Today 19.4

$\boxed{L23}$

Today 19.4

Forced vibrations

$\boxed{L23}$

Today 19.4

Tuesday 19.4, 19.5

$\boxed{L23}$

Today 19.4

Tuesday 19.4, $\underbrace{19.5}$

Damped vibrations

L23

Today 19.4

$\boxed{L23}$

Tuesday 19.4, 19.5

Thursday April 15th Review

Today 19.4

Tuesday 19.4, 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

$\boxed{L23}$

Today 19.4

$\boxed{L23}$

Tuesday 19.4, 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

Thursday April 22nd Day of reconing

ASU

$\boxed{L23}$

Today 19.4

Tuesday 19.4, 19.5

Thursday April 15th Review

Tuesday April 20th Exam 4

Thursday April 22nd Day of reconing

Will know grade for course if you decide not to take final exam

Unforced vibrations

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Unforced vibrations

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* 1st way: Use kinematics to get equation of motion into the form $\ddot{x} = -\omega_n^2 x$
* 2nd way:

Unforced vibrations

So far we have been concerned about unforced vibrations, where we displace a system from equilibrium and determine the natural frequency or natural period. Typically we proceed in one of two ways:

* 1st way: Use kinematics to get equation of motion into the form $\ddot{x} = -\omega_n^2 x$
* 2nd way: Use conservation of energy to get equation of motion into the form

$$\dot{x}^2 + \omega^2 x^2 = \text{const.}$$

Solution : $x = x_m \sin(\omega_n t + \phi)$

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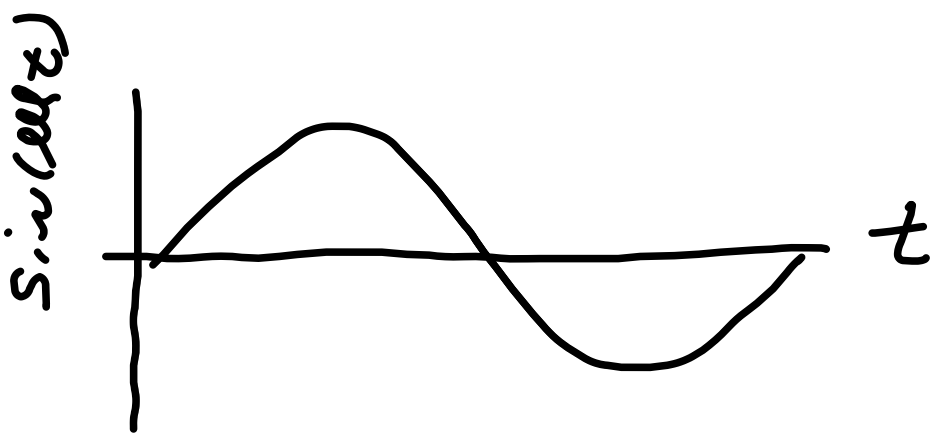
$$\dot{x}_m = \omega_n x_m \quad \& \quad \ddot{x}_m = \omega_n^2 x_m$$



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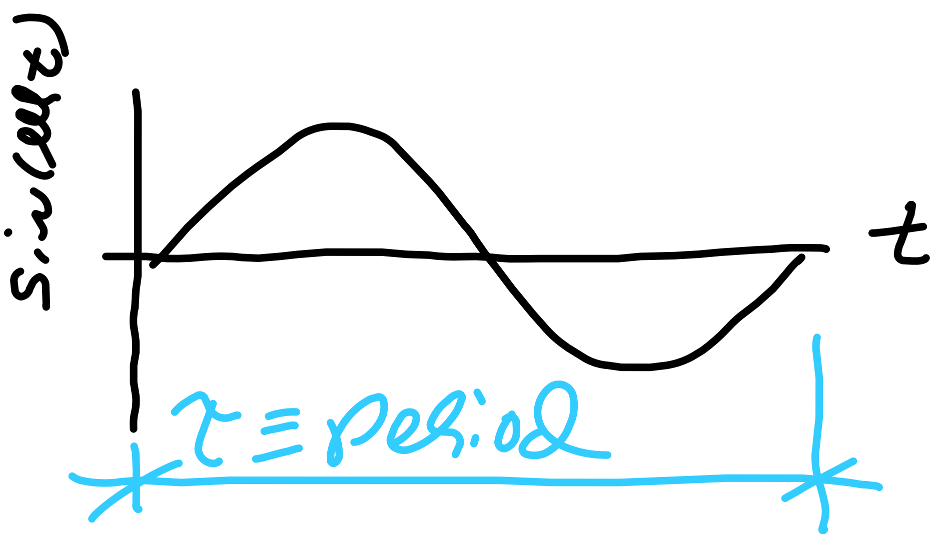

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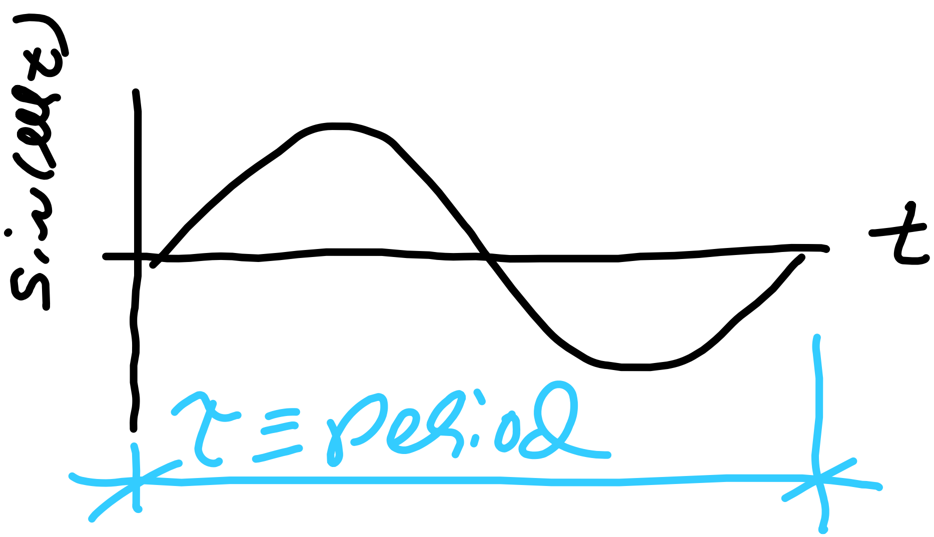

$\omega \tau = 2\pi$

Solution: $x = x_m \sin(\omega_n t + \phi) \Rightarrow$

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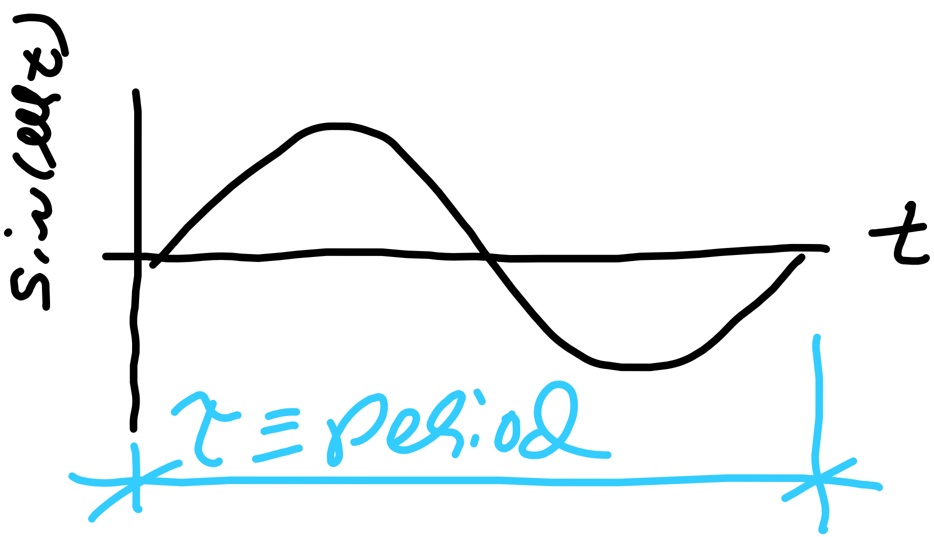

$\omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

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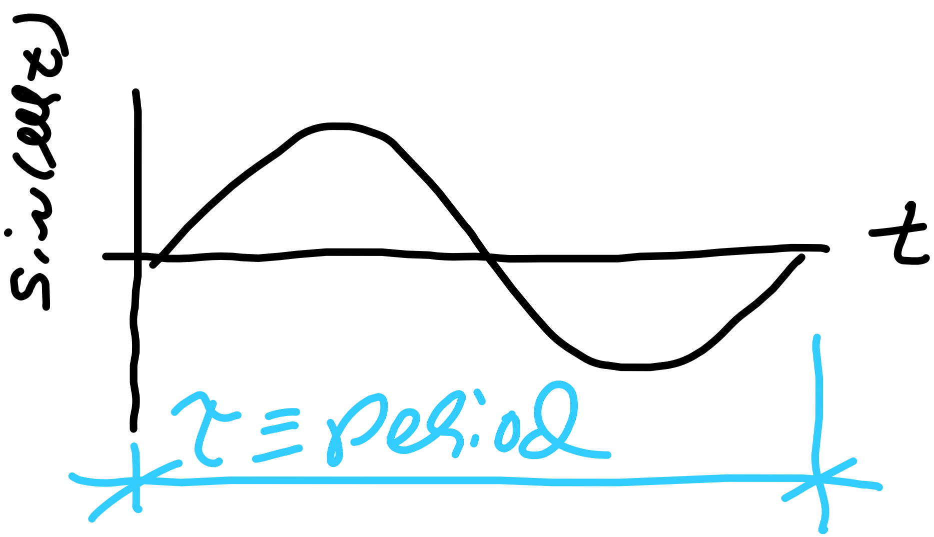

$\omega \tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

& $f = \frac{1}{\tau}$.

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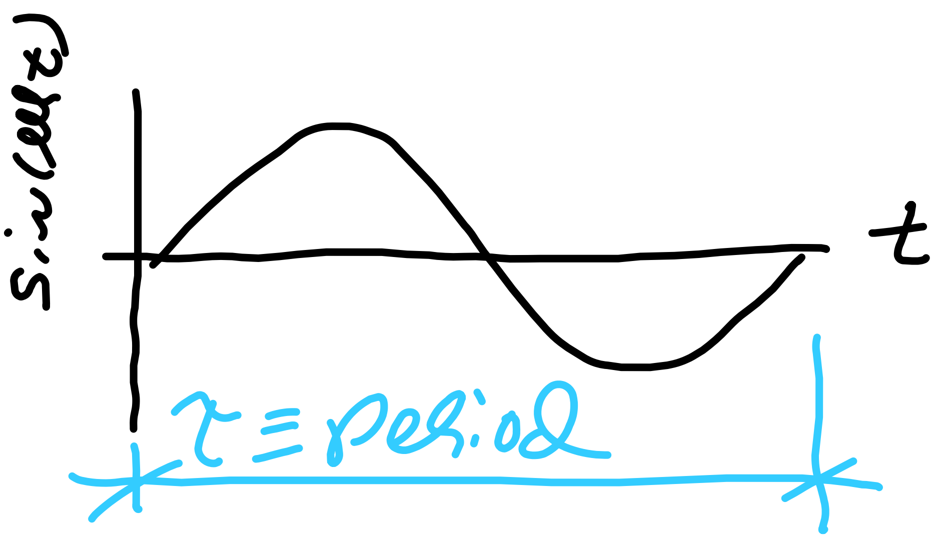

$\omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

& $f = \frac{1}{\tau}$, where

$f \equiv$ frequency

Solution: $x = X_m \sin(\omega_n t + \phi) \Rightarrow$

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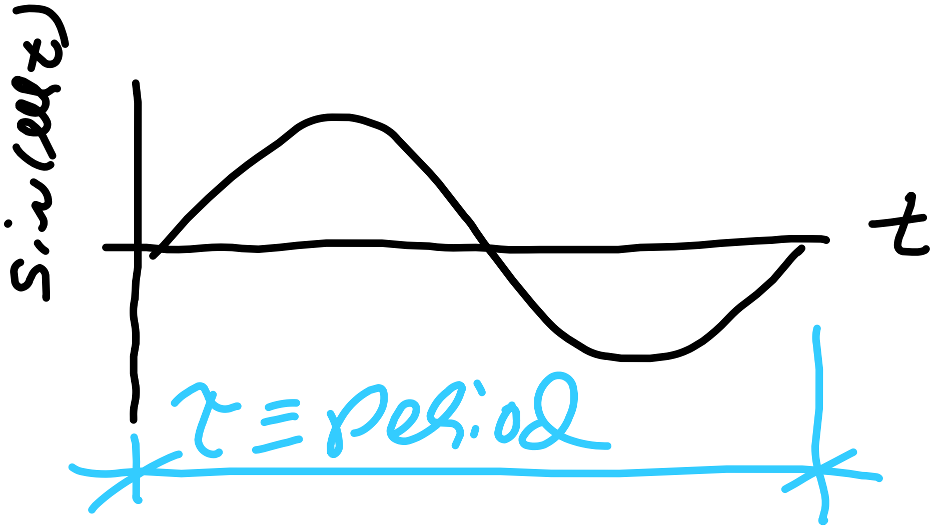

$\omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

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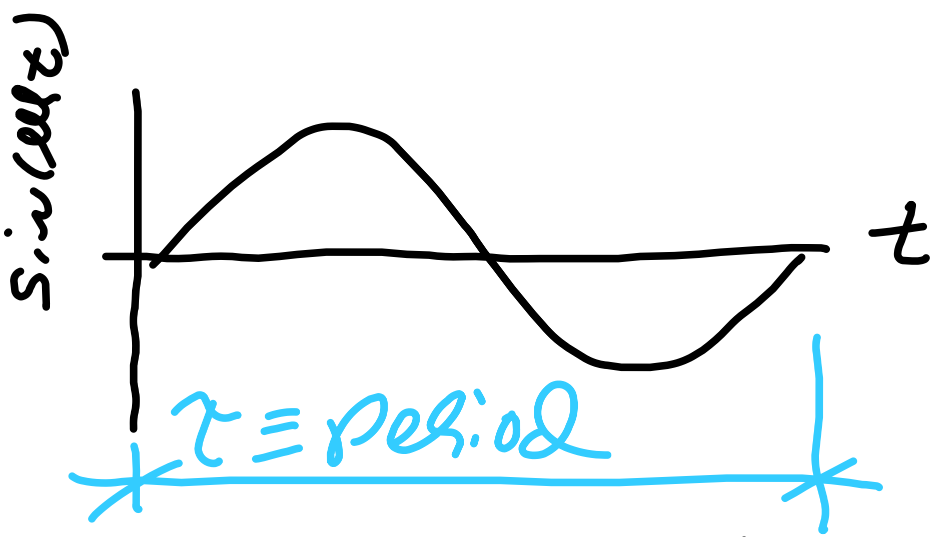
Q: What happens to X_m when the forced frequency matches the natural frequency?

<u>Solution</u>: $x = x_m \sin(\omega_n t + \phi) \Rightarrow$

$\dot{x}_m = \omega_n x_m$ & $\ddot{x}_m = \omega_n^2 x_m$



$\omega\tau = 2\pi \Rightarrow \tau = \frac{2\pi}{\omega}$

& $f = \frac{1}{\tau}$, where

$f \equiv$ frequency

What would happen if we were to force the system at the same frequency as the natural frequency

Forced vibrations – Math

Forced vibrations – Math

If we were to analyze a physical system and obtain an equation of the form

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$$A\ddot{x} + Bx = C\sin(\omega_F t)$$

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If we were to analyze a physical system and obtain an equation of the form

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Forced vibrations – Math

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$x_{part} = X_m \sin(\omega_F t)$. Now $-A\omega_F^2 X_m + BX_m = C$

Forced vibrations – Math

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$$\Rightarrow X_m = \frac{C}{-A\omega_F^2 + B}$$

Forced vibrations — Math

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ASU

Forced vibrations — Math

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$\Rightarrow \quad x_m = \dfrac{C}{-A\omega_F^2 + B} = \dfrac{(C/B)}{-A\omega_F^2/B + 1}$ But $\omega_N^2 = \frac{B}{A}$

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$\Rightarrow x_m = \dfrac{C}{-A\omega_F^2 + B} = \dfrac{(C/B)}{-A\omega_F^2/B + 1}$ But $\omega_N^2 = \dfrac{B}{A}$

So $\boxed{x_m = \dfrac{(C/B)}{1 - \omega_F^2/\omega_N^2}}$

From previous

$$X_m = \frac{(C/B)}{1-\omega_F^2/\omega_N^2}$$

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$$X_m = \frac{(C/B)}{1-\omega_f^2/\omega_n^2}$$

What happens to X_m when the forced frequency matches the natural frequency?

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What happens to x_m when the forced frequency matches the natural frequency?

See

http://www.public.asu.edu/~dugger/wine-glass-shatter.gif

https://www.youtube.com/watch?v=0FeXjhUEXlc

https://www.youtube.com/watch?v=6IJ99phNArM&feature=youtu.be&t=137

ASU

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When $X_m \to \infty$ the system is in resonance

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When $x_m \to \infty$ the system is in resonance.
Obviously x_m can <u>not</u> go infinite.

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When $x_m \to \infty$ the system is in resonance.

Obviously x_m can not go infinite. Instead, a system in resonance will eventually reach a new state that has a different natural frequency

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When $X_m \to \infty$ the system is in resonance. Obviously X_m can not go infinite. Instead, a system in resonance will eventually reach a new state that has a different natural frequency or simply destruct

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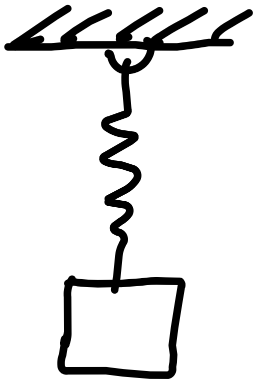
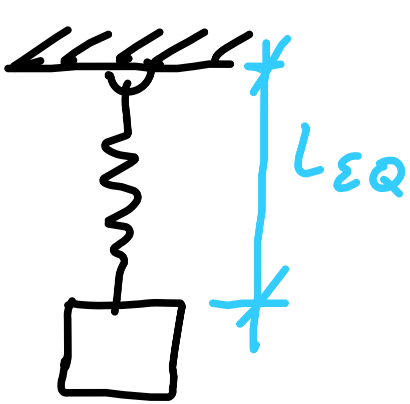
When $x_m \to \infty$ the system is in resonance. Obviously x_m can not go infinite. Instead, a system in resonance will eventually reach a new state that has a different natural frequency or simply destruct [as seen in videos]

Example



Example



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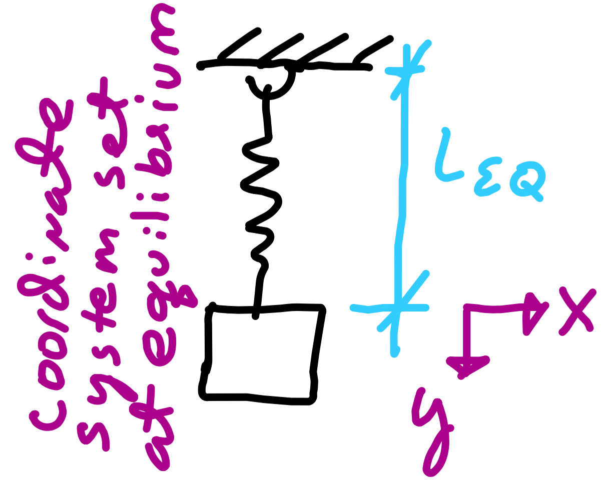

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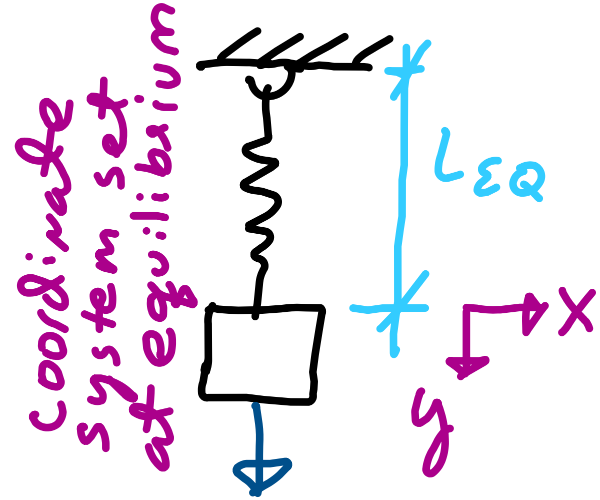
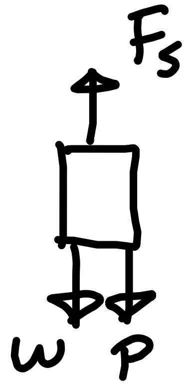
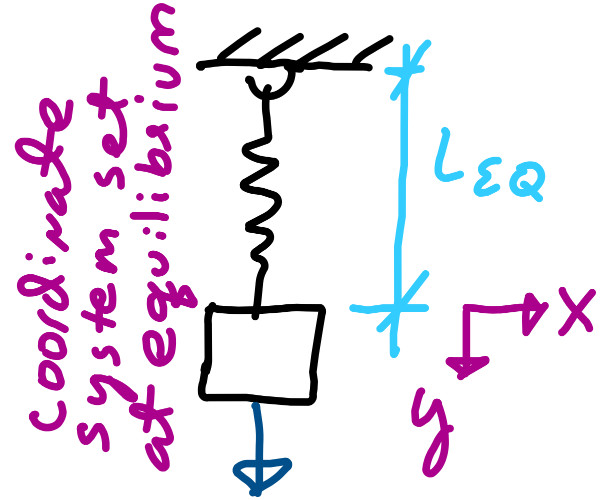


$P = P_m \sin(\omega_F t)$

Example





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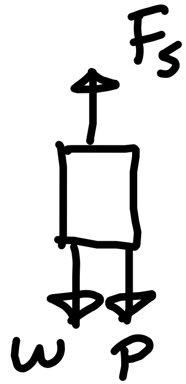
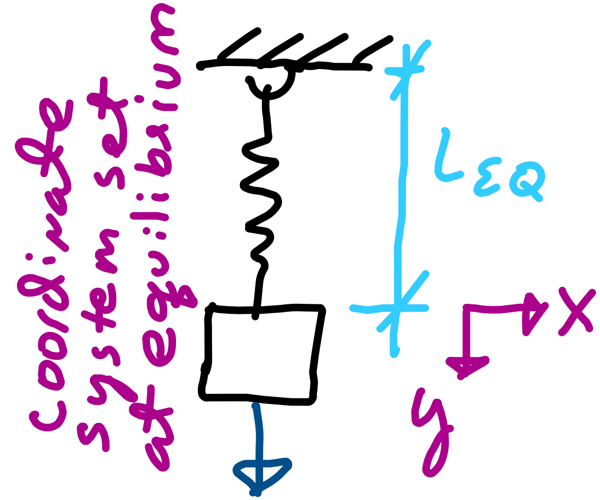


$$\Sigma F_y = m\ddot{y}$$



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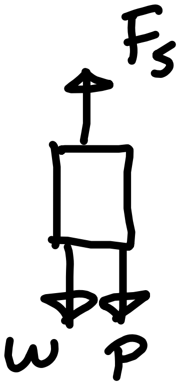
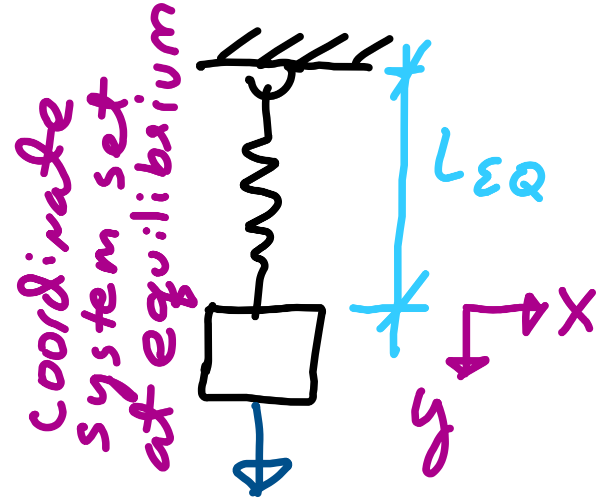


$$\Sigma F_y = m\ddot{y} \Rightarrow$$

$$-K(L-L_0)+W+P=m\ddot{y}$$



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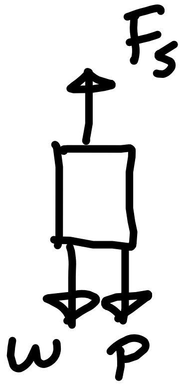

$$\Sigma F_y = m\ddot{y} \Rightarrow$$

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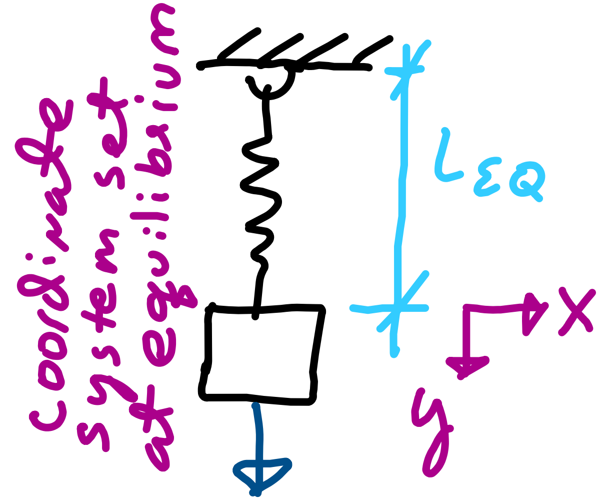
At equilibrium

$$-K(L_{EQ} - L_0) + W = 0$$



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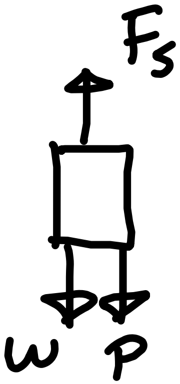

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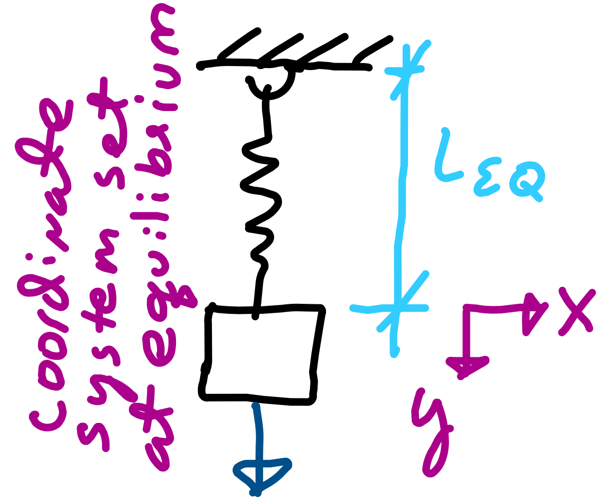
At equilibrium

$-K(L_{EQ}-L_0)+W=0$, where

$L_{EQ}=L_0+\delta$



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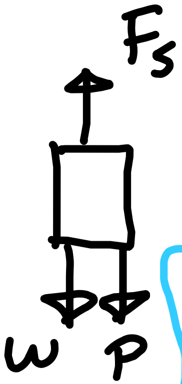

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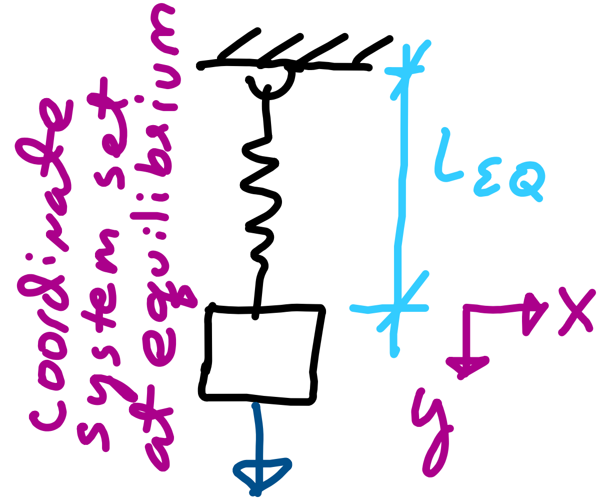

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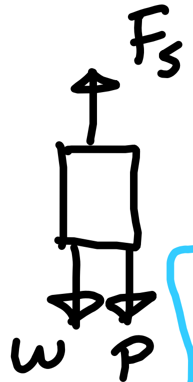

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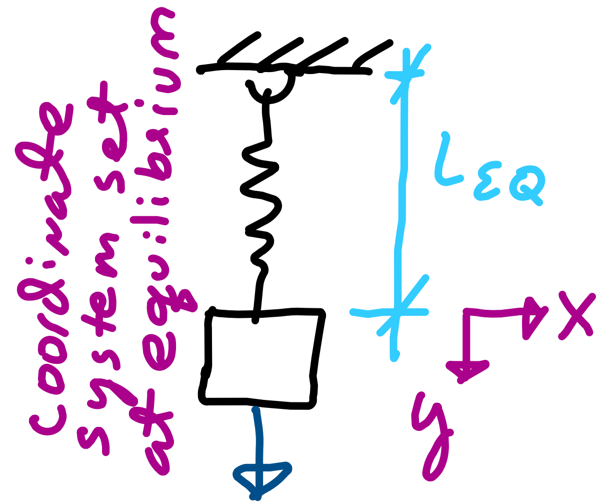
At equilibrium

$-K(L_{EQ}-L_0)+W=0$, where

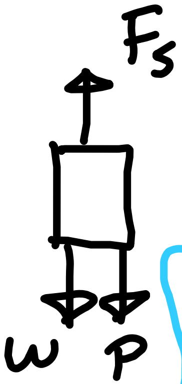
$L_{EQ}=L_0+\delta$ so $-K\delta+W=0$



$P = P_m \sin(\omega_f t)$

Now $L = L_{EQ}+y =$

Example



$$\Sigma F_y = m\ddot{y} \Rightarrow$$

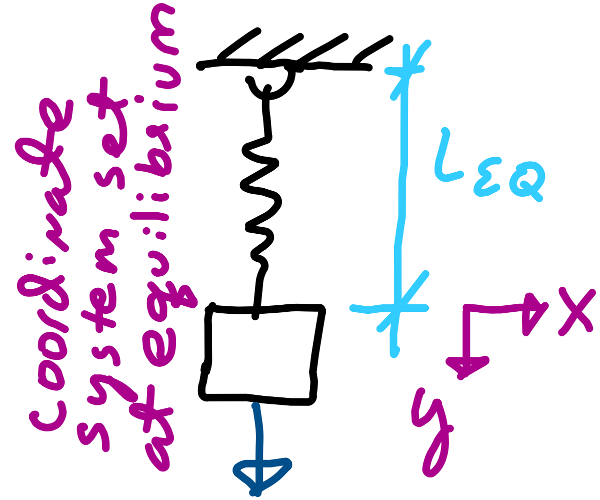
$$-K(L-L_0)+W+P=m\ddot{y}$$

At equilibrium

$$-K(L_{EQ}-L_0)+W=0,\ \text{where}$$

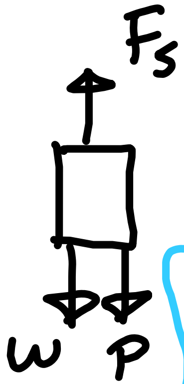
$$L_{EQ}=L_0+\delta \ \text{ so } \ -K\delta+W=0$$

Now $L = L_{EQ}+y = L_0+\delta+y$



$$P = P_m \sin(\omega_F t)$$

Example



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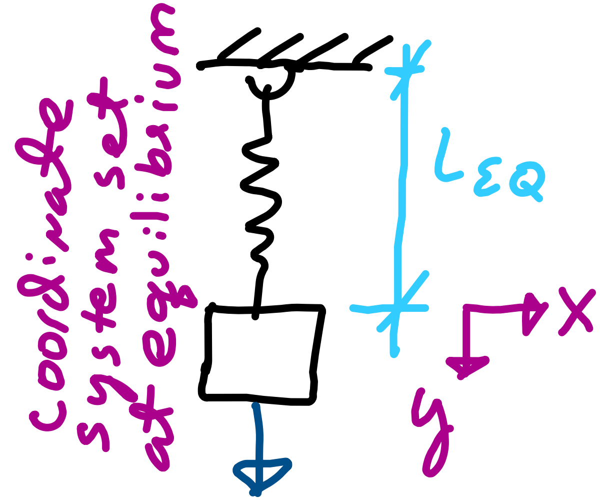

$P = P_m \sin(\omega_f t)$

At equilibrium

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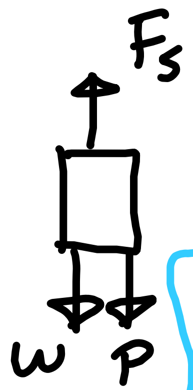
Now $L=L_{EQ}+y = L_0+\delta+y$ so $-K(L-L_0)+W+P=m\ddot{y}$

becomes

Example



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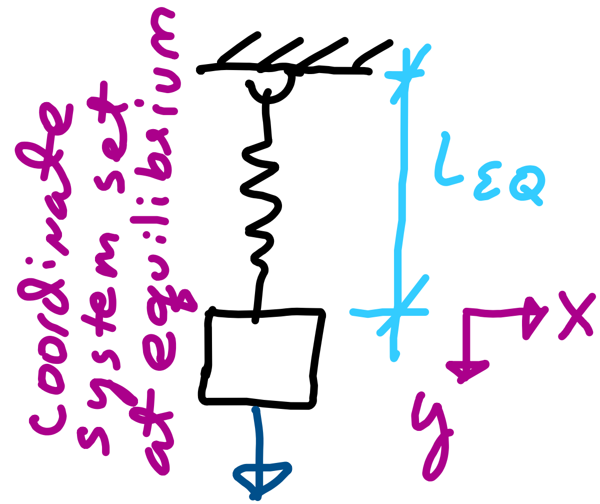

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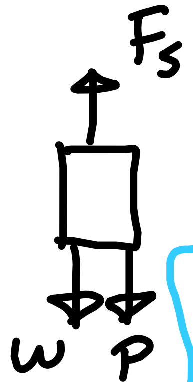
Example



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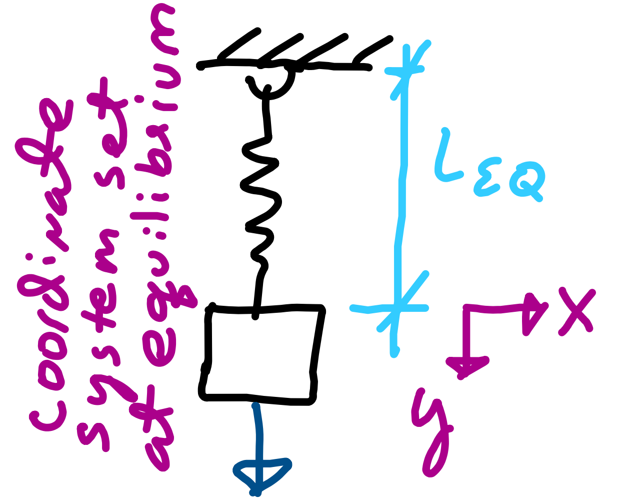

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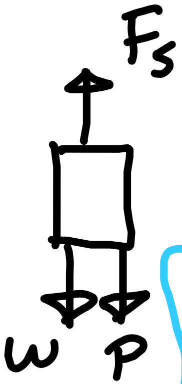
Example



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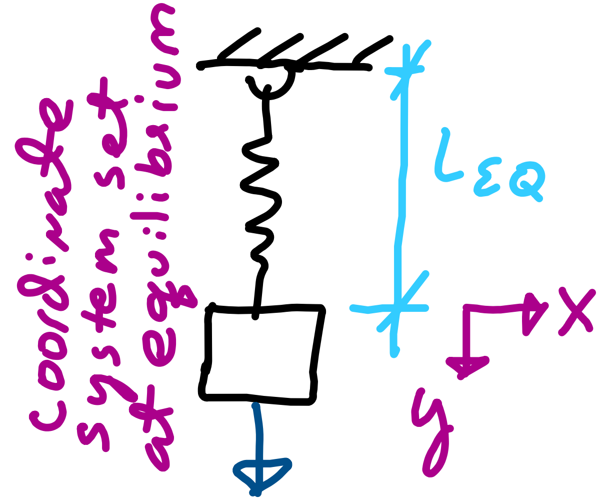

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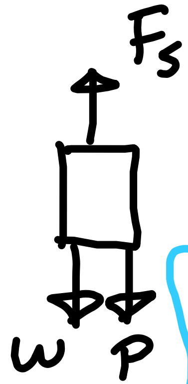
Example



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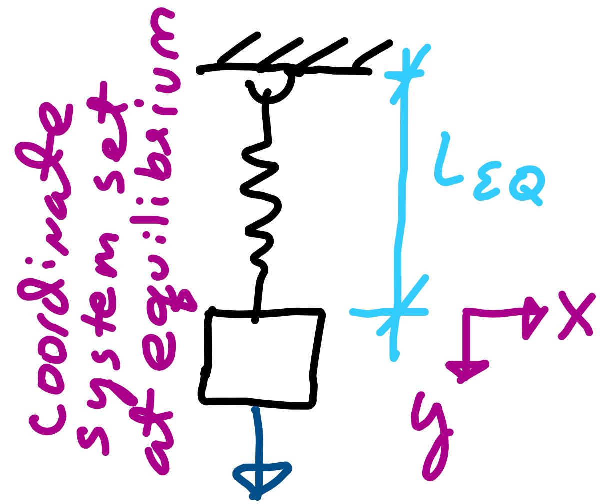

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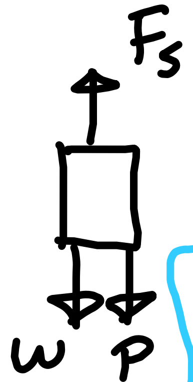
becomes $-k(L_{EQ}+y)+w+P=m\ddot{y} \Rightarrow -ky+P=m\ddot{y}$

$$\Rightarrow m\ddot{y}+ky=P$$

Example



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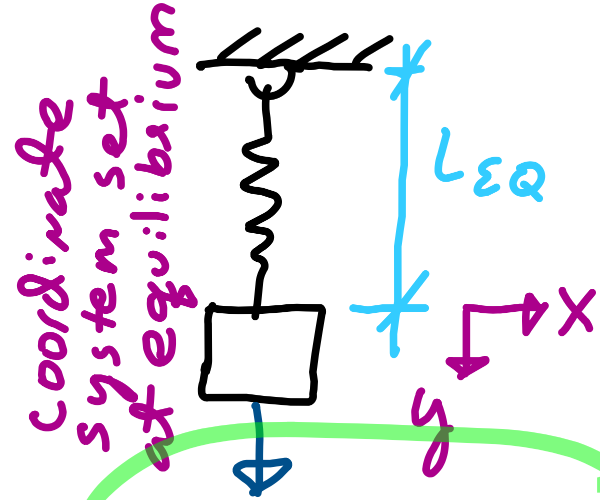

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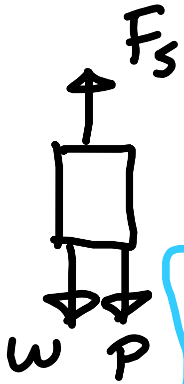

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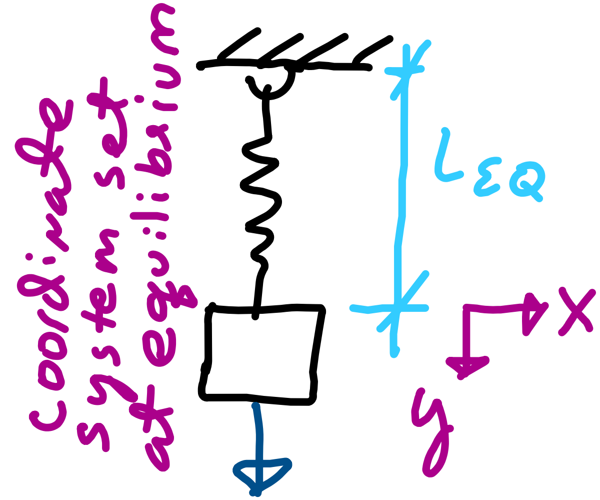

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From previous slide

$$m\ddot{y} + ky = P_m \sin(\omega_F t)$$

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Homogeneous equation:

From previous slide

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Homogeneous equation: $m\ddot{y} + ky = 0 \Rightarrow$

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From previous slide

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Homogeneous equation: $m\ddot{y} + ky = 0 \Rightarrow$

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From previous slide

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Particular solution:

From previous slide

$$m\ddot{y} + ky = P_m \sin(\omega_F t)$$

Homogeneous equation: $m\ddot{y} + ky = 0 \Rightarrow$

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Particular solution: Assume

$$y_{part} = y_m (\sin \omega_F t)$$

From previous slide

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$$y_{part} = y_m(\sin\omega_F t) \Rightarrow -m\omega_F^2 y_m + k y_m = P_m$$

From previous slide

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Particular solution: Assume

$y_{part} = y_m(\sin \omega_F t) \Rightarrow -m\omega_F^2 y_m + k y_m = P_m$

$$\Rightarrow \quad y_m = \frac{P_m}{k - m\omega_F^2}$$

From previous slide

$$m\ddot{y} + ky = P_m \sin(\omega_F t)$$

Homogeneous equation: $m\ddot{y} + ky = 0 \Rightarrow$

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Particular solution: Assume

$y_{part} = y_m (\sin \omega_F t) \Rightarrow -m\omega_F^2 y_m + k y_m = P_m$

$$\Rightarrow \quad y_m = \frac{P_m}{k - m\omega_F^2} = \frac{(P_m/k)}{1 - m\omega_F^2/k}$$

From previous slide

$m\ddot{y} + ky = P_m \sin(\omega_F t)$

Homogeneous equation: $m\ddot{y} + ky = 0 \Rightarrow$

$\ddot{y} = -\omega_n^2 y$, where $\omega_n = \sqrt{\frac{k}{m}}$

Particular solution: Assume

$y_{part} = y_m (\sin \omega_F t) \Rightarrow -m\omega_F^2 y_m + k y_m = P_m$

$\Rightarrow \quad y_m = \dfrac{P_m}{k - m\omega_F^2} = \dfrac{(P_m/k)}{1 - m\omega_F^2/k}$ But

$\frac{k}{m} = \omega_n$

From previous slide

$m\ddot{y} + ky = P_m \sin(\omega_F t)$

Homogeneous equation: $m\ddot{y} + ky = 0 \Rightarrow$

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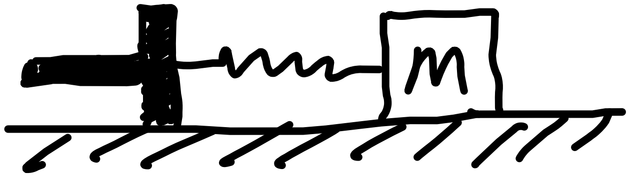
Particular solution: Assume

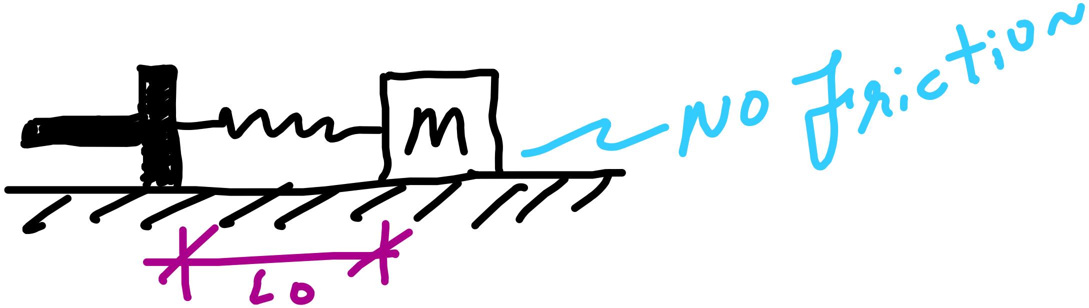
$y_{part} = y_m(\sin \omega_F t) \Rightarrow -m\omega_F^2 y_m + k y_m = P_m$

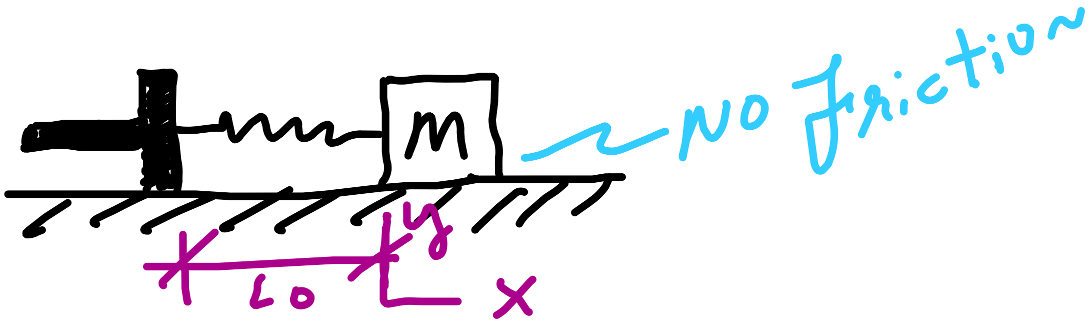
$$\Rightarrow \quad y_m = \frac{P_m}{k - m\omega_F^2} = \frac{(P_m/k)}{1 - m\omega_F^2/k} \quad \text{But}$$

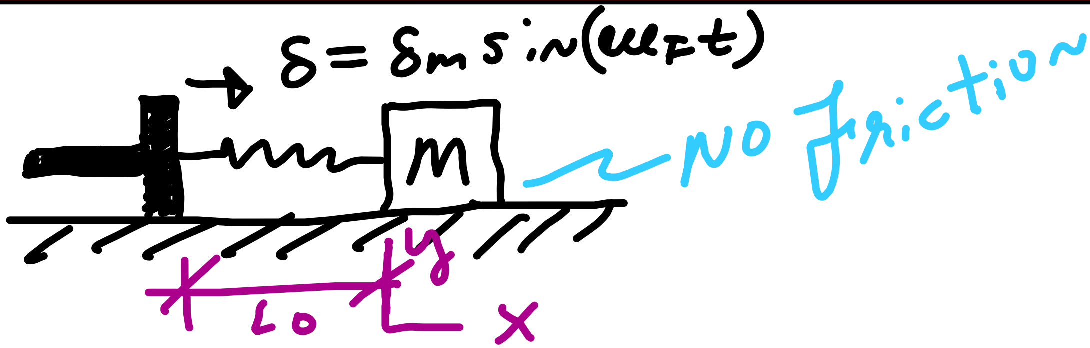
$\frac{k}{m} = \omega_n$ so

$$\boxed{y_m = \frac{(P_m/k)}{1 - \omega_F^2/\omega_n^2}}$$

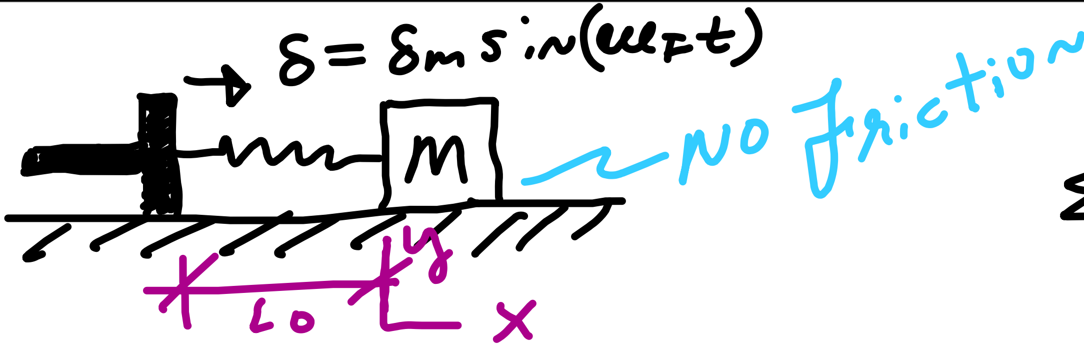

m


m
L_0
No friction


m
No friction
Lo
y
x

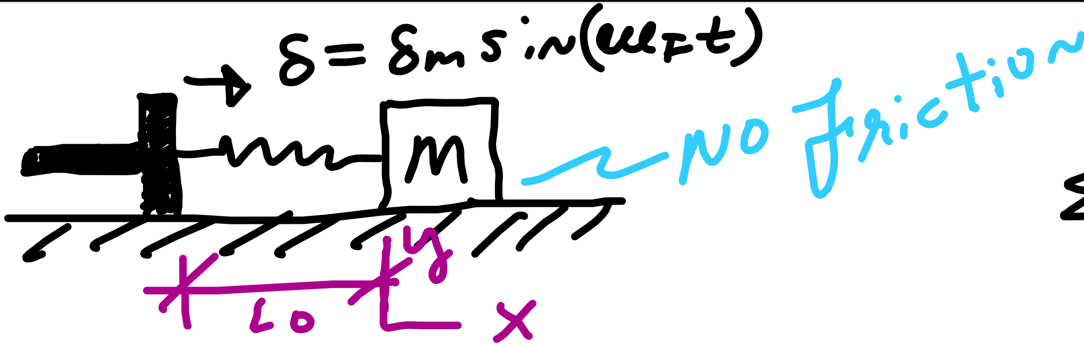


δ = δm sin(ωF t)
m
No friction
y
L0
x



At equilibrium

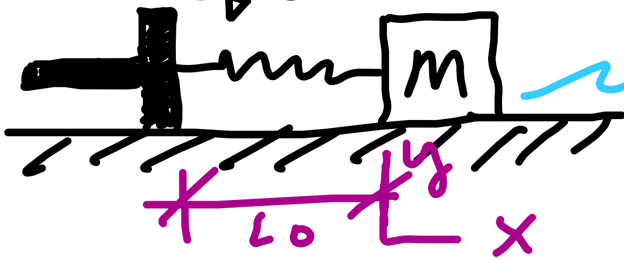
$\Sigma \vec{F}_x = 0$



At equilibrium

$$\sum \vec{F}_x = 0 \Rightarrow 0 = 0$$

$\delta = \delta_m \sin(\omega_F t)$



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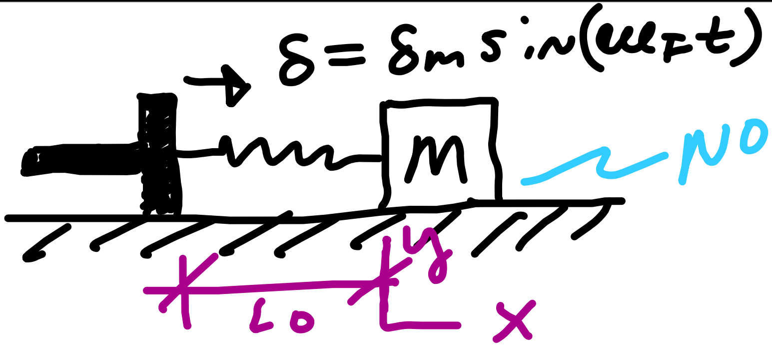

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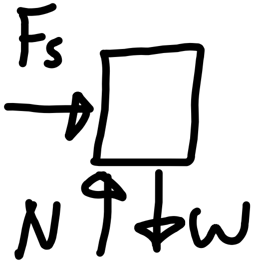
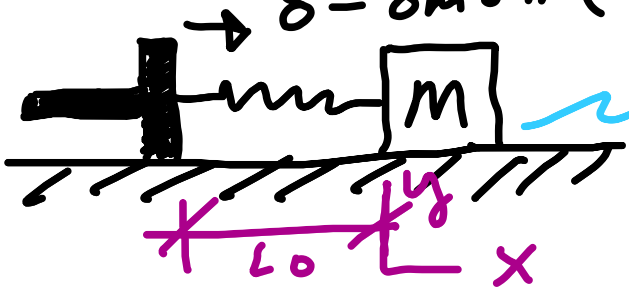


$\delta = \delta_m \sin(\omega_f t)$



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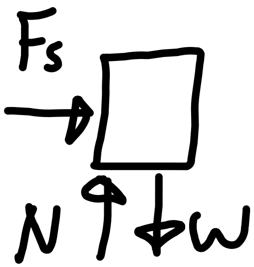

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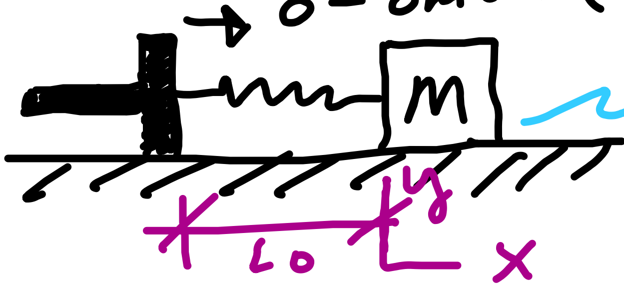

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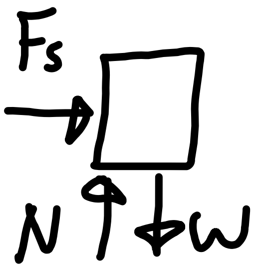

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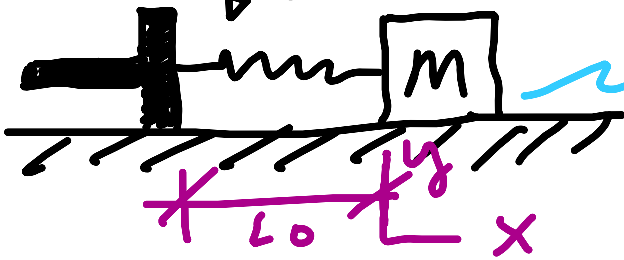

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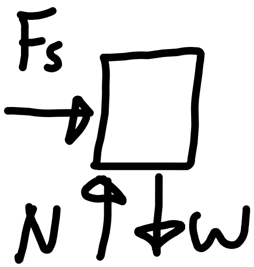
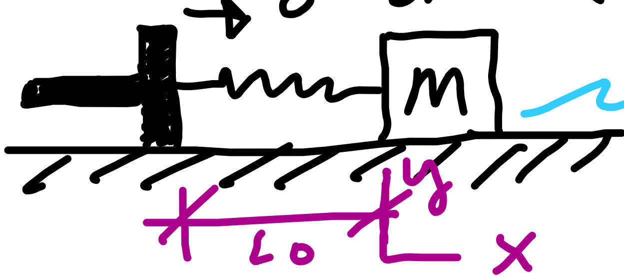


$$\Sigma F_x = m\ddot{x} \Rightarrow F_s = m\ddot{x} \; \& \; F_s = -k(l - l_0)$$

$\delta = \delta_m \sin(\omega_F t)$



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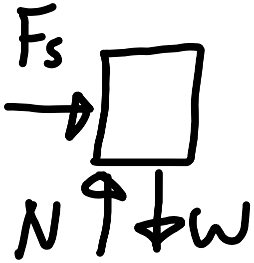
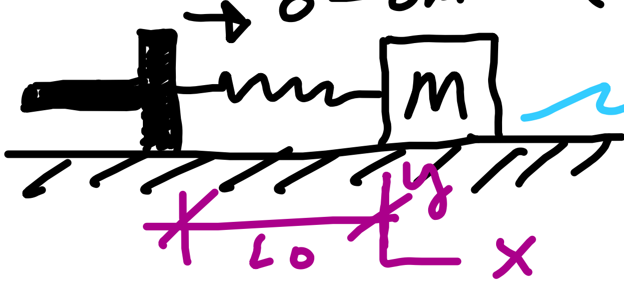


$\sum F_x = m\ddot{x} \Rightarrow F_s = m\ddot{x}$ & $F_s = -k(L - L_0)$

& $L = L_0 - \delta + x$

$\delta = \delta_m \sin(\omega_F t)$



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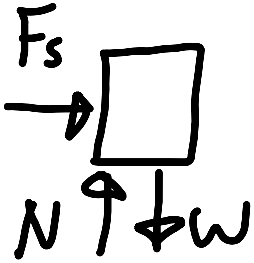

$\sum F_x = m\ddot{x} \Rightarrow F_s = m\ddot{x}$ & $F_s = -k(l - l_0)$

& $l = l_0 - \delta + x$ so $l - l_0 = -\delta + x$

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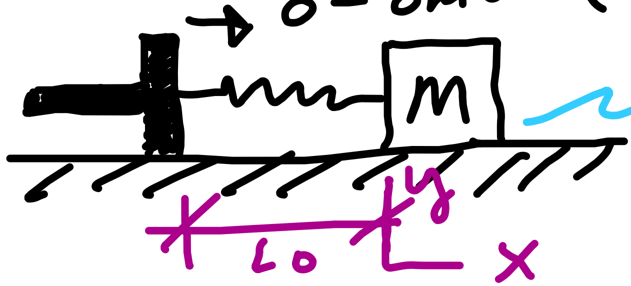

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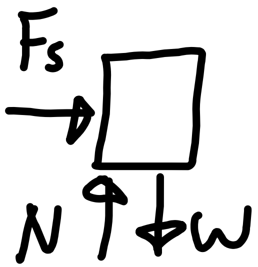

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Now $-k(x - \delta) = m\ddot{x}$

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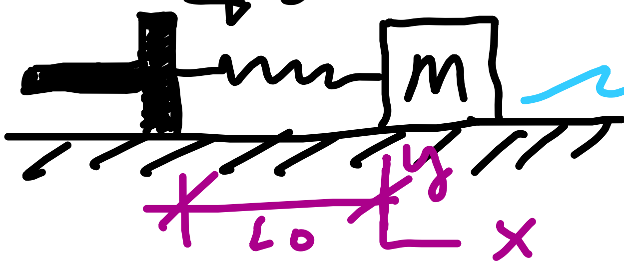

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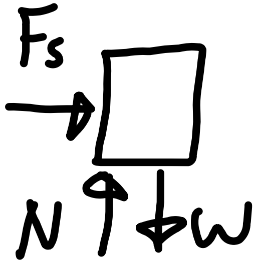


$$\sum F_x = m\ddot{x} \Rightarrow \quad F_s = m\ddot{x} \;\;\&\;\; F_s = -k(L - L_0)$$

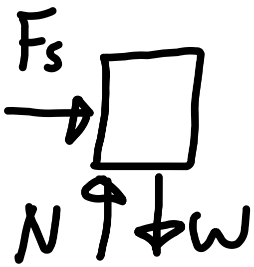
$$\& \;\; L = L_0 - \delta + x \quad \text{so} \quad L - L_0 = -\delta + x$$

Now $-k(x-\delta) = m\ddot{x} \Rightarrow \quad m\ddot{x} + kx = k\delta$

$\delta = \delta_m \sin(\omega_f t)$

At equilibrium

$\Sigma F_x = 0 \Rightarrow 0 = 0$



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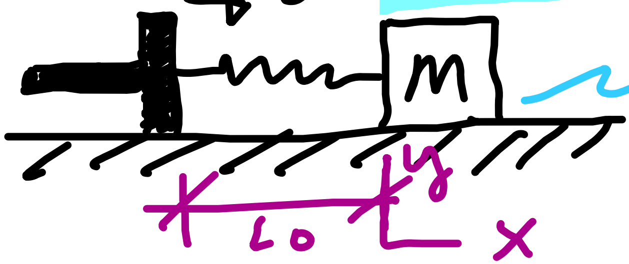
Now $-k(x-\delta) = m\ddot{x} \Rightarrow m\ddot{x} + kx = k\delta_m \sin(\omega_f t)$

$\delta = \delta_m \sin(\omega_F t)$



No friction

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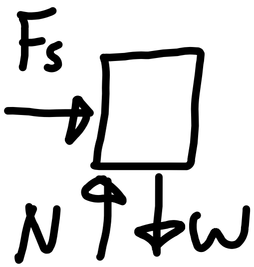

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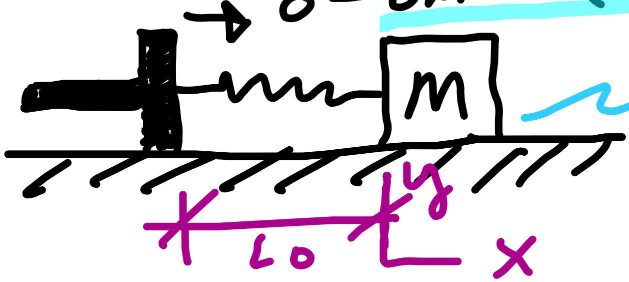

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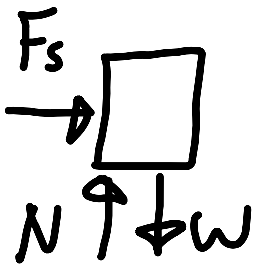

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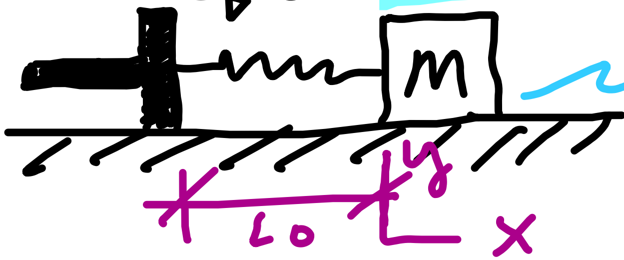

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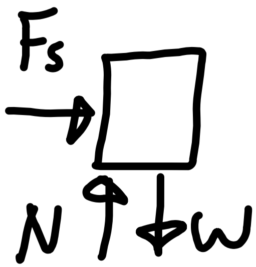

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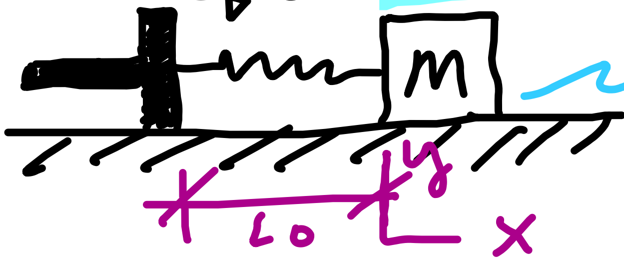

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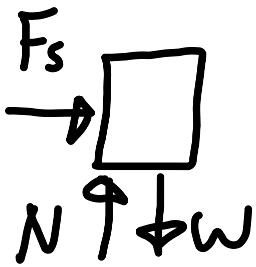

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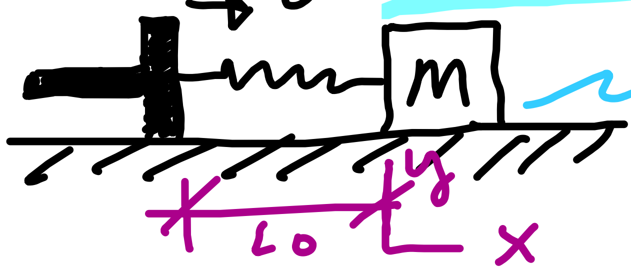

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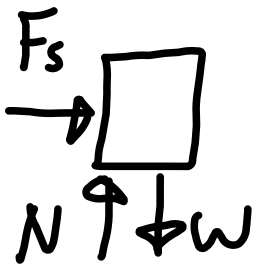

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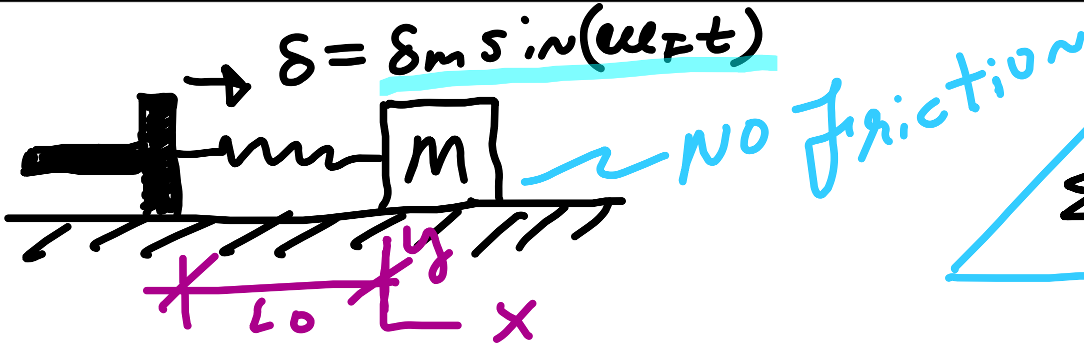

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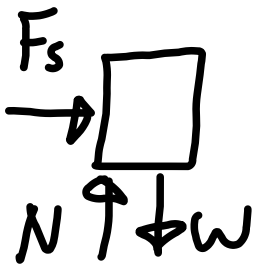

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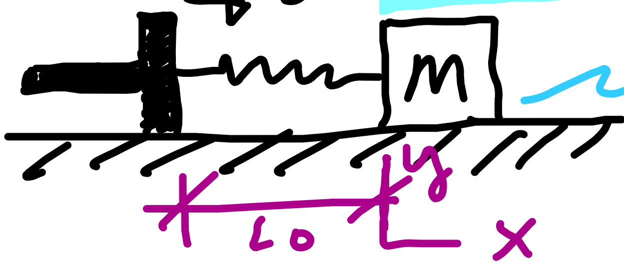

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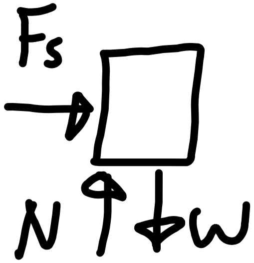

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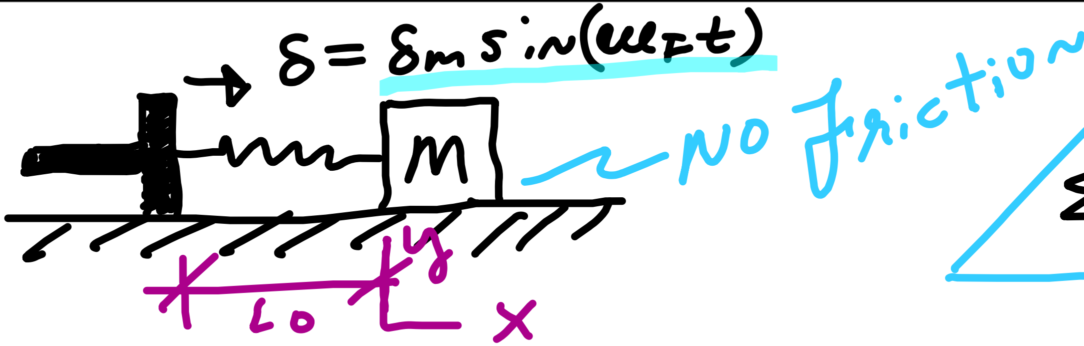

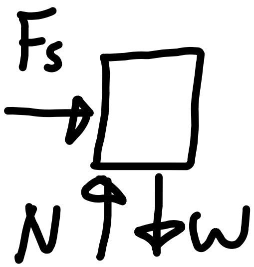

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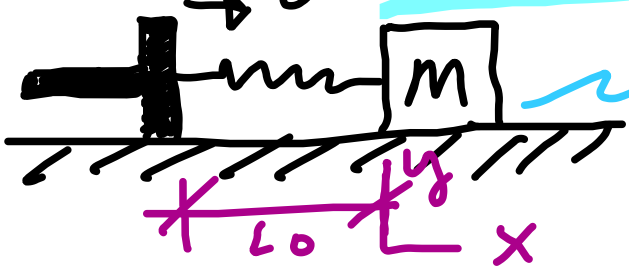

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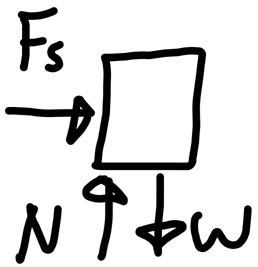

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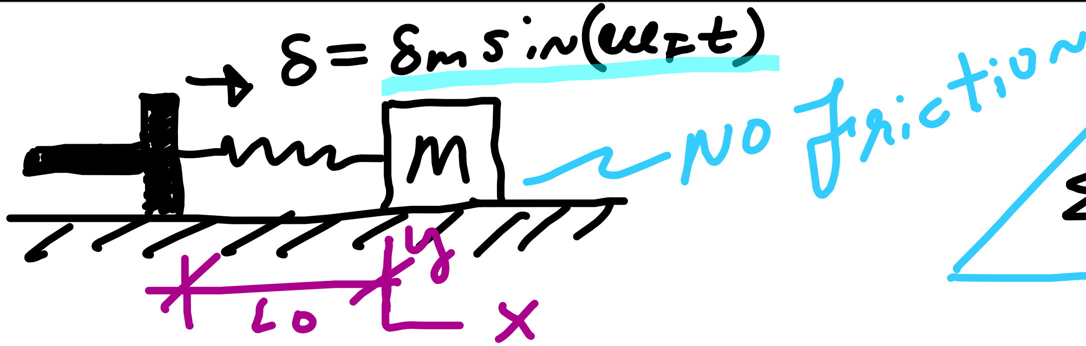

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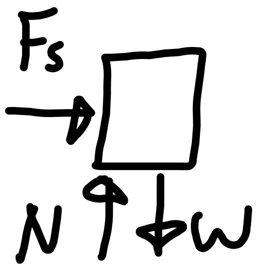

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What would happen to real spring when $\omega_F = \omega_n$?

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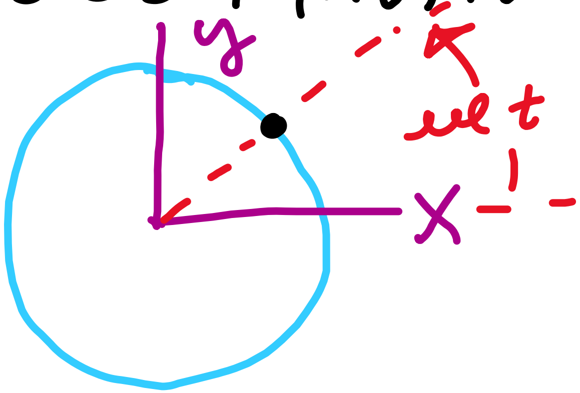

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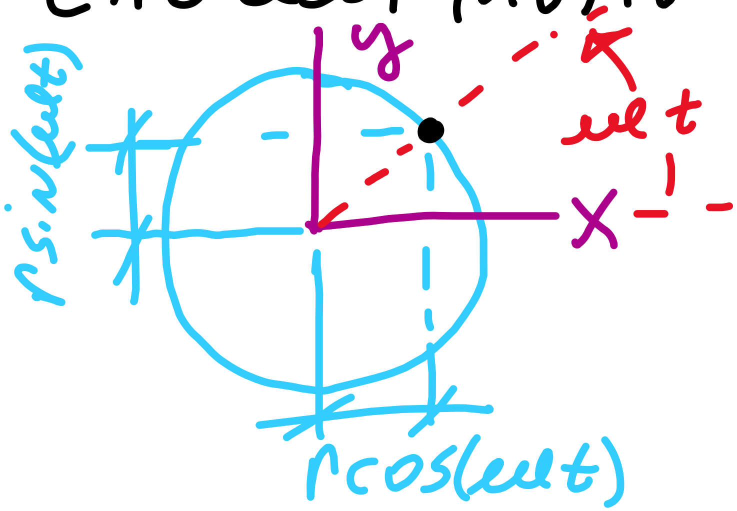

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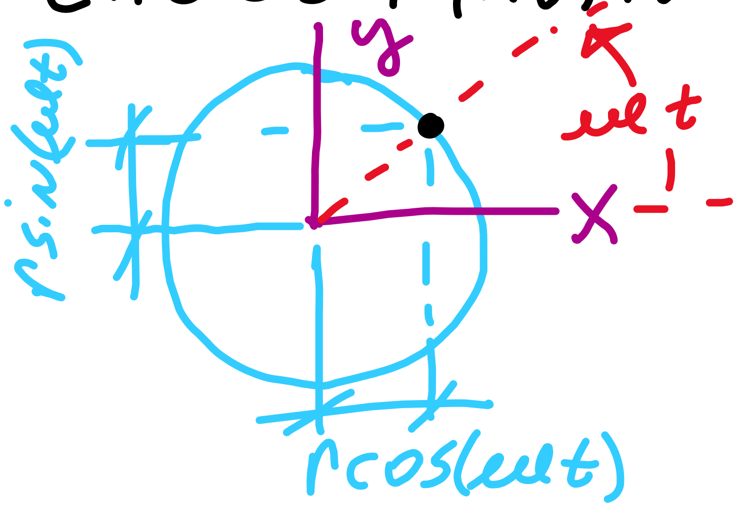

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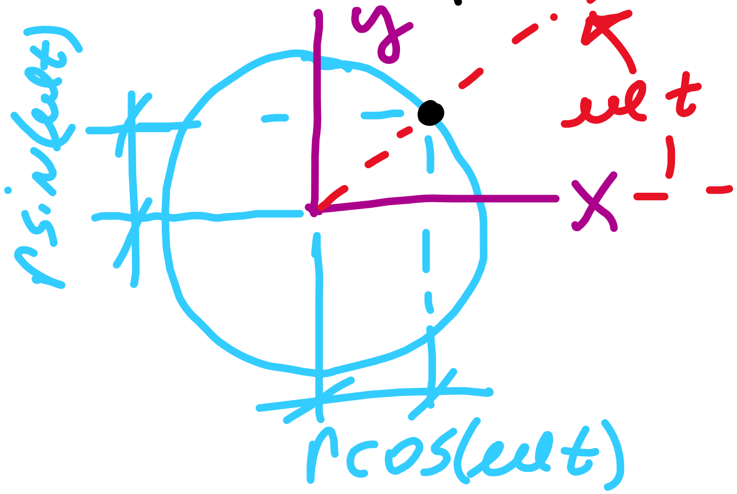

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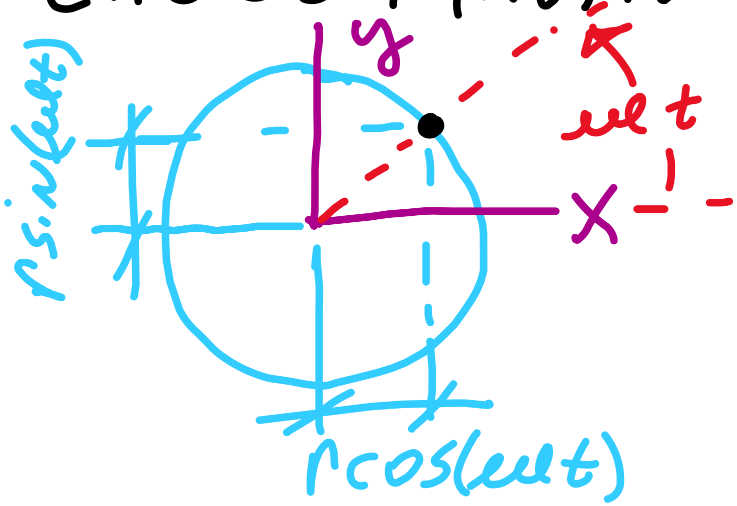

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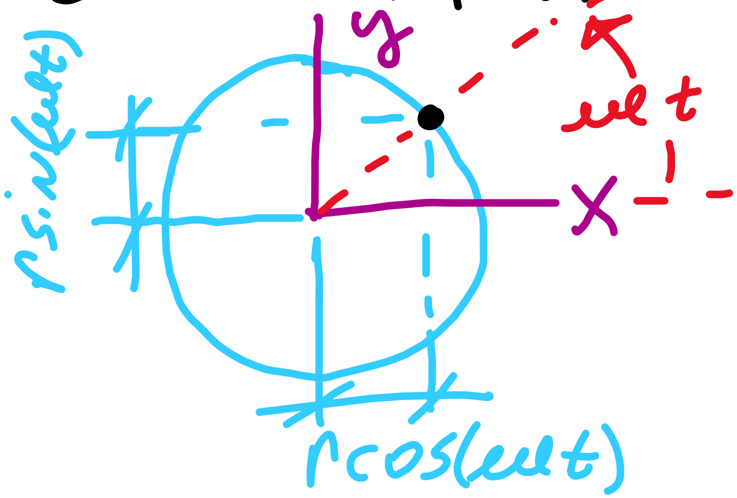

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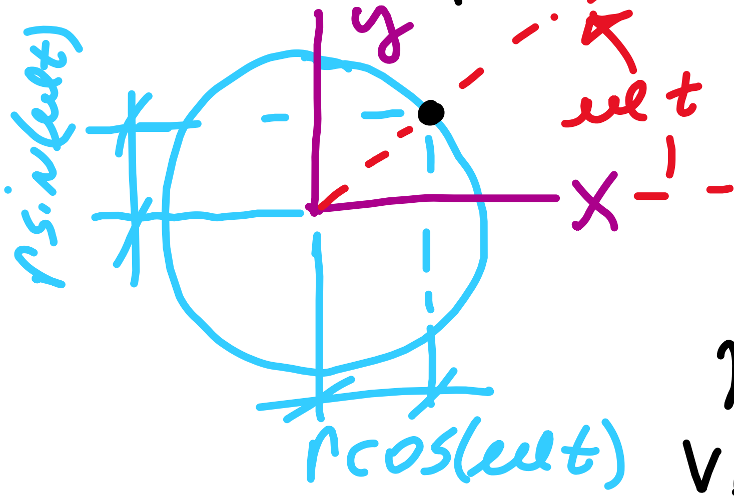

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$x_m = y_m = r$. Circular motion looks like a very particular vibration in x & y.

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When you set $e = 0.006in$, it is probably going to be easiest to work in normal and transverse components instead of cartesian coordinates