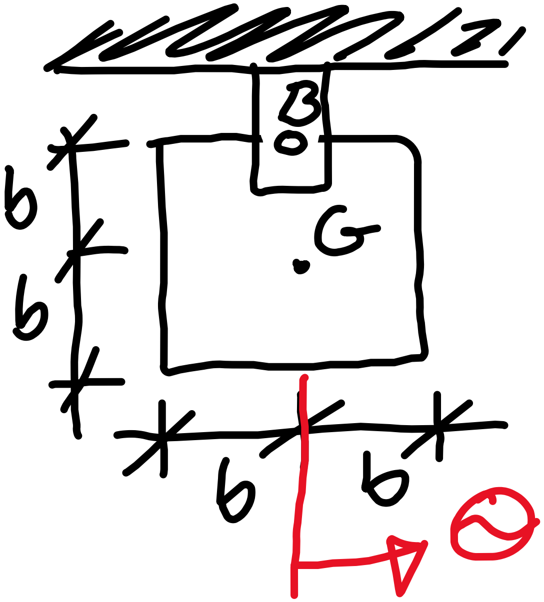


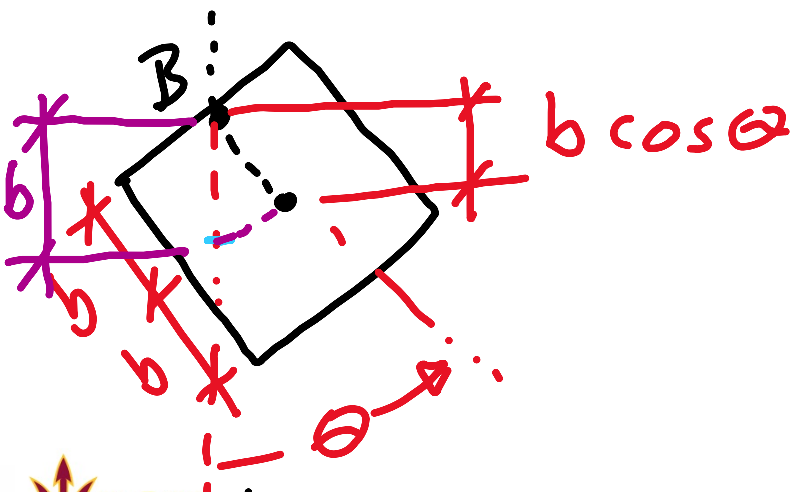
New style

$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \omega^2$ & $V = mgh$

but $h = b -$





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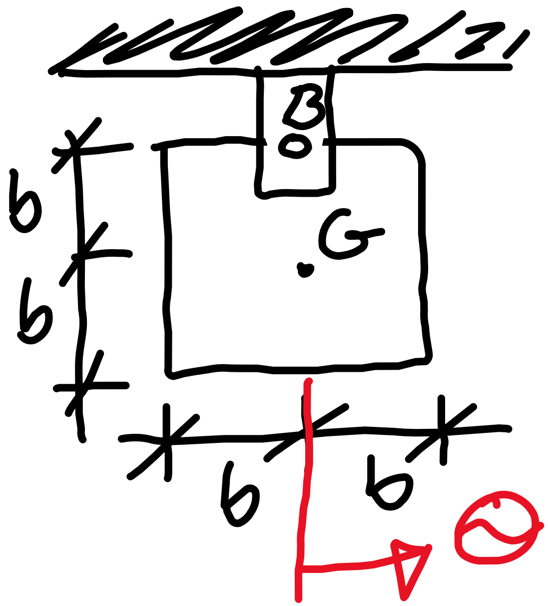
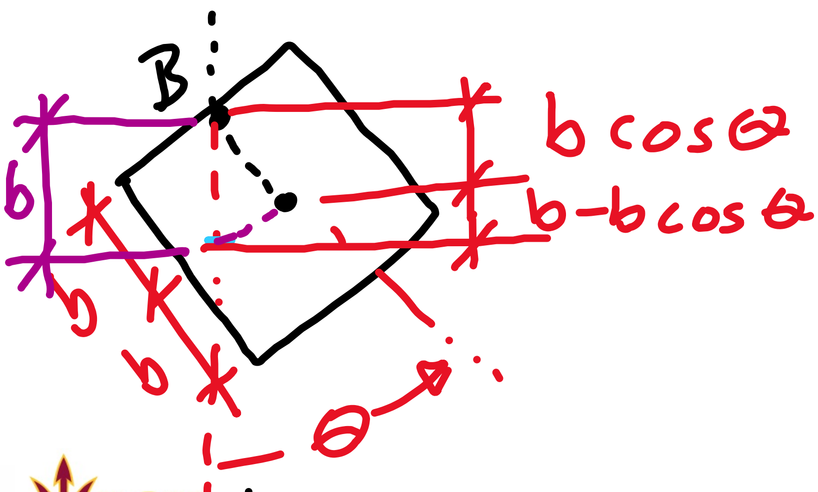


$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \omega^2$ & $V = mgh$

but $h = b - b\cos\theta$



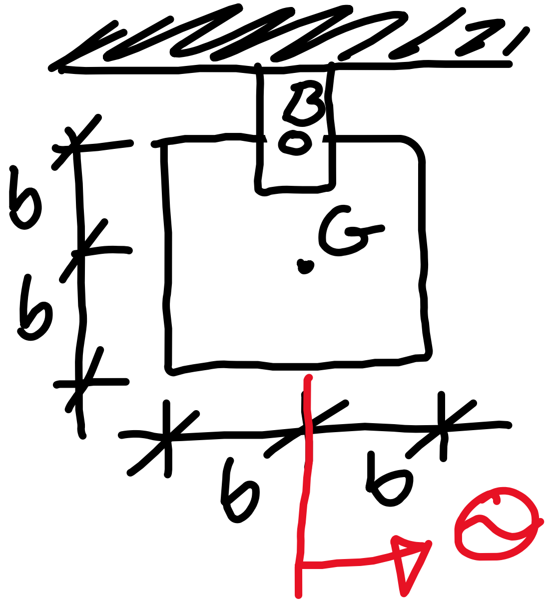
New style

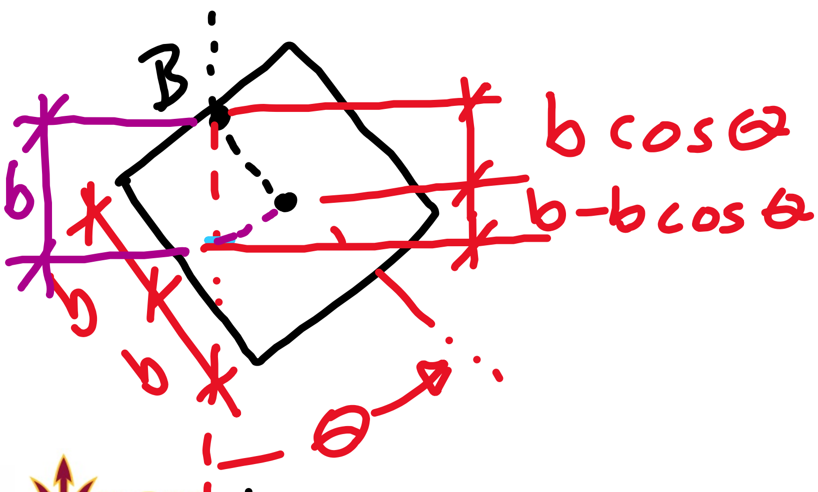
$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \dot{\theta}^2$ & $V = mgh$

but $h = b - b\cos\theta$

$= 1 - \cos\theta$





New style

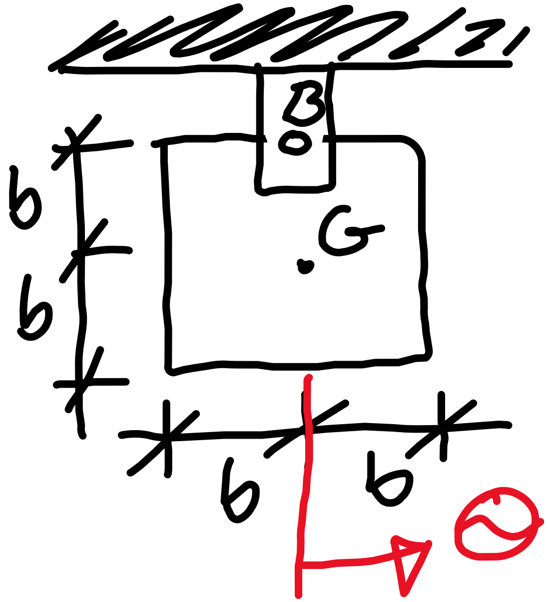
$T + V = \text{const.}$, where

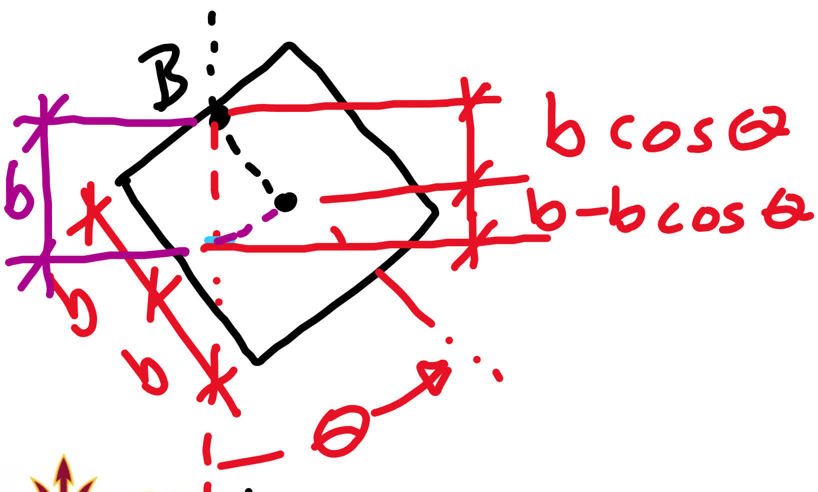
$T = \frac{1}{2} I_B \dot{\theta}^2$ & $V = mgh$

but $h = b - b\cos\theta$

$= 1 - \cos\theta$

But $\cos\theta \cong 1 - \frac{\theta^2}{2!} + \cdots$





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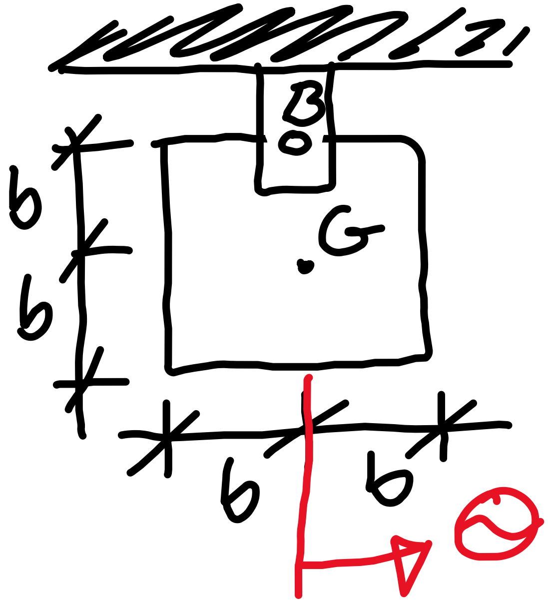


$T + V = \text{const.}$, where

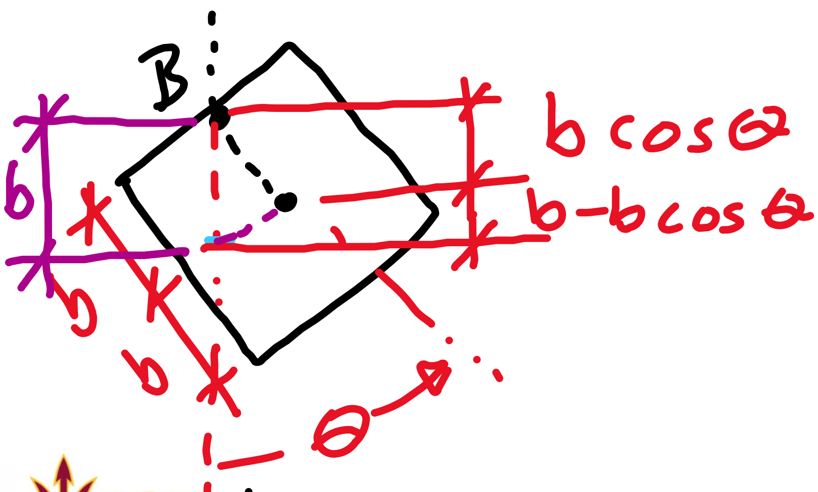
$T = \frac{1}{2} I_B \dot{\theta}^2$ & $V = mgh$

but $h = b - b\cos\theta$

$= 1 - \cos\theta$

But $\cos\theta \cong 1 - \frac{\theta^2}{2!} + \cdots$

so $h \cong \theta^2/2$



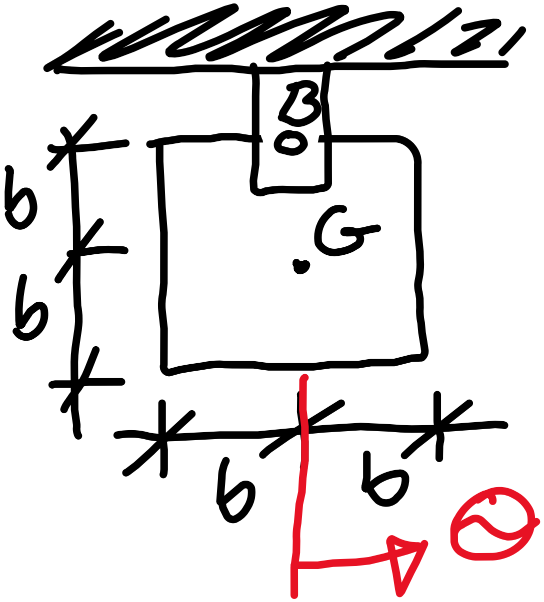
New style

$T + V = \text{const.},$ where

$T = \frac{1}{2} I_B \omega^2$ & $V = mgh$

but $h = b - b\cos\theta$

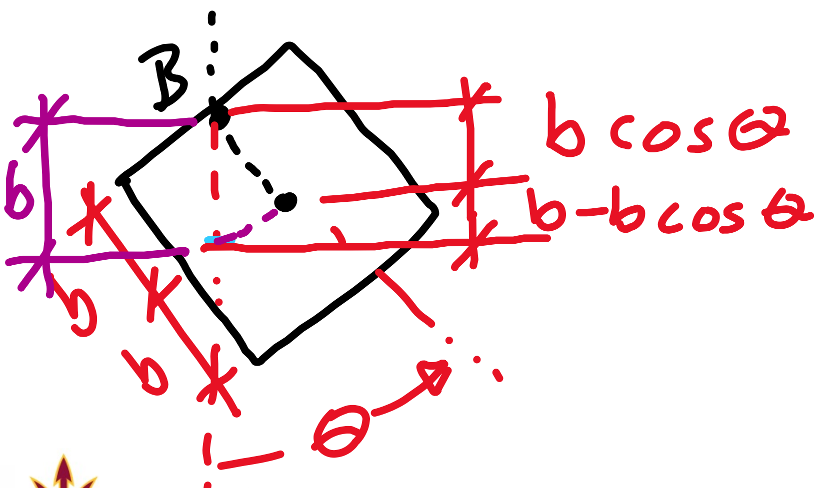
$= 1 - \cos\theta$



But $\cos\theta \cong 1 - \frac{\theta^2}{2!} + \cdots$

So $h \cong \theta^2/2$ Now

$V = mg\theta^2/2$



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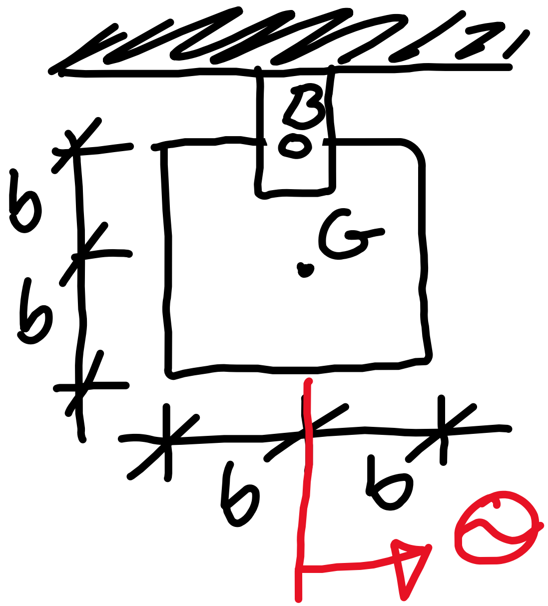


$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \omega^2$ & $V = mgh$

but $h = b - b\cos\theta$

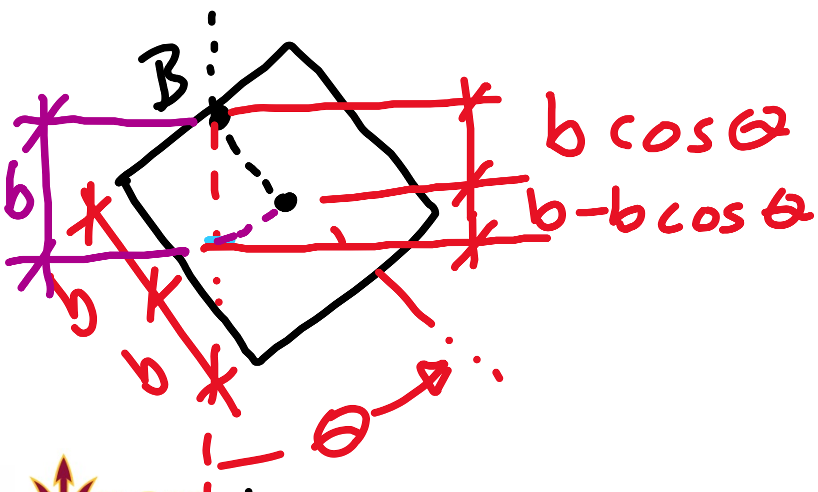
$= 1 - \cos\theta$

But $\cos\theta \cong 1 - \frac{\theta^2}{2!} + \cdots$

So $h \cong \theta^2/2$ Now

$V = mg\,\theta^2/2 \Rightarrow$

$\frac{1}{2} I_B \omega^2 + mg\,\theta^2/2 = \text{const.}$



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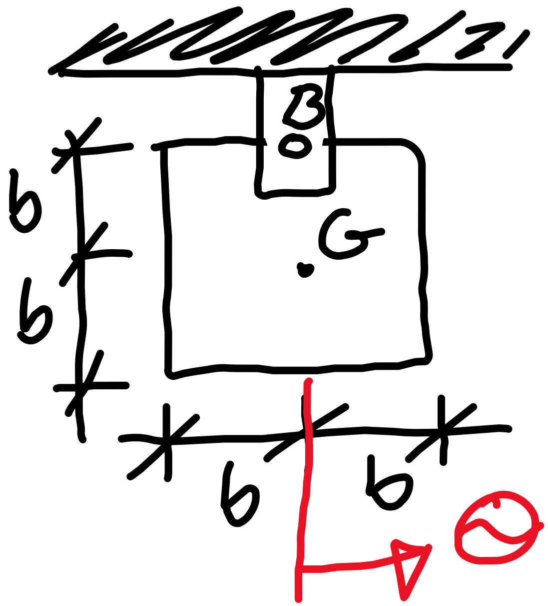


$T + V = \text{const.}$, where

$T = \frac{1}{2} I_B \omega^2$ & $V = mgh$

but $h = b - b\cos\theta$

$= 1 - \cos\theta$

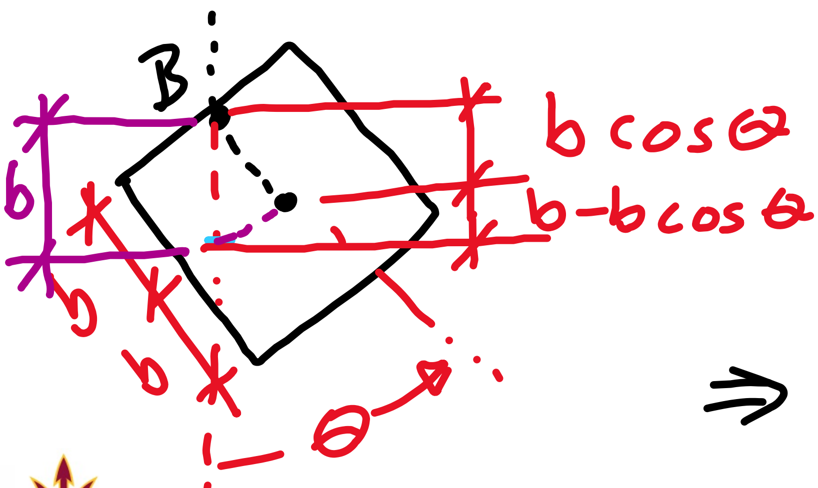
But $\cos\theta \cong 1 - \frac{\theta^2}{2!} + \cdots$

so $h \cong \theta^2/2$ Now

$V = mg\theta^2/2 \Rightarrow$

$\frac{1}{2} I_B \omega^2 + mg\theta^2/2 = \text{const.}$

$\Rightarrow \dot{\theta}^2 + \omega^2 \theta^2 = \text{const.}$



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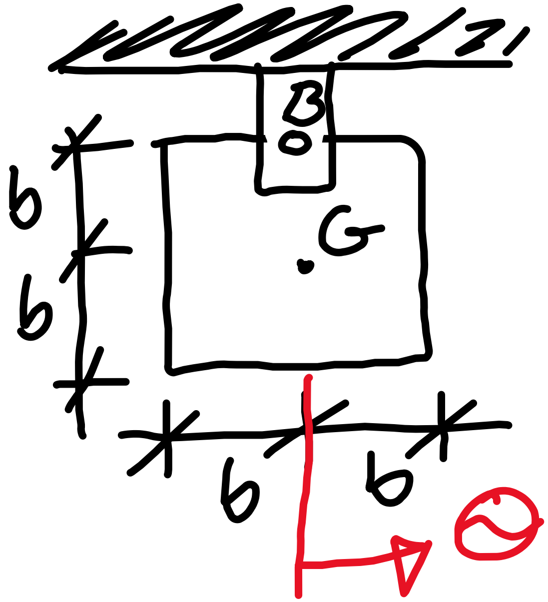


$T+V = \text{const.}$, where

$T = \frac{1}{2} I_B \omega^2$ & $V = mgh$

but $h = b - b\cos\theta$

$= 1 - \cos\theta$

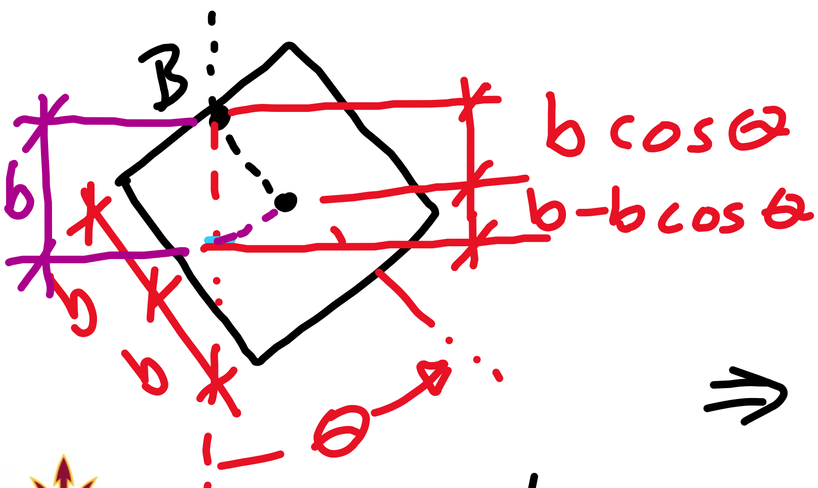
But $\cos\theta \cong 1 - \frac{\theta^2}{2!} + \cdots$

so $h \cong \theta^2/2$ Now

$V = mg\theta^2/2 \Rightarrow$

$\frac{1}{2} I_B \omega^2 + mg\theta^2/2 = \text{const.}$

$\Rightarrow \dot{\theta}^2 + \omega^2\theta^2 = \text{const.},$



where $\omega = \sqrt{\frac{Mg}{I_B}}$

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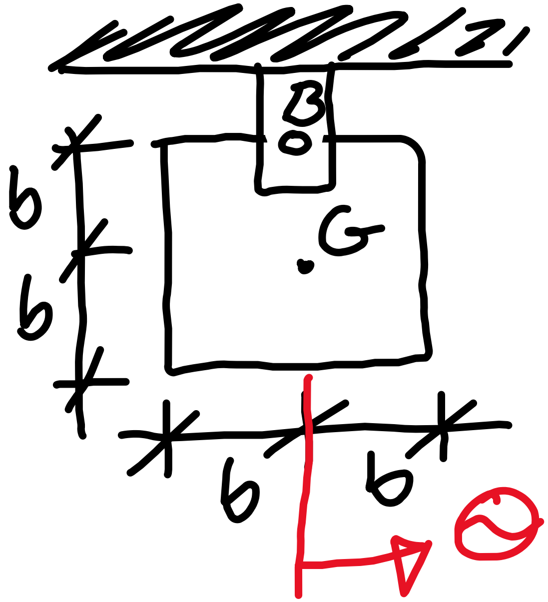


$T+V=\text{const.}$, where

$T=\frac{1}{2}I_B\omega^2$ & $V=mgh$

but $h=b-b\cos\theta$

$=1-\cos\theta$

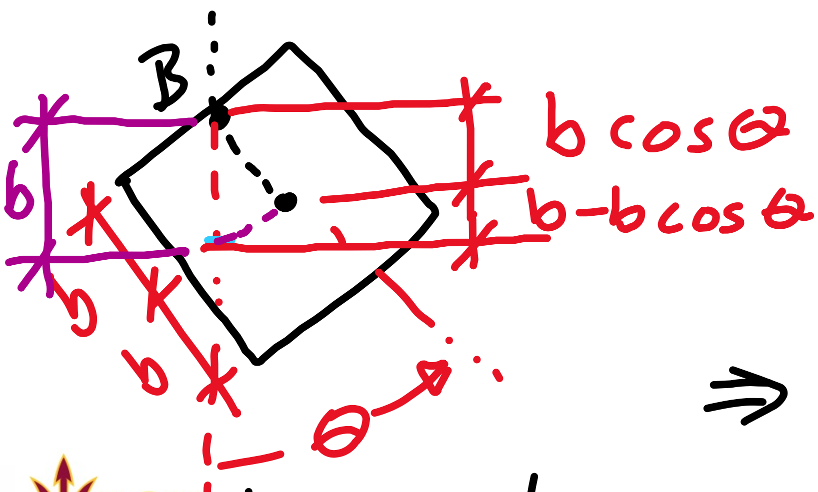
But $\cos\theta\cong 1-\frac{\theta^2}{2!}+\cdots$

so $h\cong\theta^2/2$ Now

$V=mg\theta^2/2 \Rightarrow$

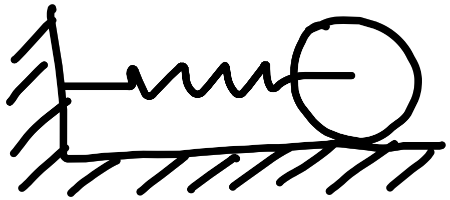
$\frac{1}{2}I_B\omega^2+mg\theta^2/2=\text{const.}$

$\Rightarrow\ \dot{\theta}^2+\omega^2\theta^2=\text{const.},$



where $\omega=\sqrt{\frac{mg}{I_B}}=\sqrt{\frac{W}{I+mb^2}}$

Old style



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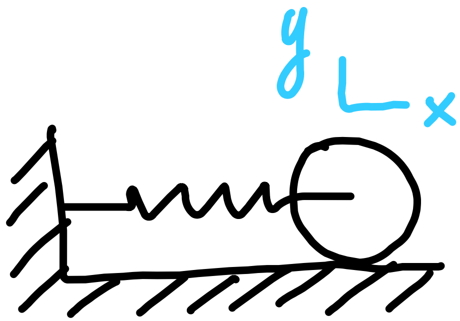

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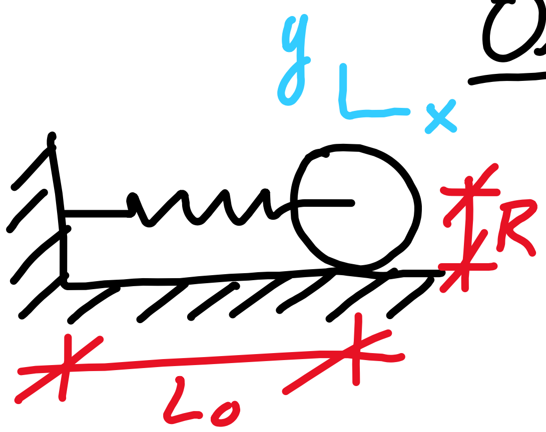

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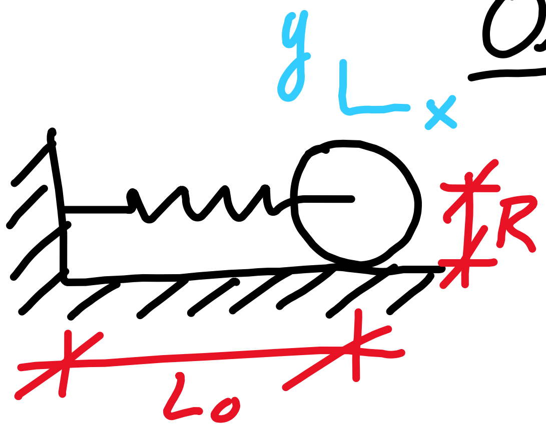
move forward



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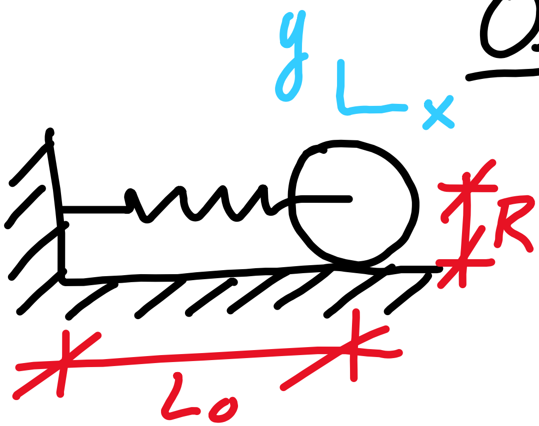

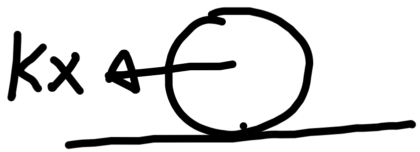

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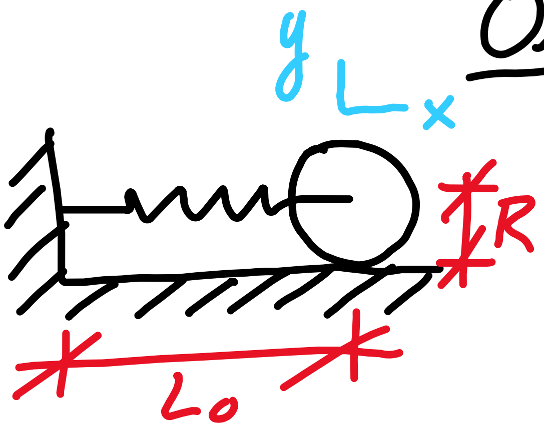

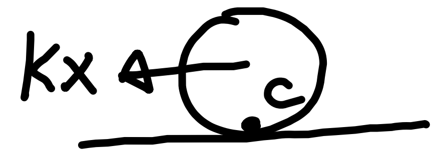

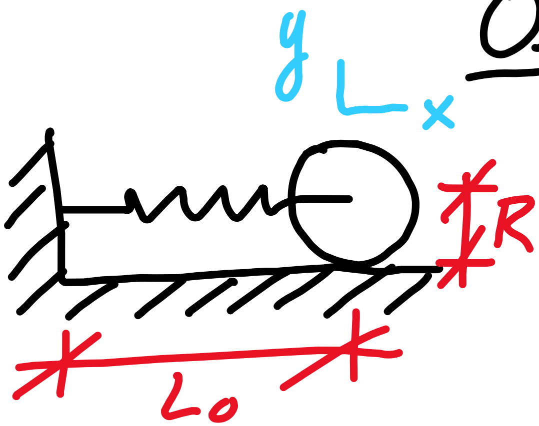

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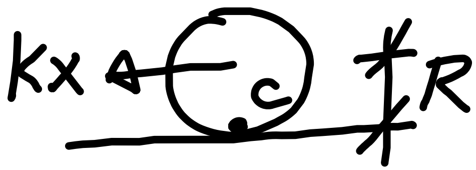

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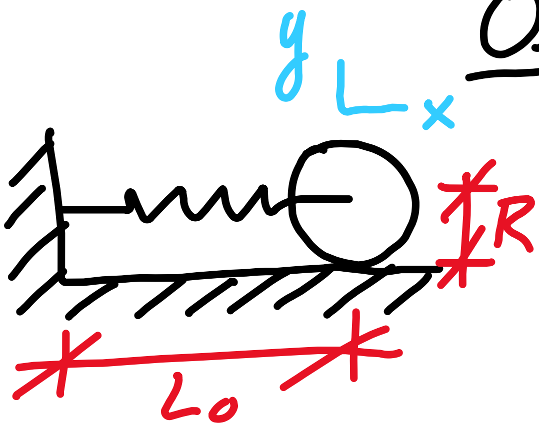

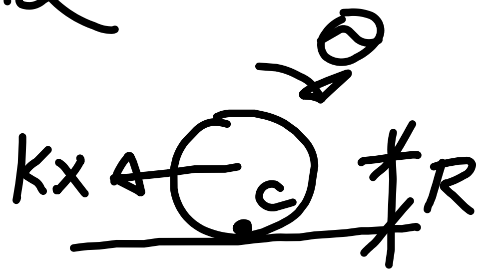

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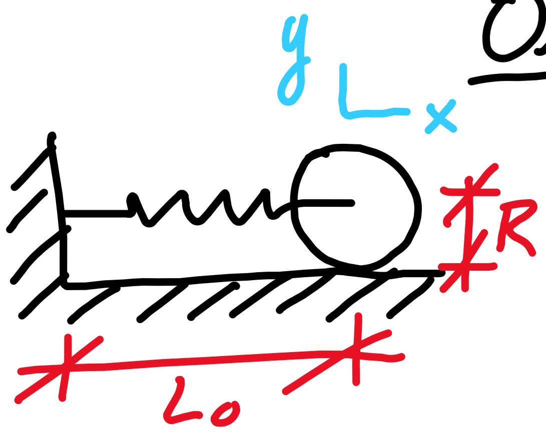

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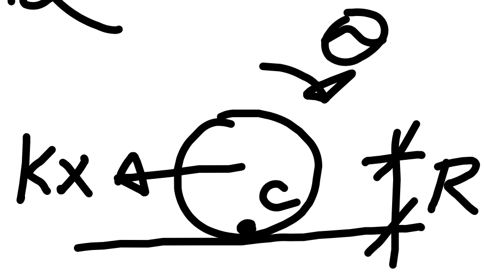
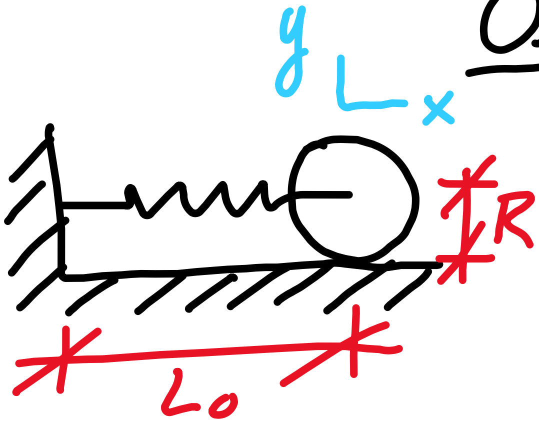
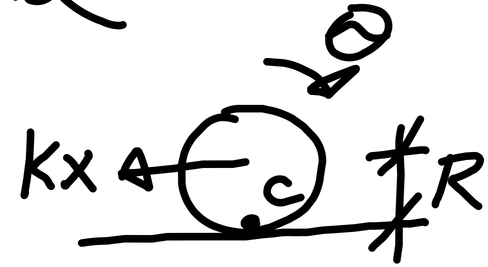


$$\sum M_c = I_c \ddot{\theta}$$

Old style

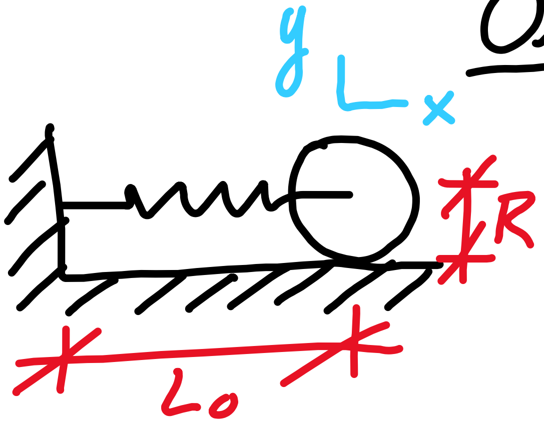
move forward





$$\circlearrowleft \sum M_C = I_C \ddot{\theta} \Rightarrow -kxR = (\bar{I} + mR^2)\ddot{\theta}$$

Old style



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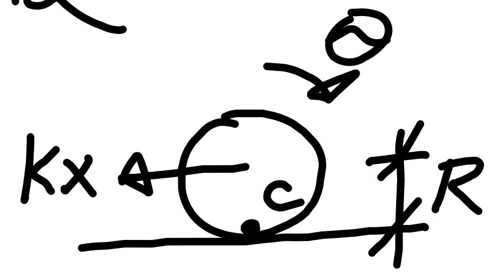
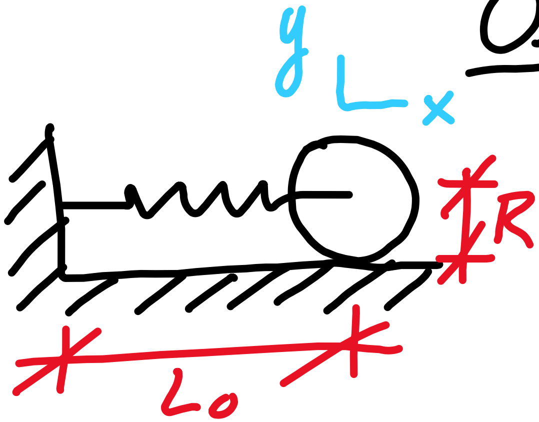


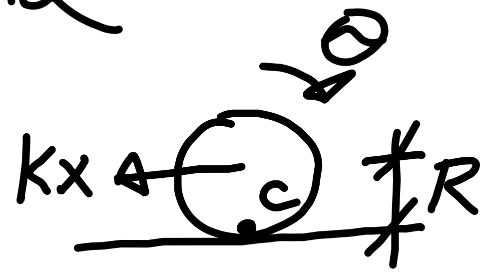
$$\circlearrowleft \sum M_c = I_c \ddot{\theta} \Rightarrow -KxR = (\bar{I} + mR^2)\ddot{\theta}$$ But

$$x = R\theta$$

Old style

move forward

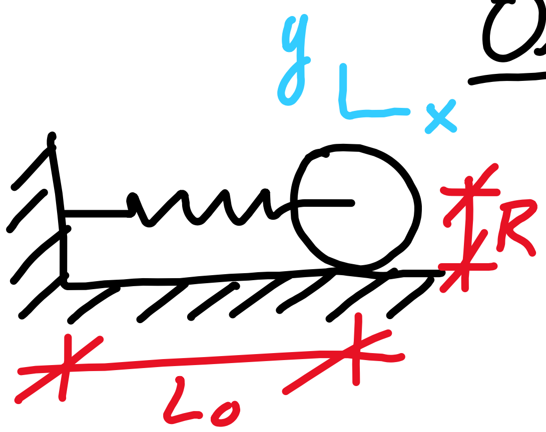




$$\circlearrowleft \sum M_c = I_c \ddot{\theta} \Rightarrow \quad -kxR = (\bar{I} + mR^2)\ddot{\theta} \quad \text{But}$$

$$x = R\theta \quad \text{so} \quad -kR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$$

Old style



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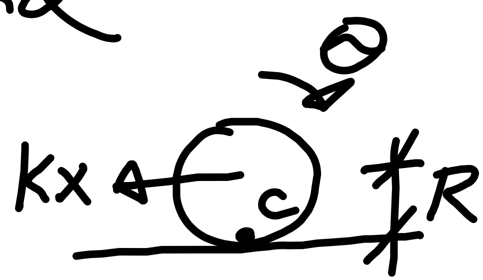


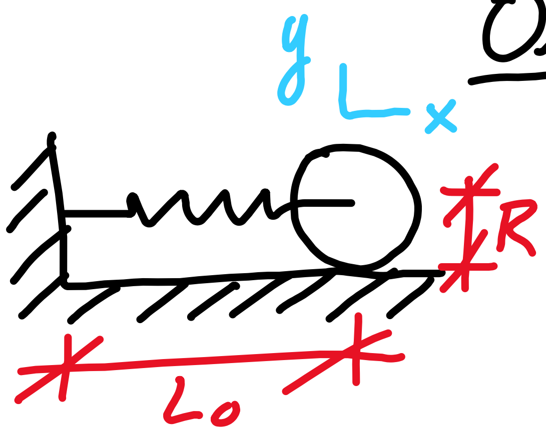
$\sum M_c = I_c \ddot{\theta} \Rightarrow -KxR = (\bar{I} + mR^2)\ddot{\theta}$ But

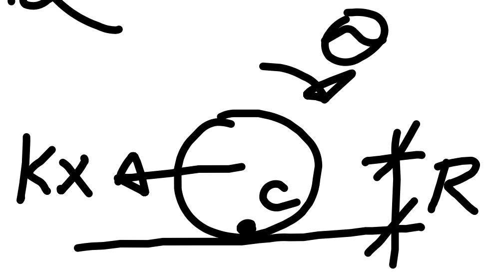
$x = R\theta$ so $-KR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$ or

$\ddot{\theta} = -\omega^2\theta$,

Old style

move forward



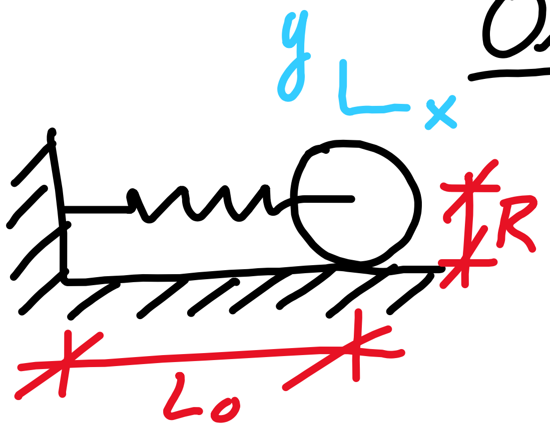


$$\sum M_c = I_c \ddot{\theta} \Rightarrow -KxR = (\bar{I} + mR^2)\ddot{\theta} \quad \text{But}$$

$$x = R\theta \quad \text{so} \quad -KR^2\theta = (\bar{I} + mR^2)\ddot{\theta} \quad \underline{\text{or}}$$

$$\ddot{\theta} = -\omega^2\theta, \quad \text{where} \quad \omega = \sqrt{\frac{KR^2}{\bar{I} + mR^2}}$$

Old style



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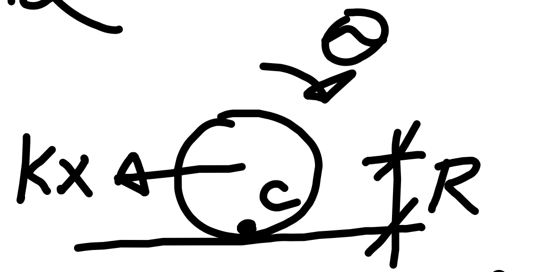


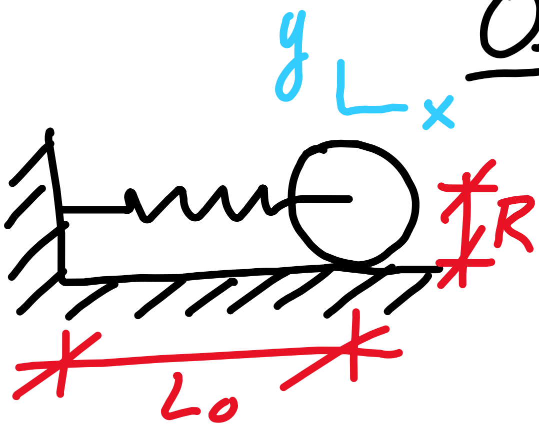
$\circlearrowleft \sum M_c = I_c \ddot{\theta} \Rightarrow -kxR = (\bar{I} + mR^2)\ddot{\theta}$ But

$x = R\theta$ so $-kR^2\theta = (\bar{I} + mR^2)\ddot{\theta}$ or

$\ddot{\theta} = -\omega^2\theta$, where $\omega = \sqrt{\dfrac{kR^2}{\bar{I} + mR^2}}$

New style

Old style



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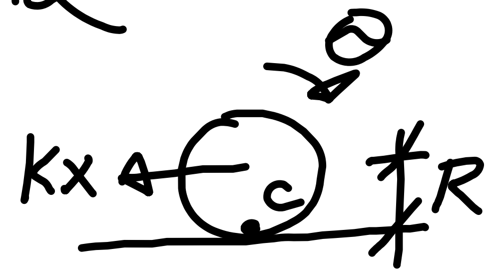


$$\circlearrowleft \sum M_c = I_c\ddot{\theta} \Rightarrow \quad -kxR = (\bar{I} + mR^2)\ddot{\theta} \quad \text{But}$$

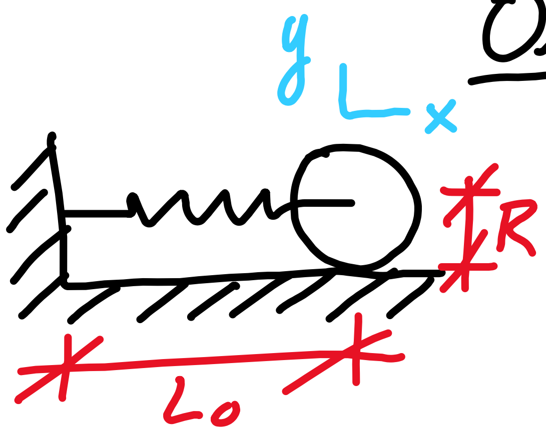
$$x = R\theta \quad \text{so} \quad -kR^2\theta = (\bar{I} + mR^2)\ddot{\theta} \quad \text{or}$$

$$\ddot{\theta} = -\omega^2\theta, \quad \text{where} \quad \omega = \sqrt{\frac{kR^2}{\bar{I} + mR^2}}$$

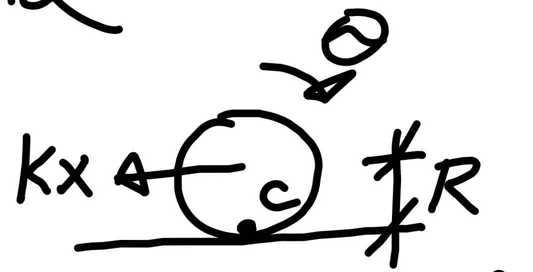
New style

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

Old style



move forward



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$$x = R\theta \quad \text{so} \quad -kR^2\theta = (\bar{I} + mR^2)\ddot{\theta} \quad \text{or}$$

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New style

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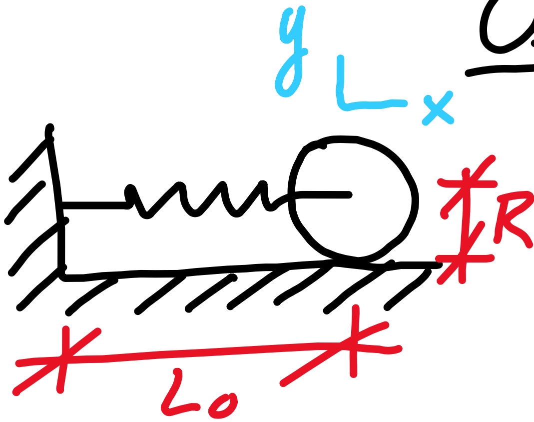

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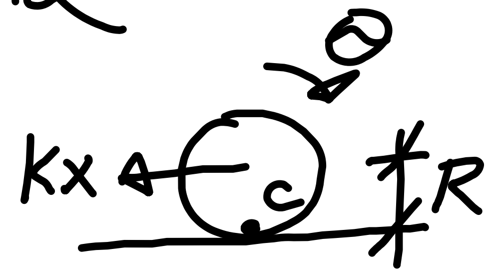

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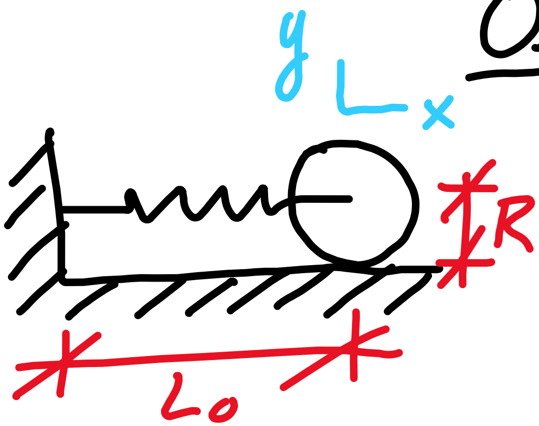

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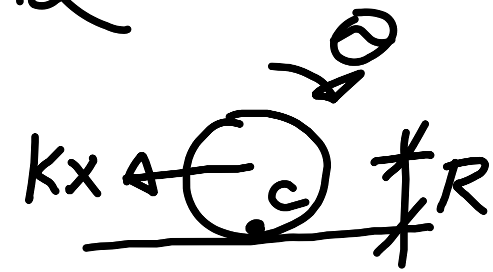

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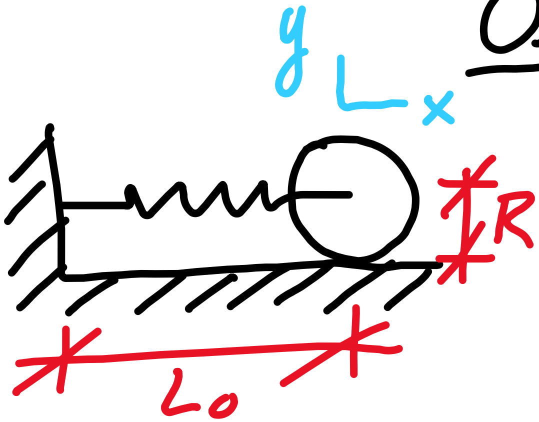

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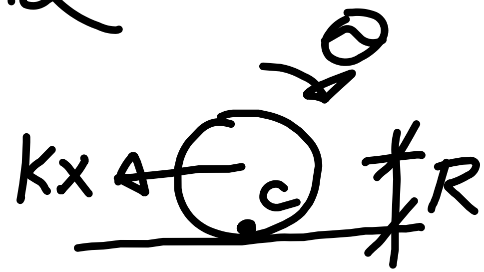

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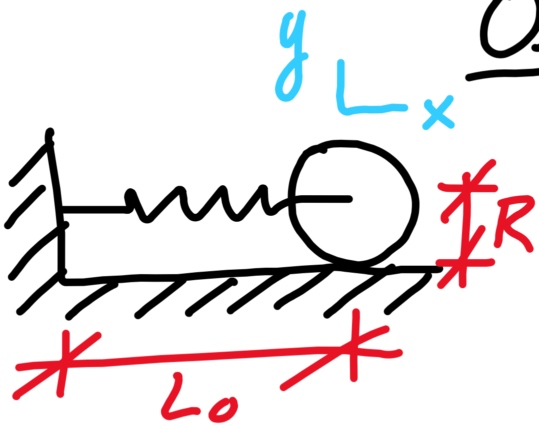

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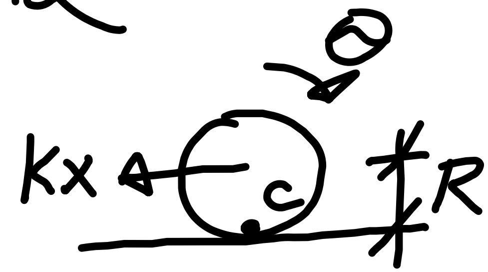

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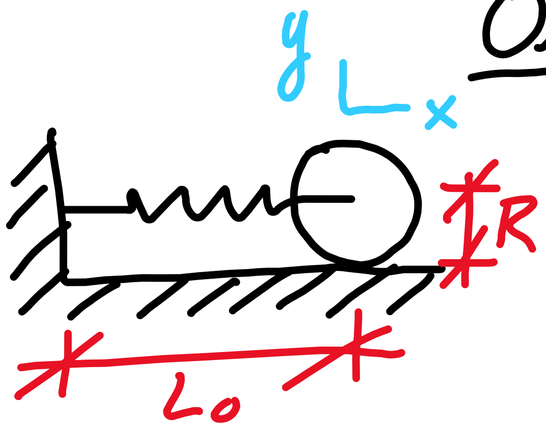

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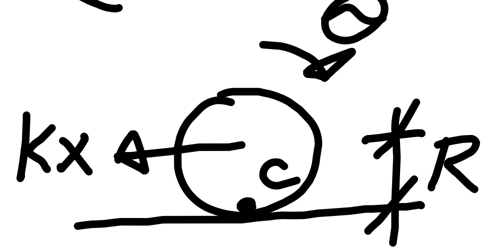

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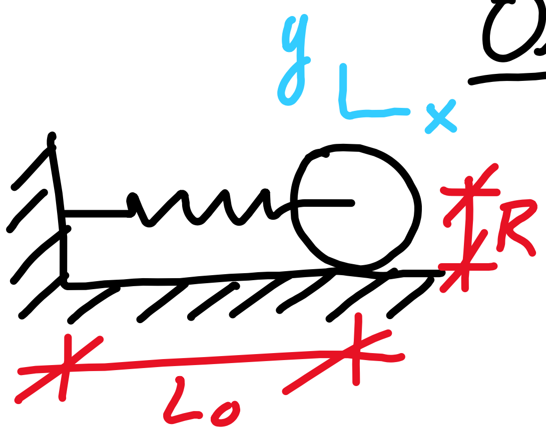
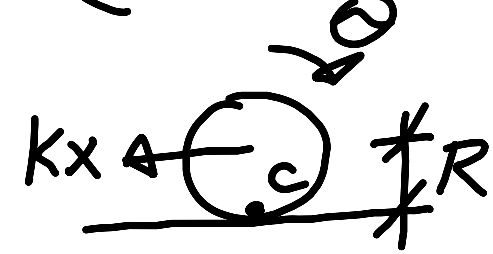
Old style



move forward



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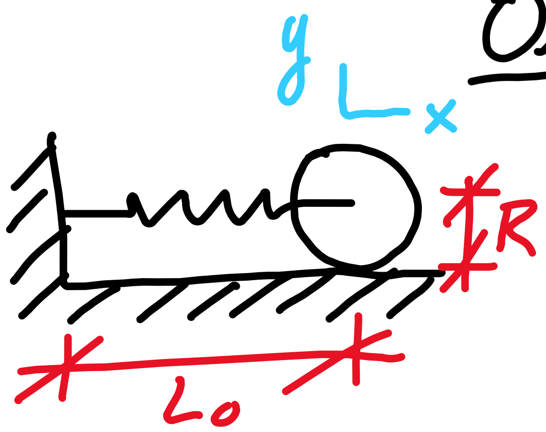
Old style



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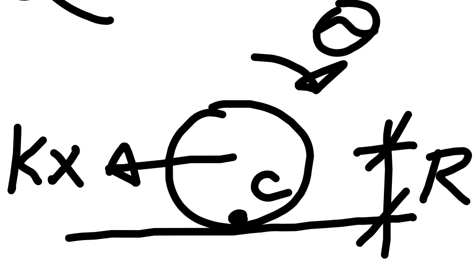

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New style

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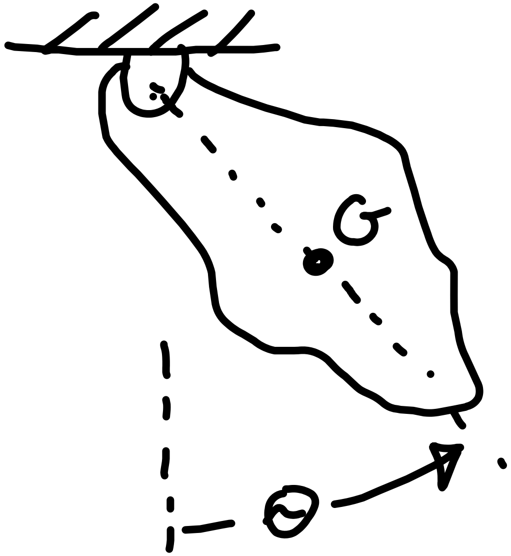
& similarly

$$\dot{\theta}_m = \theta_m \omega_n$$

For the system shown



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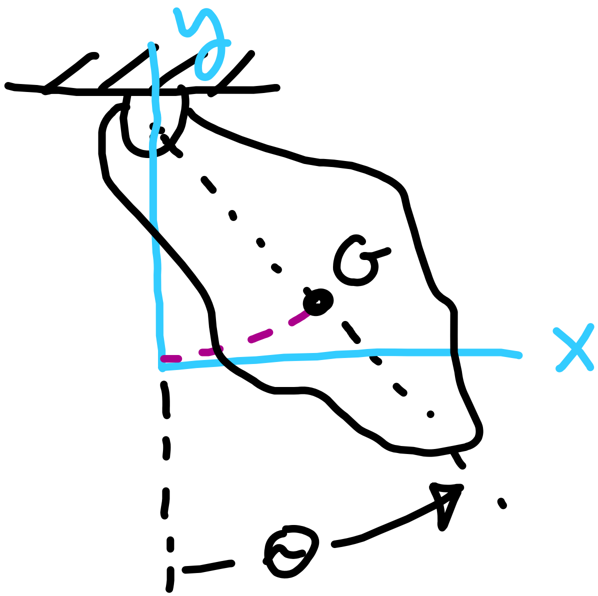

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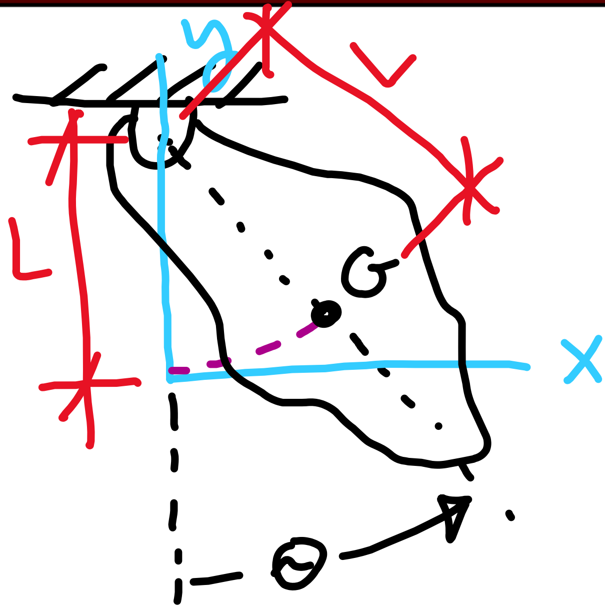

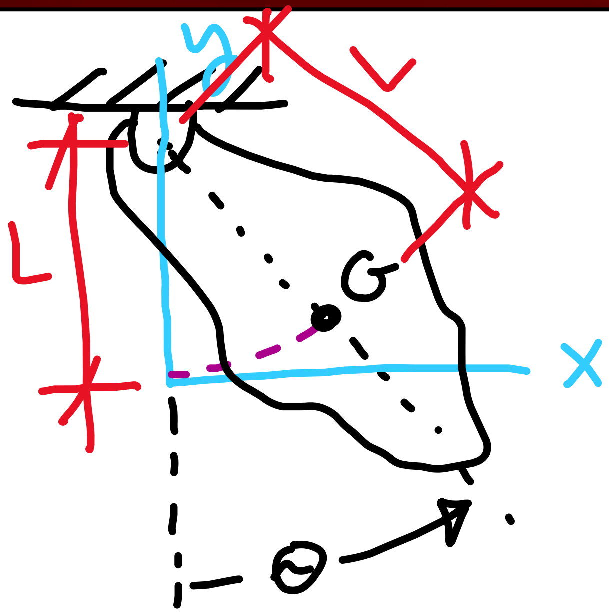

For the system shown

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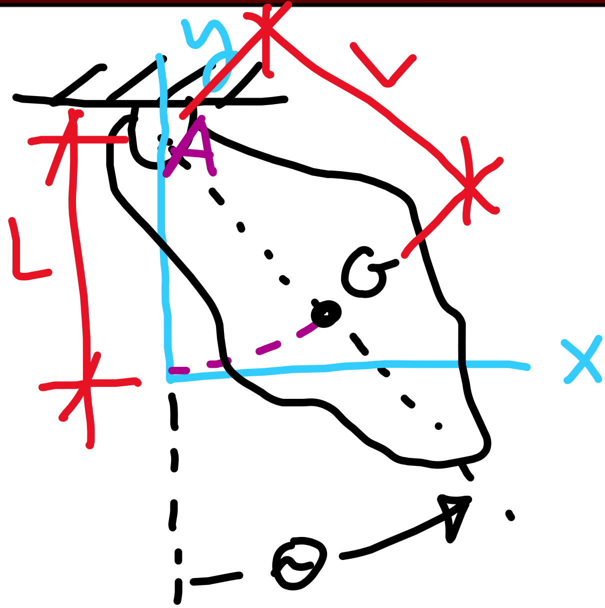

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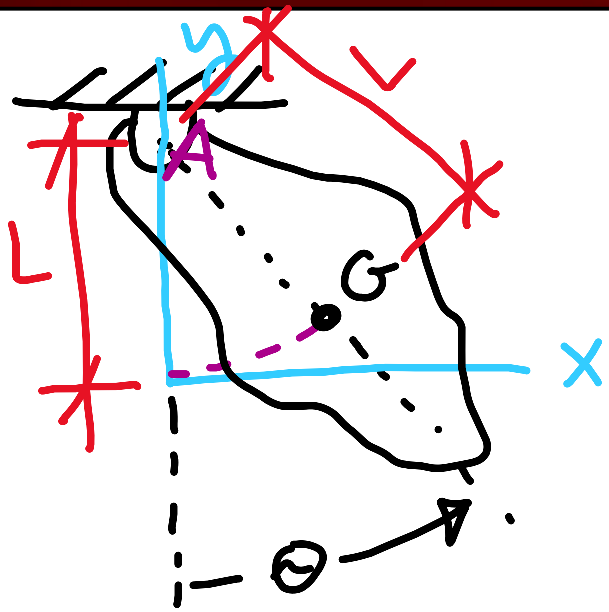

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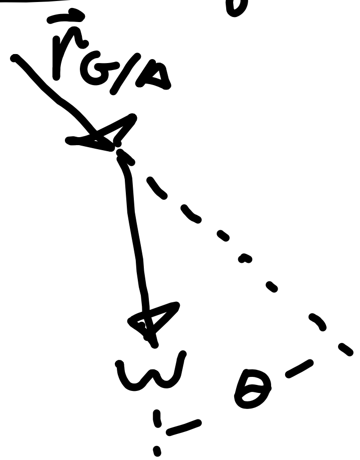

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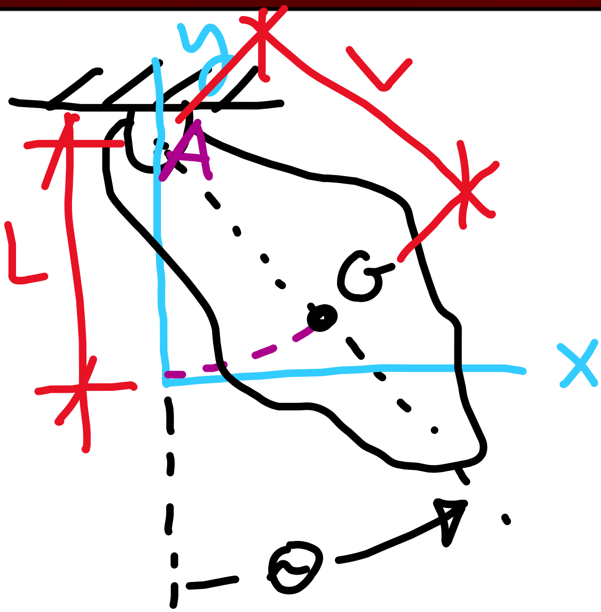

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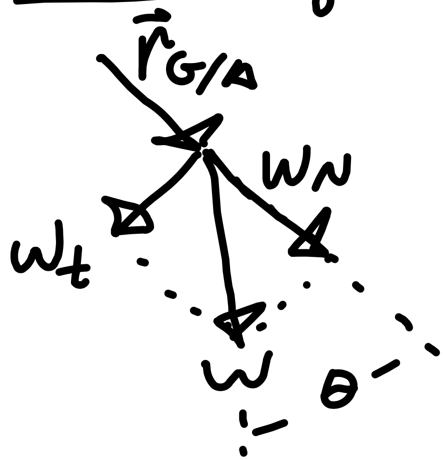

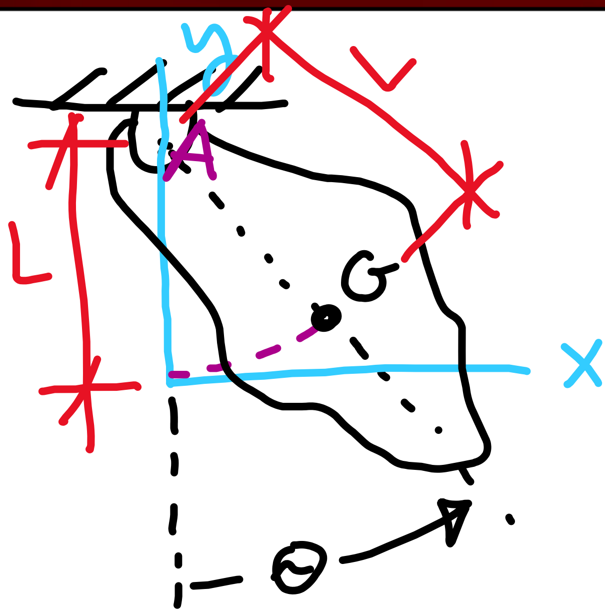

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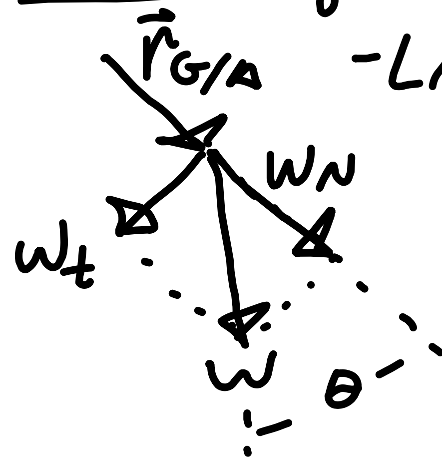

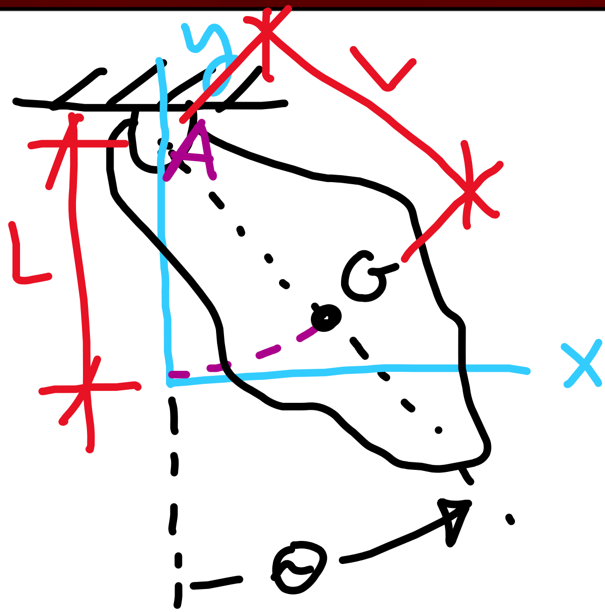

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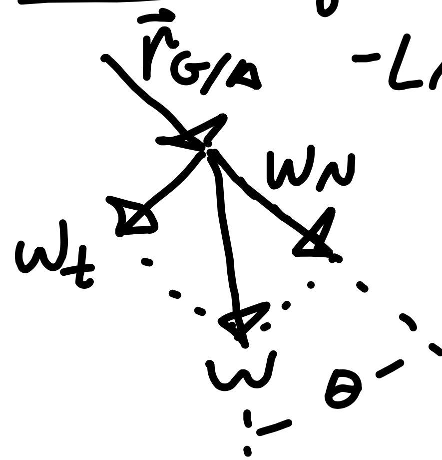

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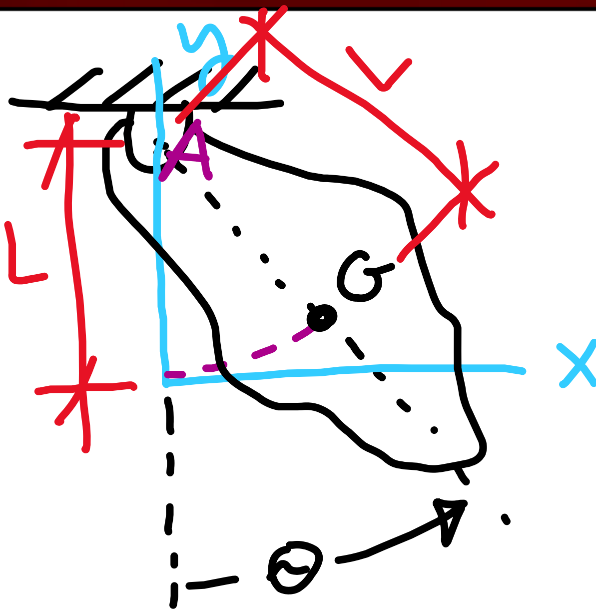

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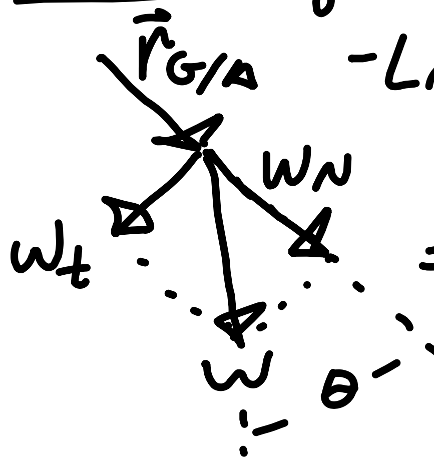

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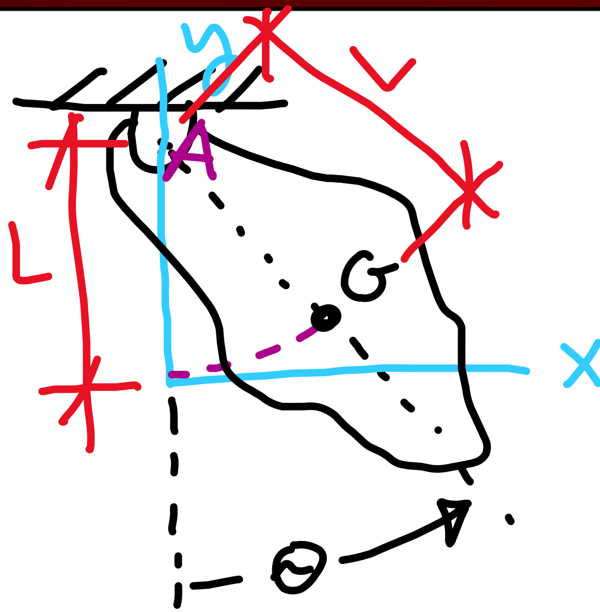

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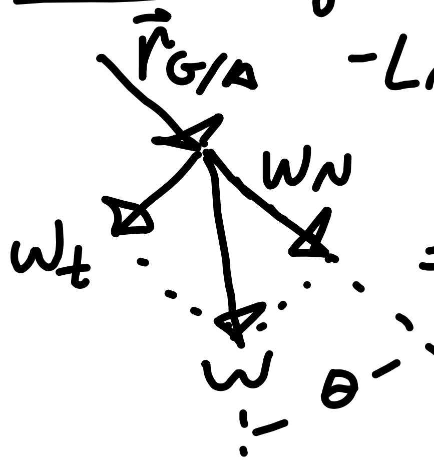

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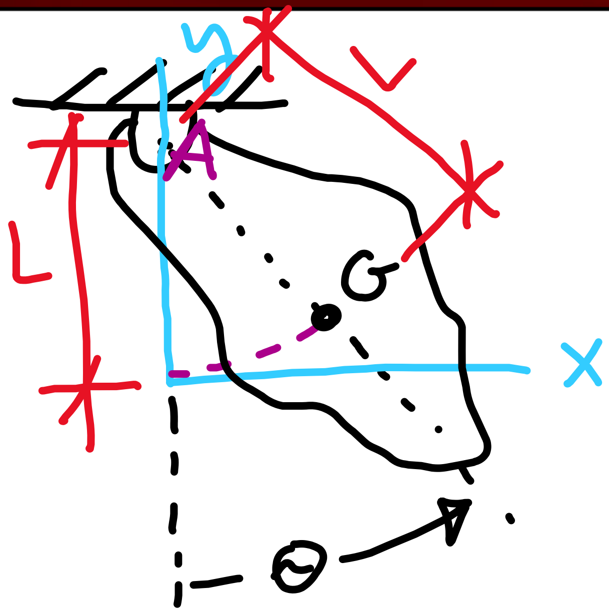

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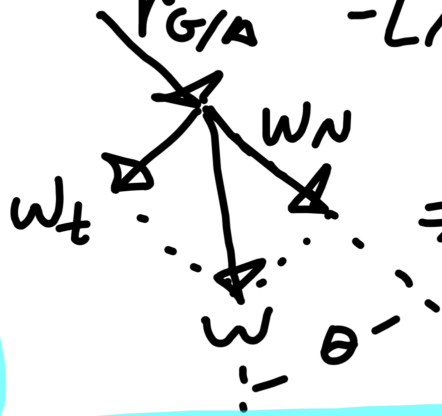

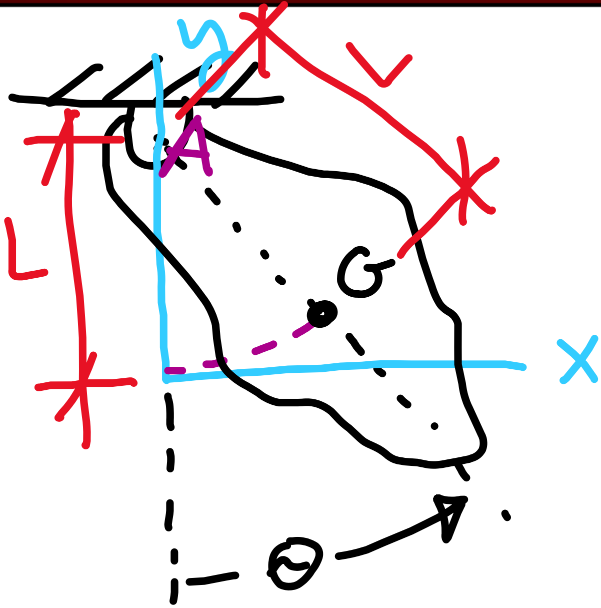

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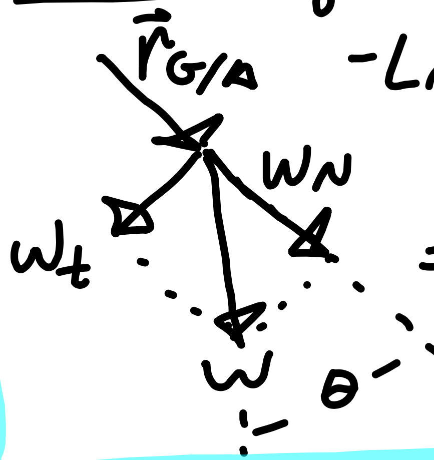

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2nd way:

For the system shown

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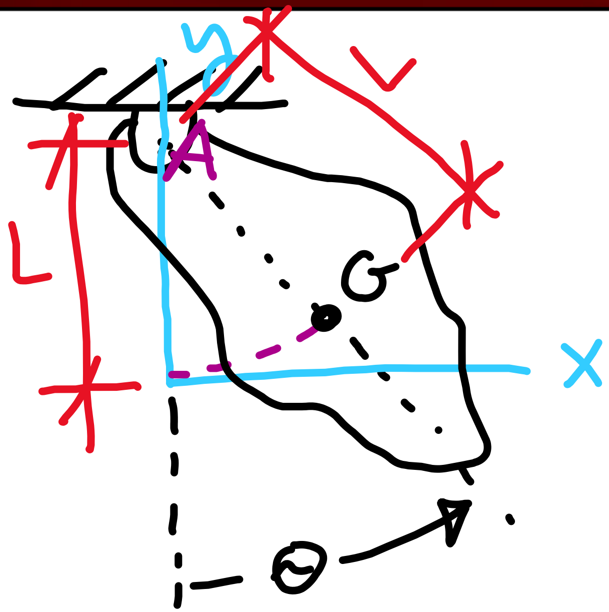

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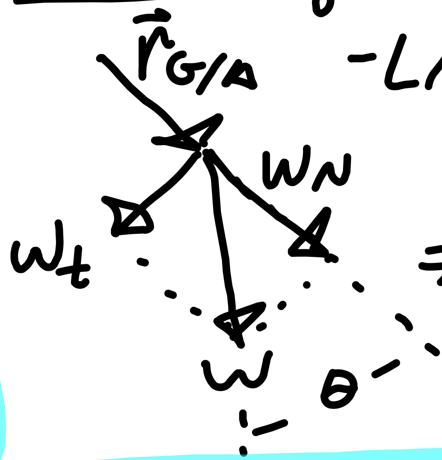

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2nd way:

$T+V=\text{const.}$

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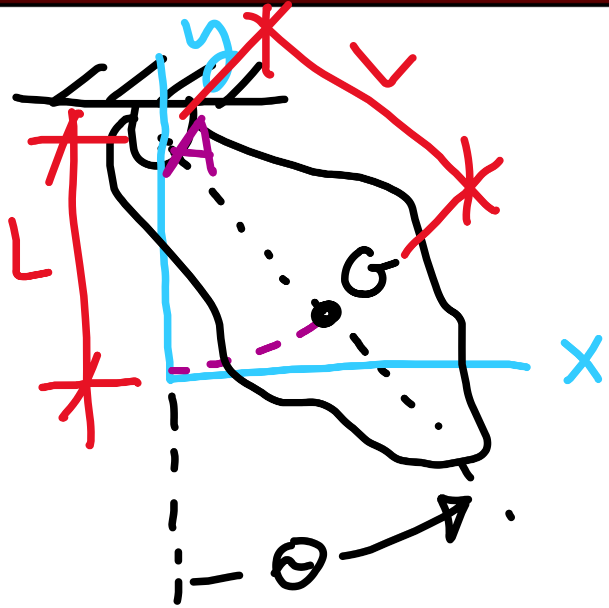

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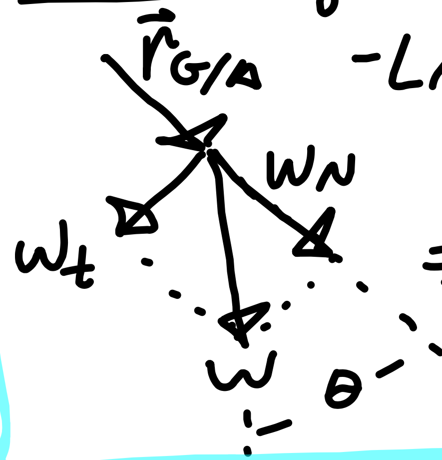

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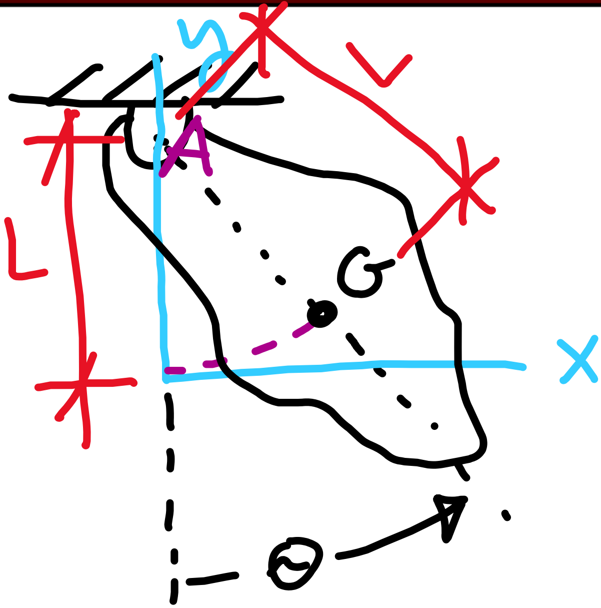

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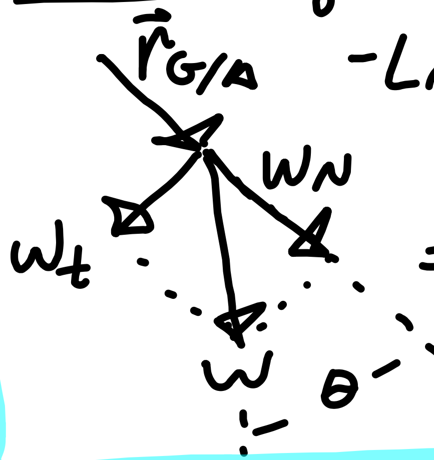

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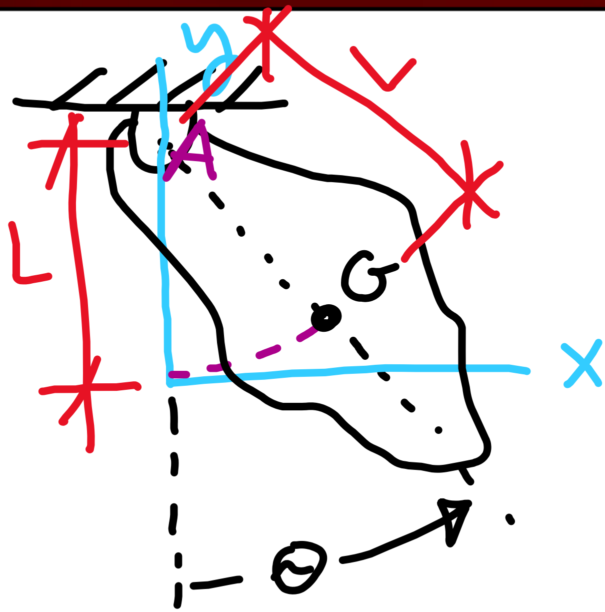

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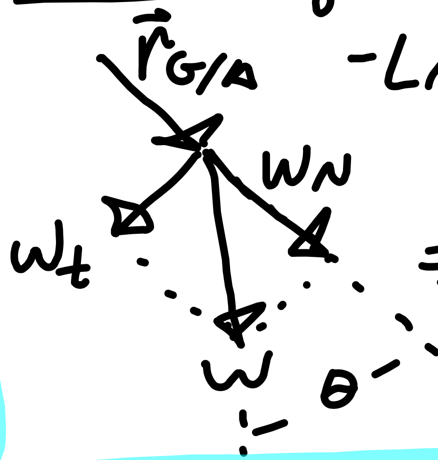

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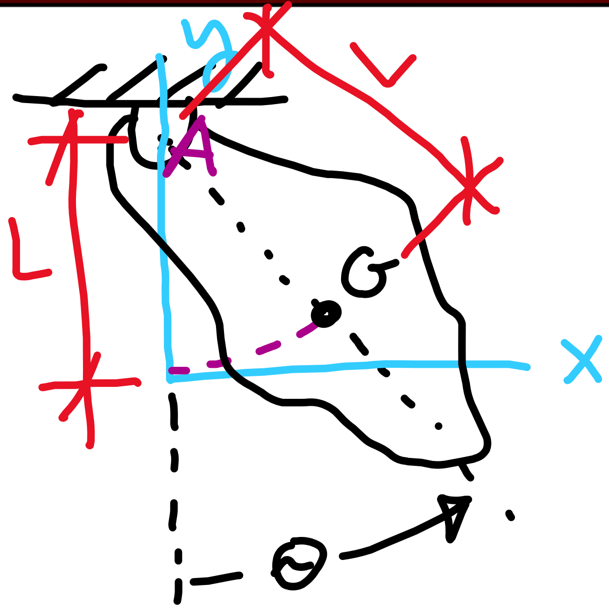

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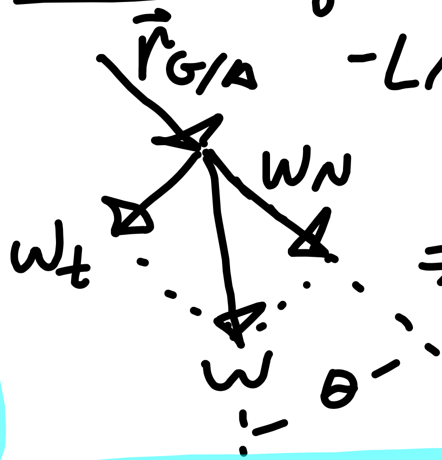

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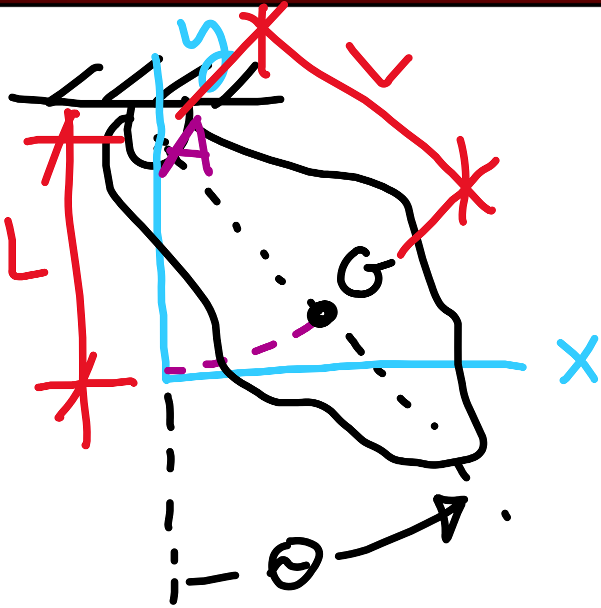

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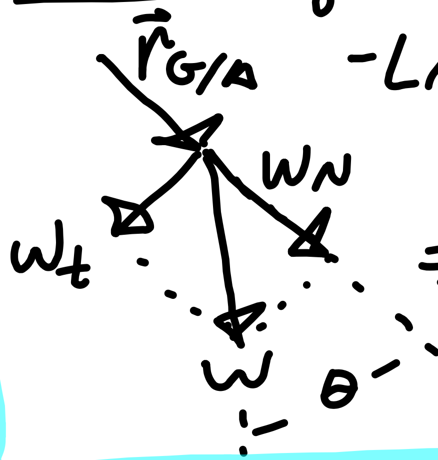

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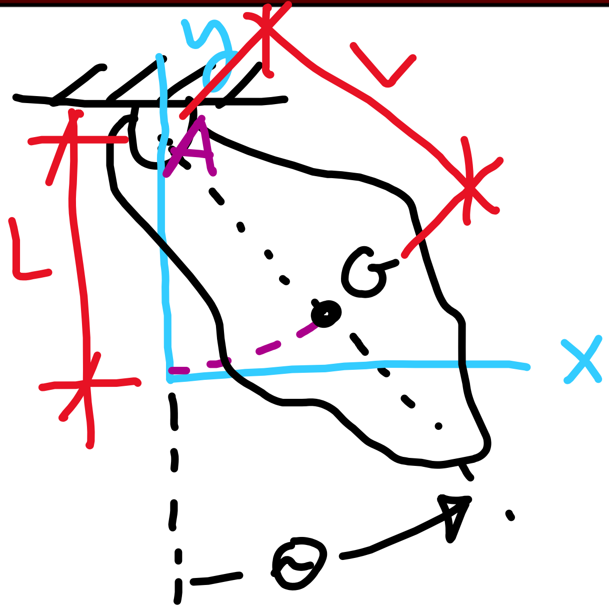

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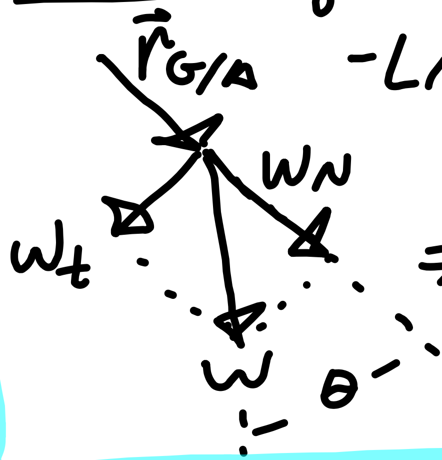

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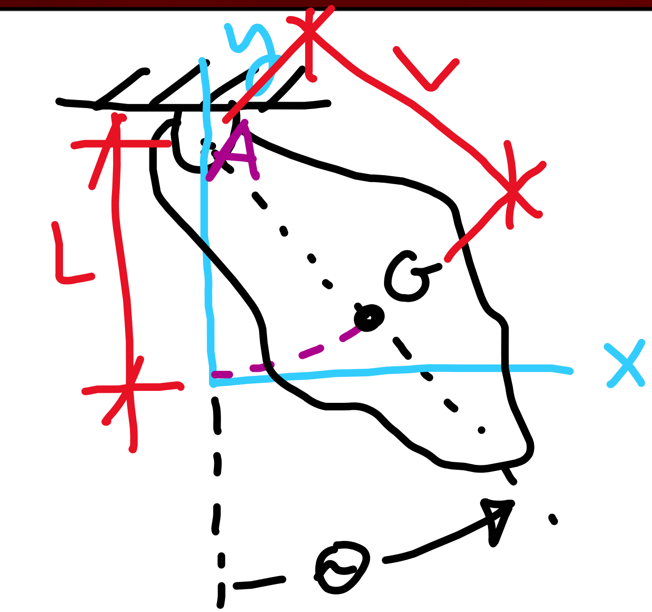

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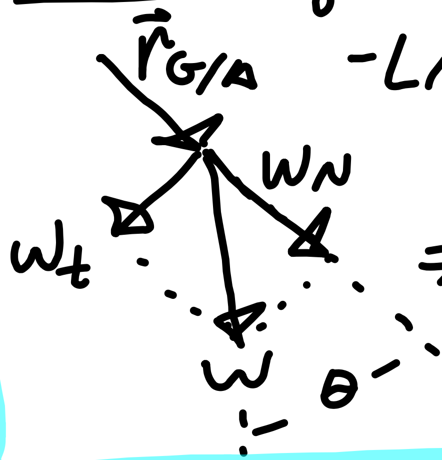

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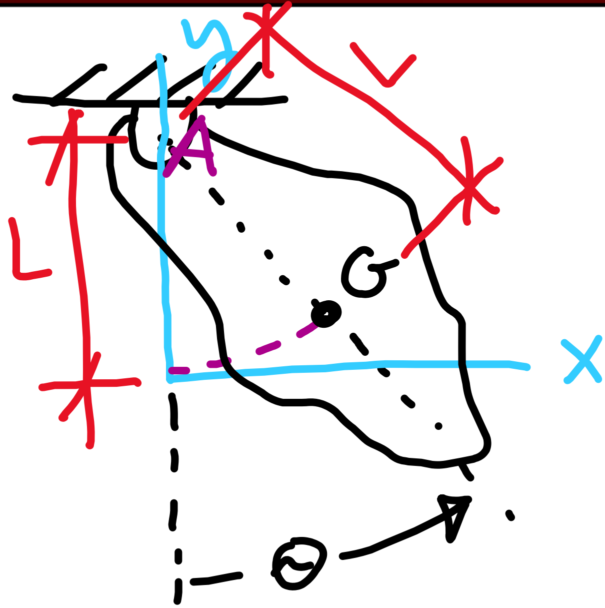

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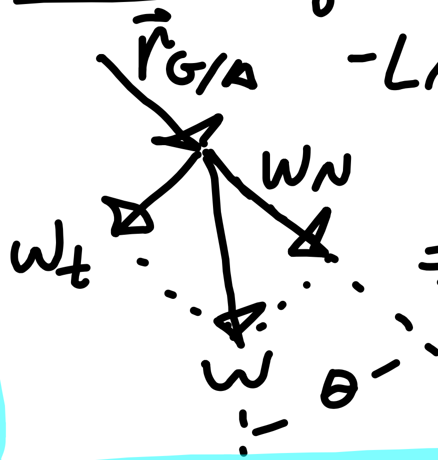

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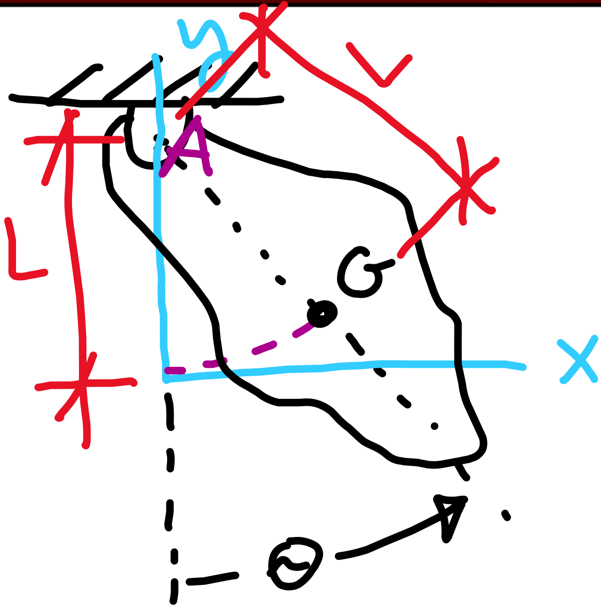

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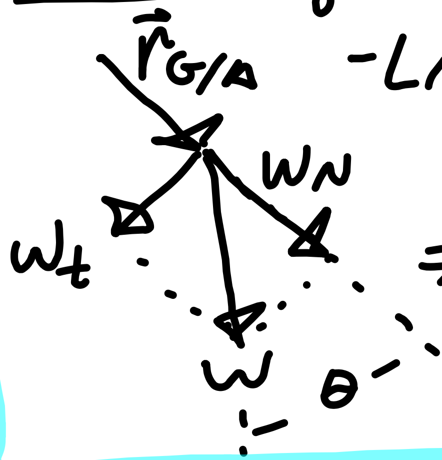

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$\underline{3^{rd} \text{ way}}$:

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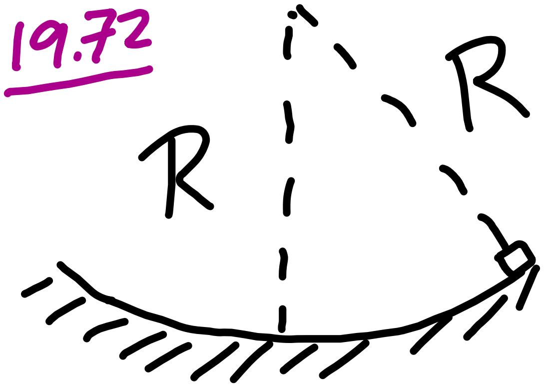
$$\Rightarrow \quad \frac{mg}{I_A} = \omega_n^2 \quad \Rightarrow \quad \boxed{\omega_n = \sqrt{\frac{mg}{I_A}}}$$

Another example

Another example [done two ways]

19.72



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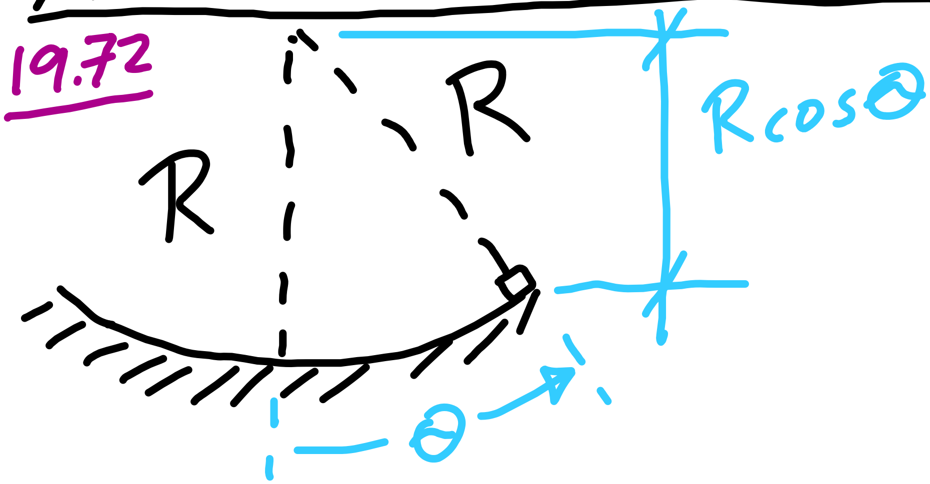

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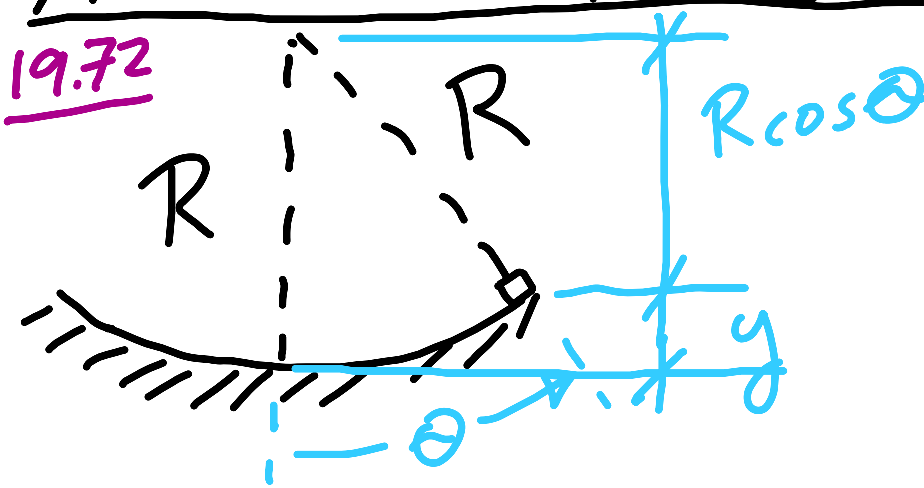

Another example [done two ways]

19.72

$\Rightarrow R = R\cos\theta + y$

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19.72

$\Rightarrow R = R\cos\theta + y \Rightarrow$

$y = R(1-\cos\theta)$

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19.72

$\Rightarrow$ $R = R\cos\theta + y \Rightarrow$

$y = R(1-\cos\theta)$ if

$\theta = \text{small}$

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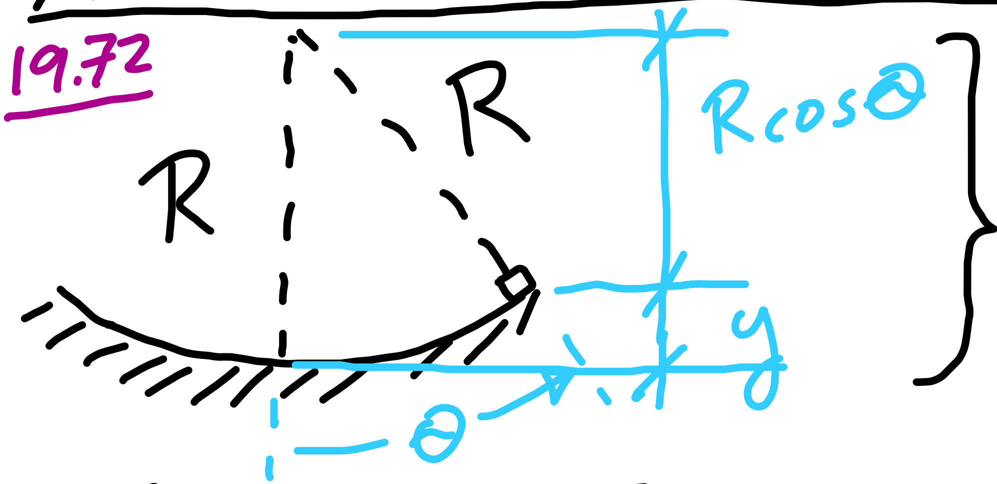

$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1-\cos\theta)$ if
θ = small then

$$y \cong R\left[1-\left(1-\frac{\theta^2}{2}\right)\right]$$

ASU

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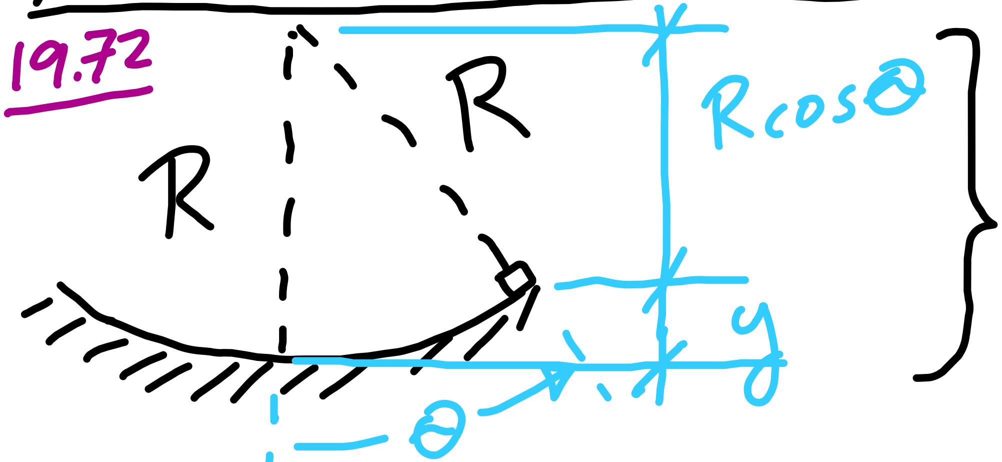
19.72



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T+V = const.

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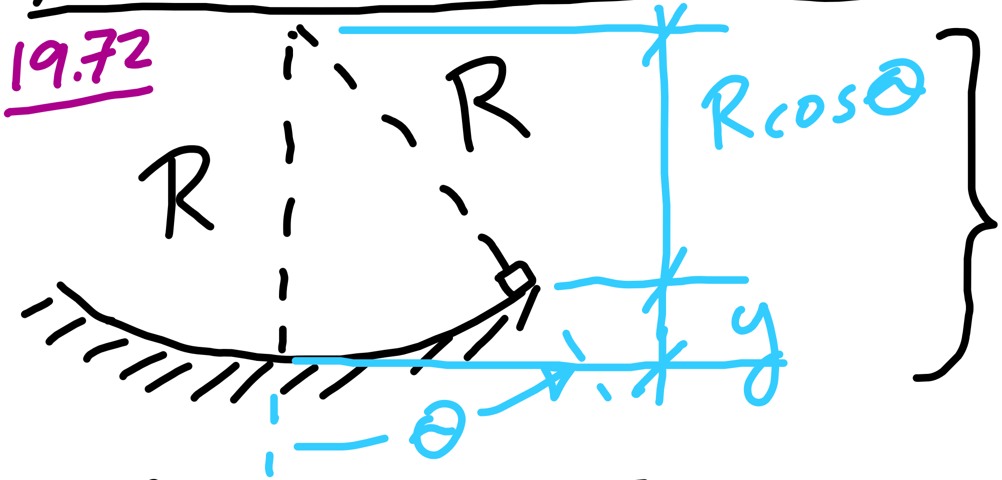
19.72



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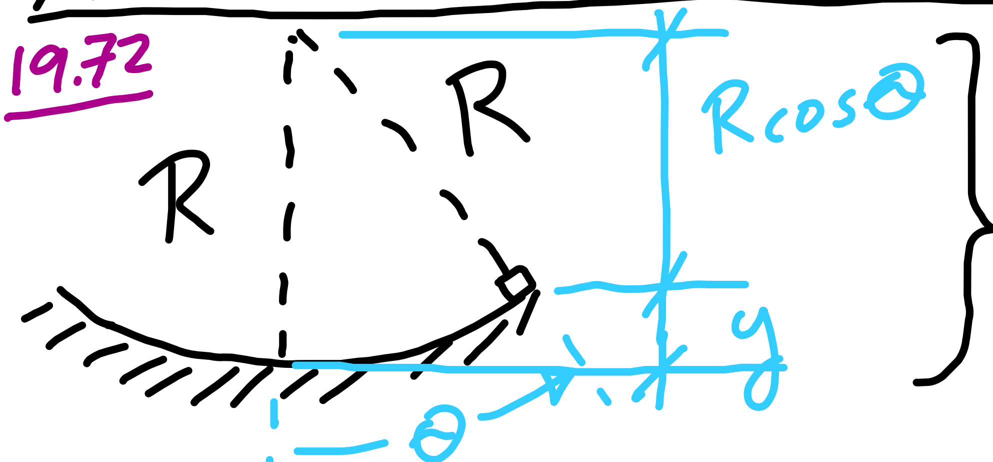

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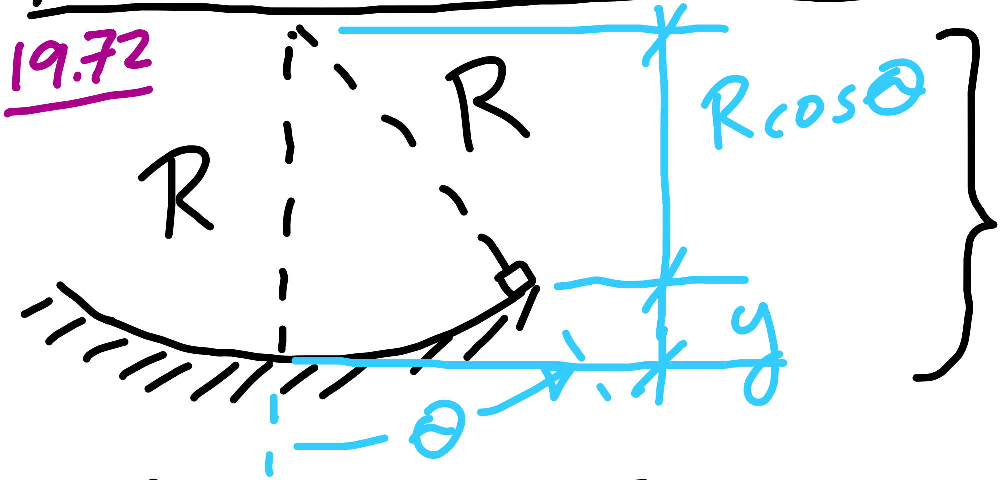
Another example [done two ways]

19.72



$\Rightarrow R = R\cos\theta + y \Rightarrow$
$y = R(1-\cos\theta)$ if
θ = small then

$y \cong R[1-(1-\frac{\theta^2}{2})] \Rightarrow y = R\theta^2/2$

1st way:

$T+V$ = const., where $T = \frac{1}{2}mv^2$ & $V = mgy$

But $v = R\dot{\theta}$

Another example [done two ways]

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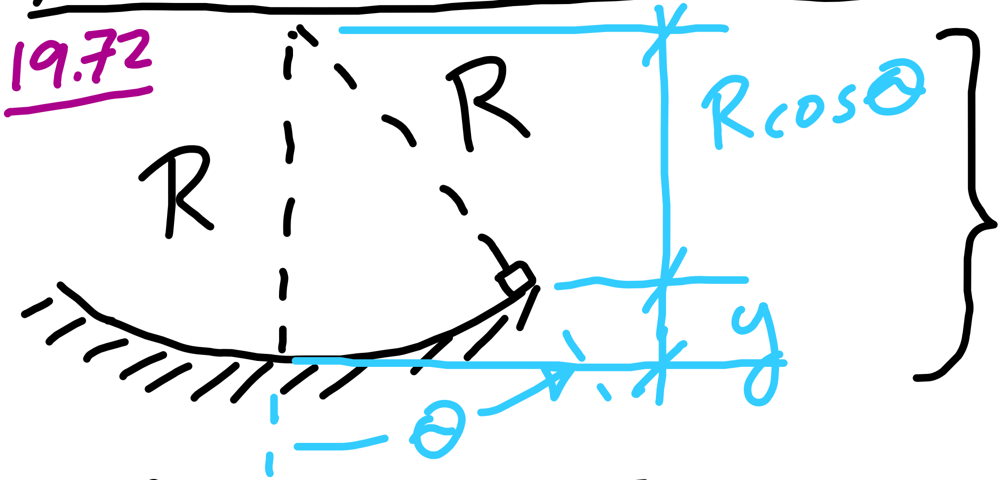

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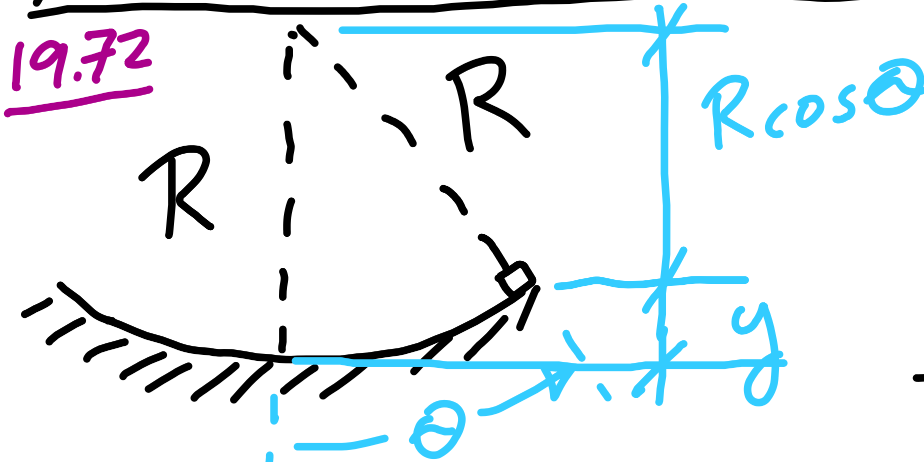

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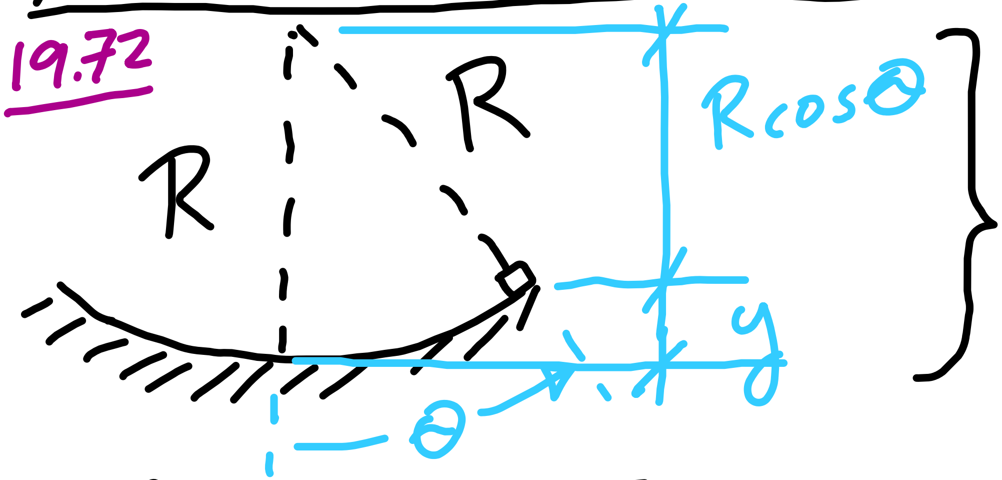

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2nd way $\rightarrow$

2nd way:

2nd way: Let $t = t_1$ when $T = \theta$

$\underline{2^{nd}\text{ way}}$: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0$

2nd way: Let $t = t_1$ when $T = 0$ & $t = t_2$ when $V = 0 \Rightarrow T_1 + V_1 = T_2 + V_2$

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__2nd way__: Let $t = t_1$ when $T = 0$

& $t = t_2$ when $V = 0 \Rightarrow \cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$

$\Rightarrow V_1 = T_2$ but $V_1 = mgR\dfrac{\theta_m^2}{2}$ &

$T_2 = \frac{1}{2}MR^2\dot{\theta}_m^2$ but $\dot{\theta}_m = \omega\theta_m$

so $T_2 = \frac{1}{2}MR^2\omega^2\theta_m^2$

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$V_1 = T_2 \Rightarrow \cancel{\tfrac{1}{2}}\cancel{m} g R \theta_m^2 = \cancel{\tfrac{1}{2}}\cancel{M} R^2 \omega^2 \theta_m^2$

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$V_1 = T_2 \Rightarrow \cancel{\tfrac{1}{2}}\cancel{m}g\cancel{R}\,\theta_m^2 = \cancel{\tfrac{1}{2}}\cancel{M}R^{\cancel{2}}\omega^2\theta_m^2$

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$$\Rightarrow \boxed{\sqrt{\frac{g}{R}} = \omega}$$