Today Review

$\boxed{L25}$

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Today Review

Tuesday Exam 4

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Thursday April 22nd Day of Reckoning

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Will know grade in course if you decide NOT to take the final exam

L25

Today Review

Tuesday Exam 4

Thursday April 22nd Day of Reckoning

Thursday April 29th Final exam from

7:30 AM to 9:20 AM

Free vibrations:

Free vibrations: If $A\ddot{x}+Bx=0$,

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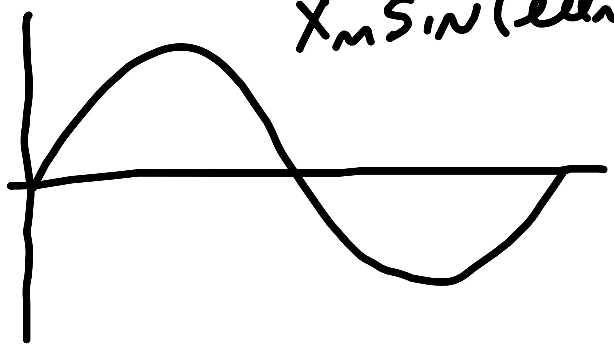
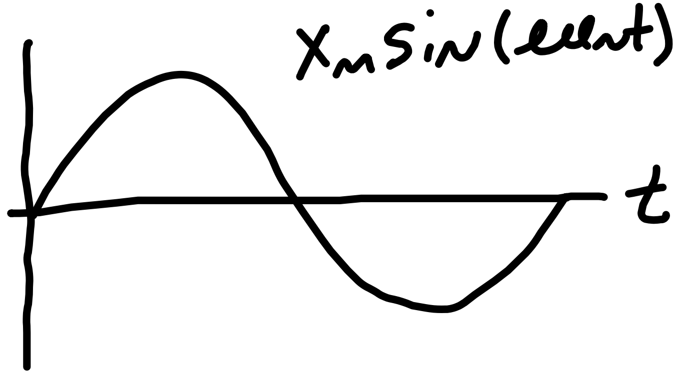


Free vibrations:

If $A\ddot{x}+Bx=0$, where A & $B \in \mathbb{R}$ & $B/A>0$, then $\ddot{x}=\omega_n^2 x$, where $\omega_n=\sqrt{B/A}$ & $x=x_m \sin(\omega_n t+\varphi)$



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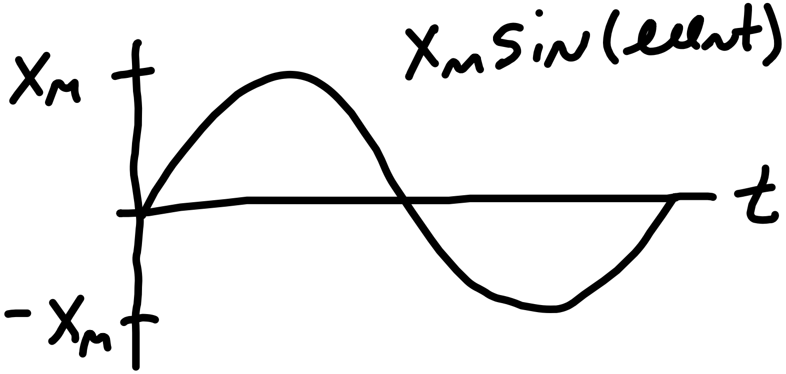
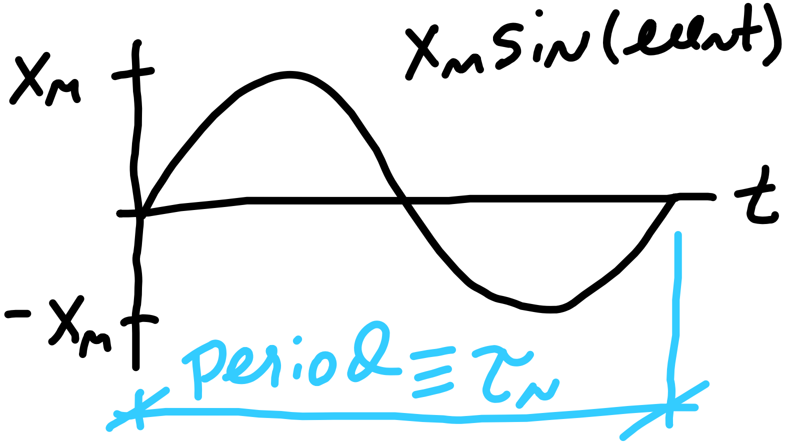


Free vibrations:

If $A\ddot{x}+Bx=0$, where A & $B\in\mathbb{R}$ & $B/A>0$, then $\ddot{x}=-\omega_n^2 x$, where $\omega_n=\sqrt{B/A}$ & $x=x_m\sin(\omega_n t+\varphi)$



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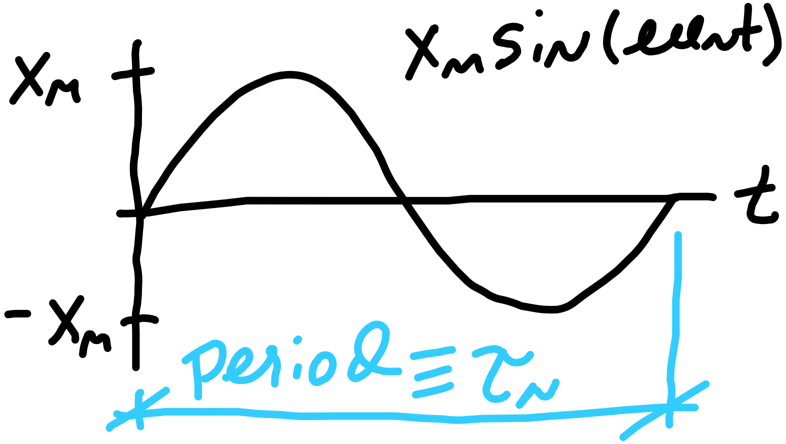

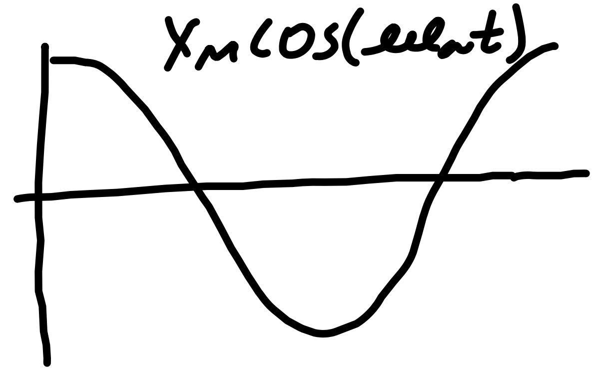

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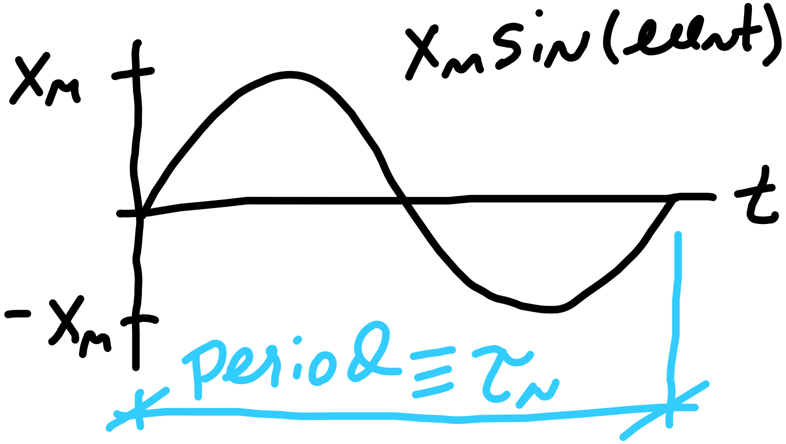

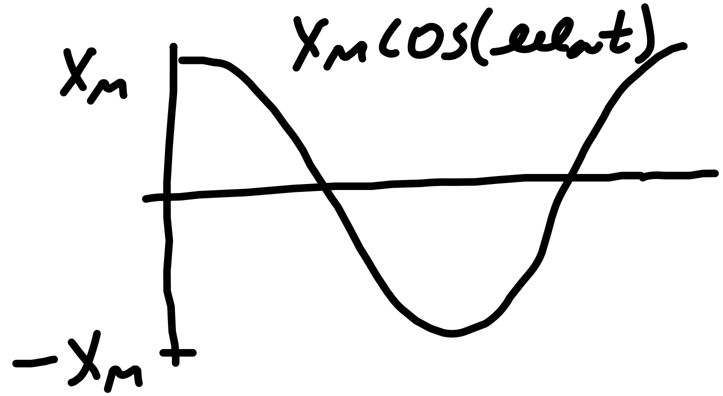

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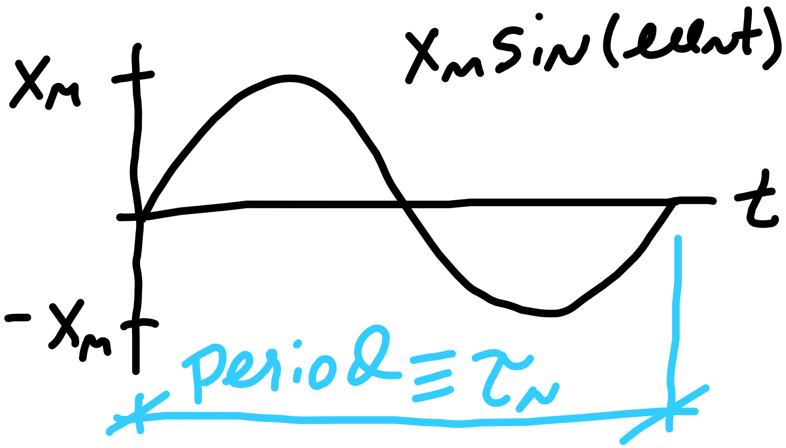

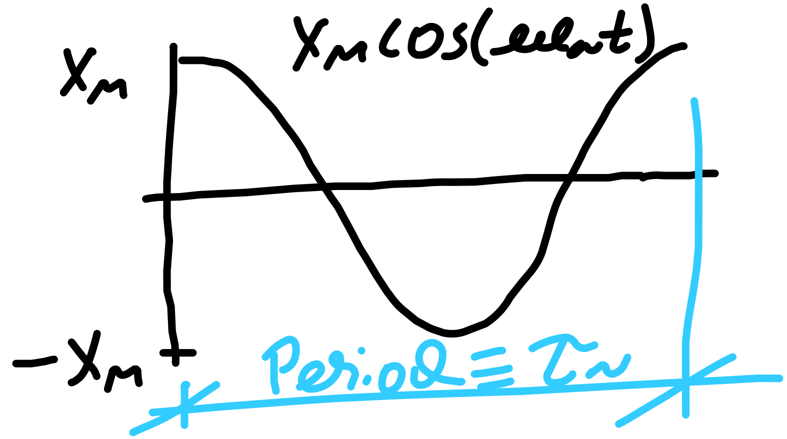

Free vibrations: If $A\ddot{x}+Bx=0$,

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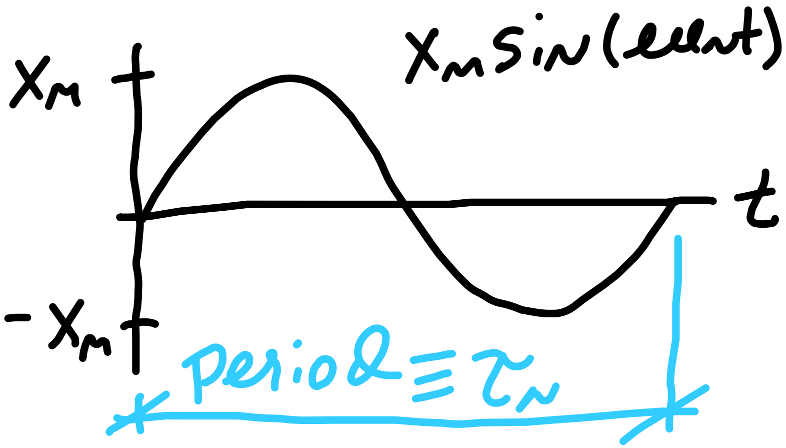

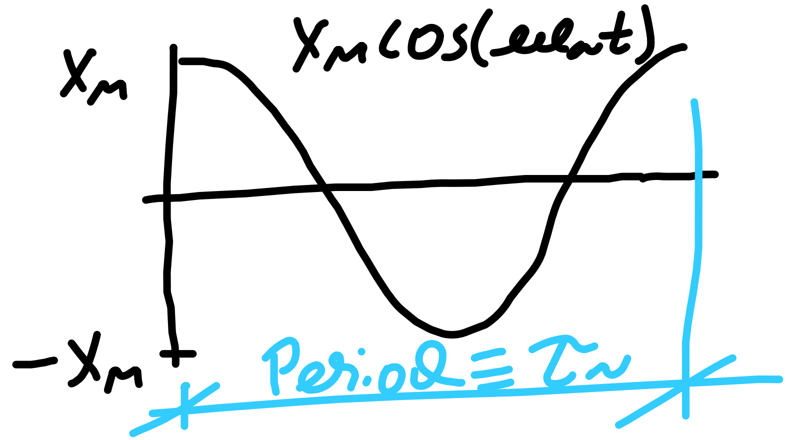

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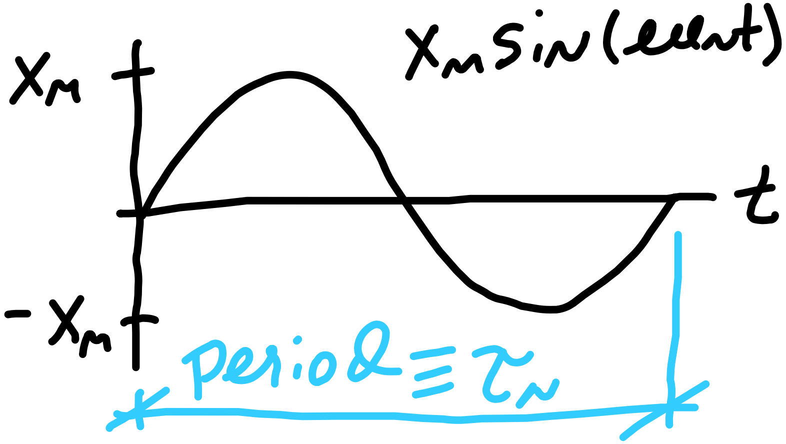

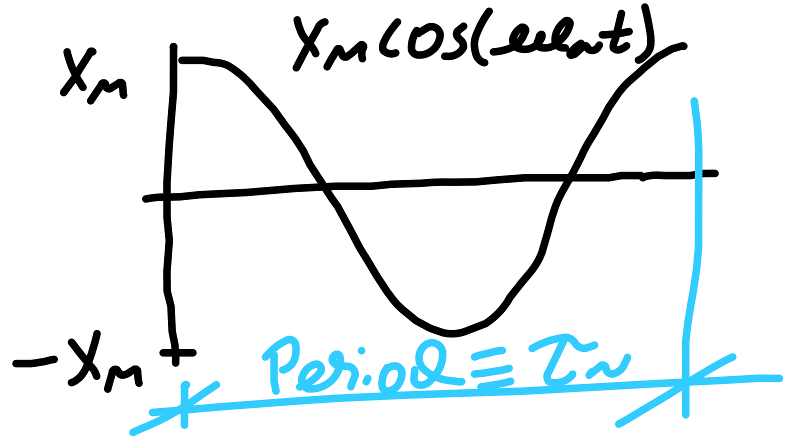

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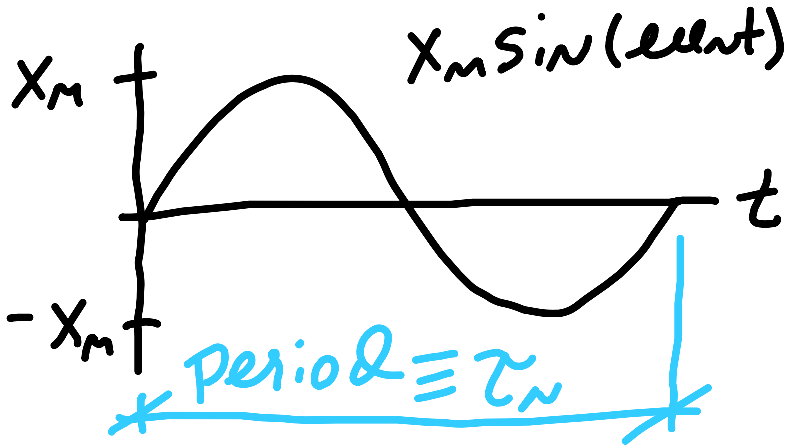

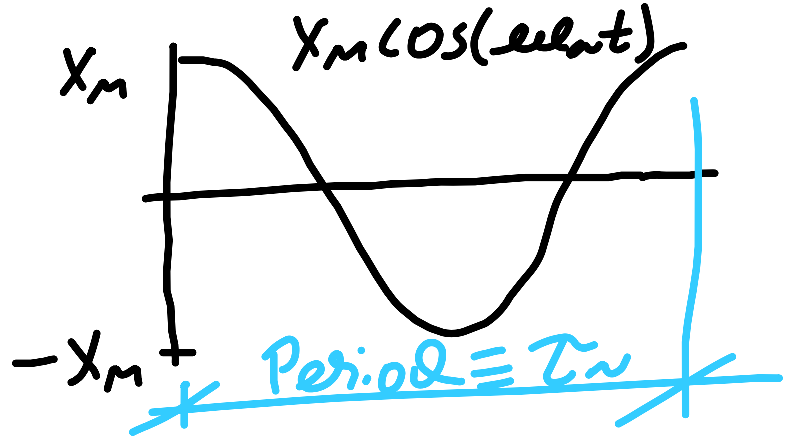

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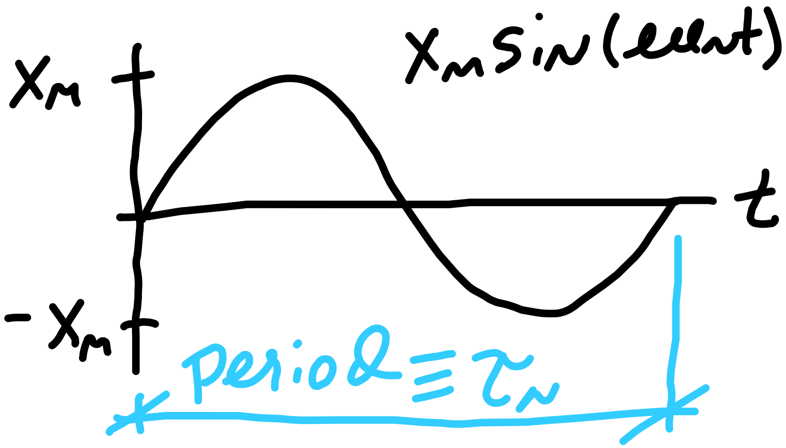

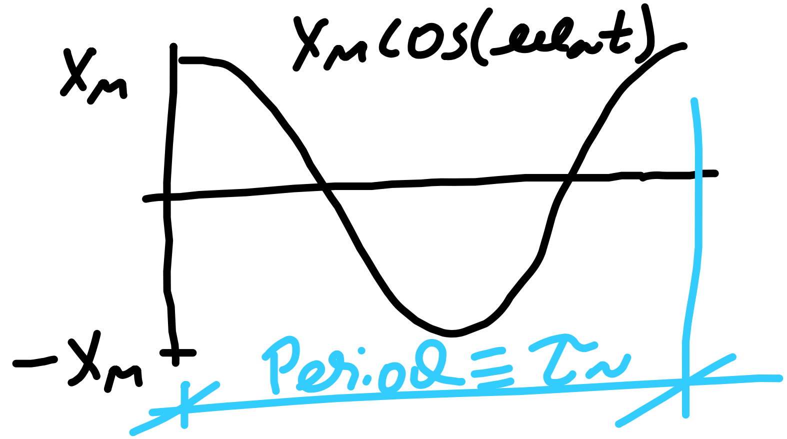

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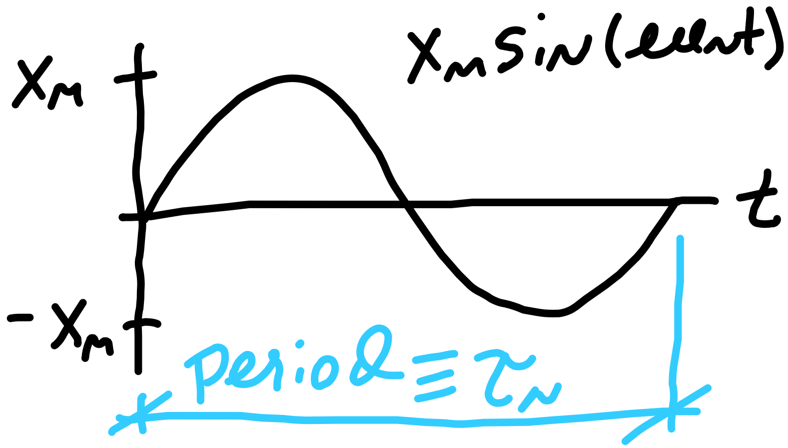

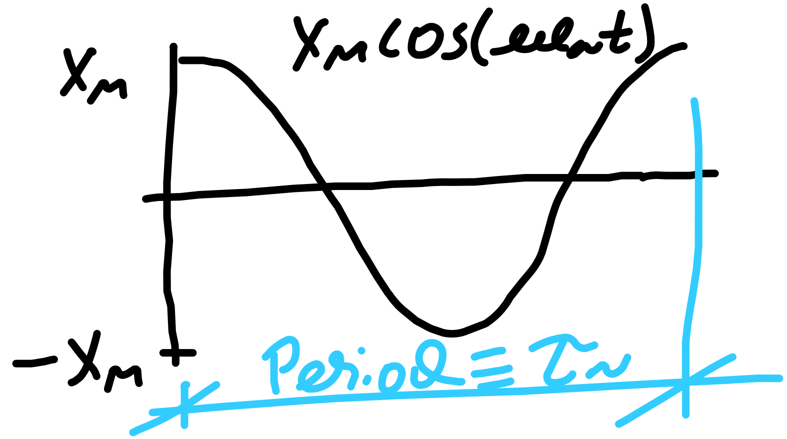

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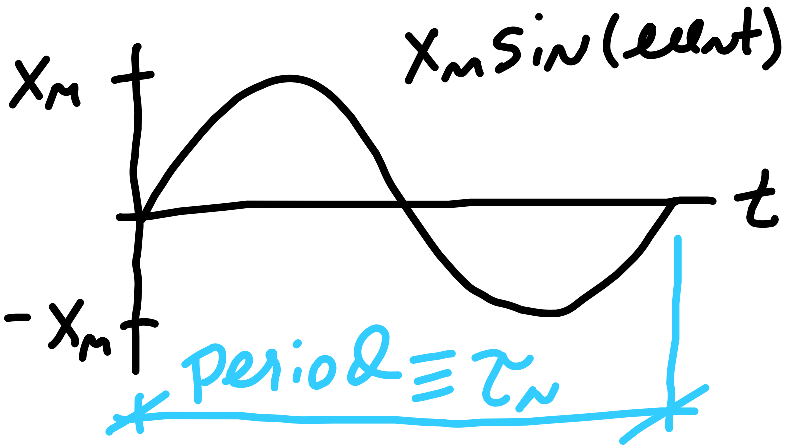

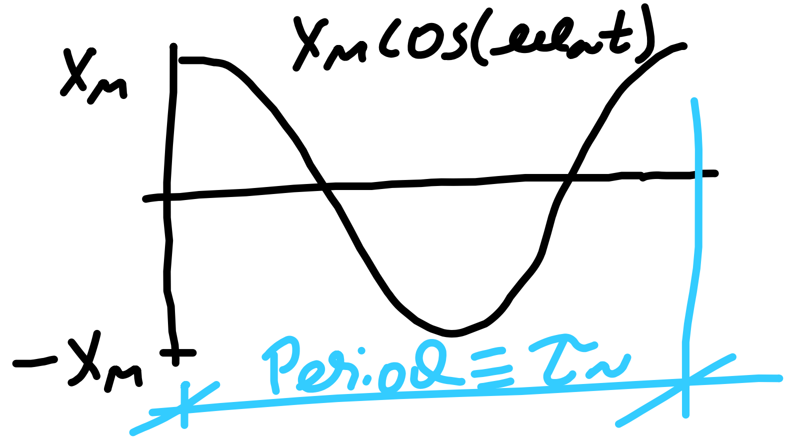

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$f\equiv\frac{1}{\tau}$

For vibration $x = x_m \sin(\omega_n t + \phi)$,

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& $a_m = x_m \omega_n^2 \Rightarrow a_m = v_m \omega_n$

Example: Spring

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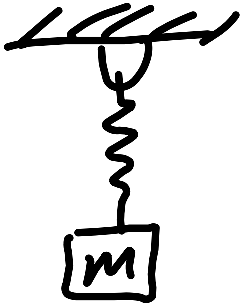

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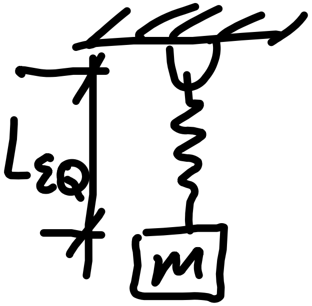

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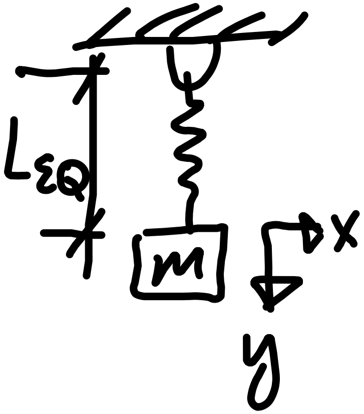
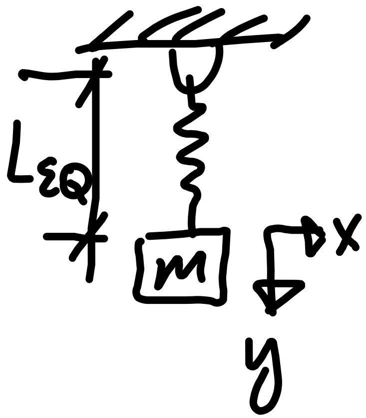


Example: Spring

Equilibrium



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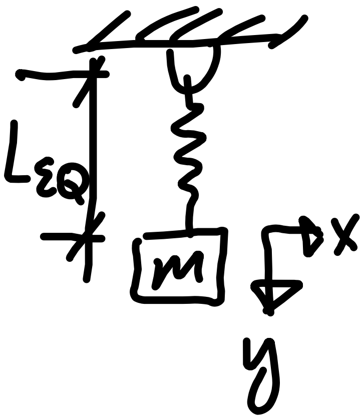
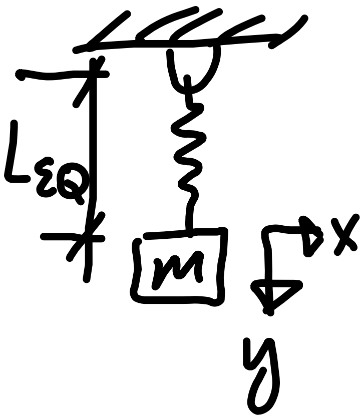


Equilibrium: $L_{EQ} = L + \delta$

Example: Spring



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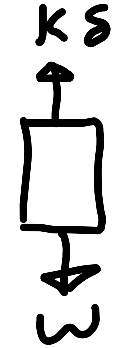

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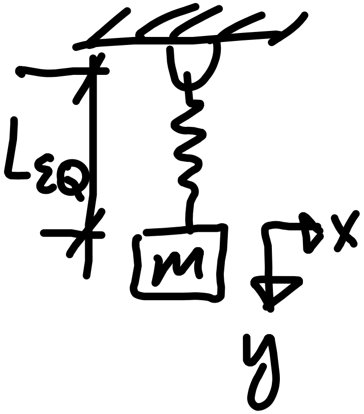
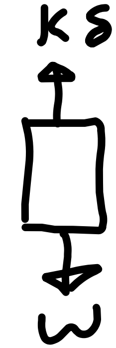


Equilibrium: $L_{EQ} = L + \delta$

$$\Sigma F = 0$$



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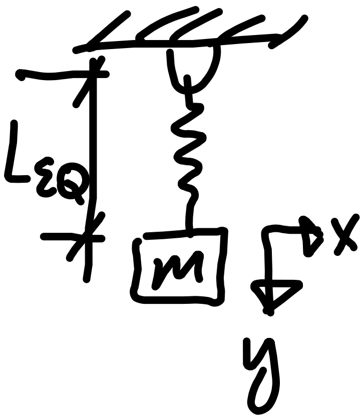

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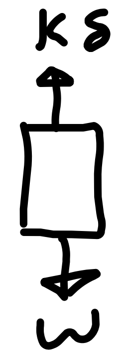

$$\Sigma F = 0 \Rightarrow W - k\delta = 0$$

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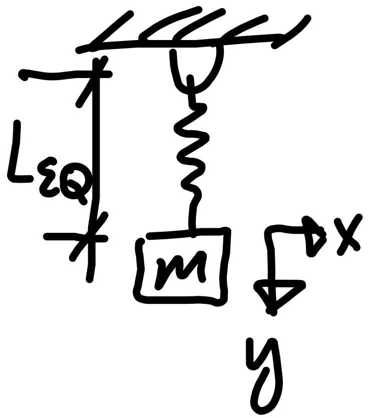

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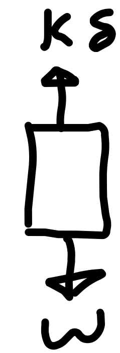
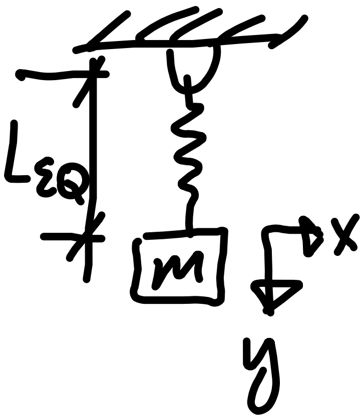
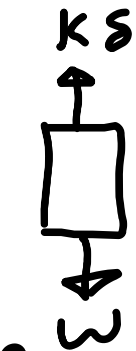


Non-equilibrium:

Example: Spring





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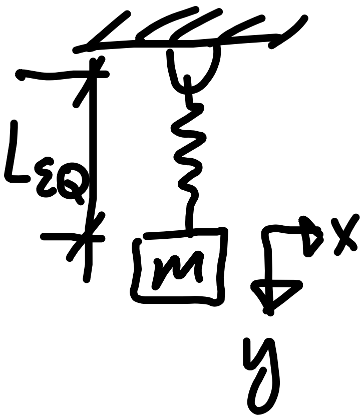
Non-equilibrium: push mass down amount y & let go.

Example: Spring



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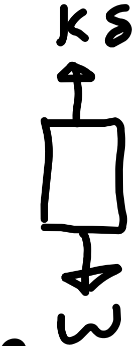

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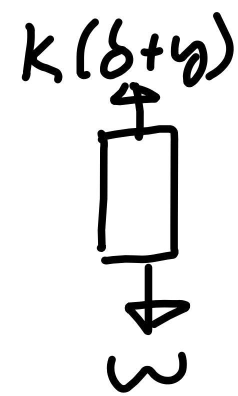

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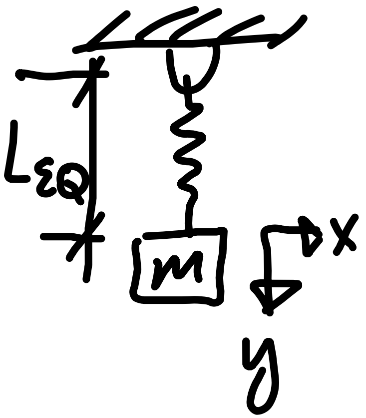

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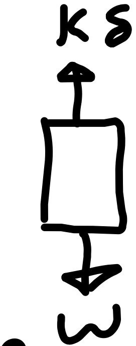

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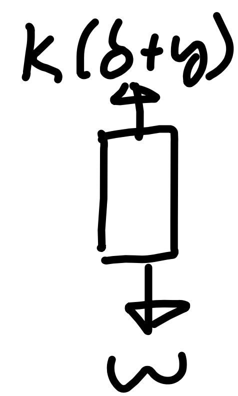

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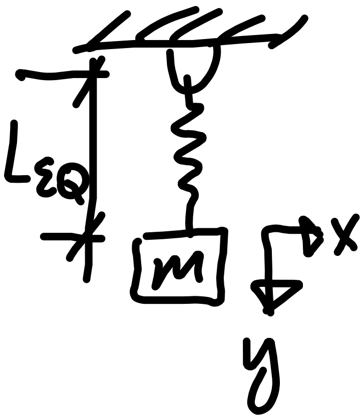

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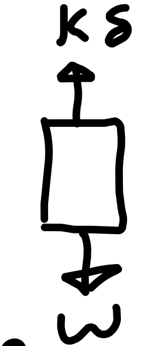
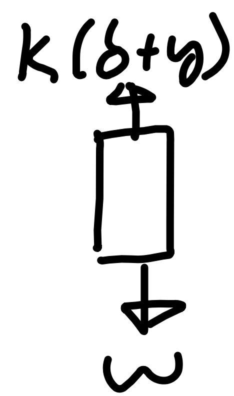


$$\Sigma F = 0 \Rightarrow w - k\delta = 0$$

Non-equilibrium: push mass down amount y & let go. Here $L = L_{EQ} + y$

$$\Sigma F_y = m\ddot{y}$$



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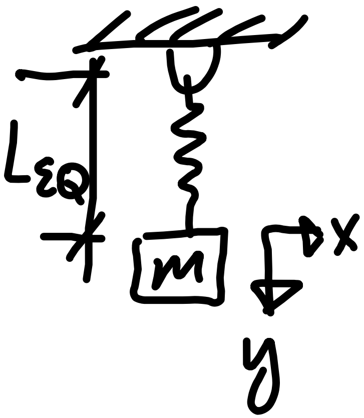

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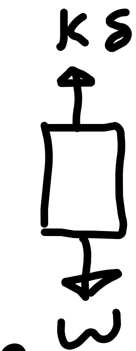

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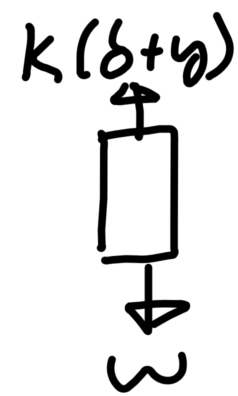
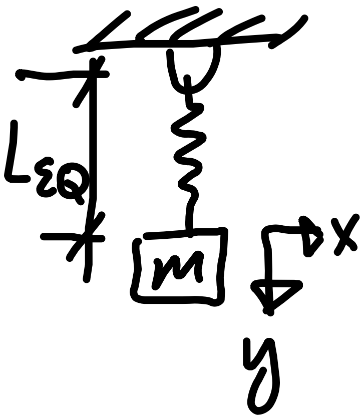
Non-equilibrium: push mass down amount y & let go. Here $L = L_{EQ} + y$

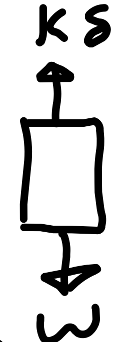
$$\Sigma F_y = m\ddot{y} \Rightarrow W - k\delta - ky = m\ddot{y}$$



Example: Spring



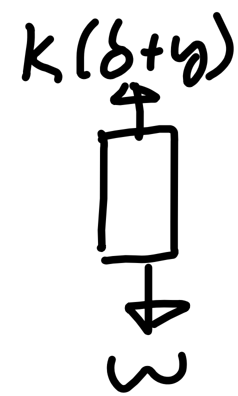
Equilibrium: $L_{EQ} = L + \delta$



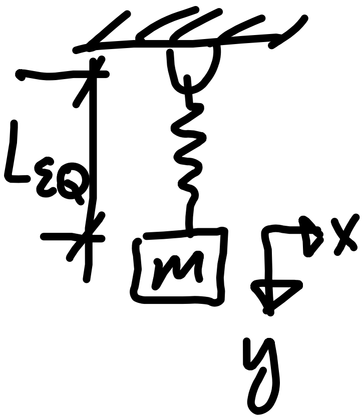
$$\Sigma F = 0 \Rightarrow \underbrace{w - k\delta = 0}$$

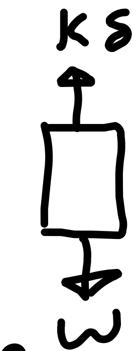
Non-equilibrium: push mass down amount y & let go. Here $L = L_{EQ} + y$

$$\Sigma F_y = m\ddot{y} \Rightarrow \underbrace{w - k\delta} - ky = m\ddot{y}$$



Example: Spring





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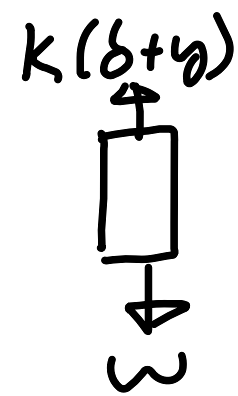

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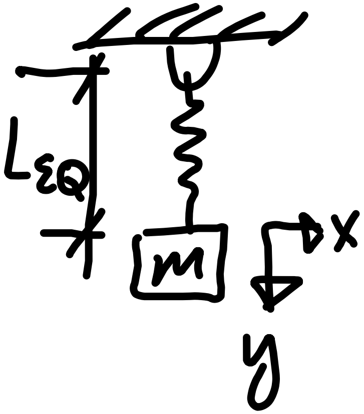

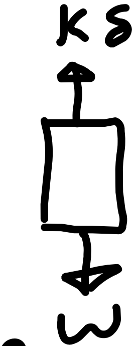

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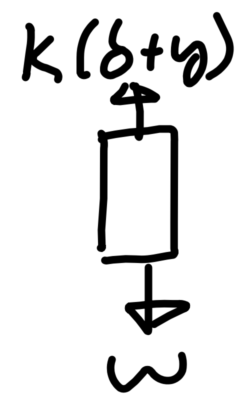
or $\ddot{y} = -\omega_n^2 y$



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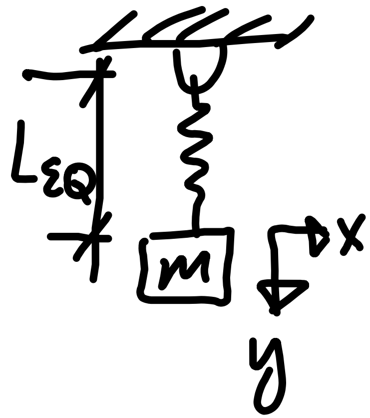

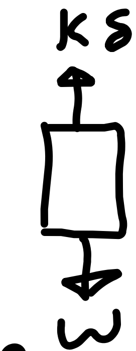

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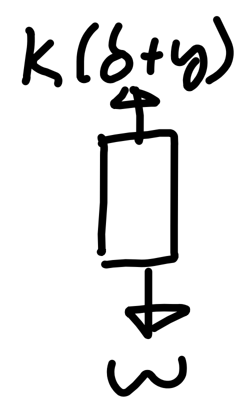
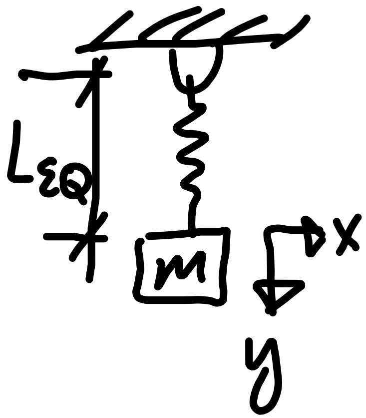
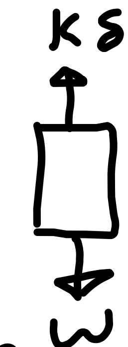




Example: Spring



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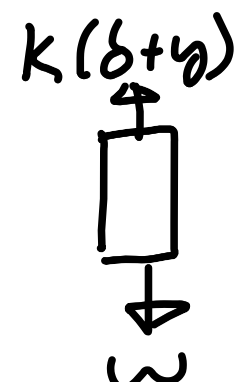
$$\tau_n = 2\pi/\omega_n$$



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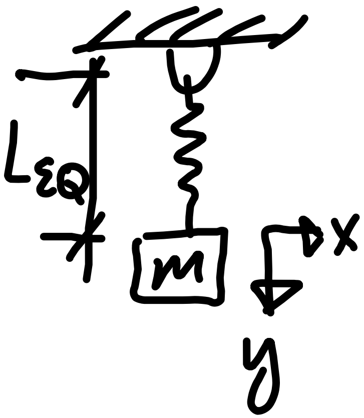

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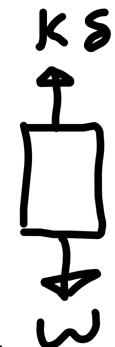

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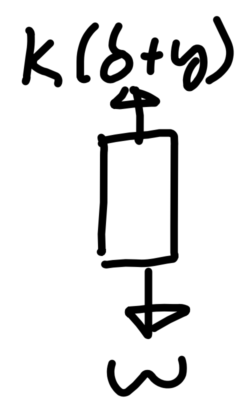

Example : Pendulum

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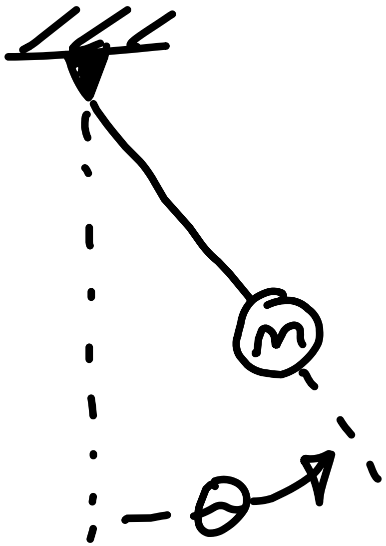
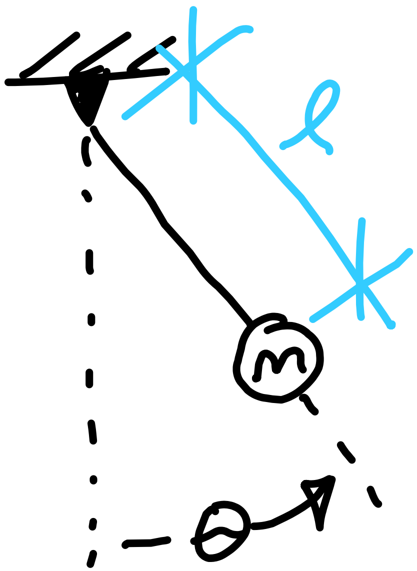
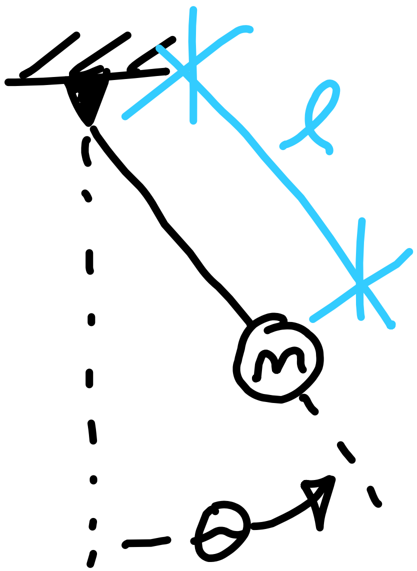
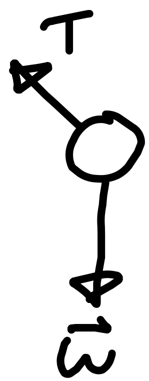


Example: Pendulum

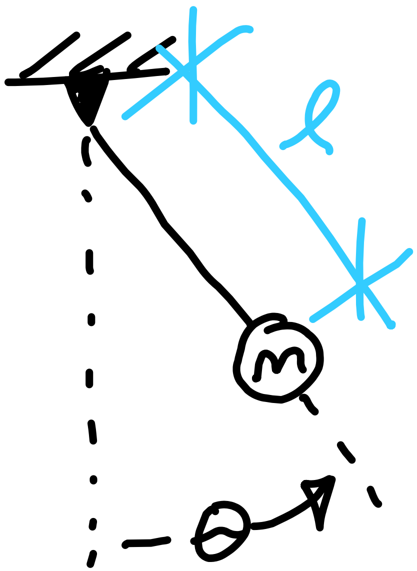


Example : Pendulum

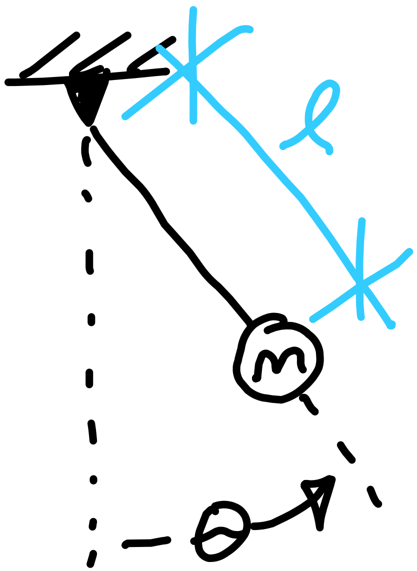




Example : Pendulum

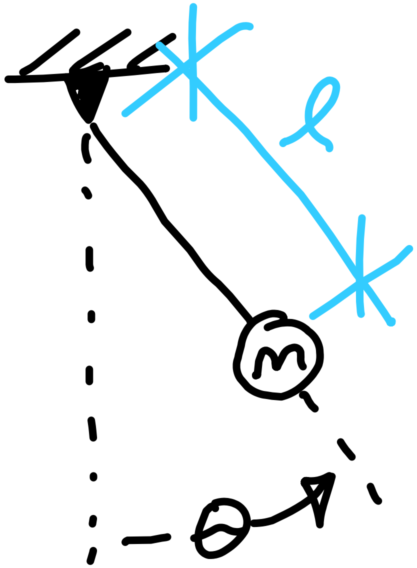


Example : Pendulum



$$\vec{w} = \vec{w}_n + \vec{w}_t$$

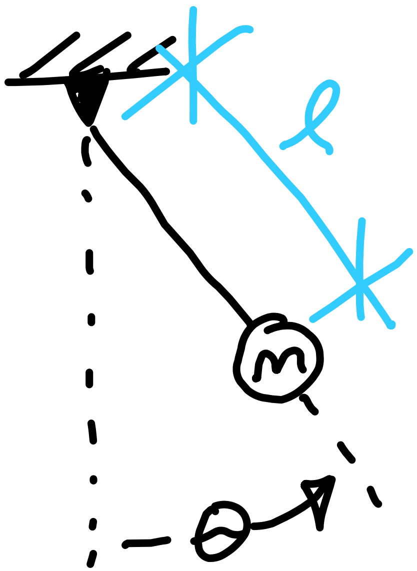
Example : Pendulum

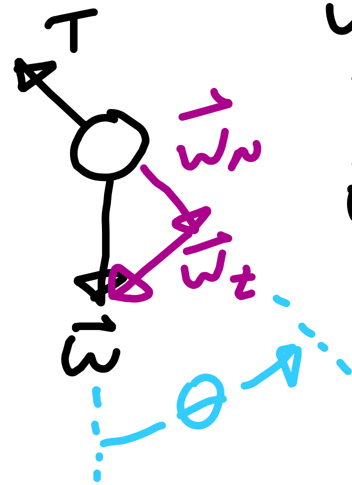


$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

Example : Pendulum





$$\vec{w} = \vec{w}_n + \vec{w}_t$$

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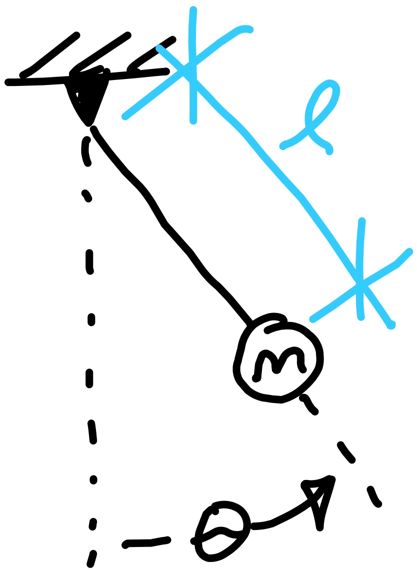
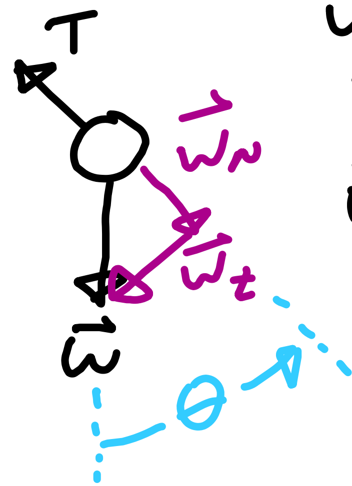
$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Example: Pendulum





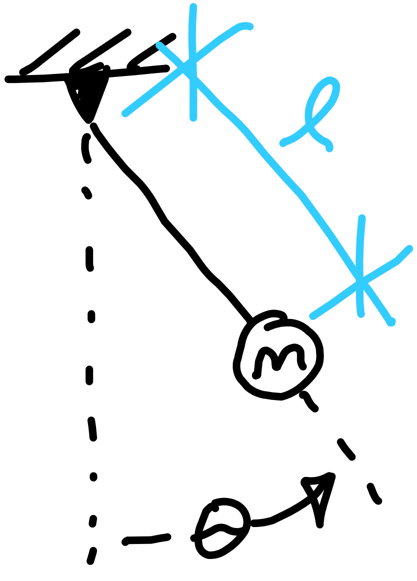
$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

Example : Pendulum



$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

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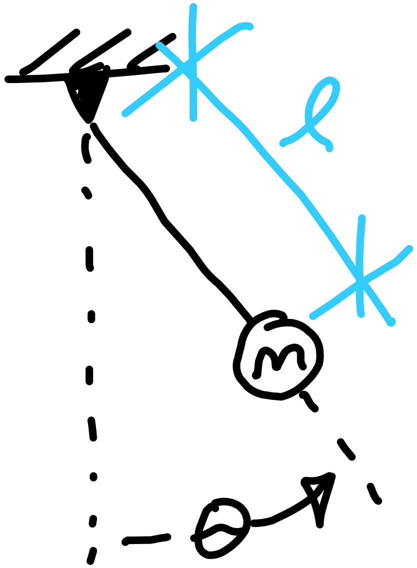
Solved 3 ways

1st way

Example : Pendulum



$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

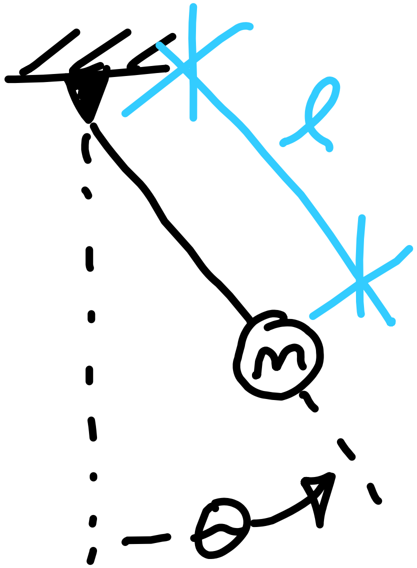
$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

1st way

$$\Sigma F_t = m a_t$$

Example : Pendulum



$$\vec{W} = \vec{W}_N + \vec{W}_t$$

$$\vec{W}_N = (W\cos\theta)(-\hat{e}_N)$$

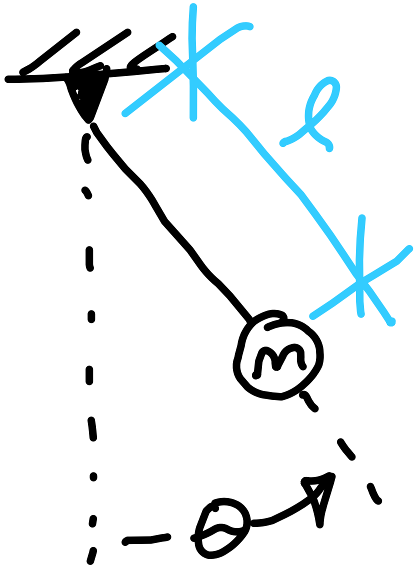
$$\vec{W}_t = (W\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

1st way

$$\Sigma F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$$

Example : Pendulum



$$\vec{W} = \vec{W}_n + \vec{W}_t$$

$$\vec{W}_n = (W\cos\theta)(-\hat{e}_n)$$

$$\vec{W}_t = (W\sin\theta)(-\hat{e}_t)$$

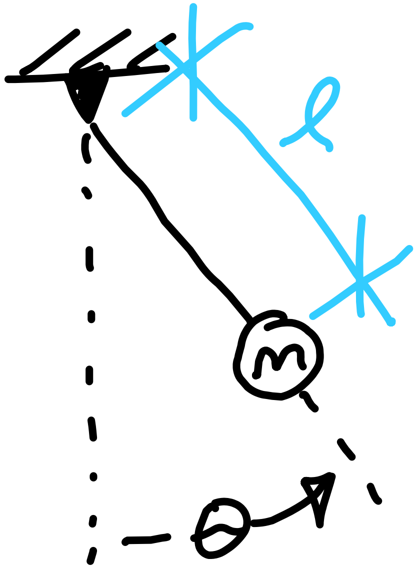
Solved 3 ways

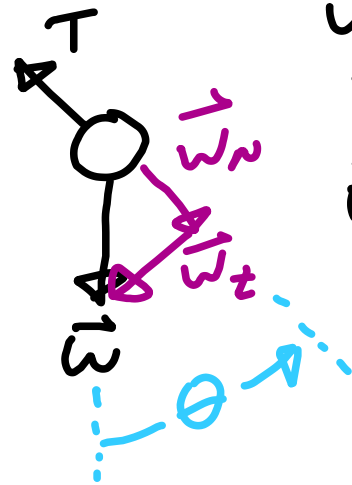
1st way

$$\Sigma F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$$

but $v = \ell\dot{\theta}$

Example : Pendulum





$\vec{w} = \vec{w}_N + \vec{w}_t$

$\vec{w}_N = (w\cos\theta)(-\hat{e}_N)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

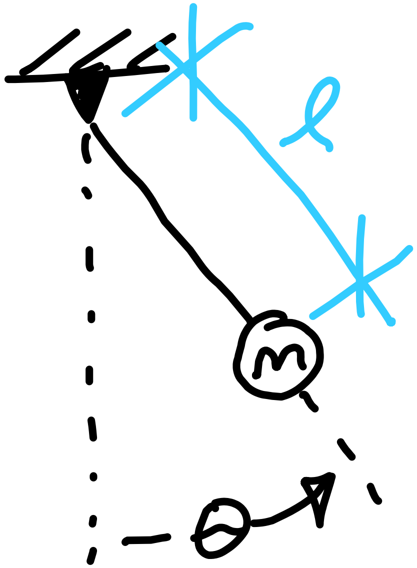
Solved 3 ways

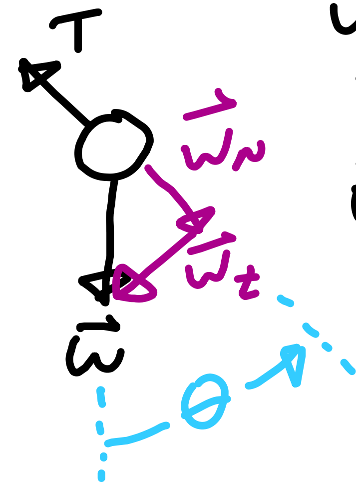
1st way

$\sum F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

but $v = l\dot{\theta}$ so $-W_t = ml\ddot{\theta}$

Example : Pendulum





$\vec{w} = \vec{w}_n + \vec{w}_t$

$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

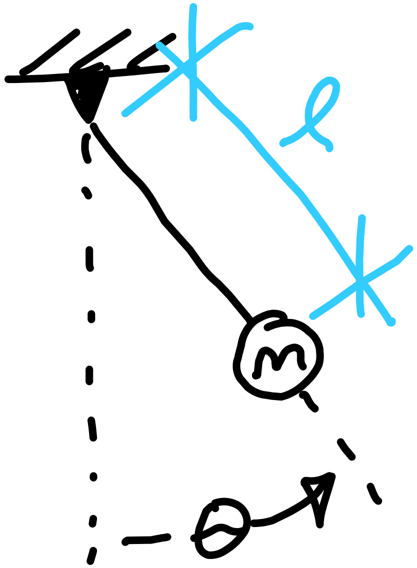
1st way

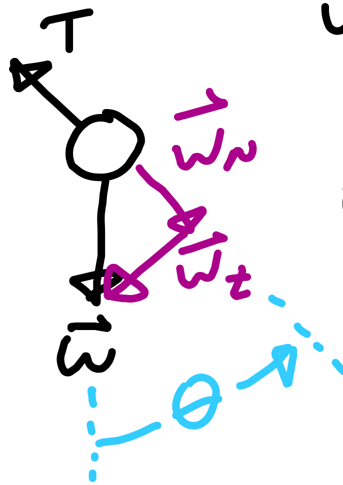
$\Sigma F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

$\Rightarrow -w\sin\theta = m\ell\ddot{\theta}$

Example : Pendulum





$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

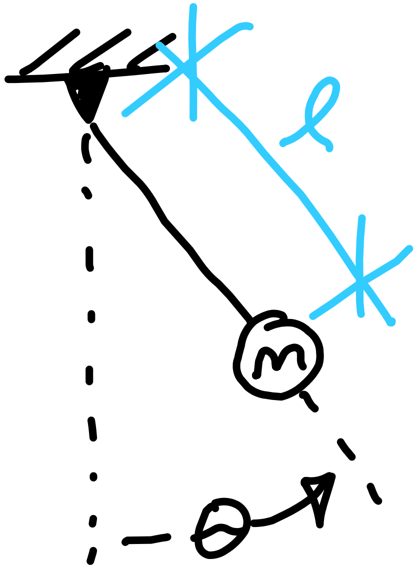
1st way

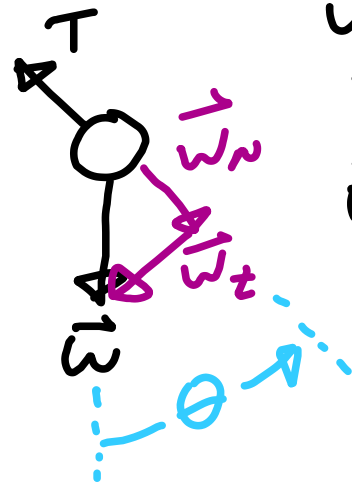
$\Sigma F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

$\Rightarrow -w\sin\theta = m\ell\ddot{\theta}$ Small angle $\sin\theta \cong \theta$

Example: Pendulum





$\vec{w} = \vec{w}_N + \vec{w}_t$

$\vec{w}_N = (w\cos\theta)(-\hat{e}_N)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

1st way

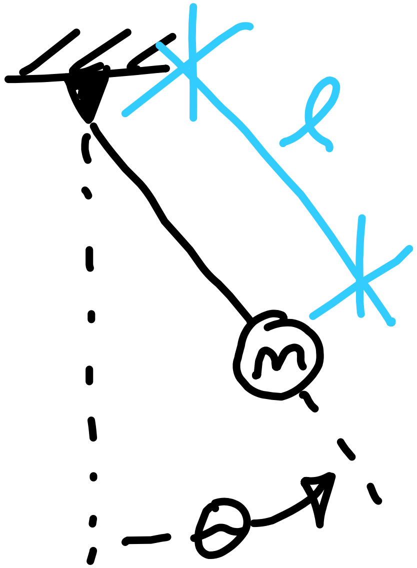
$\Sigma F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

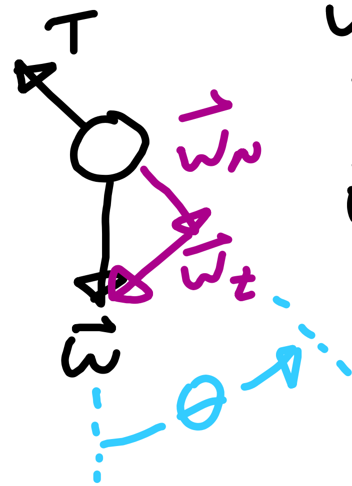
but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

$\Rightarrow -w\sin\theta = m\ell\ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-W\theta \cong m\ell\ddot{\theta}$

Example: Pendulum





$\vec{w} = \vec{w}_n + \vec{w}_t$

$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

1st way

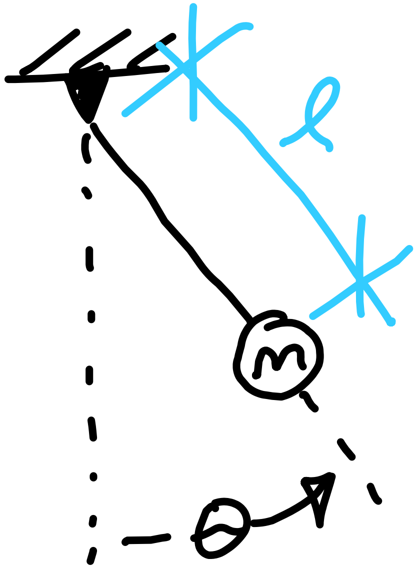
$\Sigma F_t = ma_t \Rightarrow -w_t = m\frac{dv}{dt}$

but $v = l\dot{\theta}$ so $-w_t = ml\ddot{\theta}$

$\Rightarrow -w\sin\theta = ml\ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-w\theta \cong ml\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega_n^2\theta$

Example: Pendulum



$\vec{w} = \vec{w}_n + \vec{w}_t$

$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

1st way

$\sum F_t = ma_t \Rightarrow -W_t = m\frac{dv}{dt}$

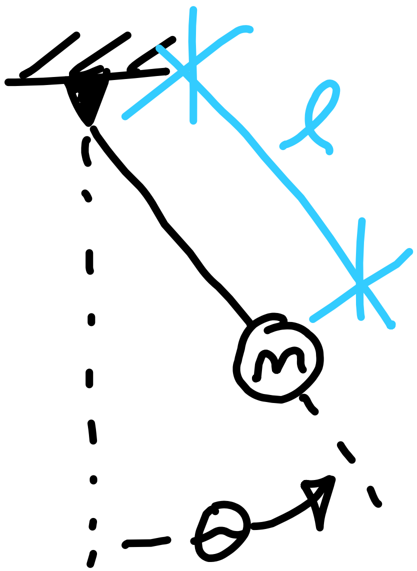
but $v = \ell\dot{\theta}$ so $-W_t = m\ell\ddot{\theta}$

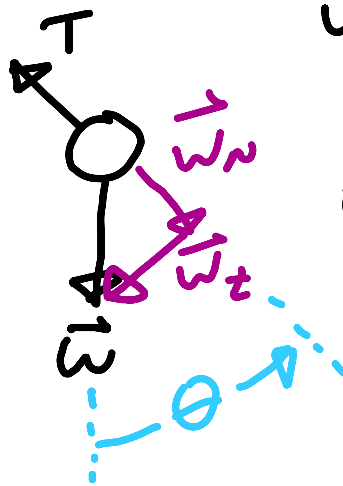
$\Rightarrow -w\sin\theta = m\ell\ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-w\theta \cong m\ell\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega_n^2\theta$,

where $\omega_n = \sqrt{\frac{mg}{m\ell}}$

Example: Pendulum





$$\vec{w} = \vec{w}_n + \vec{w}_t$$

$$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$$

$$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$$

Solved 3 ways

1st way

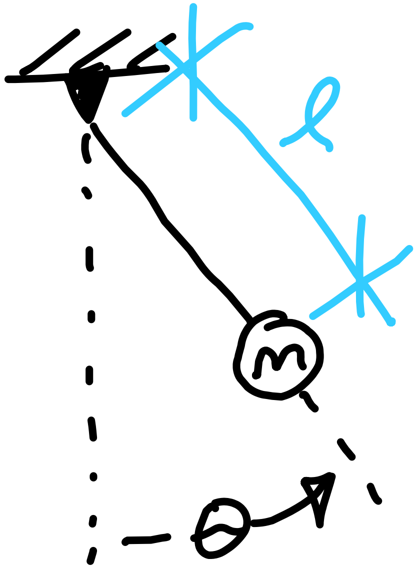
$\sum F_t = ma_t \Rightarrow -w_t = m\frac{dv}{dt}$

but $v = l\dot{\theta}$ so $-w_t = ml\ddot{\theta}$

$\Rightarrow -w\sin\theta = ml\ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-w\theta \cong ml\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega_n^2\theta$, where $\omega_n = \sqrt{\frac{mg}{ml}} = \sqrt{g/l}$

Example: Pendulum



$\vec{w} = \vec{w}_n + \vec{w}_t$

$\vec{w}_n = (w\cos\theta)(-\hat{e}_n)$

$\vec{w}_t = (w\sin\theta)(-\hat{e}_t)$

Solved 3 ways

1st way

$\sum F_t = m a_t \Rightarrow -W_t = m\frac{dv}{dt}$

but $v = l\dot{\theta}$ so $-W_t = ml\ddot{\theta}$

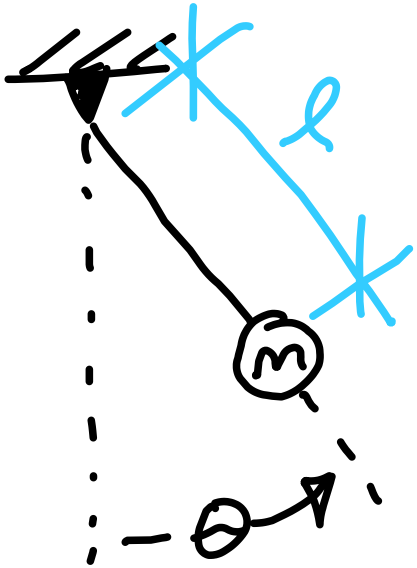
$\Rightarrow -w\sin\theta = ml\ddot{\theta}$ Small angle $\sin\theta \cong \theta$ so

$-w\theta \cong ml\ddot{\theta} \Rightarrow \ddot{\theta} = -\omega_n^2\theta$,

where $\omega_n = \sqrt{\frac{mg}{ml}} = \sqrt{g/l}$

2nd way

Example: Pendulum



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$\circlearrowleft \Sigma M_A = I_A\ddot{\theta}$

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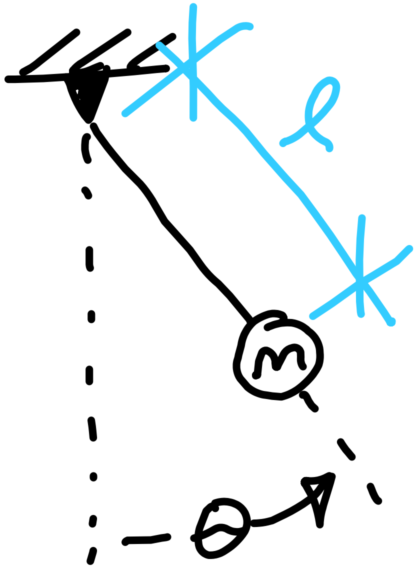

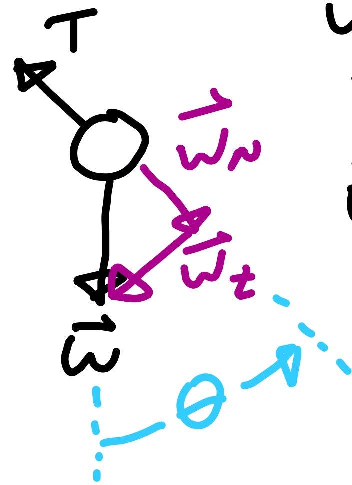

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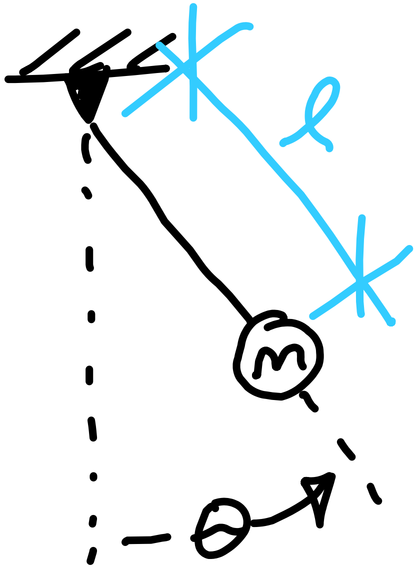

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But $I_A = m\ell^2$

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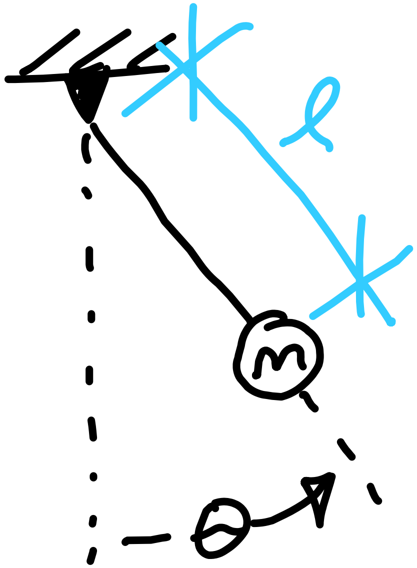

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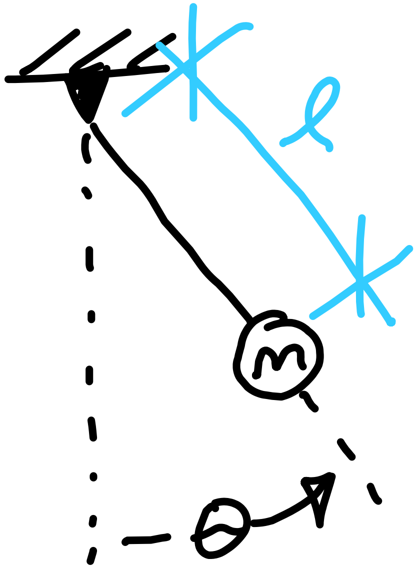

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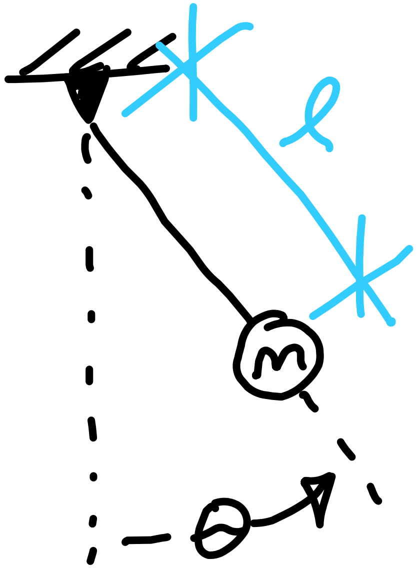

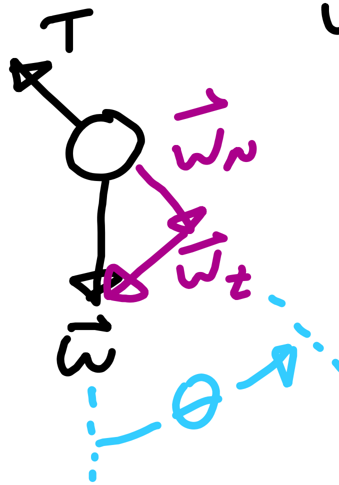

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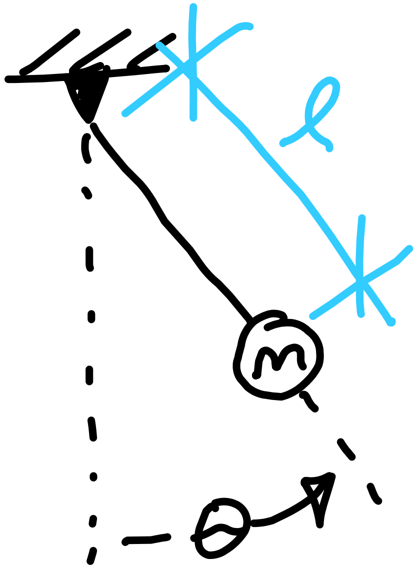

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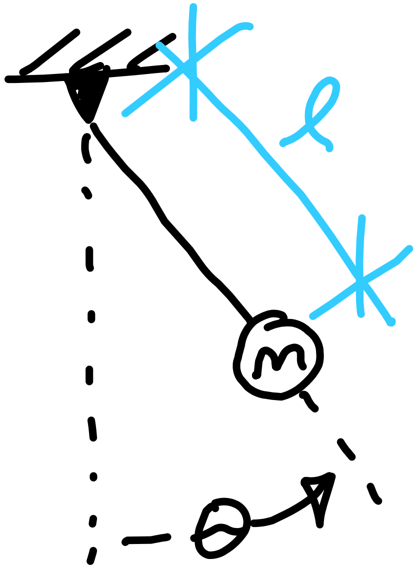

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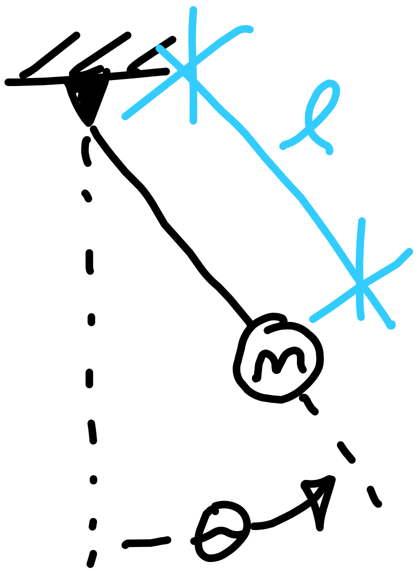

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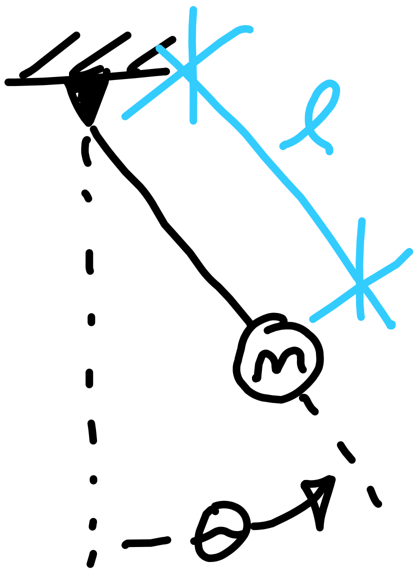

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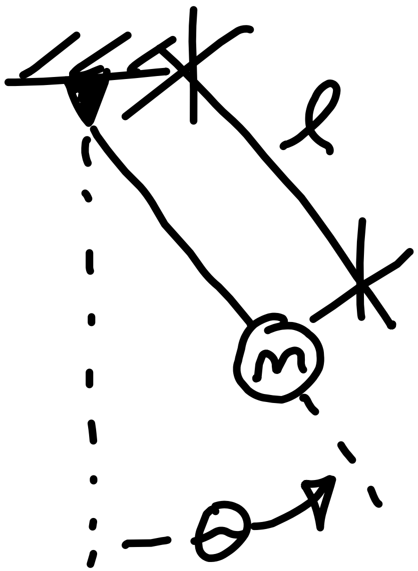
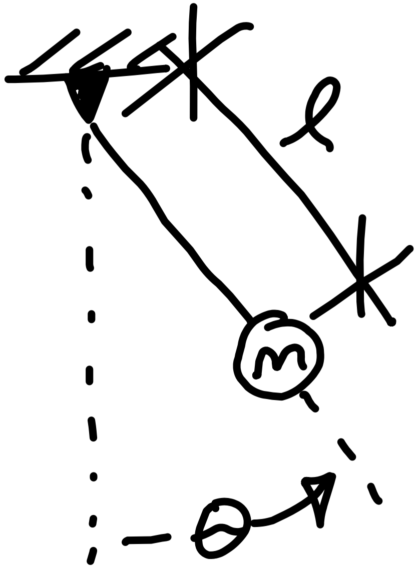
3rd way: Conservation of energy



<u>3rd way</u>: Conservation of energy & get into the form $\dot{x}^2 + \omega_n^2 x^2 = \text{const.}$



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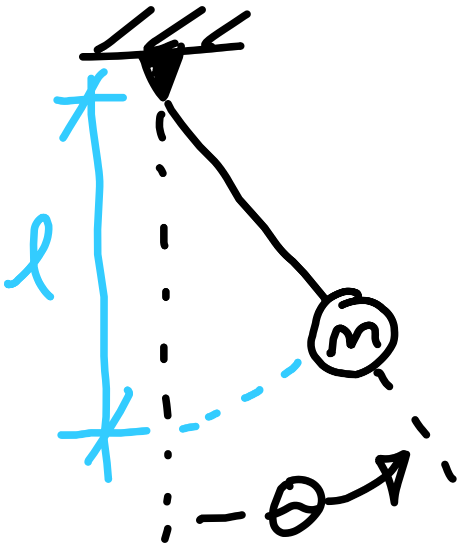

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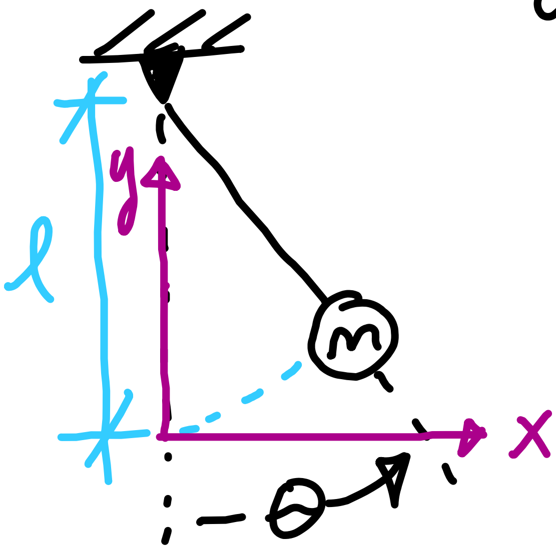

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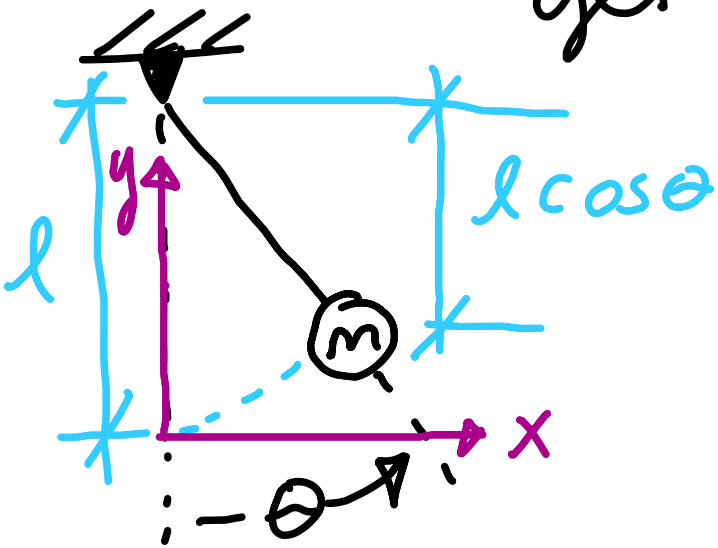

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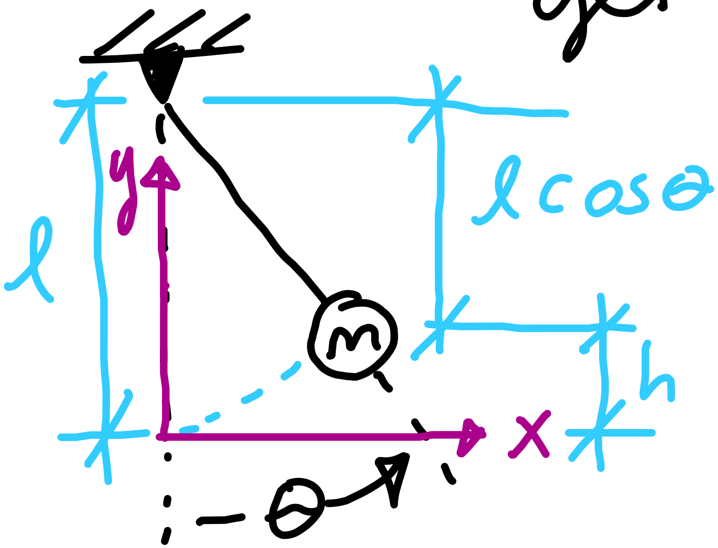

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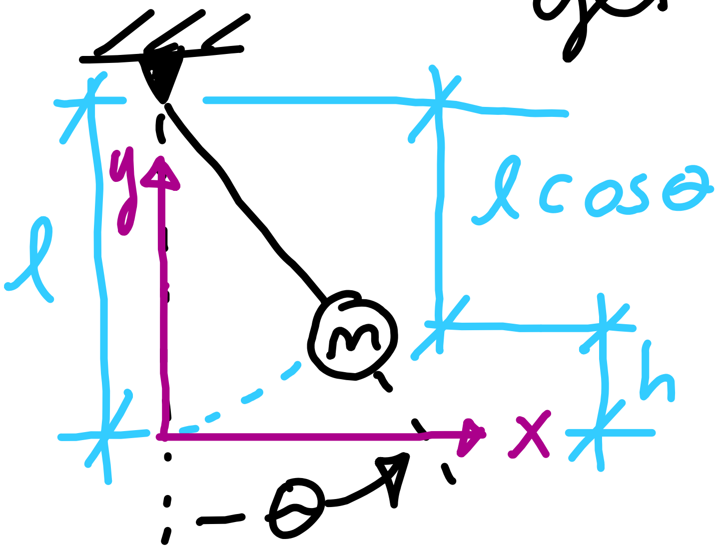
$$l = l\cos\theta + h$$



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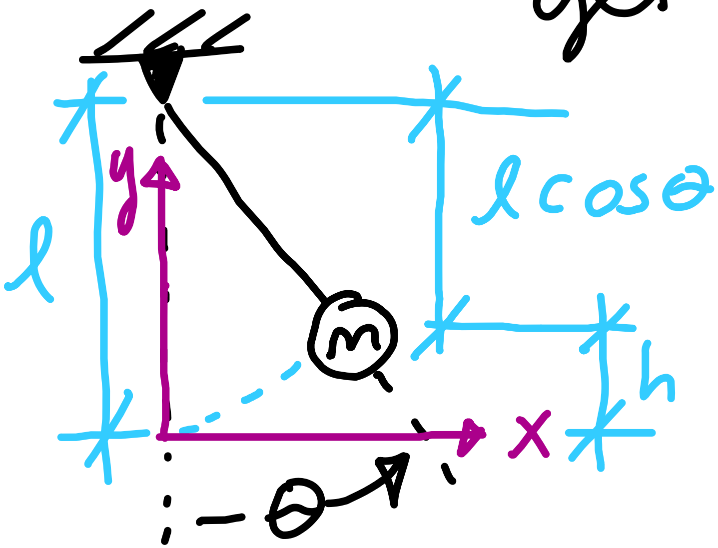

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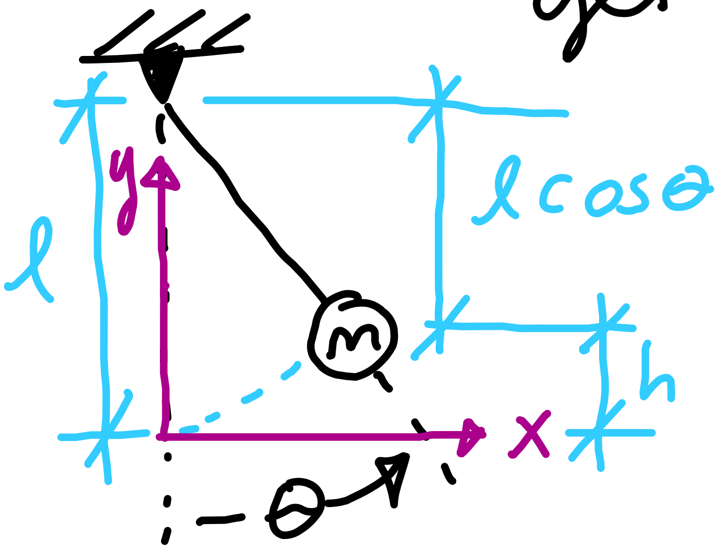

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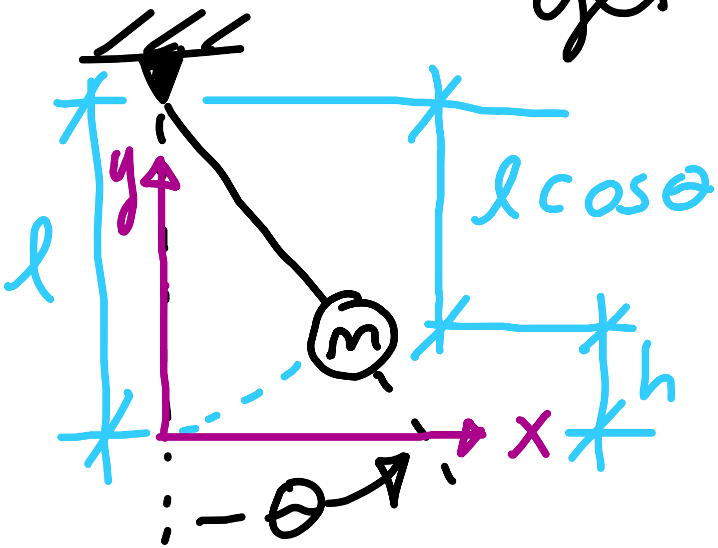

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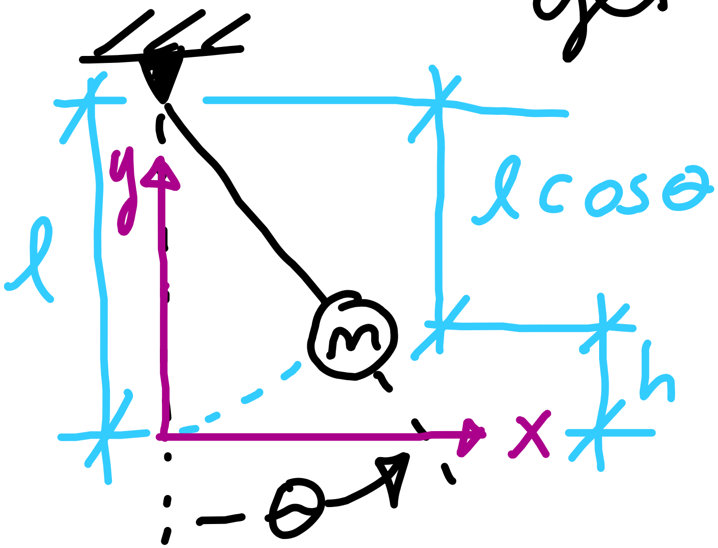

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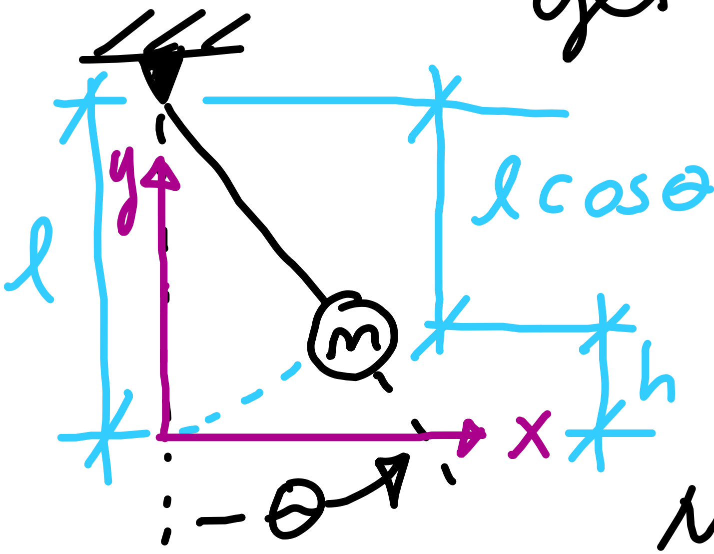

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ASU

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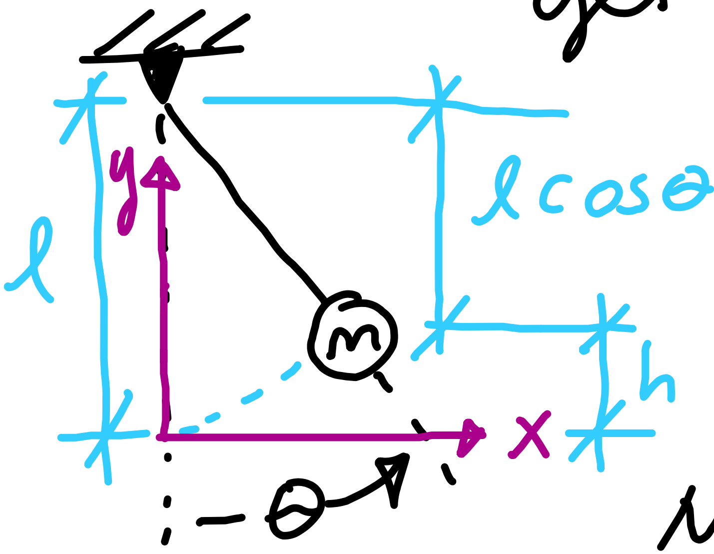

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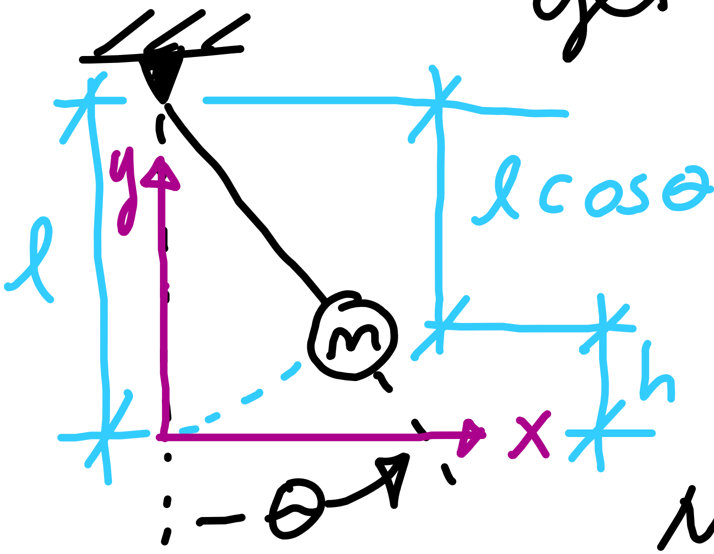

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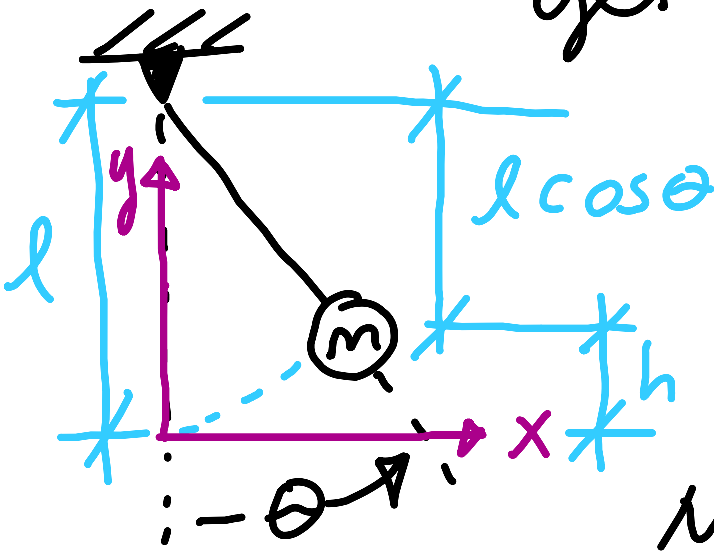

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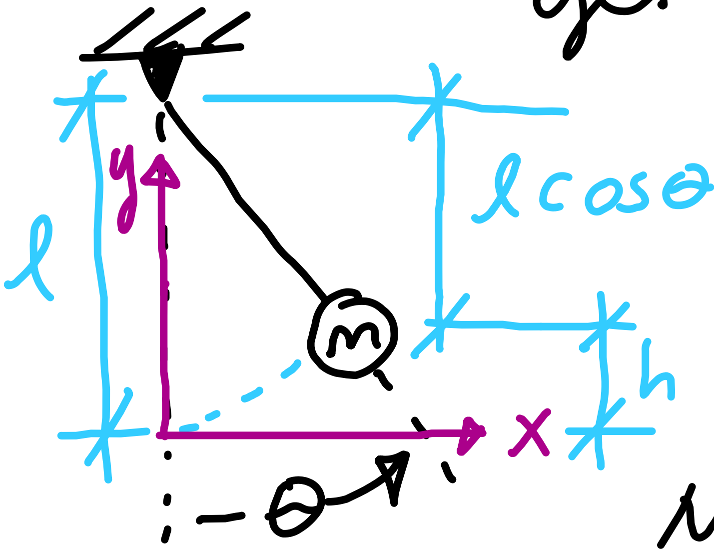

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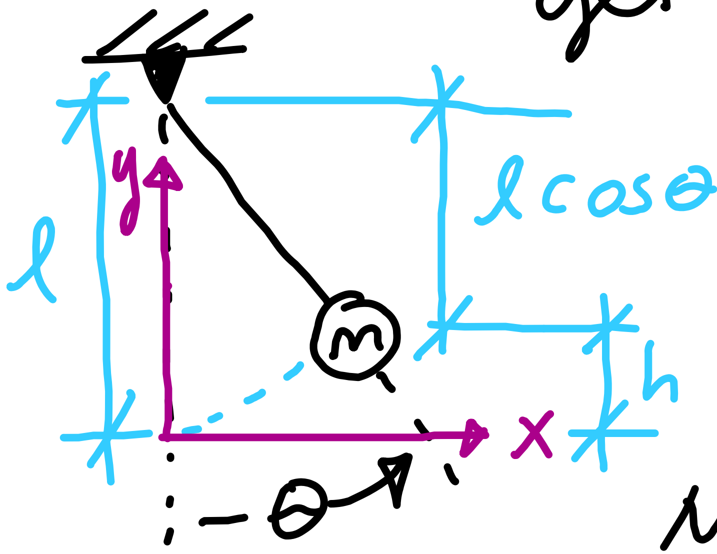

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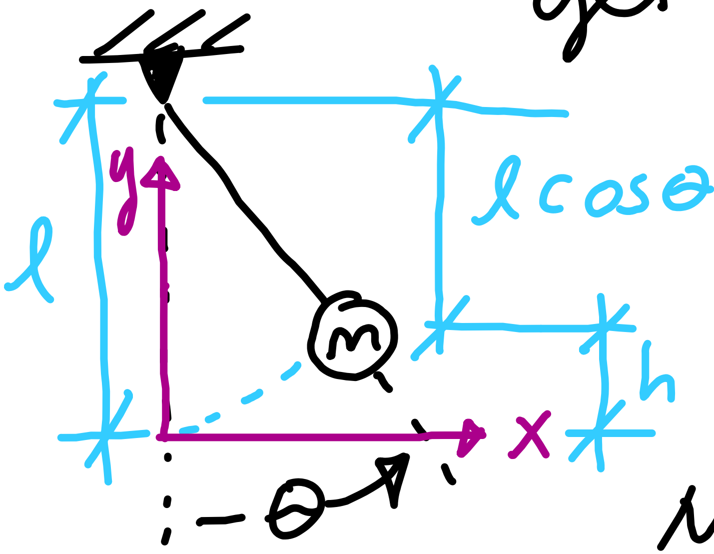

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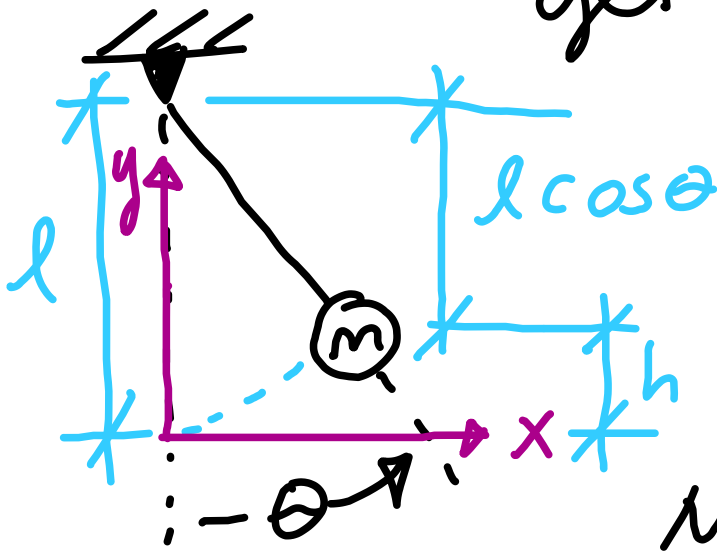
& $V=mgh$ But $v=l\dot{\theta}$ & $h=l\theta^2/2$ so

$\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}mgl\theta^2=\text{const.}$ Divide both sides by $\frac{1}{2}ml^2$

$\underline{3^{rd}\text{ way}}$: Conservation of energy & get into the form $\dot{x}^2+\omega_n^2x^2=\text{const.}$



$$l = l\cos\theta + h \text{ so}$$

$$h = l(1-\cos\theta) \text{ small } \theta$$

$$\Rightarrow \cos\theta \cong 1-\frac{\theta^2}{2} \Rightarrow$$

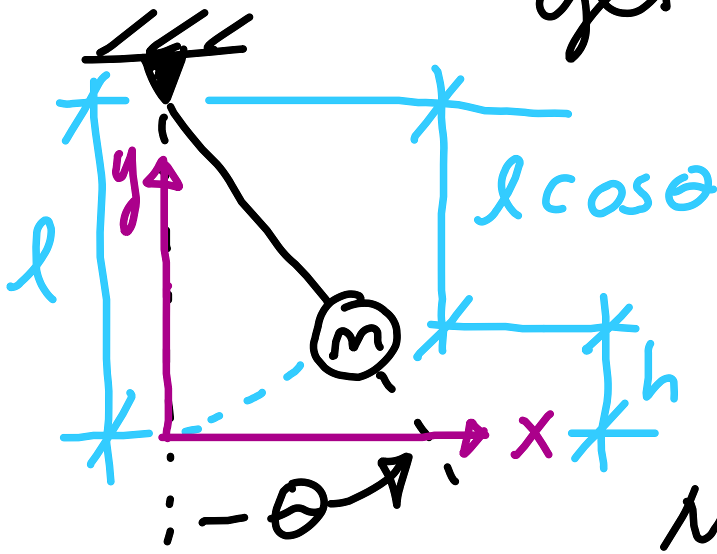
$$h \cong l\left(1-1+\frac{\theta^2}{2}\right) = l\theta^2/2$$

Now $T+V=\text{const.}$ & $T=\frac{1}{2}mv^2$

& $V=mgh$ But $v=l\dot{\theta}$ & $h=l\theta^2/2$ so

$\frac{1}{2}ml^2\dot{\theta}^2+\frac{1}{2}mgl\theta^2=\text{const.}$ Divide both sides by $\frac{1}{2}ml^2$ to get $\dot{\theta}^2+\left(\frac{g}{l}\right)\theta^2=\text{Const.}$

3rd way: Conservation of energy & get into the form $\dot{x}^2+\omega_n^2 x^2 = const.$



$l = l\cos\theta + h$ so

$h = l(1-\cos\theta)$ small θ

$\Rightarrow \quad \cos\theta \cong 1-\frac{\theta^2}{2} \quad \Rightarrow$

$h \cong l(1-1+\frac{\theta^2}{2}) = l\theta^2/2$

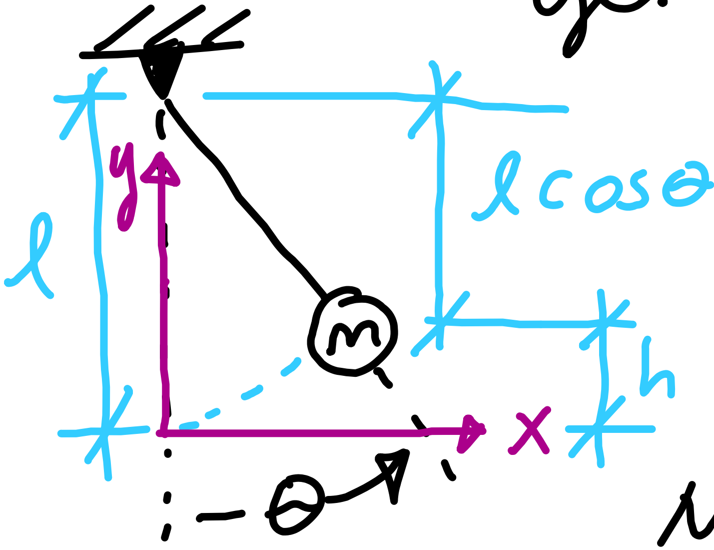
Now $T+V = const.$ & $T=\frac{1}{2}mv^2$

& $V=mgh$ But $v = l\dot{\theta}$ & $h = l\theta^2/2$ so

$\frac{1}{2}ml^2\dot{\theta}^2 + \frac{1}{2}mgl\theta^2 = const.$ Divide both sides by $\frac{1}{2}ml^2$ to get $\dot{\theta}^2 + \left(\frac{g}{l}\right)\theta^2 = Const.$

or $\quad \dot{\theta}^2 + \omega_n^2\theta^2 = const.$

$\underline{3^{rd}\text{ way}}$: Conservation of energy & get into the form $\dot{x}^2+\omega_n^2 x^2 = \text{const.}$



$$l = l\cos\theta + h \text{ so}$$

$$h = l(1-\cos\theta) \text{ small } \theta$$

$$\Rightarrow \cos\theta \cong 1 - \frac{\theta^2}{2} \Rightarrow$$

$$h \cong l\left(1-1+\frac{\theta^2}{2}\right) = l\theta^2/2$$

Now $T+V=\text{const.}$ & $T = \frac{1}{2}mv^2$ & $V = mgh$ But $v = l\dot{\theta}$ & $h = l\theta^2/2$ so

$$\frac{1}{2}ml^2\dot{\theta}^2 + \frac{1}{2}mgl\theta^2 = \text{const.}$$

Divide both sides by $\frac{1}{2}ml^2$ to get $\dot{\theta}^2 + \left(\frac{g}{l}\right)\theta^2 = \text{Const.}$

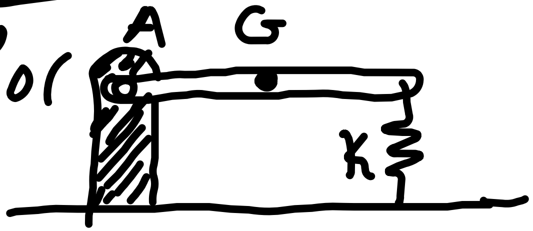
or $\dot{\theta}^2 + \omega_n^2\theta^2 = \text{const.}$ where

$$\omega_n = \sqrt{\frac{g}{l}}$$

A more complicated example

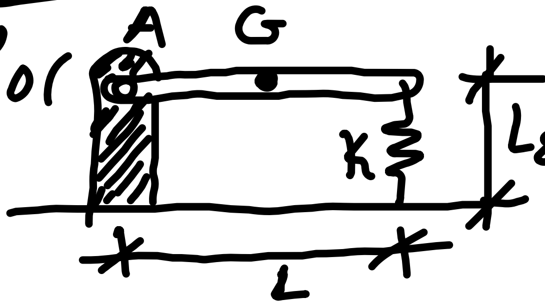
A more complicated example

Find τ_n for



A more complicated example

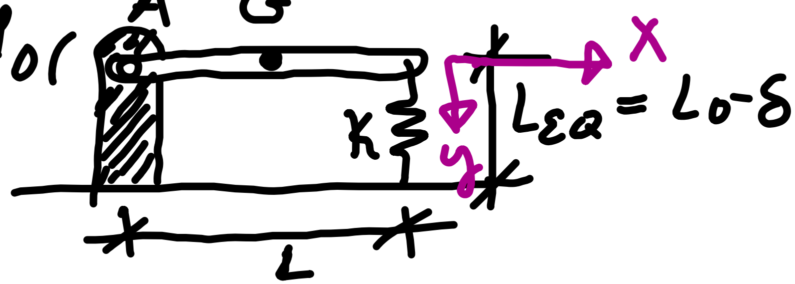
Find τ_n for



$L_{EQ} = L_0 - \delta$

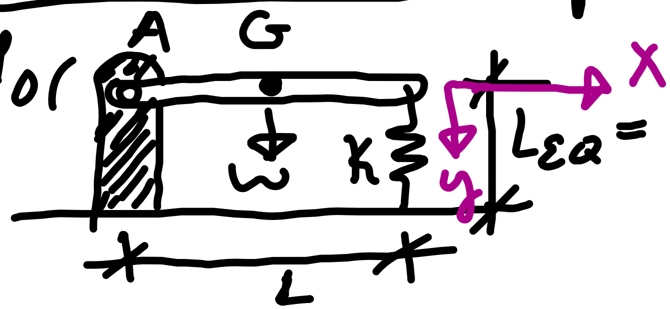
A more complicated example

Find τ_n for



A more complicated example

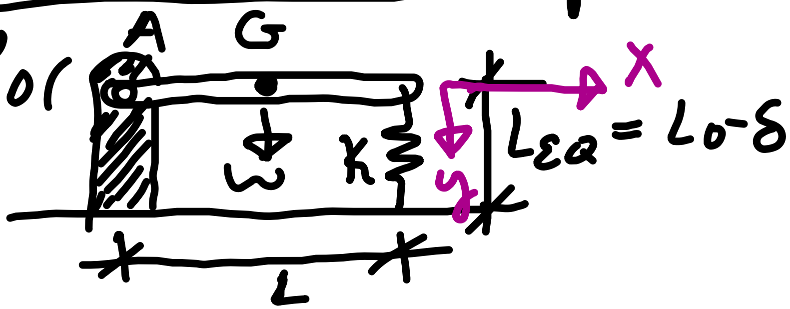
Find τ_n for



$L_{EQ} = L_0 - \delta$

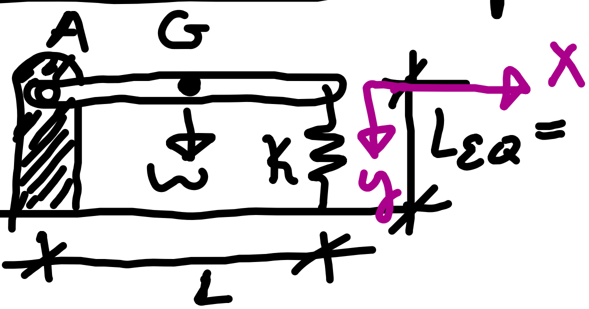
Equilibrium:

A more complicated example Find τ_n for



Equilibrium: $\Sigma M_A = 0$

A more complicated example Find τ_n for

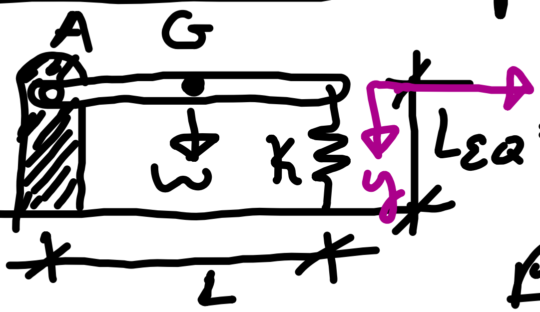


$L_{EQ} = L_0 - \delta$

Equilibrium: $\sum M_A = 0$

$\Rightarrow W\frac{L}{2} - k\delta = 0$

A more complicated example Find τ_n for

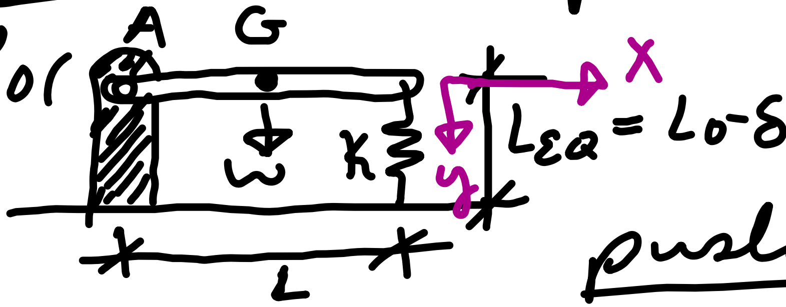


Equilibrium: $\sum M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

A more complicated example Find τ_n for



Equilibrium: $\curvearrowleft \Sigma M_A = 0$

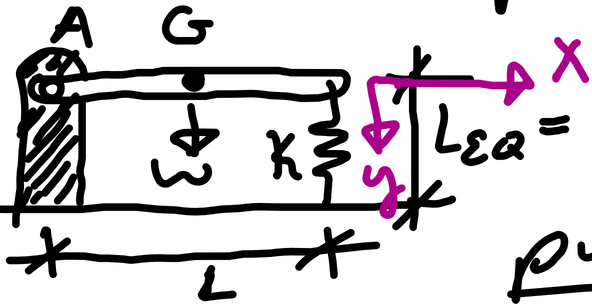
$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

$\curvearrowleft \Sigma M_A = I_A \ddot{\theta}$

A more complicated example

Find τ_n for



$L_{EQ} = L_0 - \delta$

Equilibrium: $\circlearrowright \Sigma M_A = 0$

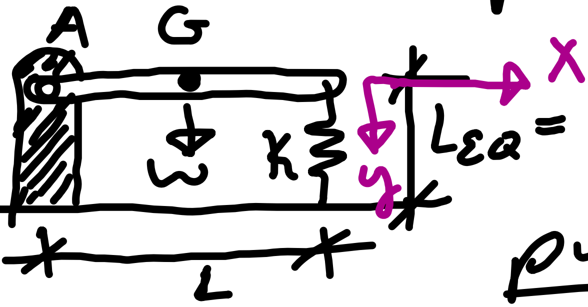
$\Rightarrow W\frac{L}{2} - k\delta = 0$

push down small amount y:

$\circlearrowright \Sigma M_A = I_A \ddot{\theta} \Rightarrow W\frac{L}{2} - k\delta - ky = I_A \ddot{\theta}$

A more complicated example

Find τ_n for



$L_{EQ} = L_0 - \delta$

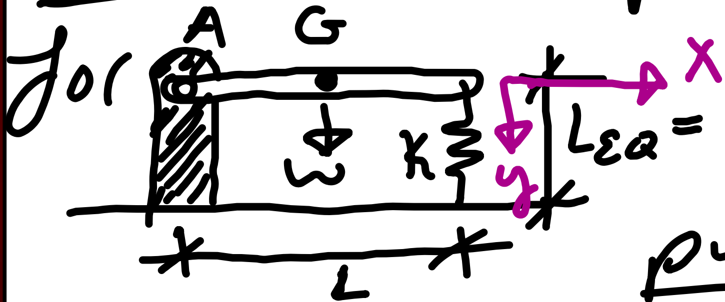
Equilibrium: $\circlearrowleft \Sigma M_A = 0$

$\Rightarrow w\frac{L}{2} - k\delta = 0$

push down small amount y:

$\circlearrowleft \Sigma M_A = I_A\ddot{\theta} \Rightarrow \overbrace{w\frac{L}{2} - k\delta}^{0} - ky = I_A\ddot{\theta}$

A more complicated example Find τ_n for



$L_{EQ} = L_0 - \delta$

Equilibrium: $\curvearrowleft \sum M_A = 0$

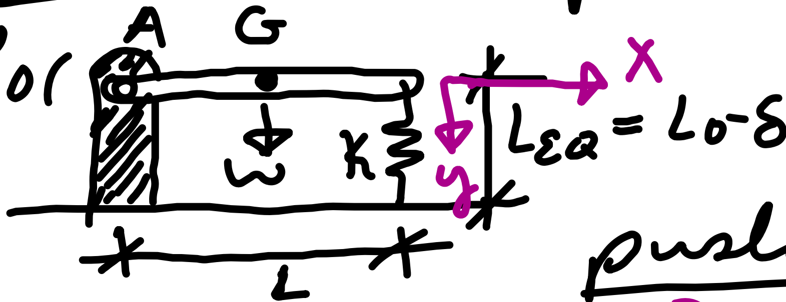
$\Rightarrow W\frac{L}{2} - k\delta = 0$

push down small amount y:

$\curvearrowleft \sum M_A = I_A\ddot{\theta} \Rightarrow \underbrace{W\frac{L}{2} - k\delta}_{0} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$

A more complicated example

Find τ_n for



Equilibrium: $\circlearrowleft \sum M_A = 0$

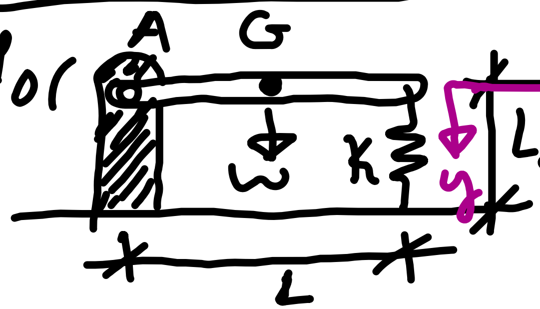
$\Rightarrow \underline{W\frac{L}{2} - k\delta = 0}$

push down small amount y:

$$\circlearrowleft \sum M_A = I_A\ddot{\theta} \Rightarrow \underbrace{W\frac{L}{2} - k\delta}_{=0} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$$

But $y = L\sin\theta$

A more complicated example Find τ_n for



Equilibrium: $\circlearrowleft \sum M_A = 0$

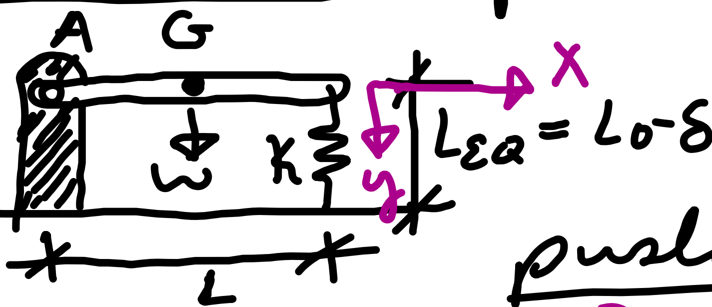
$\Rightarrow W\frac{L}{2} - k\delta = 0$

push down small amount y:

$\circlearrowleft \sum M_A = I_A \ddot{\theta} \Rightarrow \underbrace{W\frac{L}{2} - k\delta}_{0} - ky = I_A \ddot{\theta} \Rightarrow -ky = I_A \ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta$

A more complicated example Find τ_n for



Equilibrium: $\curvearrowright \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

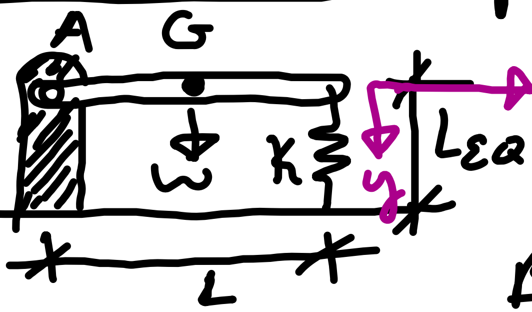
$\curvearrowright \Sigma M_A = I_A \ddot{\theta} \Rightarrow \underbrace{W\frac{L}{2} - K\delta}_{0} - Ky = I_A\ddot{\theta} \Rightarrow \quad -Ky = I_A\ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A\ddot{\theta}$

A more complicated example

Find τ_n for



Equilibrium: $\curvearrowright \Sigma M_A = 0 \Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

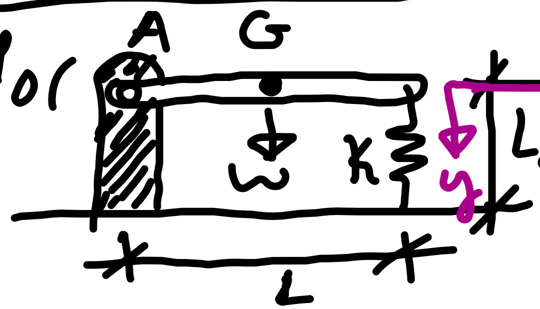
$\curvearrowright \Sigma M_A = I_A \ddot{\theta} \Rightarrow \cancel{W\frac{L}{2} - K\delta} - Ky = I_A \ddot{\theta} \Rightarrow -Ky = I_A \ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A \ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2 \theta$

A more complicated example

Find ω_n for



Equilibrium: $\circlearrowleft \sum M_A = 0$

$\Rightarrow w\frac{L}{2} - k\delta = 0$

push down small amount y:

$$\circlearrowleft \sum M_A = I_A\ddot{\theta} \Rightarrow \underbrace{w\frac{L}{2} - k\delta}_{0} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$$

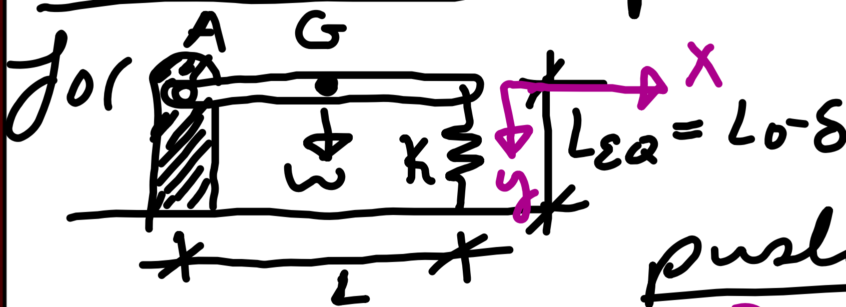
But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta \cong I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$$\omega_n = \sqrt{\frac{KL^2}{I_A}}$$

A more complicated example Find τ_n

for



Equilibrium: $\circlearrowleft \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

$$\circlearrowleft \Sigma M_A = I_A\ddot{\theta} \Rightarrow \cancel{W\frac{L}{2} - K\delta} - Ky = I_A\ddot{\theta} \Rightarrow -Ky = I_A\ddot{\theta}$$

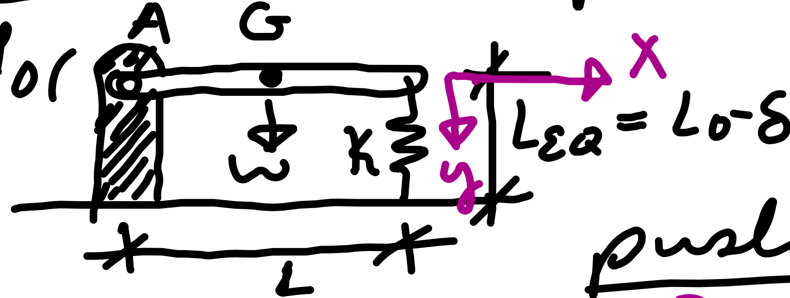
But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{KL^2}{I_A}}$ But $I_A = \bar{I} + m\left(\frac{L}{2}\right)^2$

A more complicated example

Find ω_n for



Equilibrium: $\sum M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

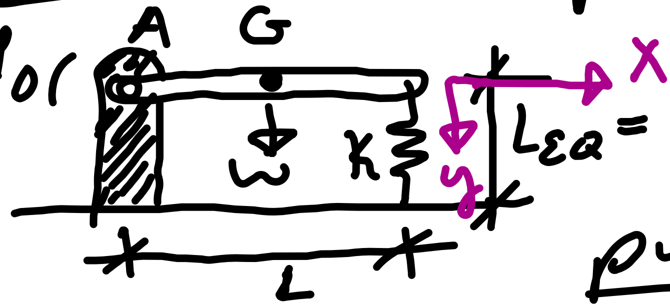
$\sum M_A = I_A\ddot{\theta} \Rightarrow \cancelto{0}{W\frac{L}{2} - K\delta} - Ky = I_A\ddot{\theta} \Rightarrow \quad -Ky = I_A\ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{KL^2}{I_A}}$ But $I_A = \bar{I} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4}$

A more complicated example Find τ_n for



Equilibrium: $\circlearrowright \Sigma M_A = 0$

$$\Rightarrow \; W\frac{L}{2} - k\delta = 0$$

push down small amount y:

$$\circlearrowright \Sigma M_A = I_A\ddot{\theta} \;\Rightarrow\; \underbrace{W\frac{L}{2} - k\delta}_{=0} - ky = I_A\ddot{\theta} \;\Rightarrow\; -ky = I_A\ddot{\theta}$$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

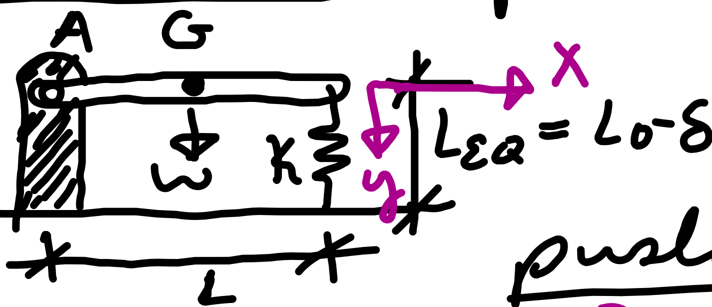
$-kL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\dfrac{kL^2}{I_A}}$ But $I_A = \bar{I} + m\left(\frac{L}{2}\right)^2 = \dfrac{mL^2}{12} + \dfrac{mL^2}{4}$

$$\Rightarrow \; I_A = mL^2\left(\frac{1}{12} + \frac{3}{12}\right)$$

A more complicated example

Find τ_n for



Equilibrium: $\circlearrowleft \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - K\delta = 0$

push down small amount y:

$$\circlearrowleft \Sigma M_A = I_A\ddot{\theta} \Rightarrow \underbrace{\cancel{W\frac{L}{2} - K\delta}}_{}\overset{0}{} - Ky = I_A\ddot{\theta} \Rightarrow -Ky = I_A\ddot{\theta}$$

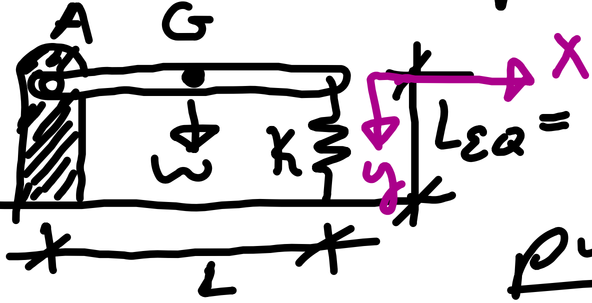
But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{KL^2}{I_A}}$ But $I_A = \bar{I} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4}$

$\Rightarrow I_A = ML^2\left(\frac{1}{12} + \frac{3}{12}\right) = ML^2/3$

A more complicated example Find ω_n for



Equilibrium: $\curvearrowleft \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - k\delta = 0$

push down small amount y:

$\curvearrowleft \Sigma M_A = I_A\ddot{\theta} \Rightarrow \underbrace{W\frac{L}{2} - k\delta}_{=0} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-kL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

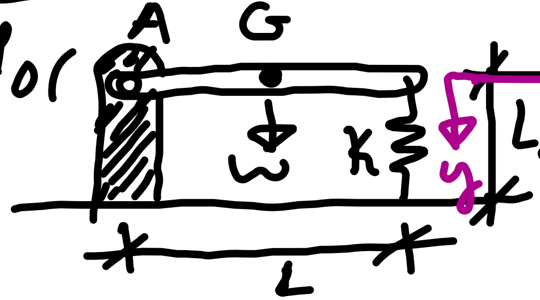
$\omega_n = \sqrt{\frac{kL^2}{I_A}}$ But $I_A = \bar{I} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{12} + \frac{mL^2}{4}$

$\Rightarrow I_A = mL^2\left(\frac{1}{12} + \frac{3}{12}\right) = mL^2/3$. Now

$$\omega_n = \sqrt{\frac{kL^2}{mL^2/3}}$$

A more complicated example

Find ω_n for



Equilibrium: $\curvearrowright \sum M_A = 0$

$\Rightarrow W\frac{L}{2} - k\delta = 0$

push down small amount y:

$\curvearrowright \sum M_A = I_A\ddot{\theta} \Rightarrow \overset{0}{\cancel{W\frac{L}{2} - k\delta}} - ky = I_A\ddot{\theta} \Rightarrow \quad -ky = I_A\ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

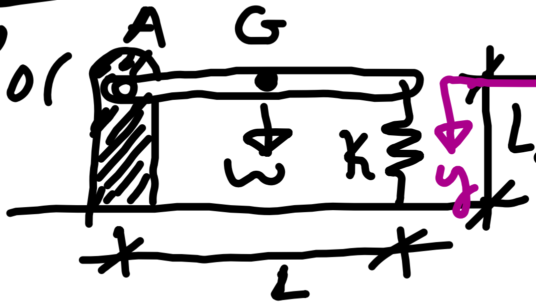
$-KL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{KL^2}{I_A}}$ But $I_A = \bar{I} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4}$

$\Rightarrow I_A = ML^2\left(\frac{1}{12} + \frac{3}{12}\right) = ML^2/3$. Now

$$\omega_n = \sqrt{\frac{KL^2}{ML^2/3}} = \sqrt{\frac{3K}{M}}$$

A more complicated example. Find τ_N

For


Equilibrium: $\circlearrowleft \Sigma M_A = 0$

$\Rightarrow W\frac{L}{2} - k\delta = 0$

push down small amount y:

$\circlearrowleft \Sigma M_A = I_A \ddot{\theta} \Rightarrow \underbrace{W\frac{L}{2} - k\delta}_{0} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-KL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

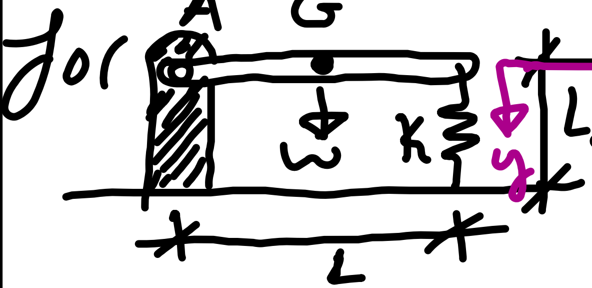
$\omega_n = \sqrt{\frac{KL^2}{I_A}}$ But $I_A = \bar{I} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4}$

$\Rightarrow I_A = ML^2\left(\frac{1}{12} + \frac{3}{12}\right) = ML^2/3$. Now

$\omega_N = \sqrt{\frac{KL^2}{ML^2/3}} = \sqrt{\frac{3k}{M}}$ & since $\tau_N = \frac{2\pi}{\omega_N}$

A more complicated example

Find τ_N for



Equilibrium: $\circlearrowright \Sigma M_A = 0$

$\Rightarrow w\frac{L}{2} - k\delta = 0$

push down small amount y:

$$\circlearrowright \Sigma M_A = I_A\ddot{\theta} \Rightarrow \cancel{w\frac{L}{2} - k\delta} - ky = I_A\ddot{\theta} \Rightarrow -ky = I_A\ddot{\theta}$$

But $y = L\sin\theta \Rightarrow$ for small θ: $y \cong L\theta \Rightarrow$

$-kL\theta = I_A\ddot{\theta}$ or $\ddot{\theta} = -\omega_n^2\theta$, where

$\omega_n = \sqrt{\frac{kL^2}{I_A}}$ But $I_A = \bar{I} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{12} + \frac{mL^2}{4}$

$\Rightarrow I_A = mL^2\left(\frac{1}{12} + \frac{3}{12}\right) = mL^2/3$. Now

$\omega_N = \sqrt{\frac{kL^2}{mL^2/3}} = \sqrt{\frac{3k}{m}}$ & since $\tau_N = \frac{2\pi}{\omega_N}$

then $\boxed{\tau_N = 2\pi\sqrt{\frac{m}{3k}}}$

Forced vibrations: Get into the form $A\ddot{x}+Bx = C\sin(\omega_F t)$

Forced vibrations: Get into the

form $A\ddot{x} + Bx = \underbrace{C\sin(\omega_F t)}_{\text{forcing term}}$

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* Homogeneous solution when $C=0$

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* Particular solution:

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Form $A\ddot{x}+Bx=\underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$

* Homogeneous solution when $C=0$

* Particular solution: Assume $x_p = X_m \sin(\omega_F t)$

Forced vibrations: Get into the

Form $A\ddot{x}+Bx=\underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$

* Homogeneous solution when $C=0$

* Particular solution: Assume $x_p = X_m \sin(\omega_F t)$

Example:

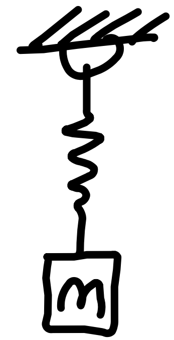
Forced vibrations: Get into the

form $A\ddot{x} + Bx = \underbrace{C\sin(\omega_F t)}_{\text{forcing term}}$

* Homogeneous solution when $C = 0$

* Particular solution: Assume $x_p = X_m \sin(\omega_F t)$

Example:



Forced vibrations: Get into the

Form $\quad A\ddot{x} + Bx = \underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$

* Homogeneous solution when $C = 0$

* Particular solution: Assume $x_p = X_m \sin(\omega_F t)$

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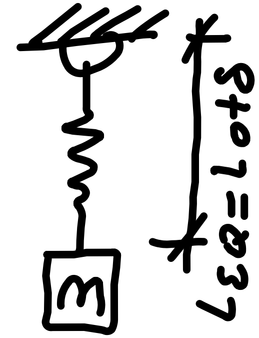

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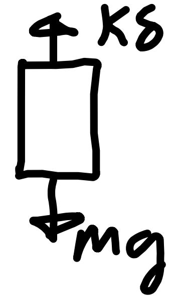

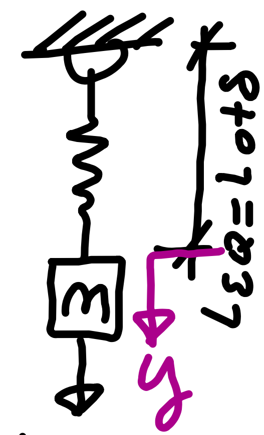

$$P = P_m \sin(\omega_F t)$$

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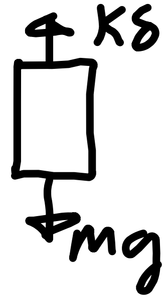
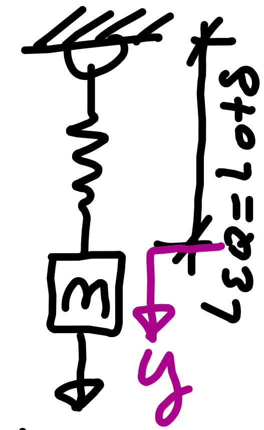


$\Sigma F_y = 0$



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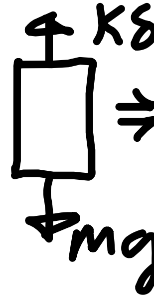

$\sum F_y = 0$

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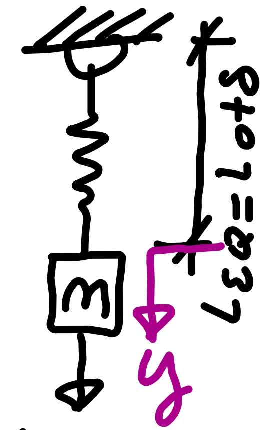

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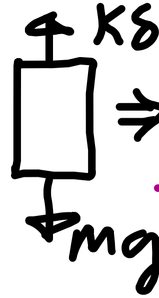

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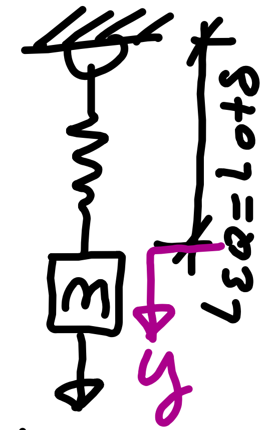

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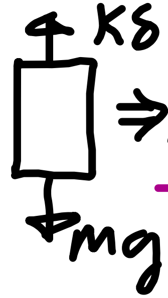

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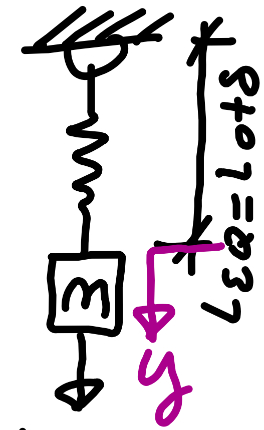
Non-Equilibrium: $\Sigma F_y = m\ddot{y}$



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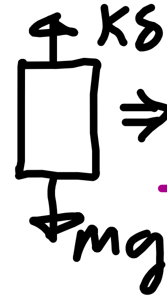

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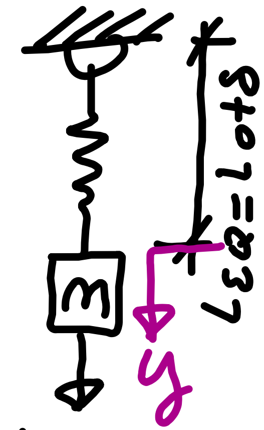

Non-Equilibrium: $\Sigma F_y = m\ddot{y}$

$\Rightarrow \quad mg - k\delta - ky + P = m\ddot{y}$

$P = P_m \sin(\omega_F t)$

Forced vibrations: Get into the form $A\ddot{x}+Bx=\underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$

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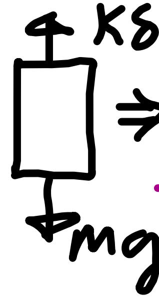

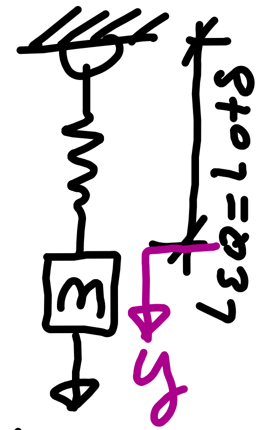

$P=P_m\sin(\omega_F t)$

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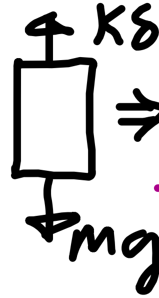

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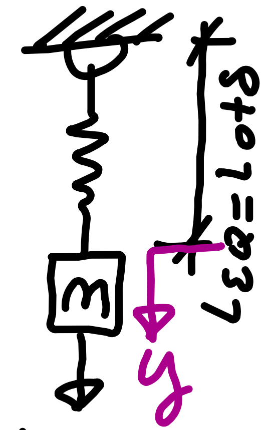

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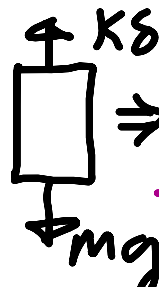

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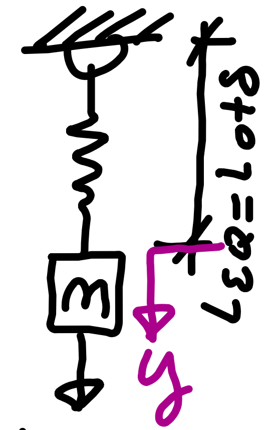

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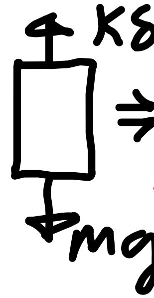

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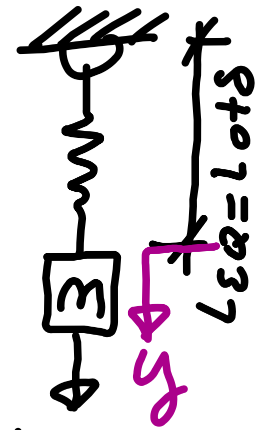

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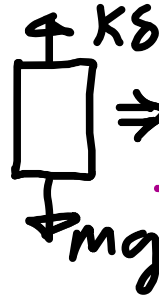

$\sum F_y = 0$

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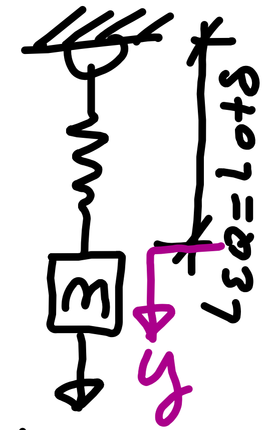

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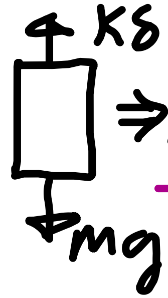

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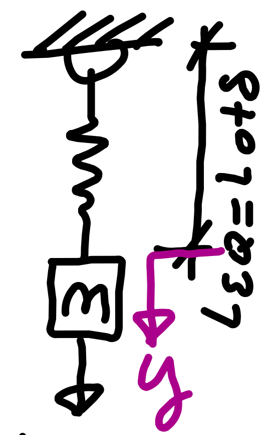

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$P = P_m \sin(\omega_F t)$

Homogeneous Part: Take $P_m = 0$

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Forced vibrations: Get into the form

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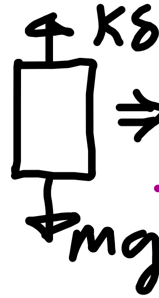

$\Sigma F_y = 0$
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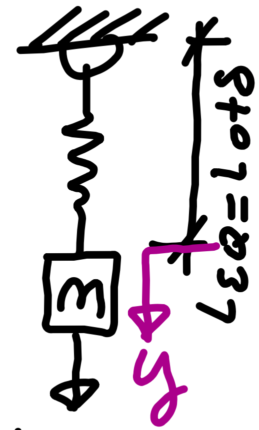

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Homogeneous Part: Take $P_m = 0$

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ASU

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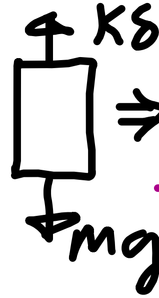

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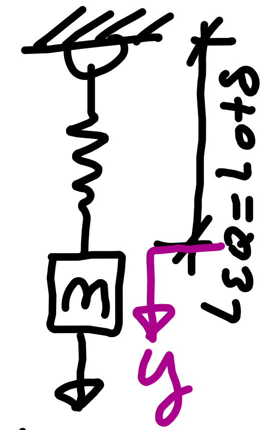

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Forced vibrations: Get into the form

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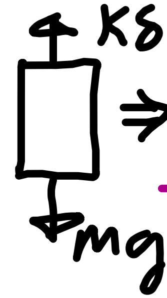

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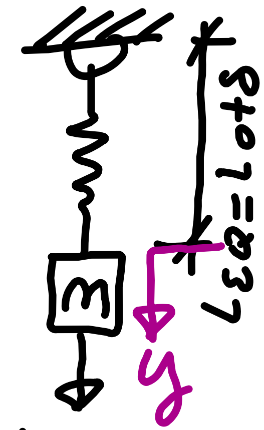

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$P=P_m\sin(\omega_F t)$

Homogeneous part: Take $P_m=0$

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Particular part: Assume $y=y_m\sin(\omega_F t)$

Forced vibrations: Get into the form $A\ddot{x} + Bx = \underbrace{C\sin(\omega_F t)}_{\text{Forcing term}}$

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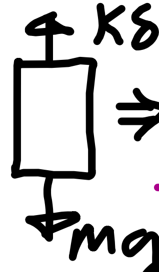

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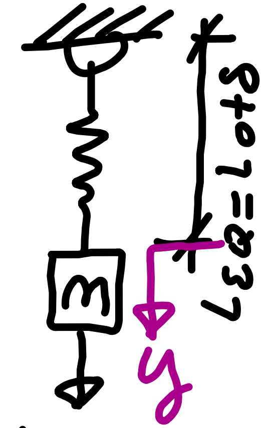

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$P = P_m \sin(\omega_F t)$

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Particular part: Assume $y = y_m \sin(\omega_F t)$

$\Rightarrow -m\omega_F^2 y_m + k y_m = P_m$

From previous $-m y_n \omega_F^2 + k y_n = P_n$

From previous $-m y_n \omega_F^2 + k y_n = P_n$

$\Rightarrow \quad y_n(k - m\omega_F^2) = P_n$

From previous $\quad -m y_m \omega_F^2 + k y_m = P_m$

$$\Rightarrow \quad y_m (k - m\omega_F^2) = P_m \Rightarrow y_m = \frac{P_m}{k - m\omega_F^2}$$

From previous $\quad -m y_m \omega_F^2 + k y_m = P_m$

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From previous $\quad -m y_n \omega_F^2 + k y_n = P_n$

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$$\Rightarrow \quad y_n = \frac{(P_n/k)}{1 - \left(\frac{m}{k}\right)\omega_F^2}$$

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$$\Rightarrow \quad y_m = \frac{(P_m/k)}{1 - \left(\frac{m}{k}\right)\omega_F^2} \qquad \text{But } \omega_N^2 = \frac{k}{m}$$

From previous $\quad -m y_m \omega_F^2 + k y_m = P_m$

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$$\Rightarrow \quad y_m = \frac{(P_m/k)}{1 - \left(\frac{m}{k}\right)\omega_F^2}$$ But $\omega_n^2 = \frac{k}{m}$ so

$$y_m = \frac{(P_m/A)}{1 - \omega_F^2/\omega_n^2}$$

From previous $-m y_m \omega_F^2 + k y_m = P_m$

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$$\boxed{y_m = \frac{(P_m/A)}{1 - \omega_F^2/\omega_n^2}}$$

From previous $-my_m\omega_F^2 + ky_m = P_m$

$$\Rightarrow \quad y_m(k - m\omega_F^2) = P_m \Rightarrow y_m = \left[\frac{P_m}{k - m\omega_F^2}\right]\left(\frac{1/k}{1/k}\right)$$

$$\Rightarrow \quad y_m = \frac{(P_m/k)}{1 - \left(\frac{m}{k}\right)\omega_F^2}$$ But $\omega_N^2 = \frac{k}{m}$ so

$$\boxed{y_m = \frac{(P_m/A)}{1 - \omega_F^2/\omega_N^2}}$$

& resonance when $\omega_F = \omega_N$

From previous $-m y_m \omega_F^2 + k y_m = P_m$

$\Rightarrow \quad y_m (k - m\omega_F^2) = P_m \Rightarrow \quad y_m = \left[\dfrac{P_m}{k - m\omega_F^2}\right]\left(\dfrac{1/k}{1/k}\right)$

$\Rightarrow \quad y_m = \dfrac{(P_m/k)}{1 - \left(\frac{m}{k}\right)\omega_F^2}$ But $\omega_n^2 = \dfrac{k}{m}$ so

$$\boxed{y_m = \frac{(P_m/k)}{1 - \omega_F^2/\omega_n^2}}$$

& resonance when $\omega_F = \omega_n$

Unforced damped vibrations:

Unforced damped vibrations: Form is

$$m\ddot{x}+c\dot{x}+kx = \cancel{0}$$

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$m\ddot{x} + c\dot{x} + kx = 0$, Assume $x e^{\lambda t}$

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$m\ddot{x} + c\dot{x} + kx = 0$, Assume $x e^{\lambda t} \Rightarrow$

$\lambda^2 m + \lambda c + k = 0$

Unforced damped vibrations: Form is

$m\ddot{x}+c\dot{x}+kx=0$, Assume $x\,e^{\lambda t} \Rightarrow$

$$\underbrace{\lambda^2 m+\lambda c+k=0}_{\text{Characteristic equation}}$$

Unforced damped vibrations: Form is

$$m\ddot{x}+c\dot{x}+kx=0, \text{ Assume } x\,e^{\lambda t} \Rightarrow$$

$$\underbrace{\lambda^2 m+\lambda c+k=0}_{\text{Characteristic equation}} \Rightarrow \lambda = \frac{c \pm \sqrt{c^2-4mk}}{2m}$$

Unforced damped vibrations: Form is

$m\ddot{x}+c\dot{x}+kx=0$, Assume $x\,e^{\lambda t}$ $\Rightarrow$

$\underbrace{\lambda^2 m+\lambda c+k=0}_{\text{Characteristic equation}}$ $\Rightarrow$ $\lambda=\dfrac{c\pm\sqrt{c^2-4mk}}{2m}$

* Critically damped when $c^2-4mk=0$

Unforced damped vibrations: Form is

$m\ddot{x} + c\dot{x} + kx = 0$, Assume $x e^{\lambda t} \Rightarrow$

$$\underbrace{\lambda^2 m + \lambda c + k = 0}_{\text{Characteristic equation}} \Rightarrow \lambda = \frac{c \pm \sqrt{c^2 - 4mk}}{2m}$$

* Critically damped when $c^2 - 4mk = 0$

{No vibrations, gets to equilibrium fastest}

Unforced damped vibrations: Form is

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* Critically damped when $c^2-4mk=0$

{No vibrations, gets to equilibrium fastest}

* Underdamped when $c^2-4mk<0$

Unforced damped vibrations: Form is

$m\ddot{x}+c\dot{x}+kx=0$, Assume $x e^{\lambda t} \Rightarrow$

$$\underbrace{\lambda^2 m+\lambda c+k=0}_{\text{Characteristic equation}} \Rightarrow \lambda=\frac{c\pm\sqrt{c^2-4mk}}{2m}$$

* Critically damped when $c^2-4mk=0$

{No vibrations, gets to equilibrium fastest}

* Underdamped when $c^2-4mk<0$

{vibrates}

Unforced damped vibrations: Form is

$m\ddot{x} + c\dot{x} + kx = 0$, Assume $x\,e^{\lambda t} \Rightarrow$

$$\underbrace{\lambda^2 m + \lambda c + k = 0}_{\text{Characteristic equation}} \Rightarrow \lambda = \frac{c \pm \sqrt{c^2 - 4mk}}{2m}$$

* Critically damped when $c^2 - 4mk = 0$
  {No vibrations, gets to equilibrium fastest}
* Underdamped when $c^2 - 4mk < 0$
  {vibrates}
* Overdamped when $c^2 - 4mk > 0$

ASU

Unforced damped vibrations: Form is

$m\ddot{x} + c\dot{x} + kx = 0$, Assume $x\, e^{\lambda t}$ $\Rightarrow$

$$\underbrace{\lambda^2 m + \lambda c + k = 0}_{\text{Characteristic equation}} \Rightarrow \lambda = \frac{c \pm \sqrt{c^2 - 4mk}}{2m}$$

* Critically damped when $c^2 - 4mk = 0$

{No vibrations, gets to equilibrium fastest}

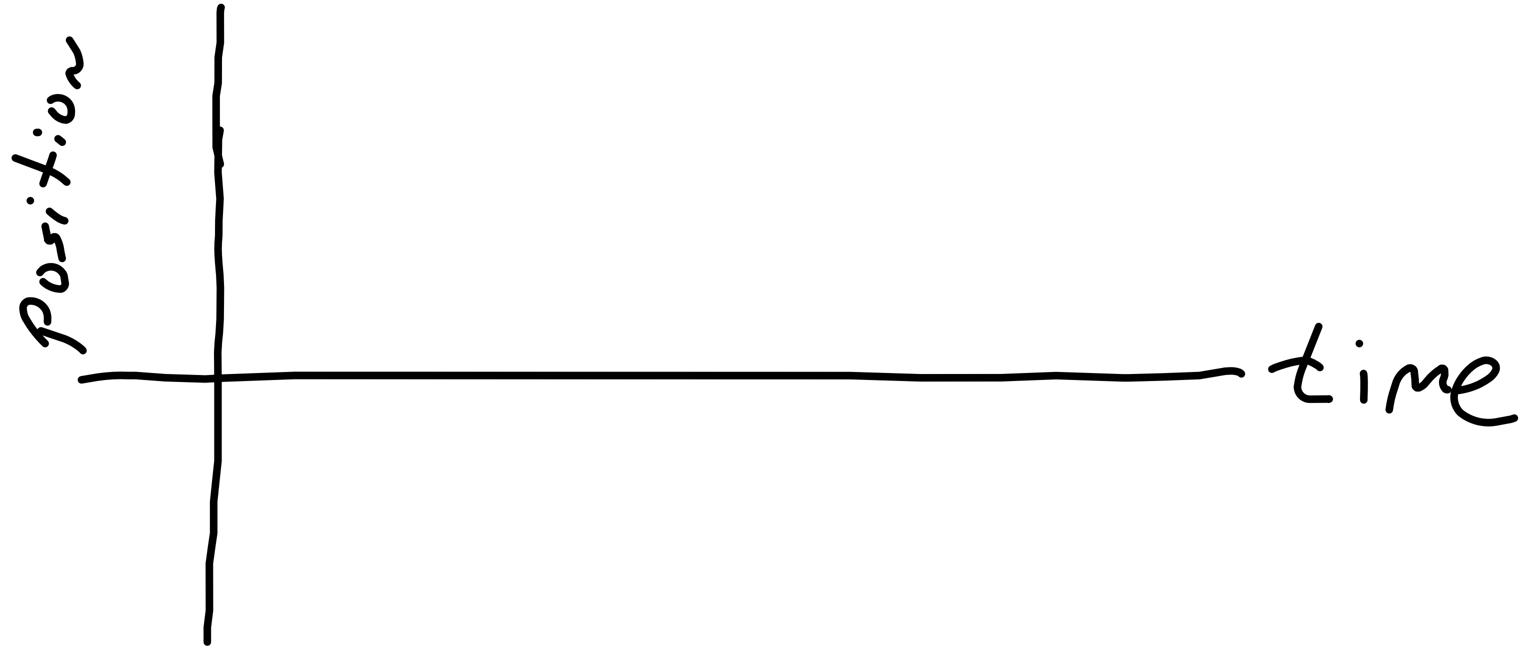
* Underdamped when $c^2 - 4mk < 0$

{vibrates}

* Overdamped when $c^2 - 4mk > 0$

{No Vibration}

ASU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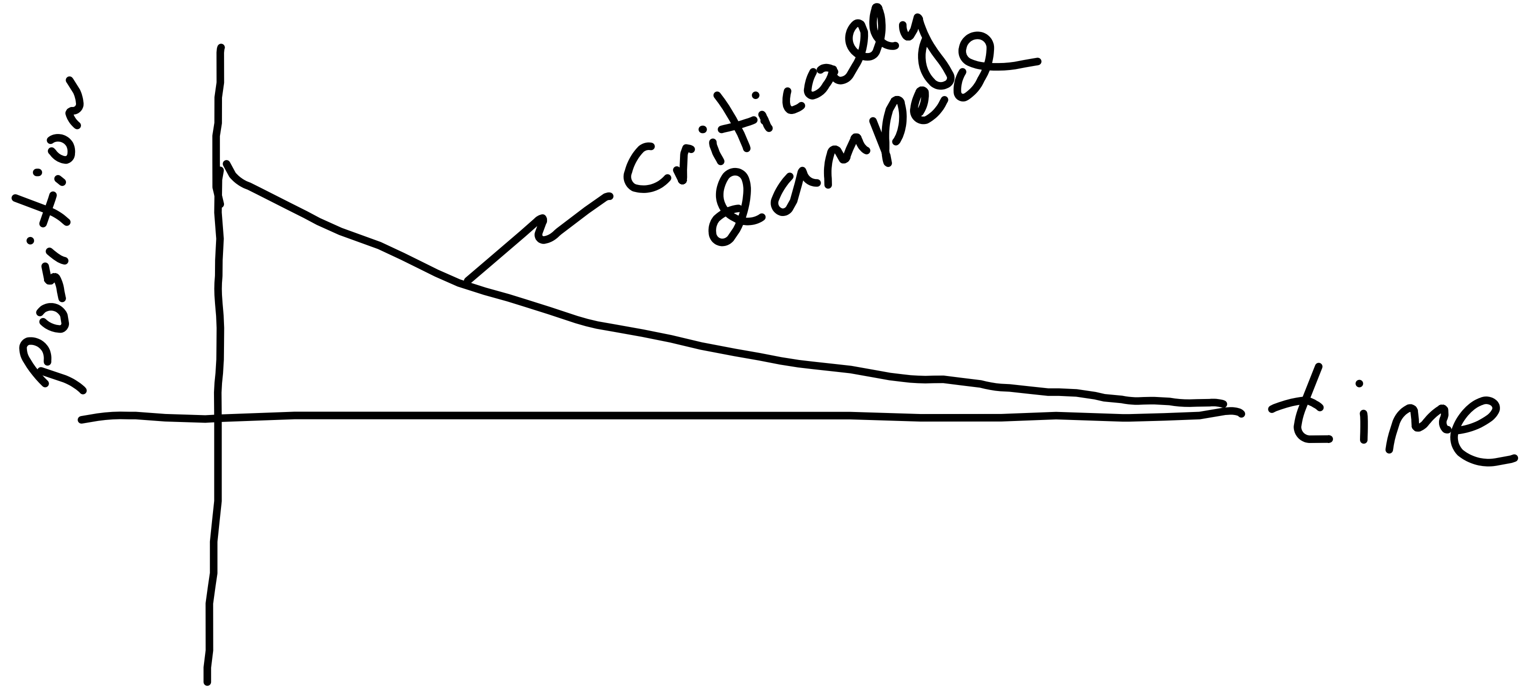

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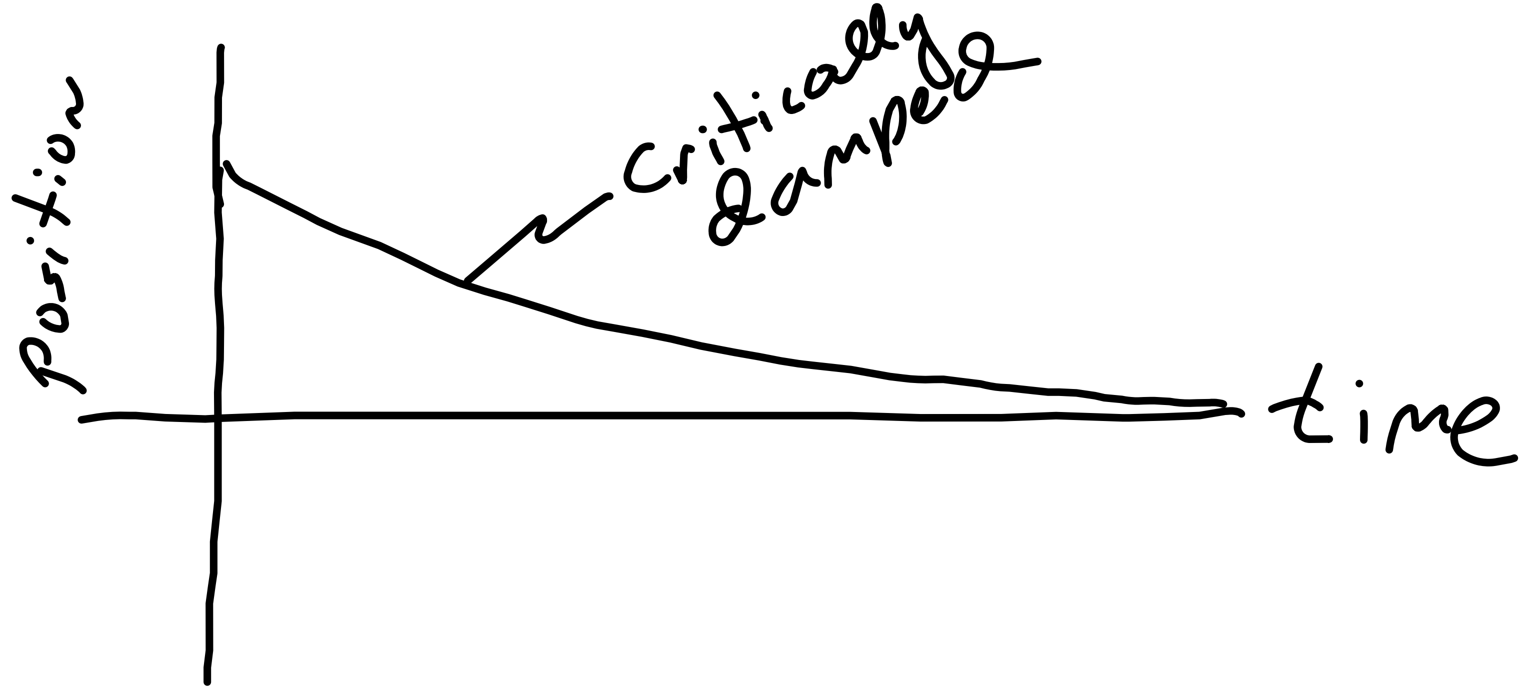
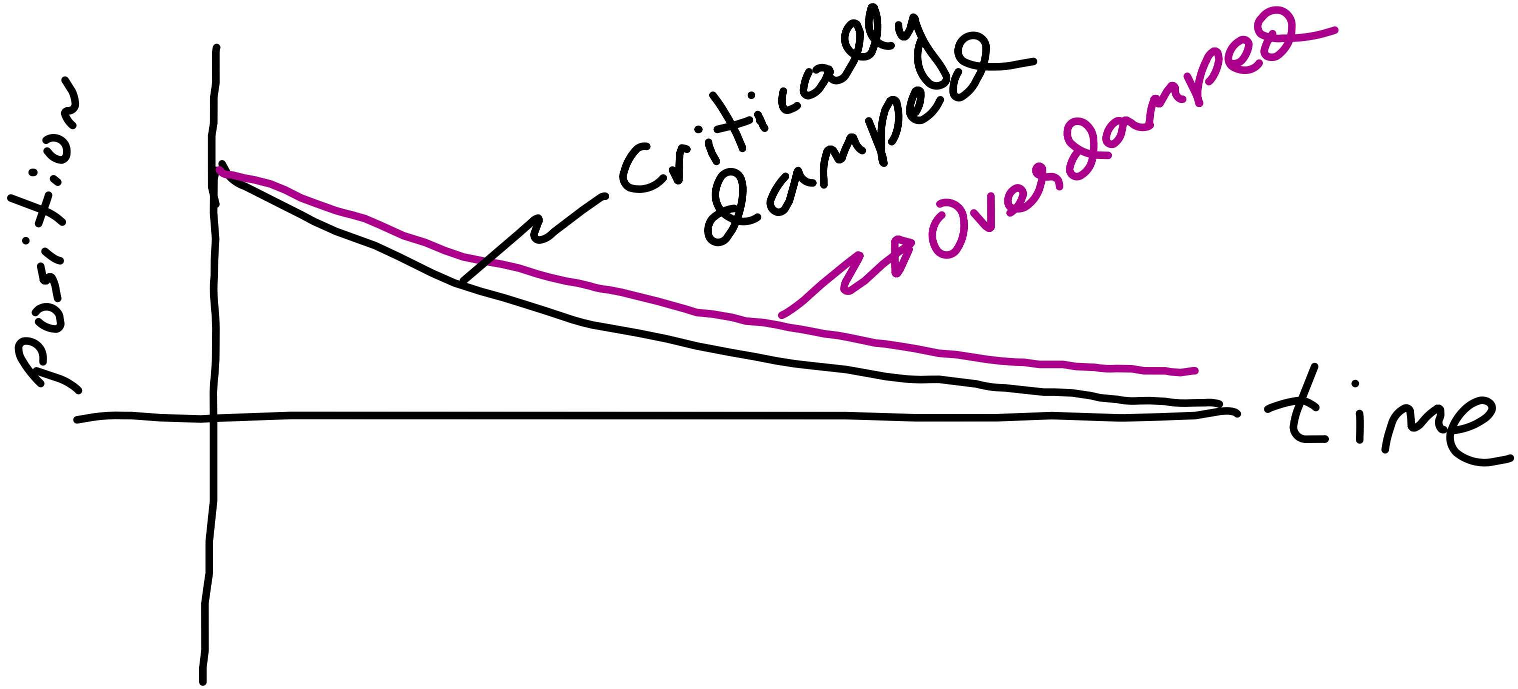
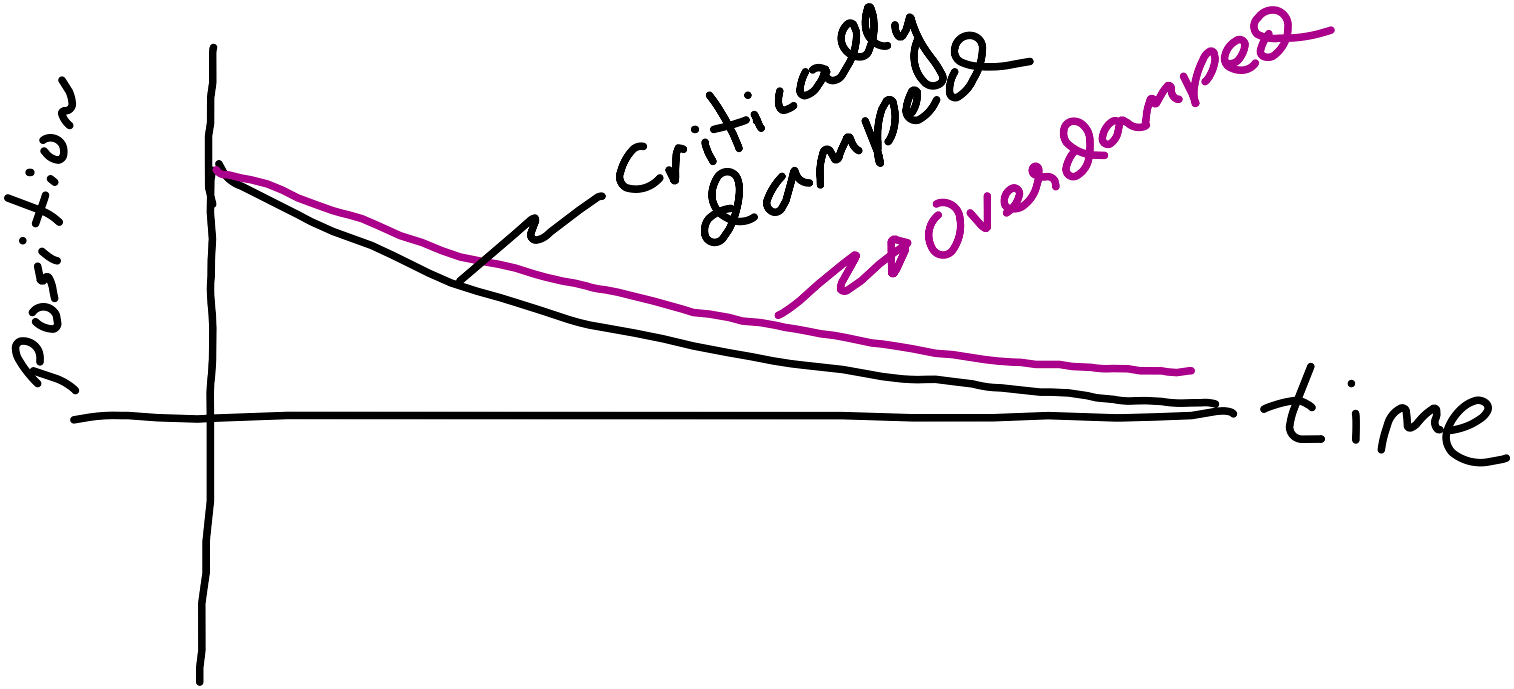


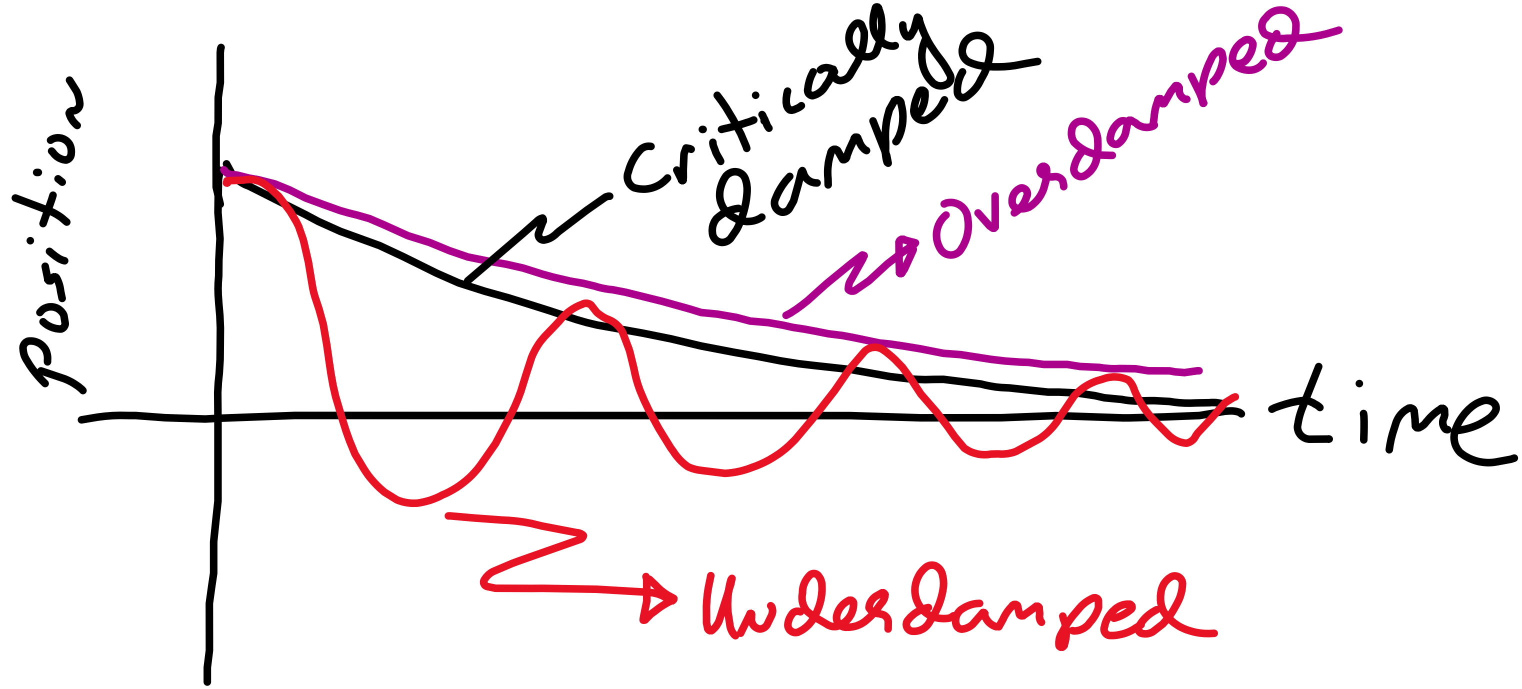
* No Vibration
* Gets to equilibrium fastest

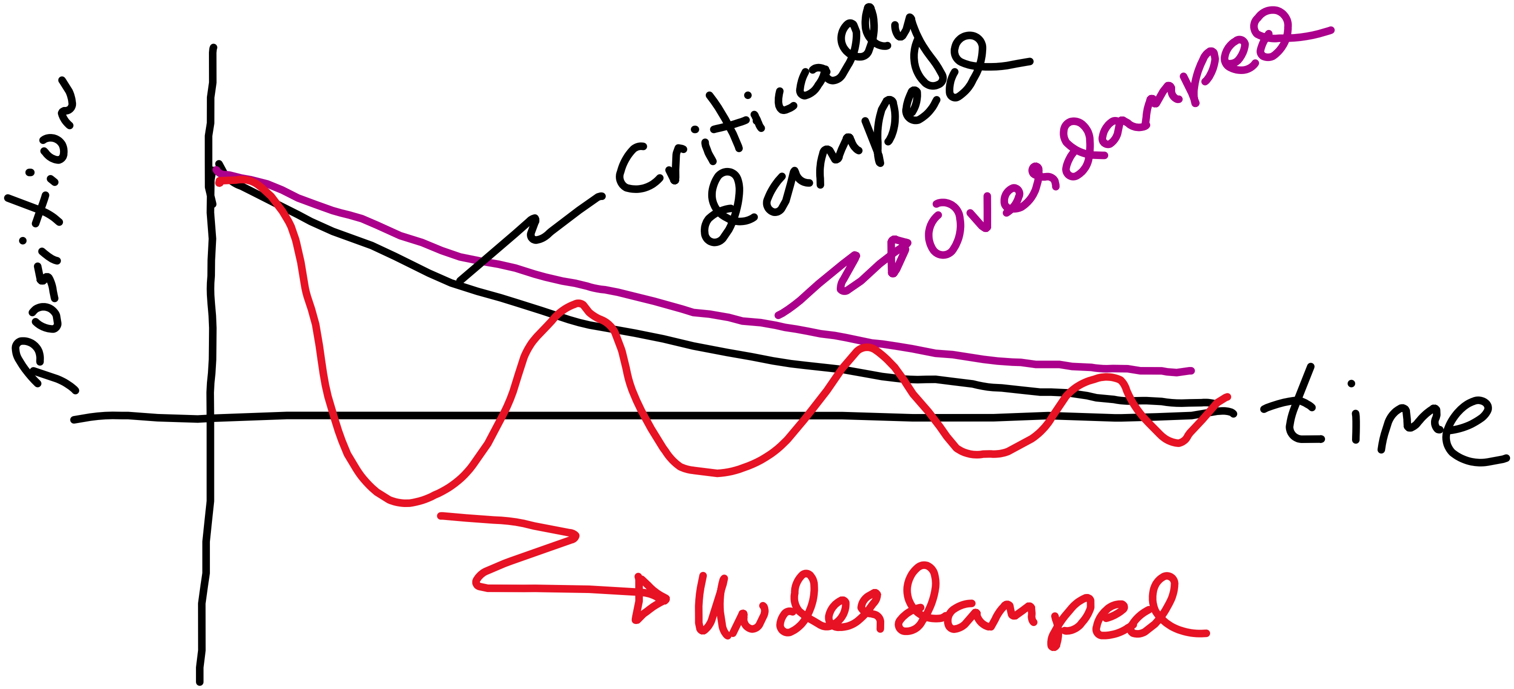


Position
time
Critically damped
Overdamped



* No vibration


Position
time
Critically damped
Overdamped
Underdamped



* Vibrates

The End