Today: 11.4 → Curvilinear Motion of particles

$\boxed{L4}$

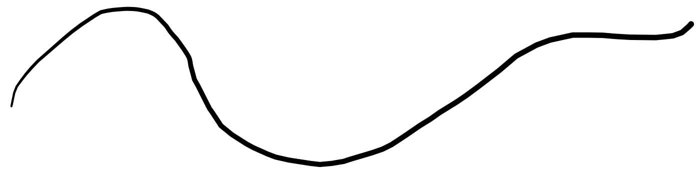
L4

Today: 11.4

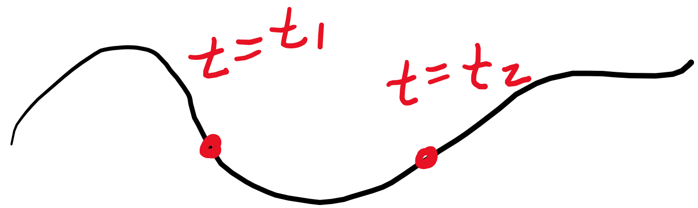
Thursday: 11.5 → Non-Rectangular Components

Now moving from 1d to 3d

Now moving from 1d to 3d

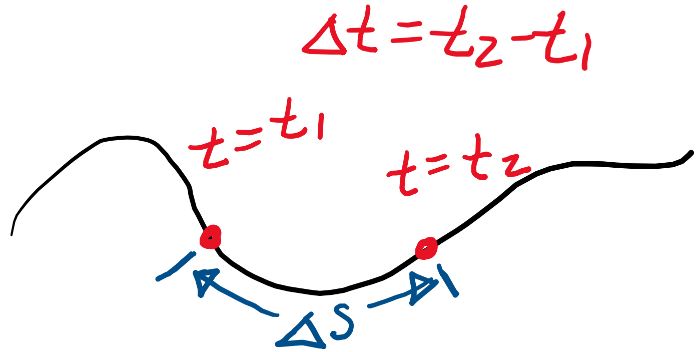


Now moving from 1d to 3d



path of some particle

Now moving from 1d to 3d

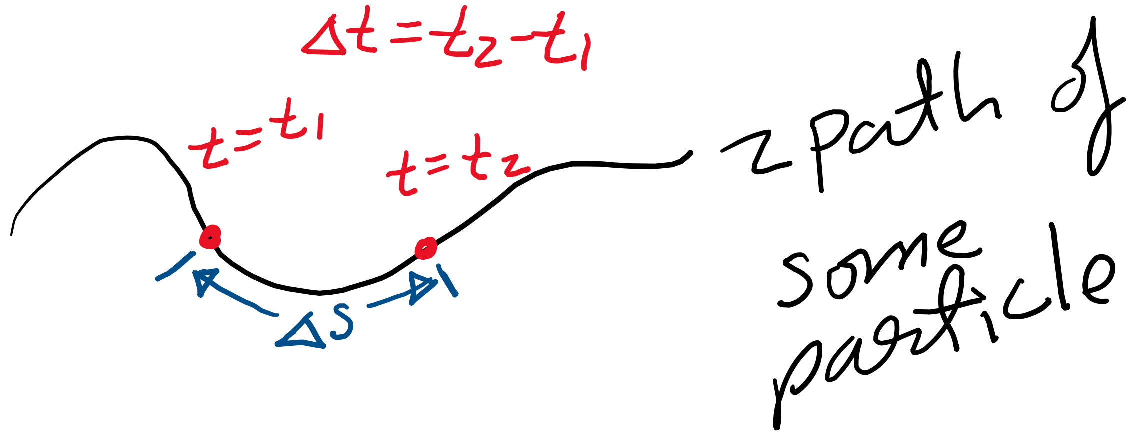


path of some particle

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$

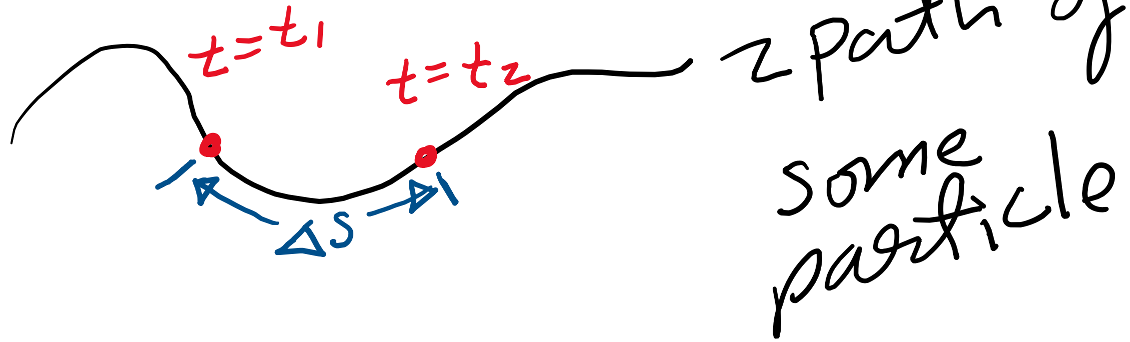


Now moving from 1d to 3d

$$v_{AVE} = \frac{\Delta s}{\Delta t}$$

$$\Rightarrow v = \frac{ds}{dt}$$

$\Delta t = t_2 - t_1$

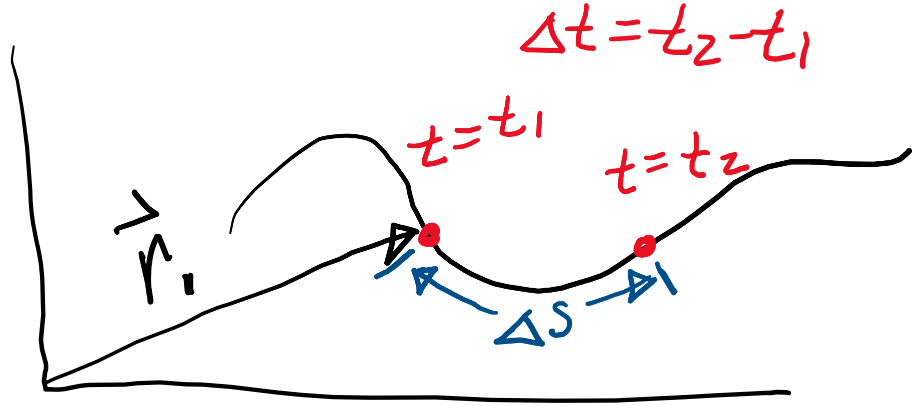


Note: scalar $v = |\vec{v}|$

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



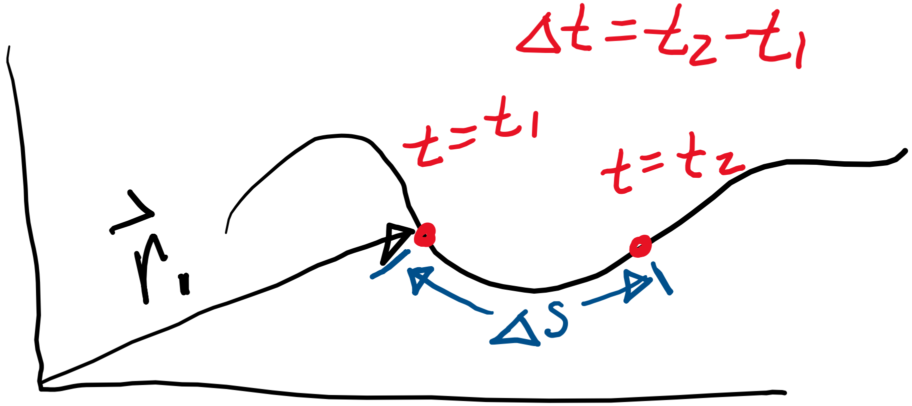
path of some particle

Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt}$

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



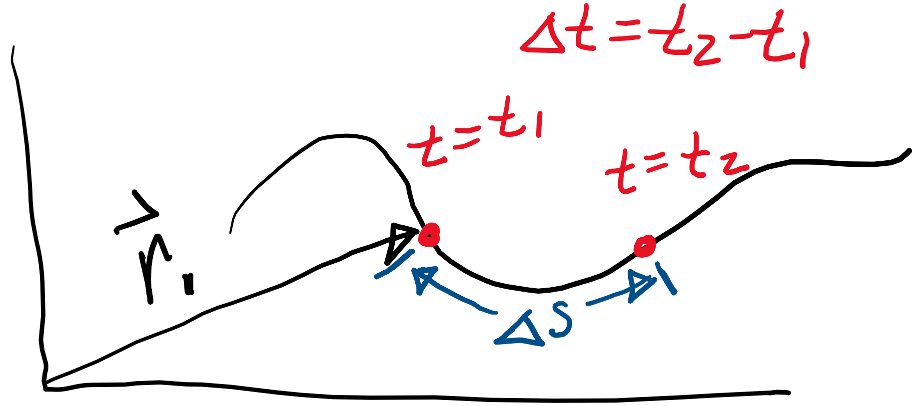
← path of some particle

Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

Now moving from 1d to 3d

$V_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow V = \frac{ds}{dt}$



← path of some particle

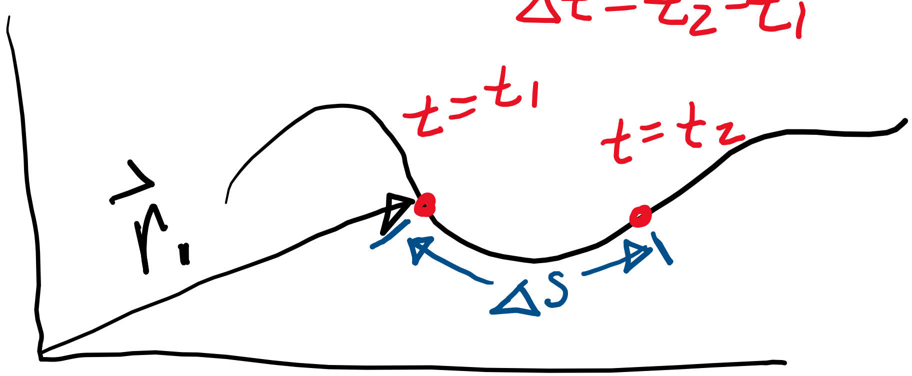
Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



path of some particle

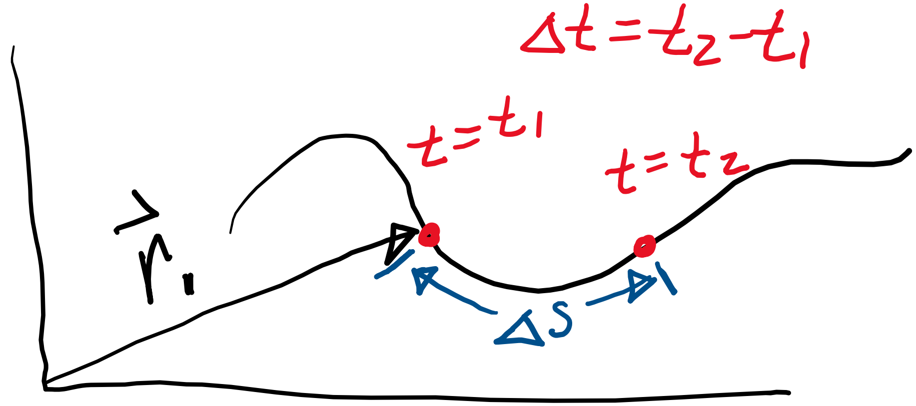
Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different: The first is a scalar.

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



path of some particle

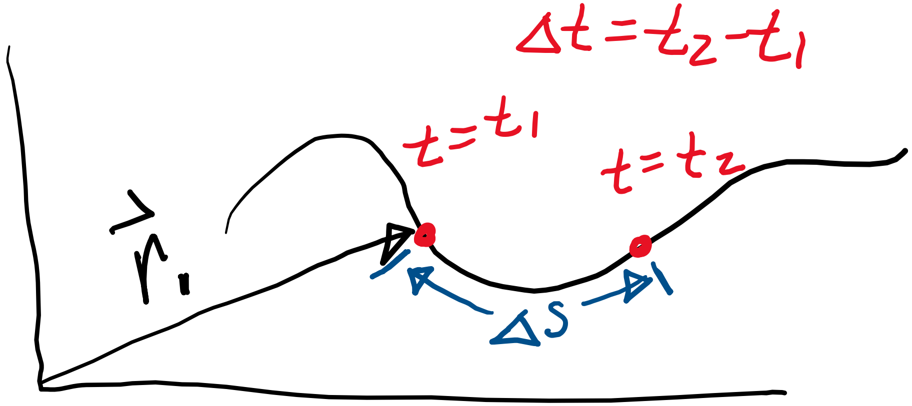
Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different: The first is a scalar & the second is a vector.

Now moving from 1d to 3d

$v_{AVE} = \frac{\Delta s}{\Delta t}$

$\Rightarrow v = \frac{ds}{dt}$



← path of some particle

Note: scalar $v = |\vec{v}|$ & vector $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt}$

make sure to notice that $v = \frac{ds}{dt}$ and $\vec{v} = \frac{d\vec{r}}{dt}$ are different: The first is a scalar & the second is a vector.

Unit vectors:

$\hat{i} \equiv$ one unit in x-direction

$\hat{j} \equiv$ one unit in y-direction

$\hat{k} \equiv$ one unit in z-direction

Other common notation

$$\hat{i} = \hat{x} = \hat{x}_1$$
$$\hat{j} = \hat{y} = \hat{x}_2$$
$$\hat{k} = \hat{z} = \hat{x}_3$$

Other common notation

$\hat{i} = \hat{x} = \hat{x}_1$

$\hat{j} = \hat{y} = \hat{x}_2$

$\hat{k} = \hat{z} = \hat{x}_3$

Position vector:

$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$

Other common notation

$\hat{i} = \hat{x} = \hat{x}_1$
$\hat{j} = \hat{y} = \hat{x}_2$
$\hat{k} = \hat{z} = \hat{x}_3$

Position vector:

$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$ or

$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$

Other common notation

$$\hat{i} = \hat{x} = \hat{x}_1$$
$$\hat{j} = \hat{y} = \hat{x}_2$$
$$\hat{k} = \hat{z} = \hat{x}_3$$

Position vector:

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \quad \text{or}$$
$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k} \quad \text{or}$$
$$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$$

Other common notation

$\hat{i} = \hat{x} = \hat{x}_1$

$\hat{j} = \hat{y} = \hat{x}_2$

$\hat{k} = \hat{z} = \hat{x}_3$

Position vector:

$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$ or

$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$ or

$\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$

Just needs to be clear and understandable

Other common notation

$\hat{i} = \hat{x} = \hat{x}_1$

$\hat{j} = \hat{y} = \hat{x}_2$

$\hat{k} = \hat{z} = \hat{x}_3$

Position vector:

$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}$ or

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ or

$\vec{r} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z}$

Velocity vector: $\vec{V} = \frac{dr_x}{dt}\hat{i} + \frac{dr_y}{dt}\hat{j} + \frac{dr_z}{dt}\hat{k}$

Other common notation

$\hat{i} = \hat{x} = \hat{x}_1$

$\hat{j} = \hat{y} = \hat{x}_2$

$\hat{k} = \hat{z} = \hat{x}_3$

Position vector:

$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}$ or

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ or

$\vec{r} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z}$

Velocity vector: $\vec{V} = \frac{dr_x}{dt}\hat{i} + \frac{dr_y}{dt}\hat{j} + \frac{dr_z}{dt}\hat{k}$

Acceleration vector:

$$\vec{a} = \frac{d^2r_x}{dt^2}\hat{i} + \frac{d^2r_y}{dt^2}\hat{j} + \frac{d^2r_z}{dt^2}\hat{k}$$

Let time derivitive be given by an over-dot

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k}$$

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Two over-dots for 2nd time derivitives

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Dot product:

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

<u>Dot product:</u> $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

<u>Magnitude-squared:</u>

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

<u>Dot product</u>: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

<u>Magnitude-squared</u>: $\vec{r}^2 \equiv \vec{r} \cdot \vec{r} = r_x^2 + r_y^2 + r_z^2$

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

<u>Dot product</u>: $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

<u>Magnitude-squared</u>: $\vec{r}^{\,2} = \vec{r}\cdot\vec{r} = r_x^2 + r_y^2 + r_z^2$

<u>Magnitude</u>:

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

Dot product: $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared: $\vec{r}^{\,2} \equiv \vec{r}\cdot\vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude: $r \equiv |\vec{r}| \equiv (\vec{r}\cdot\vec{r})^{1/2} = [r_x^2 + r_y^2 + r_z^2]^{1/2}$

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{\imath} + \dot{r}_y \hat{\jmath} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Magnitude-squared: $\vec{r}^{\,2} \equiv \vec{r} \cdot \vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude: $r \equiv |\vec{r}| \equiv (\vec{r} \cdot \vec{r})^{1/2} = [r_x^2 + r_y^2 + r_z^2]^{1/2}$

Cross product:

ASU

Let time derivitive be given by an over-dot such that

$$\vec{v} = \dot{r}_x \hat{\imath} + \dot{r}_y \hat{\jmath} + \dot{r}_z \hat{k} \quad \text{or} \quad \vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$$

& similarly $\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$

Dot product: $\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

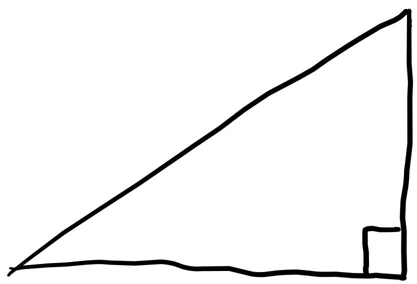
Magnitude-squared: $\vec{r}^{\,2} \equiv \vec{r}\cdot\vec{r} = r_x^2 + r_y^2 + r_z^2$

Magnitude: $r \equiv |\vec{r}| \equiv (\vec{r}\cdot\vec{r})^{1/2} = [r_x^2 + r_y^2 + r_z^2]^{1/2}$

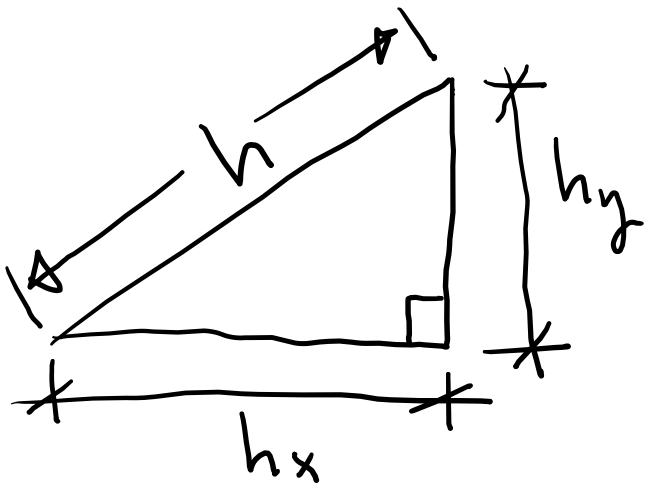
Cross product: $\vec{A}\times\vec{B} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$

Super quick trig review:

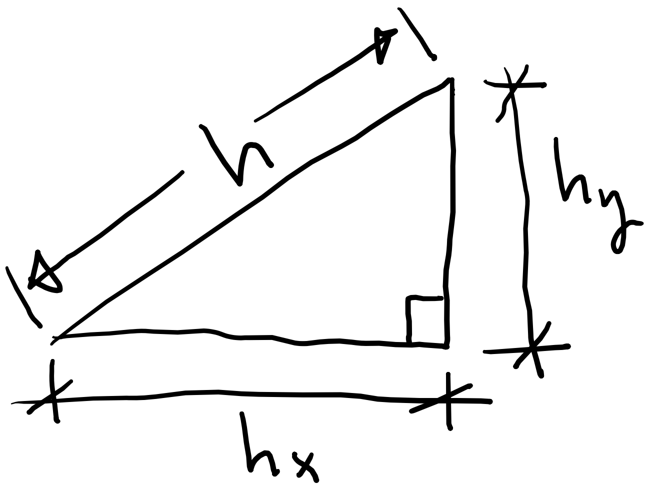
Super quick trig review:



Super quick trig review:

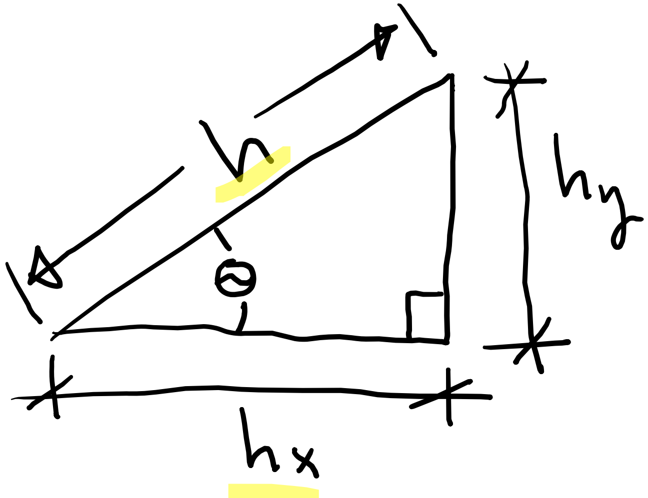


Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

Super quick trig review:

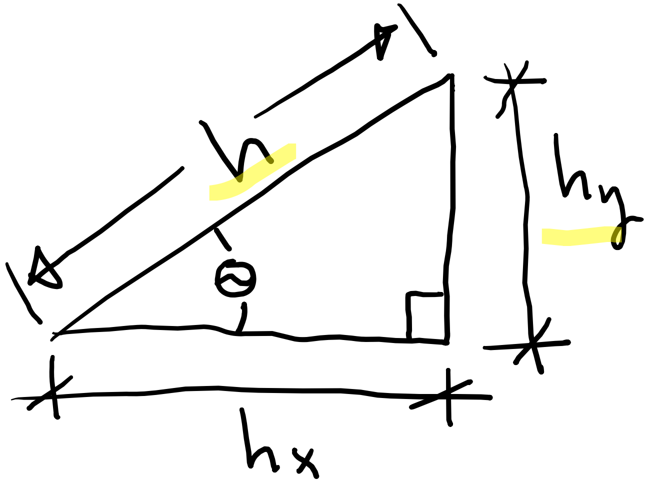


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{h_x}{h}$$

Super quick trig review:

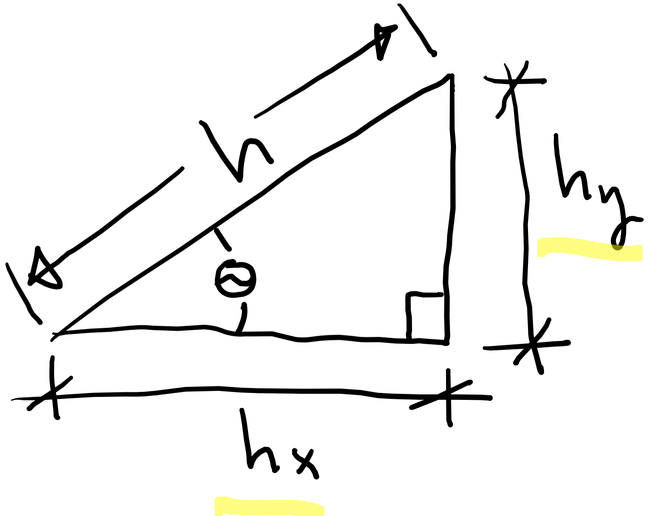


$$h^2 = h_x^2 + h_y^2$$

$$\cos\Theta = \frac{h_x}{h}, \quad \sin\Theta = \frac{h_y}{h}$$

$$\sin\Theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{h_y}{h}$$

Super quick trig review:

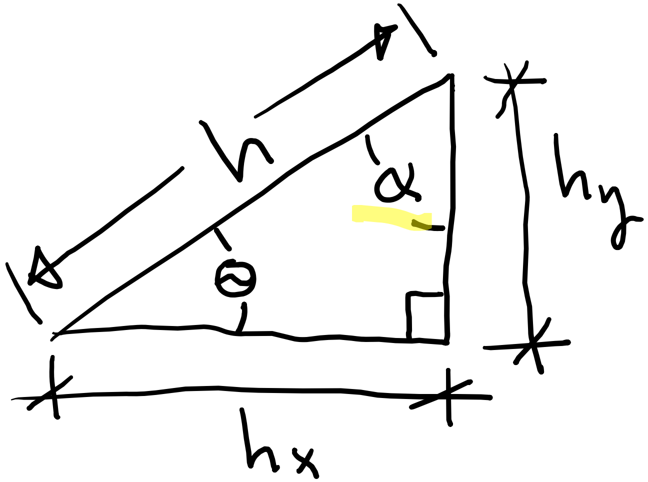


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\tan\theta = \frac{\text{opposite}}{\text{Adjacent}} = \frac{h_y}{h_x}$$

Super quick trig review:

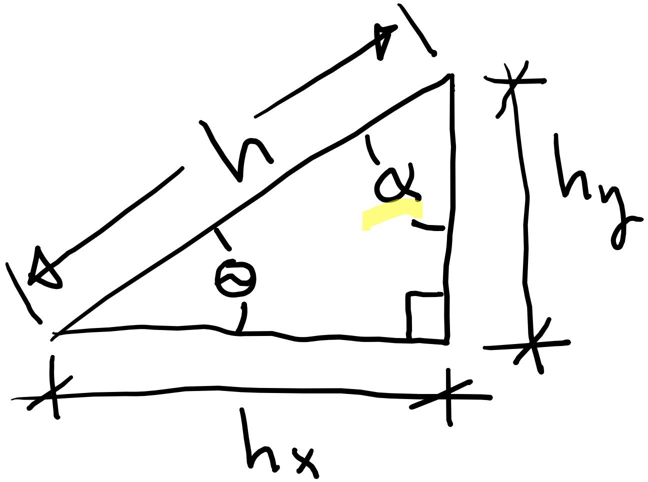


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}$$

Super quick trig review:

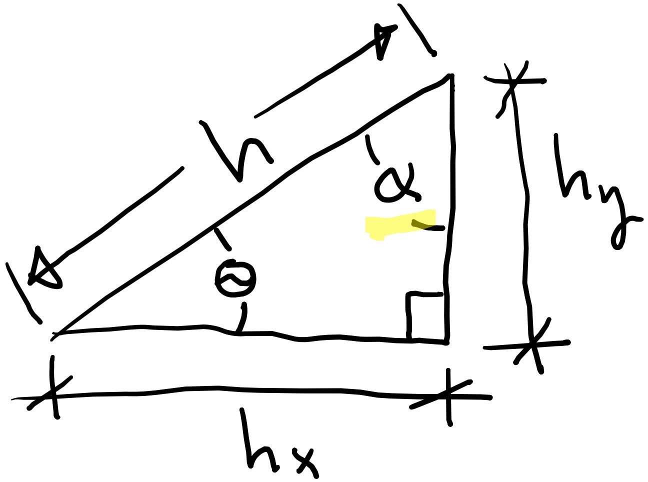


$$h^2 = h_x^2 + h_y^2$$

$$\cos\Theta = \frac{h_x}{h}, \quad \sin\Theta = \frac{h_y}{h}, \quad \tan\Theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}$$

Super quick trig review:

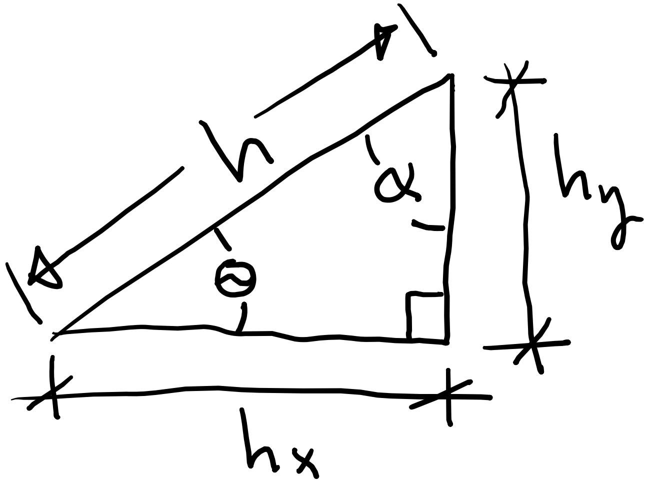


$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \sin\theta = \frac{h_y}{h}, \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \sin\alpha = \frac{h_x}{h}, \tan\alpha = \frac{h_x}{h_y}$$

Super quick trig review:



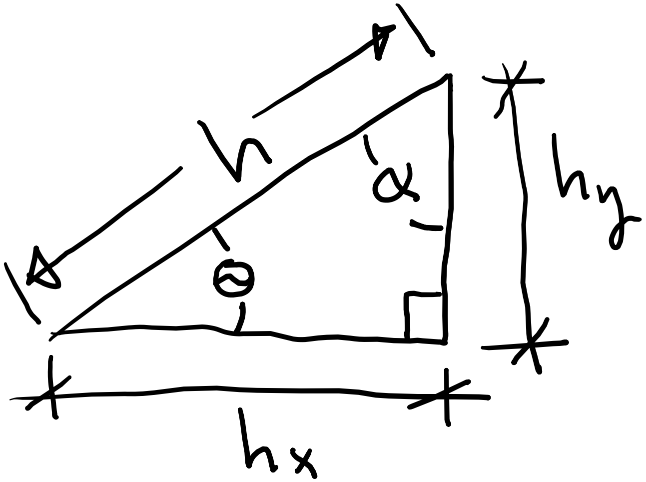
$$h^2 = h_x^2 + h_y^2$$

$$\cos\Theta = \frac{h_x}{h}, \sin\Theta = \frac{h_y}{h}, \tan\Theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \sin\alpha = \frac{h_x}{h}, \tan\alpha = \frac{h_x}{h_y}$$

$$\Theta + \alpha = 90^\circ$$

Super quick trig review:



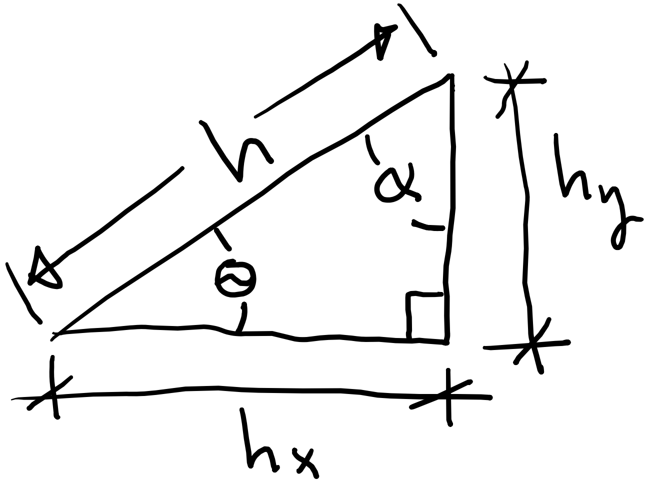
$$h^2 = h_x^2 + h_y^2$$

$$\cos\Theta = \frac{h_x}{h}, \quad \sin\Theta = \frac{h_y}{h}, \quad \tan\Theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

$$\Theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \Theta$$

Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \sin\theta = \frac{h_y}{h}, \tan\theta = \frac{h_y}{h_x}$$

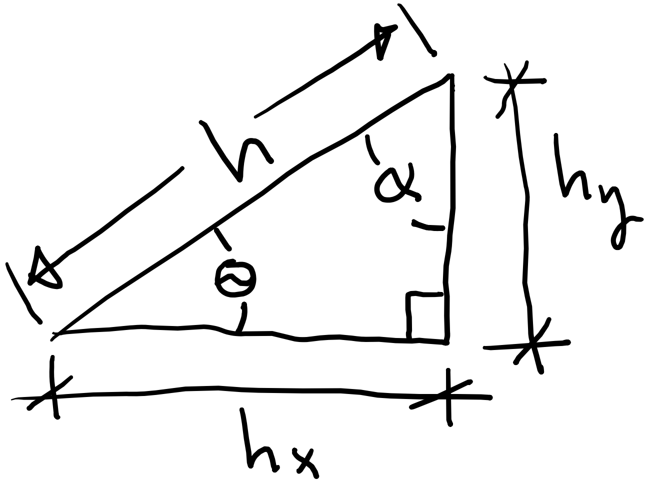
$$\cos\alpha = \frac{h_y}{h}, \sin\alpha = \frac{h_x}{h}, \tan\alpha = \frac{h_x}{h_y}$$

$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

$$\& \ \sin(90^\circ - \theta) = \cos\theta$$

Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h},\ \sin\theta = \frac{h_y}{h},\ \tan\theta = \frac{h_y}{h_x}$$

$$\cos\alpha = \frac{h_y}{h},\ \sin\alpha = \frac{h_x}{h},\ \tan\alpha = \frac{h_x}{h_y}$$

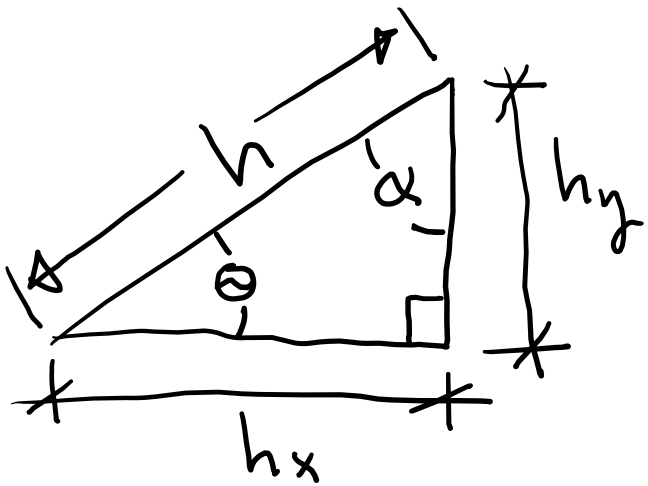
$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

$$\&\ \sin(90^\circ - \theta) = \cos\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1$$

Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

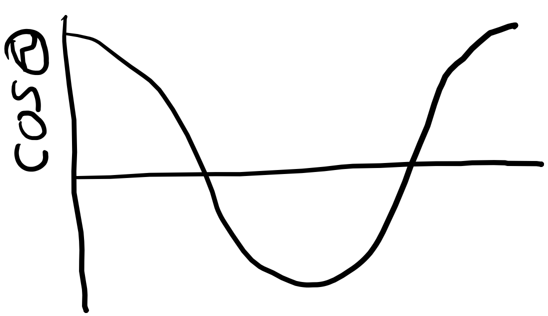
$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

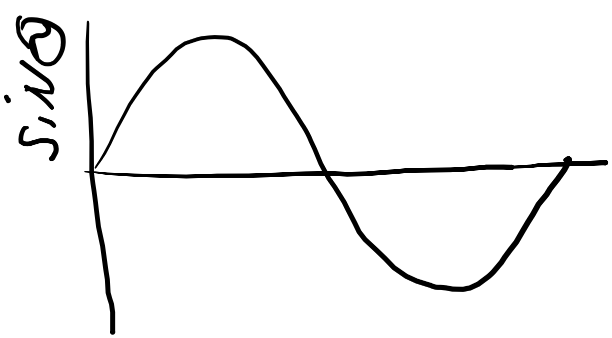
$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

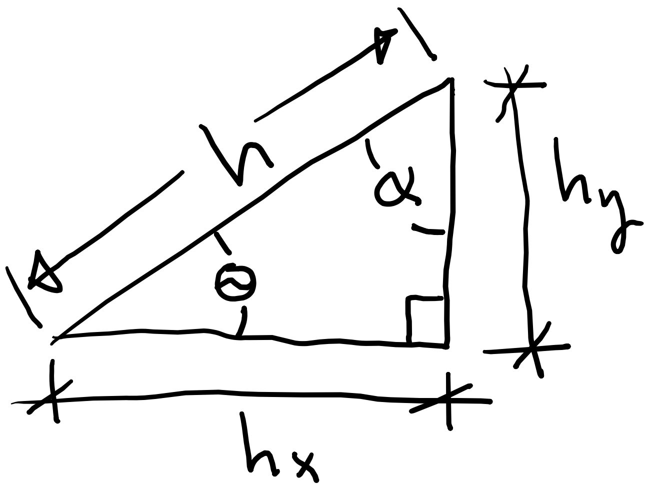
$$\& \ \sin(90^\circ - \theta) = \cos\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1$$





Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \ \sin\theta = \frac{h_y}{h}, \ \tan\theta = \frac{h_y}{h_x}$$

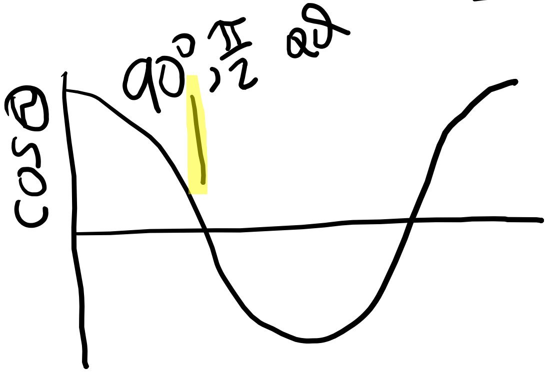
$$\cos\alpha = \frac{h_y}{h}, \ \sin\alpha = \frac{h_x}{h}, \ \tan\alpha = \frac{h_x}{h_y}$$

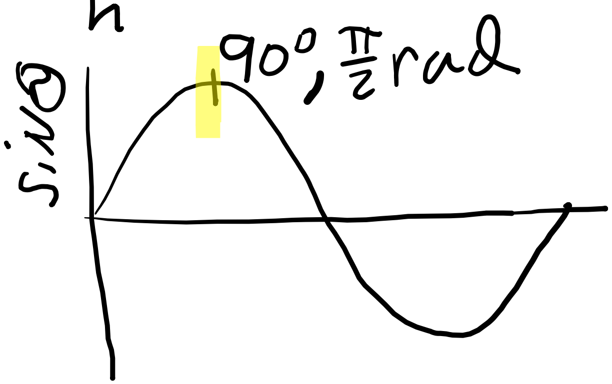
$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

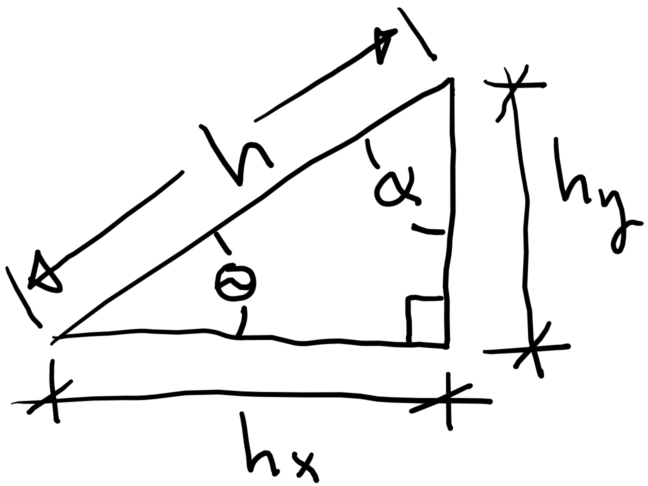
& $$\sin(90^\circ - \theta) = \cos\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1$$





Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

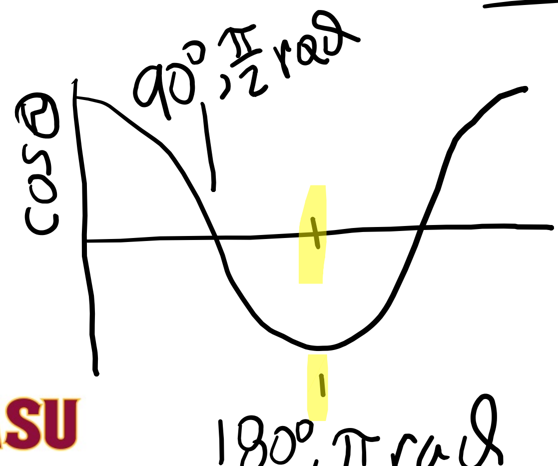
$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

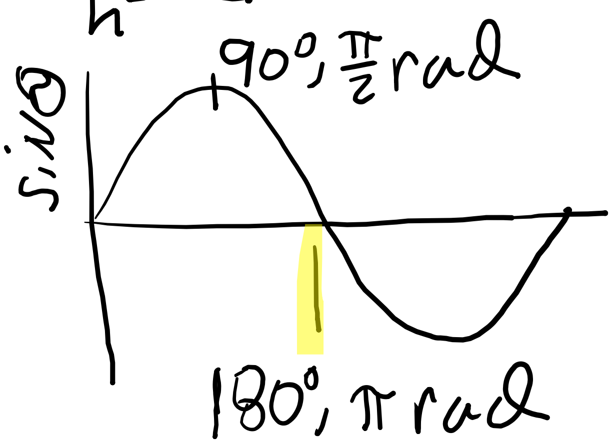
$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

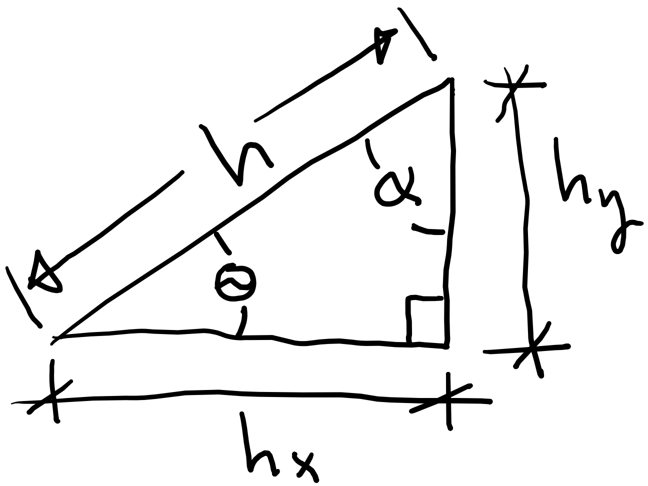
$$\& \ \sin(90^\circ - \theta) = \cos\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1$$





Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \ \sin\theta = \frac{h_y}{h}, \ \tan\theta = \frac{h_y}{h_x}$$

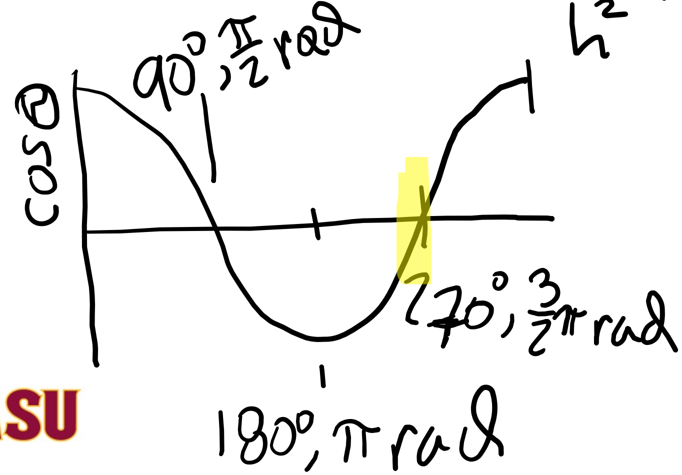
$$\cos\alpha = \frac{h_y}{h}, \ \sin\alpha = \frac{h_x}{h}, \ \tan\alpha = \frac{h_x}{h_y}$$

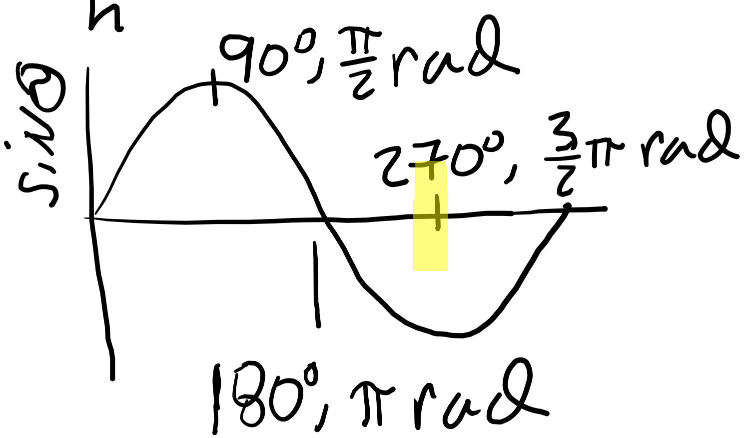
$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

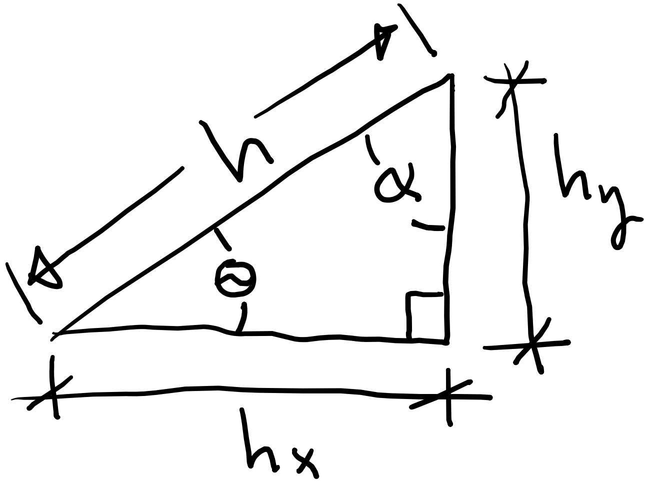
$$\& \ \sin(90^\circ - \theta) = \cos\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1$$





Super quick trig review:



$$h^2 = h_x^2 + h_y^2$$

$$\cos\theta = \frac{h_x}{h}, \quad \sin\theta = \frac{h_y}{h}, \quad \tan\theta = \frac{h_y}{h_x}$$

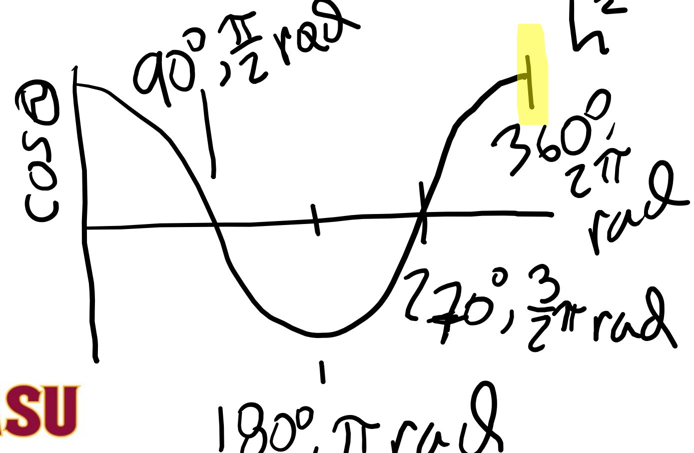
$$\cos\alpha = \frac{h_y}{h}, \quad \sin\alpha = \frac{h_x}{h}, \quad \tan\alpha = \frac{h_x}{h_y}$$

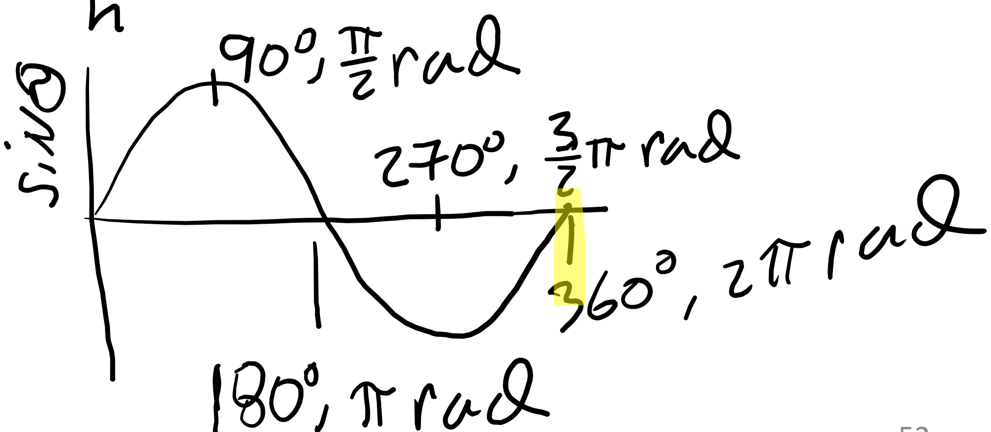
$$\theta + \alpha = 90^\circ \Rightarrow \alpha = 90^\circ - \theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

$$\& \ \sin(90^\circ - \theta) = \cos\theta$$

$$\cos^2\theta + \sin^2\theta = \frac{h_x^2 + h_y^2}{h^2} = \frac{h^2}{h^2} = 1$$





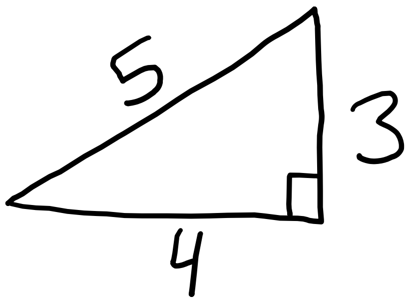
Special triangle [my favorite]

Special triangle [my favorite]

3, 4, 5 triangle

Special triangle [my favorite]

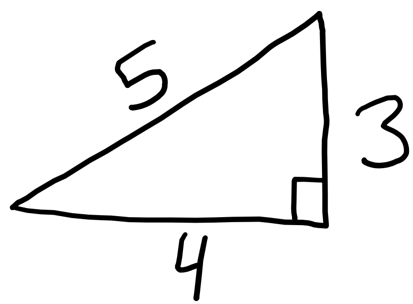
3, 4, 5 triangle



Special triangle [my favorite]

3, 4, 5 triangle: $h = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$

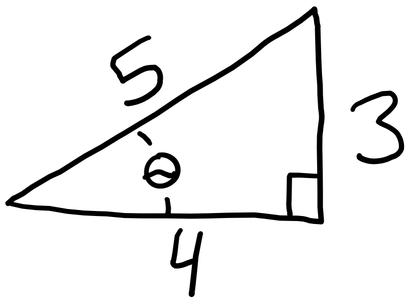
$\Rightarrow h = 5$



Special triangle [my favorite]

3, 4, 5 triangle: $h = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$

$\Rightarrow h = 5$

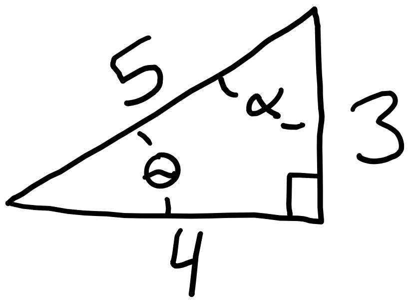


$\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$, $\tan\theta = \frac{3}{4}$

Special triangle [my favorite]

3, 4, 5 triangle: $h = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$

$\Rightarrow h = 5$



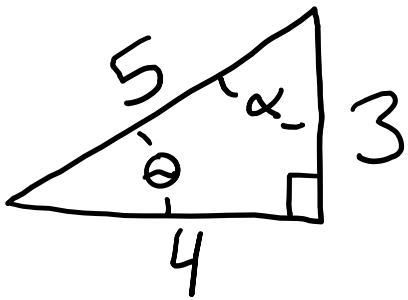
$\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$, $\tan\theta = \frac{3}{4}$

$\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$, $\tan\alpha = \frac{4}{3}$

Special triangle [my favorite]

3, 4, 5 triangle: $h = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$

$\Rightarrow h = 5$



$\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$, $\tan\theta = \frac{3}{4}$

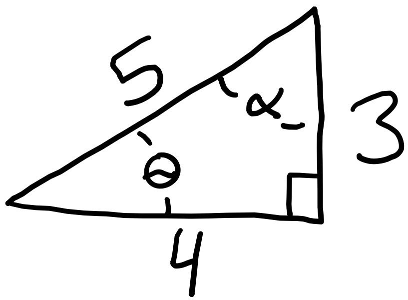
$\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$, $\tan\alpha = \frac{4}{3}$

Vector derivitives

Special triangle [my favorite]

3, 4, 5 triangle: $h = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$

$\Rightarrow h = 5$



$\cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}, \tan\theta = \frac{3}{4}$

$\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5}, \tan\alpha = \frac{4}{3}$

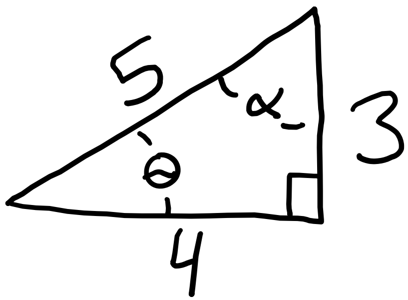
Vector derivitives

If $\vec{P}(u)$ & $\vec{Q}(u)$ are vector functions of scalar variable u

Special triangle [my favorite]

3, 4, 5 triangle: $h = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$

$\Rightarrow h = 5$



$\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$, $\tan\theta = \frac{3}{4}$

$\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$, $\tan\alpha = \frac{4}{3}$

Vector derivitives

If $\vec{P}(u)$ & $\vec{Q}(u)$ are vector functions of scalar variable u, then

$$\frac{d}{du}\left[\vec{P}+\vec{Q}\right] = \frac{d}{du}\vec{P} + \frac{d}{du}\vec{Q}$$

ASU

Vector derivitives [continued]

If $f(u)$ is a scalar function of variable u

Vector derivitives [continued]

If $f(u)$ is a scalar function of variable u, then

$$\frac{d}{du}(f\vec{P}) = \left(\frac{df}{du}\right)\vec{P} + f\left(\frac{d\vec{P}}{du}\right)$$

Vector derivitives [continued]

If $f(u)$ is a scalar function of variable u, then

$$\frac{d}{du}(f\vec{P}) = \left(\frac{df}{du}\right)\vec{P} + f\left(\frac{d\vec{P}}{du}\right)$$

& $$\frac{d}{du}(\vec{P}\cdot\vec{Q}) = \left(\frac{d\vec{P}}{du}\right)\cdot\vec{Q} + \vec{P}\cdot\left(\frac{dQ}{du}\right)$$

ASU

Vector derivitives [continued]

If $f(u)$ is a scalar function of variable u, then

$$\frac{d}{du}\left(f\vec{P}\right) = \left(\frac{df}{du}\right)\vec{P} + f\left(\frac{d\vec{P}}{du}\right)$$

& $$\frac{d}{du}\left(\vec{P}\cdot\vec{Q}\right) = \left(\frac{d\vec{P}}{du}\right)\cdot\vec{Q} + \vec{P}\cdot\left(\frac{d\vec{Q}}{du}\right)$$

& $$\frac{d}{du}\left(\vec{P}\times\vec{Q}\right) = \left(\frac{d\vec{P}}{du}\right)\times\vec{Q} + \vec{P}\times\left(\frac{d\vec{Q}}{du}\right)$$

ASU

Vector derivitives [continued]

If $f(u)$ is a scalar function of variable u, then

$$\frac{d}{du}(f\vec{P}) = \left(\frac{df}{du}\right)\vec{P} + f\left(\frac{d\vec{P}}{du}\right)$$

& $$\frac{d}{du}(\vec{P}\cdot\vec{Q}) = \left(\frac{d\vec{P}}{du}\right)\cdot\vec{Q} + \vec{P}\cdot\left(\frac{d\vec{Q}}{du}\right)$$

& $$\frac{d}{du}(\vec{P}\times\vec{Q}) = \left(\frac{d\vec{P}}{du}\right)\times\vec{Q} + \vec{P}\times\left(\frac{d\vec{Q}}{du}\right)$$

with $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

ASU

Vector derivitives [continued]

If $f(u)$ is a scalar function of variable u, then

$$\frac{d}{du}\left(f\vec{P}\right) = \left(\frac{df}{du}\right)\vec{P} + f\left(\frac{d\vec{P}}{du}\right)$$

& $$\frac{d}{du}\left(\vec{P}\cdot\vec{Q}\right) = \left(\frac{d\vec{P}}{du}\right)\cdot\vec{Q} + \vec{P}\cdot\left(\frac{d\vec{Q}}{du}\right)$$

& $$\frac{d}{du}\left(\vec{P}\times\vec{Q}\right) = \left(\frac{d\vec{P}}{du}\right)\times\vec{Q} + \vec{P}\times\left(\frac{d\vec{Q}}{du}\right)$$

with $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k} \Rightarrow$

$$\frac{d\vec{P}}{du} = \left(\frac{dP_x}{du}\right)\hat{i} + \left(\frac{dP_y}{du}\right)\hat{j} + \left(\frac{dP_z}{du}\right)\hat{k}$$

ASU

Vector derivitives [continued]

If $f(u)$ is a scalar function of variable u, then

$$\frac{d}{du}\left(f\vec{P}\right)=\left(\frac{df}{du}\right)\vec{P}+f\left(\frac{d\vec{P}}{du}\right)$$

& $$\frac{d}{du}\left(\vec{P}\cdot\vec{Q}\right)=\left(\frac{d\vec{P}}{du}\right)\cdot\vec{Q}+\vec{P}\cdot\left(\frac{d\vec{Q}}{du}\right)$$

& $$\frac{d}{du}\left(\vec{P}\times\vec{Q}\right)=\left(\frac{d\vec{P}}{du}\right)\times\vec{Q}+\vec{P}\times\left(\frac{d\vec{Q}}{du}\right)$$

with $\vec{P}=P_x\hat{i}+P_y\hat{j}+P_z\hat{k}\Rightarrow$

$$\frac{d\vec{P}}{du}=\left(\frac{dP_x}{du}\right)\hat{i}+\left(\frac{dP_y}{du}\right)\hat{j}+\left(\frac{dP_z}{du}\right)\hat{k}$$

Notation: $\frac{d}{dt}\vec{P}\equiv\dot{\vec{P}}=\dot{P}_x\hat{i}+\dot{P}_y\hat{j}+\dot{P}_z\hat{k}$

Relative motion

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0$$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$y = \left(-\frac{g}{2}\right)t^2 + V_{0y}t + y_0$ & $x = V_{0x}t + x_0$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t + y_0$ & $x = V_{ox}t + x_0$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t + y_0 \quad \& \quad x = V_{ox}t + x_0$$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$

$$\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$$

ASU

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t + y_0$ & $X = V_{ox}t + X_0$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$

$\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $X = V_{ox}t$

ASU

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\Rightarrow \vec{V}_{B/A} = \vec{V}_B - \vec{V}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Special case of projectile motion:

$y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t + y_o$ & $X = V_{ox}t + X_o$

Many times it is easiest to choose coordinates such that $X_o = y_o = 0$

$\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $X = V_{ox}t$

We have y as a function of t

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$$\Rightarrow \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t + y_0 \quad \& \quad x = V_{ox}t + x_0$$

Many times it is easiest to choose coordinates such that $x_0 = y_0 = 0$

$$\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t \quad \& \quad x = V_{ox}t$$

We have y as a function of t & x as a function of t.

ASU

Relative motion: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$$\Rightarrow \vec{V}_{B/A} = \vec{V}_B - \vec{V}_A \Rightarrow \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

Special case of projectile motion:

$$y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t + y_o \quad \& \quad x = V_{ox}t + x_o$$

Many times it is easiest to choose coordinates such that $x_o = y_o = 0$

$$\Rightarrow y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t \quad \& \quad x = V_{ox}t$$

We have y as a function of t & x as a function of t. We can get rid of the t variable & obtain an equation of y as a function of x.

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{0y}t$ & $x_0 = V_{0x}t$

We have (for $x_0 = y_0 = 0$) $y = (-\frac{g}{2})t^2 + V_{0y}t$ & $x_0 = V_{0x}t$

To get rid of t: First obtain t as a function of x

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_0 = V_{ox}t$

To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_0 = V_{ox}t$

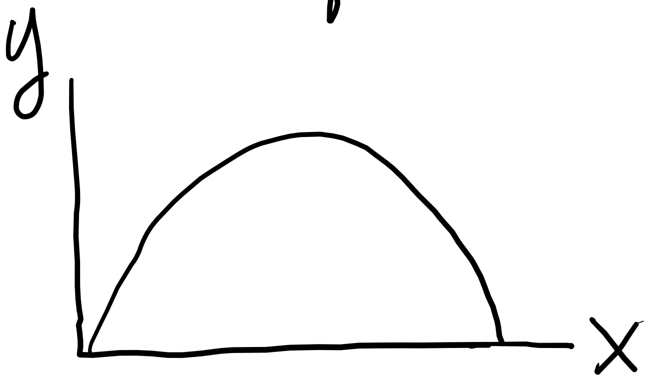
To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$. Now substitute into equation for y:

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_0 = V_{ox}t$

To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$. Now substitute into equation for y: $y = \left(\frac{-g}{2V_{ox}^2}\right)x^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{0y}t$ & $x_0 = V_{0x}t$

To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{0x}}$. Now substitute into equation for y: $y = \left(-\frac{g}{2V_{0x}^2}\right)x^2 + \left(\frac{V_{0y}}{V_{0x}}\right)x$

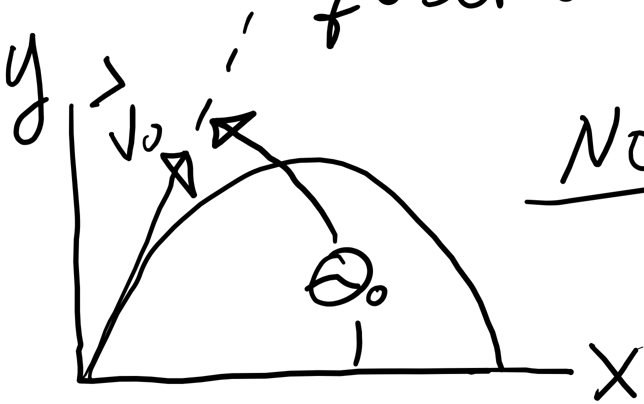


We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{0y}t$ & $x_0 = V_{0x}t$

To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{0x}}$. Now substitute into equation for y: $y = \left(-\frac{g}{2V_{0x}^2}\right)x^2 + \left(\frac{V_{0y}}{V_{0x}}\right)x$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_o = V_{ox}t$

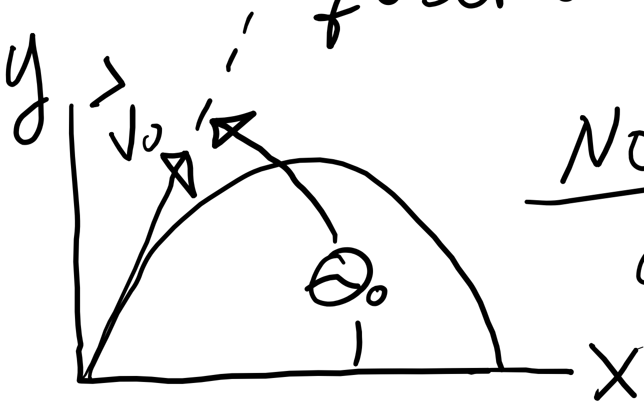
To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$. Now substitute into equation for y: $y = \left(-\frac{g}{2V_{ox}^2}\right)x^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$



<u>Note</u>: The initial velocity

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_0 = V_{ox}t$

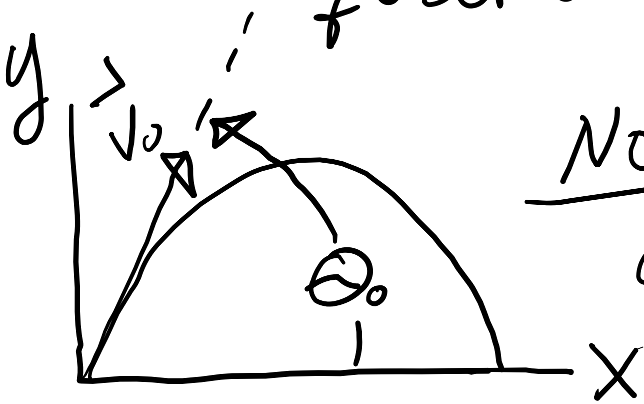
To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$. Now substitute into equation for y: $y = \left(\frac{-g}{2V_{ox}^2}\right)x^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$



Note: The initial velocity given by $\vec{V}_0 = V_{ox}\hat{i} + V_{oy}\hat{j}$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_0 = V_{ox}t$

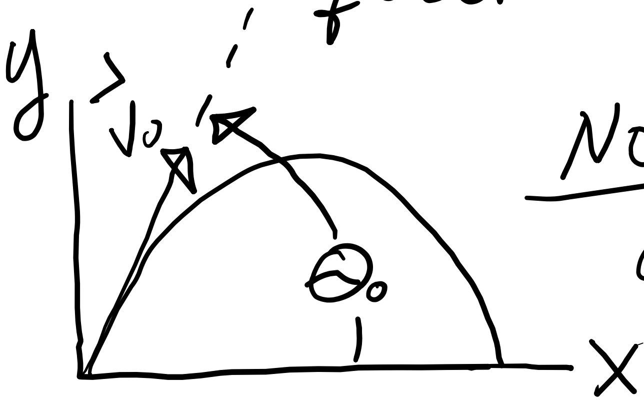
To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$. Now substitute into equation for y: $y = \left(\frac{-g}{2V_{ox}^2}\right)x^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$



<u>Note</u>: The initial velocity given by $\vec{V}_0 = V_{ox}\hat{i} + V_{oy}\hat{j}$ starts out at an angle θ_0 with respect to the x-axis

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_0 = V_{ox}t$

To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$. Now substitute into equation for y: $y = \left(\frac{-g}{2V_{ox}^2}\right)x^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$



Note: The initial velocity given by $\vec{V}_0 = V_{ox}\hat{i} + V_{oy}\hat{j}$ starts out at an angle θ_0 with respect to the x-axis

$\Rightarrow V_{ox} = V_0 \cos\theta_0$ & $V_{oy} = V_0 \sin\theta_0$

ASU

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_0 = V_{ox}t$

To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$. Now substitute into equation for y: $y = \left(-\frac{g}{2V_{ox}^2}\right)x^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$

Note: The initial velocity given by $\vec{V}_0 = V_{ox}\hat{i} + V_{oy}\hat{j}$ starts out at an angle θ_0 with respect to the x-axis

$\Rightarrow V_{ox} = V_0 \cos\theta_0$ & $V_{oy} = V_0 \sin\theta_0 \Rightarrow$

$$\frac{V_{oy}}{V_{ox}} = \frac{\sin\theta_0}{\cos\theta_0} = \tan\theta_0$$

We have (for $x_0 = y_0 = 0$) $y = \left(-\frac{g}{2}\right)t^2 + V_{oy}t$ & $x_0 = V_{ox}t$

To get rid of t: First obtain t as a function of x: $t = \frac{x}{V_{ox}}$. Now substitute into equation for y: $y = \left(\frac{-g}{2V_{ox}^2}\right)x^2 + \left(\frac{V_{oy}}{V_{ox}}\right)x$

Note: The initial velocity given by $\vec{V}_0 = V_{ox}\hat{i} + V_{oy}\hat{j}$ starts out at an angle θ_0 with respect to the x-axis

$\Rightarrow V_{ox} = V_0 \cos\theta_0$ & $V_{oy} = V_0 \sin\theta_0 \Rightarrow$

$\frac{V_{oy}}{V_{ox}} = \frac{\sin\theta_0}{\cos\theta_0} = \tan\theta_0 \Rightarrow y = \left(\frac{-g}{2V_{ox}^2}\right)x^2 + x\tan\theta_0$

ASU

We have $y = \left(\frac{-g}{2v_{0x}^2}\right)x^2 + x\tan\theta_0$

We have $y = \left(\frac{-g}{2V_{0x}^2}\right)x^2 + x\tan\theta_0$, but

$$\frac{1}{V_{0x}^2} = \left(\frac{1}{V_0^2}\right)\left(\frac{1}{\cos^2\theta_0}\right)$$

We have $y = \left(\frac{-g}{2V_{0x}^2}\right)x^2 + x\tan\theta_0$, but

$\frac{1}{V_{0x}^2} = \left(\frac{1}{V_0^2}\right)\left(\frac{1}{\cos^2\theta_0}\right)$ so $y = \left[\frac{-g}{2V_0^2\cos^2\theta_0}\right]x^2 + x\tan\theta_0$

We have $y = \left(\frac{-g}{2V_{ox}^2}\right)x^2 + x\tan\theta_0$, but

$\frac{1}{V_{ox}^2} = \left(\frac{1}{V_0^2}\right)\left(\frac{1}{\cos^2\theta_0}\right)$ so $y = \left[\frac{-g}{2V_0^2\cos^2\theta_0}\right]x^2 + x\tan\theta_0$

We can write $\frac{1}{\cos^2\theta_0} = \tan^2\theta_0 + 1$ by

Noticing that $\sin^2\theta_0 + \cos^2\theta_0 = 1 \Rightarrow$

$$\frac{1}{\cos^2\theta_0} = \frac{\sin^2\theta_0 + \cos^2\theta_0}{\cos^2\theta_0} = \frac{\sin^2\theta_0}{\cos^2\theta_0} + 1 = \tan^2\theta_0 + 1$$

We have $y=\left(\frac{-g}{2V_{0x}^2}\right)x^2+x\tan\theta_0$, but

$\frac{1}{V_{0x}^2}=\left(\frac{1}{V_0^2}\right)\left(\frac{1}{\cos^2\theta_0}\right)$ so $y=\left[\frac{-g}{2V_0^2\cos^2\theta_0}\right]x^2+x\tan\theta_0$

We can write $\frac{1}{\cos^2\theta_0}=\tan^2\theta_0+1$ by

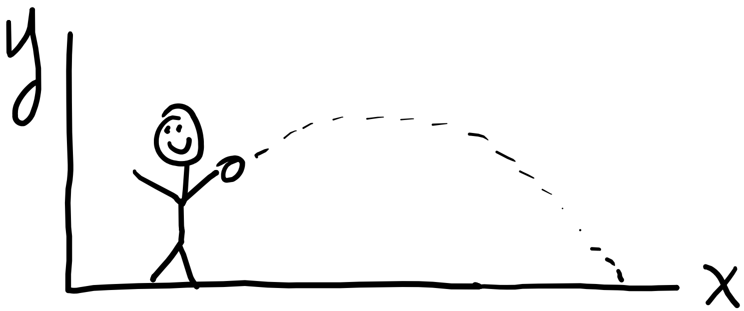
Noticing that $\sin^2\theta_0+\cos^2\theta_0=1 \Rightarrow$

$$\frac{1}{\cos^2\theta_0}=\frac{\sin^2\theta_0+\cos^2\theta_0}{\cos^2\theta_0}=\frac{\sin^2\theta_0}{\cos^2\theta_0}+1=\tan^2\theta_0+1$$

So $y=\left(\frac{-g}{2V_0^2}\right)x^2\left(\tan^2\theta_0+1\right)+x\tan\theta_0$

ASU

Notes on problem $\boxed{11.89}$



Ball thrown such that

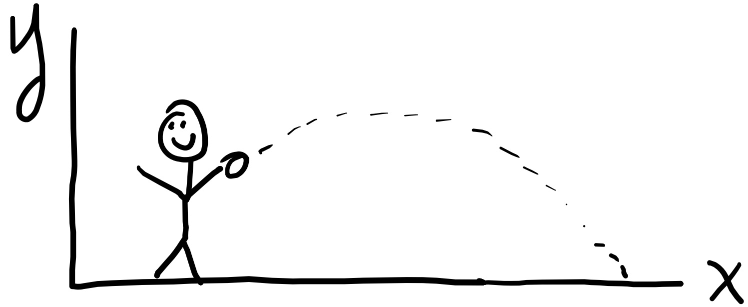
$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{v}|$ at $t = 1s$

11.89 A ball is thrown so that the motion is defined by the equations $x = 5t$ and $y = 2 + 6t - 4.9t^2$, where x and y are expressed in meters and t is expressed in seconds. Determine (*a*) the velocity at $t = 1$ s, (*b*) the horizontal distance the ball travels before hitting the ground.

Fig. P11.89

Notes on problem $\boxed{11.89}$



Ball thrown such that

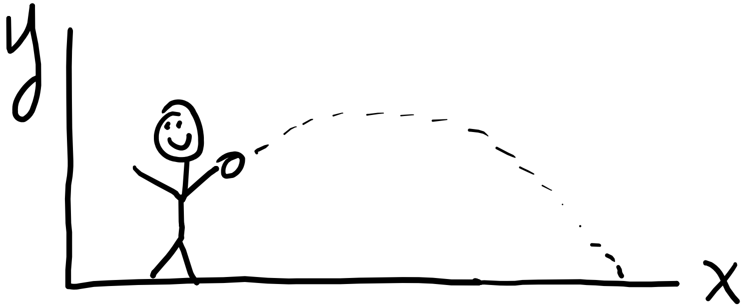
$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{v}|$ at $t = 1s$

Note: for some vector $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$B = |\vec{B}| = \left[B_x^2 + B_y^2 + B_z^2\right]^{1/2}.$$

Notes on problem $\boxed{11.89}$



Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{V}|$ at $t = 1s$

Note: for some vector $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

Notes on problem $\boxed{11.89}$

Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

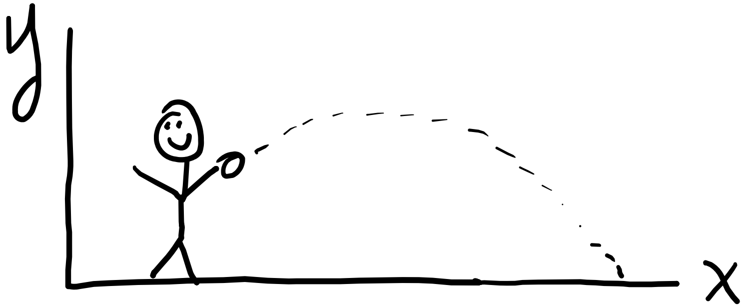
(a) Find $|\vec{v}|$ at $t = 1s$

Note: For some vector $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{v}| = [v_x^2 + v_y^2]^{1/2}$, where $v_x = \dot{x}$ & $v_y = \dot{y}$

Notes on problem $\boxed{11.89}$



Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{v}|$ at $t = 1s$

Note: For some vector $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

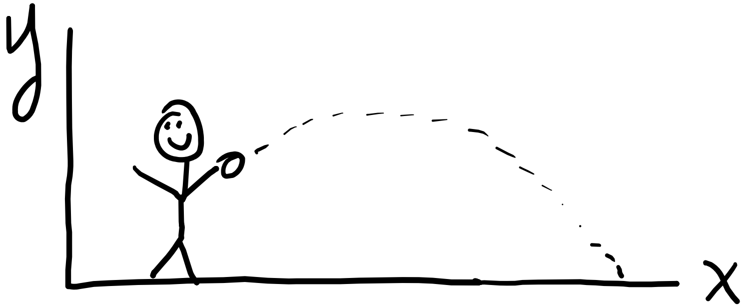
$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{v}| = [v_x^2 + v_y^2]^{1/2}$, where $v_x = \dot{x}$ & $v_y = \dot{y}$

just need to find $v_x(t=1s)$ & $v_y(t=1s)$ to determine $|\vec{v}|$ at 1s

ASU

Notes on problem $\boxed{11.89}$



Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{v}|$ at $t = 1s$

Note: For some vector $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

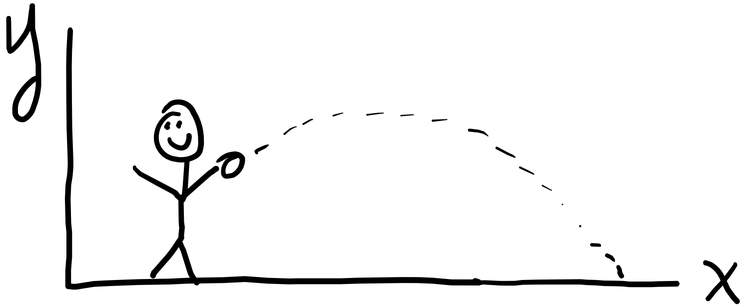
$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{v}| = [v_x^2 + v_y^2]^{1/2}$, where $v_x = \dot{x}$ & $v_y = \dot{y}$

just need to find $v_x(t=1s)$ & $v_y(t=1s)$ to determine $|\vec{v}|$ at 1s

(b) Find $x(t_h)$, where $t_h \equiv$ time of hit

Notes on problem $\boxed{11.89}$



Ball thrown such that

$y = 2 + 6t - 4.9t^2$ & $x = 5t$

(a) Find $|\vec{V}|$ at $t = 1s$

Note: For some vector $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

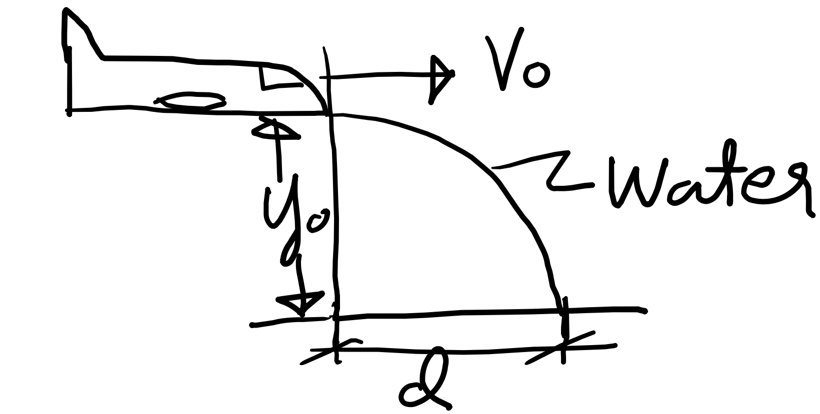
$B = |\vec{B}| = [B_x^2 + B_y^2 + B_z^2]^{1/2}$. Back to problem:

$|\vec{V}| = [V_x^2 + V_y^2]^{1/2}$, where $V_x = \dot{x}$ & $V_y = \dot{y}$

just need to find $V_x(t=1s)$ & $V_y(t=1s)$ to determine $|\vec{V}|$ at 1s

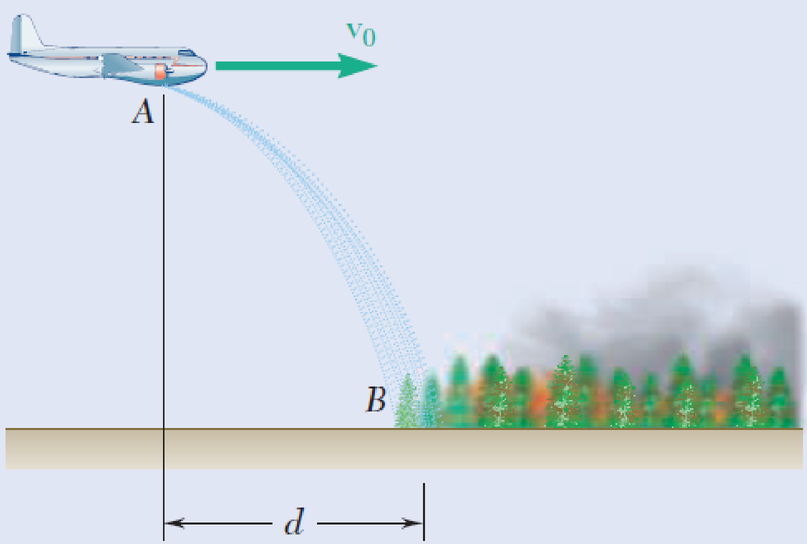
(b) Find $x(t_h)$, where $t_h \equiv$ time of hit: Couple of ways to do this. One way is to set $y(t_h) = 0$ & get t_h [quadratic formula] & then find $x(t_h)$

Notes on $\boxed{11.97}$

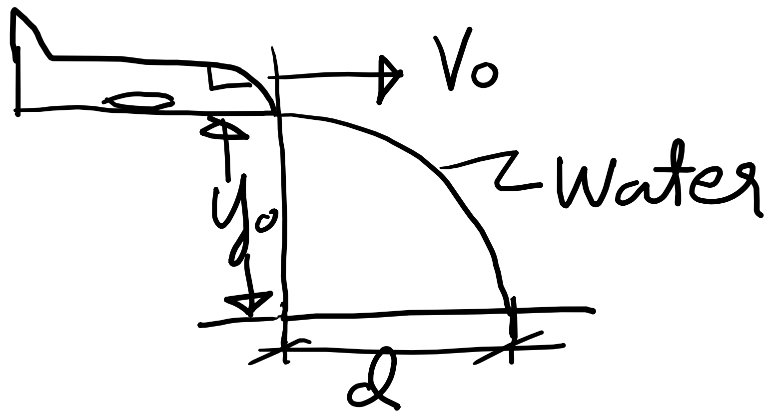


Given:
$$\begin{cases} V_0 = 180\,\frac{mi}{hr} \\ y_0 = 300\,ft \\ V_{y_0} = 0 \end{cases}$$

11.97 An airplane used to drop water on brushfires is flying horizontally in a straight line at 180 mi/h at an altitude of 300 ft. Determine the distance d at which the pilot should release the water so that it will hit the fire at B.



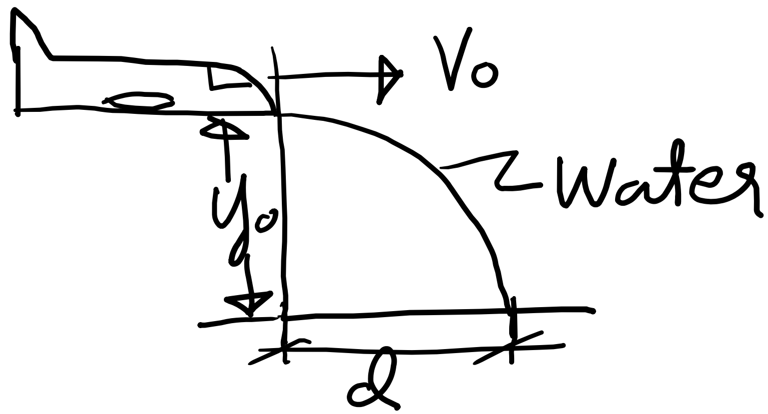
Notes on $\boxed{11.97}$



Given: $\begin{cases} V_0 = 180 \frac{mi}{hr} \\ y_0 = 300\,ft \\ V_{y_0} = 0 \end{cases}$

$$X = V_0 t \Rightarrow t = \frac{X}{V_0}$$

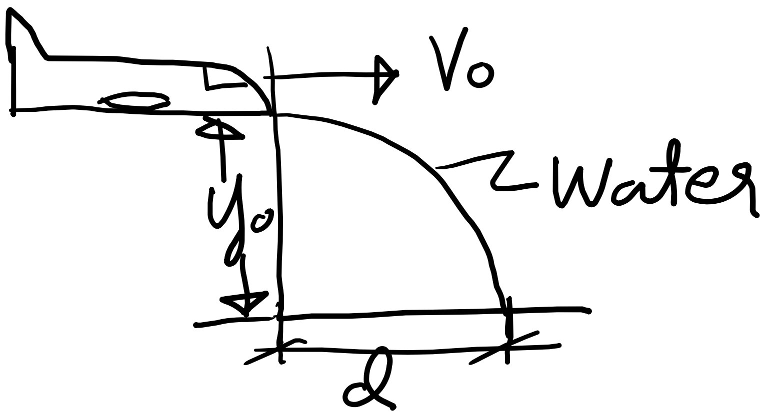
Notes on $\boxed{11.97}$



Given: $\begin{cases} V_0 = 180 \frac{mi}{hr} \\ y_0 = 300\, ft \\ V_{y_0} = 0 \end{cases}$

$x = V_0 t \Rightarrow t = \frac{x}{V_0}$ so when water hits ground $x = d \Rightarrow t_{hit} = \frac{d}{V_0}$

Notes on $\boxed{11.97}$

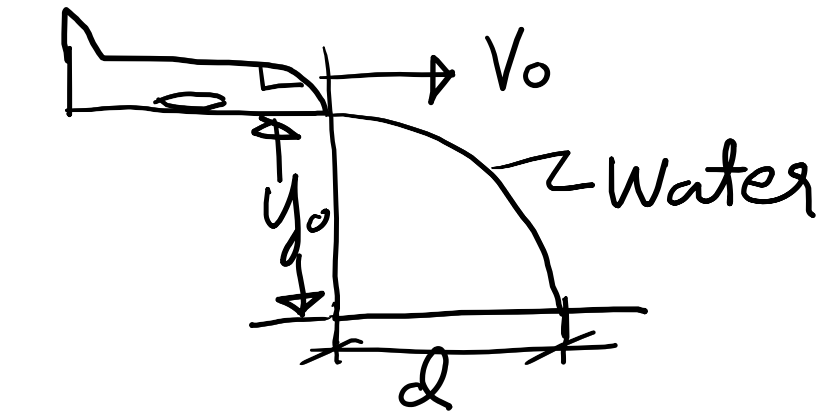


Given: $\begin{cases} V_0 = 180 \frac{mi}{hr} \\ y_0 = 300\,ft \\ V_{y_0} = \theta \end{cases}$

$X = V_0 t \Rightarrow t = \frac{X}{V_0}$ so when water hits ground $x = d \Rightarrow t_{hit} = \frac{d}{V_0}$

Also $y = -\frac{g}{2}t^2 + V_{0y}t + y_0$

Notes on $\boxed{11.97}$



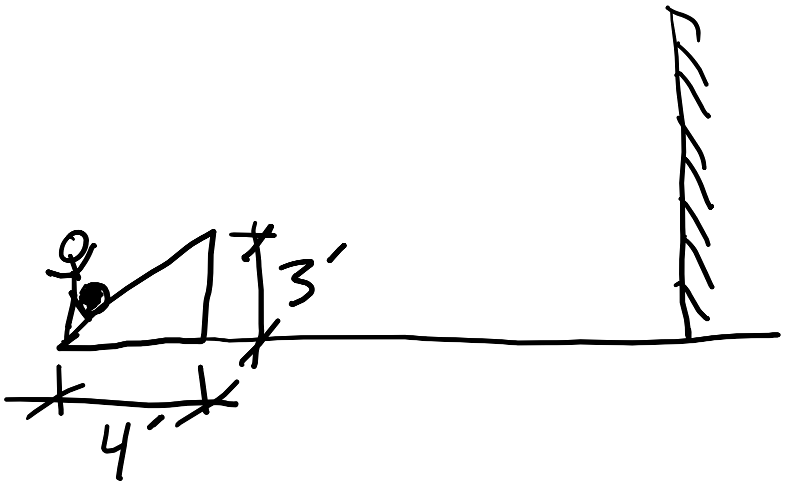
Given: $\begin{cases} V_o = 180 \frac{mi}{hr} \\ y_o = 300\,ft \\ V_{y_o} = 0 \end{cases}$

$X = V_o t \Rightarrow t = \frac{x}{V_o}$ so when water hits ground $x = d \Rightarrow t_{hit} = \frac{d}{V_o}$

Also $y = -\frac{g}{2}t^2 + \cancel{V_{oy}t}^{\,0} + y_o \Rightarrow y_{hit} = -\frac{g}{2}t_{hit}^2 + y_o$

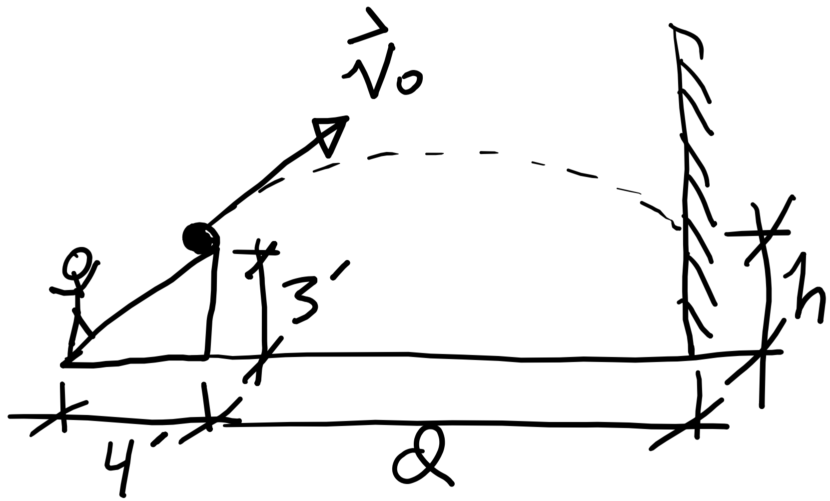
But $y_{hit} = 0$

Example similar to problem 11.105



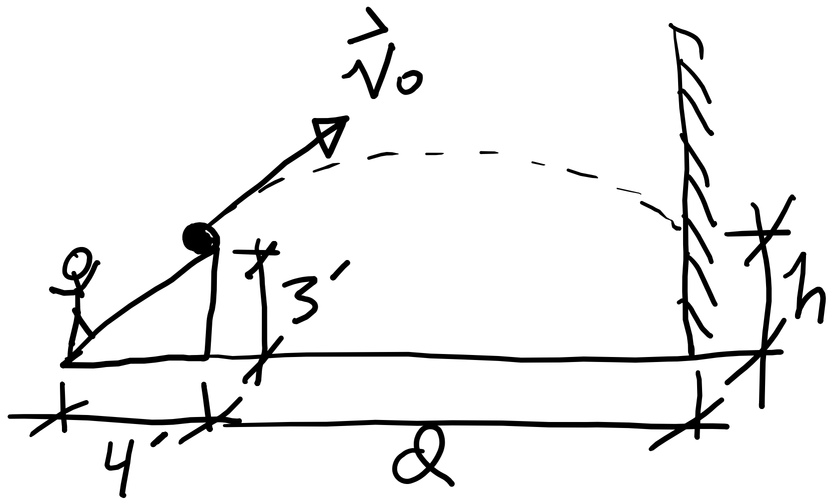
Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp

Example similar to problem 11.105



Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Example similar to problem 11.105



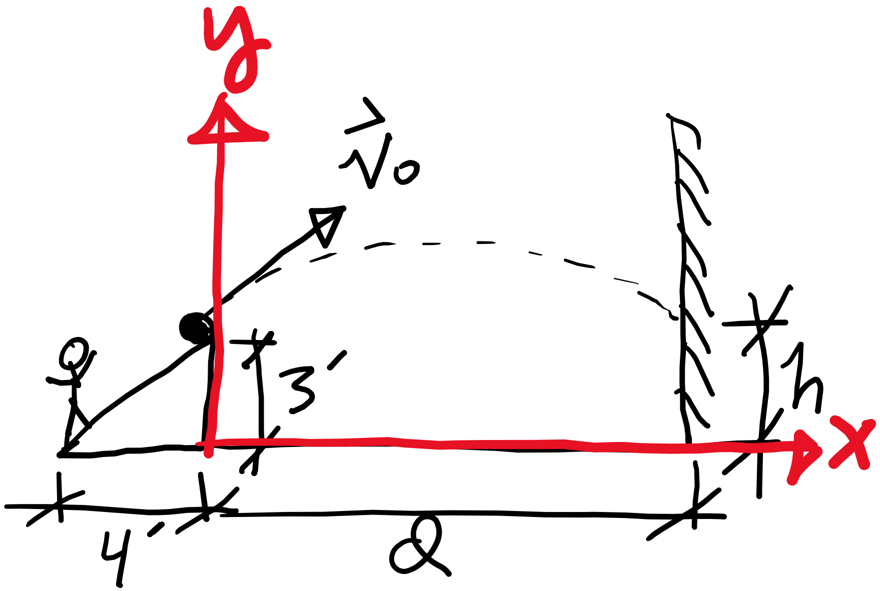
Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\ ft/s$, $d = 16\ ft$ & taking $g = 32\ ft/s^2$

Instead of $32.2\ ft/s^2$

[to make life easier]

Example similar to problem 11.105

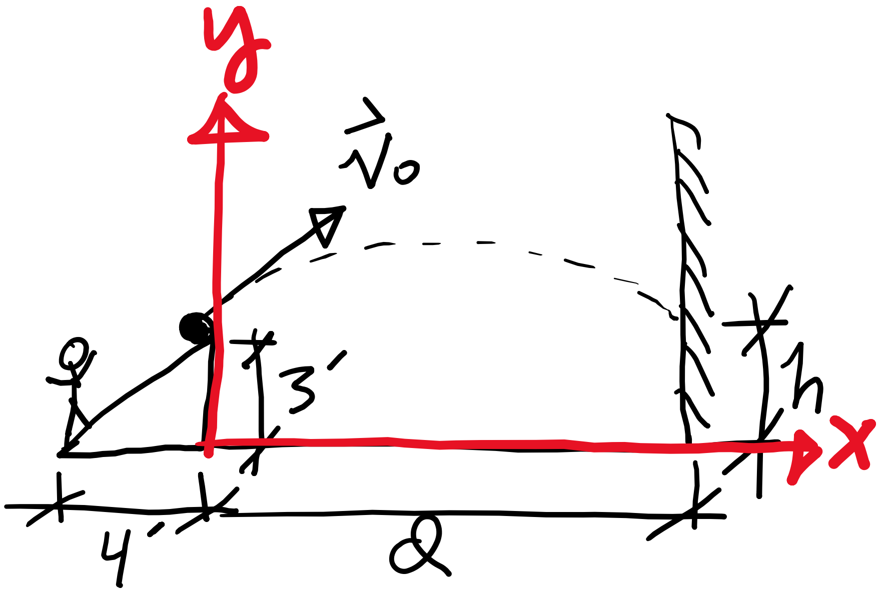


Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\,\text{ft/s}$, $d = 16\,\text{ft}$ & taking $g = 32\,\text{ft/s}^2$

Using coordinate system shown above

Example similar to problem 11.105

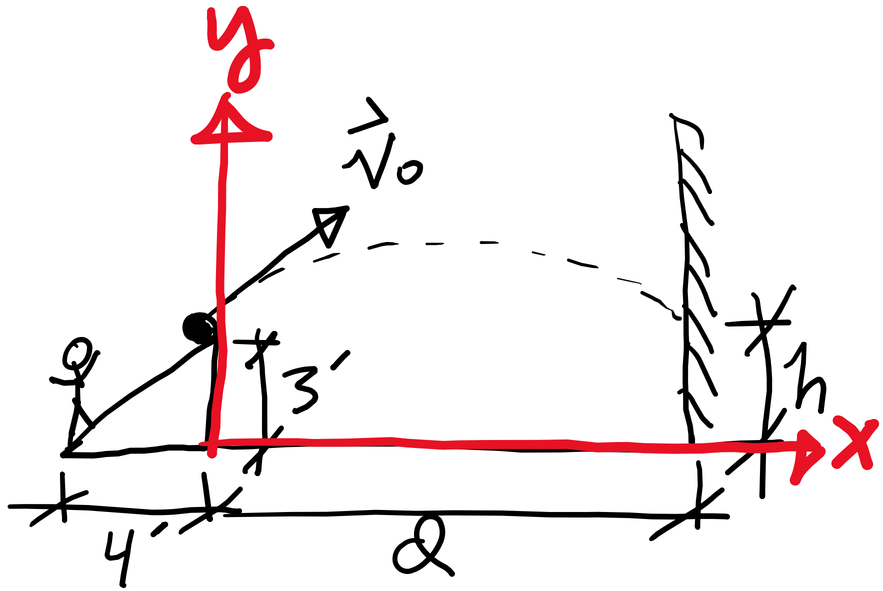


Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\,ft/s$, $d = 16\,ft$ & taking $g = 32\,ft/s^2$

We have $\quad x = v_{x_0} t \Rightarrow t_h = d/v_{x_0}$

Example similar to problem 11.105



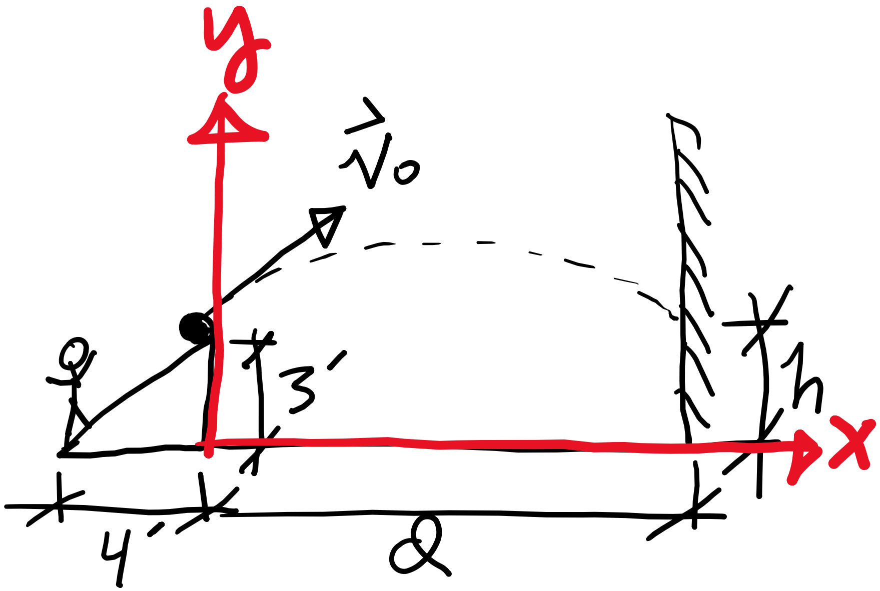
Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\, ft/s$, $d = 16\, ft$ & taking $g = 32\, ft/s^2$

We have $\quad x = v_{x_0} t \Rightarrow t_h = d/v_{x_0}$ &

$$y = \left(-\frac{g}{2}\right)t^2 + v_{oy}t + y_0$$

Example similar to problem 11.105



Kid kicks ball off ramp. Ball hits wall at height h a distance d from ramp.

Given $v_0 = 32\,ft/s$, $d = 16\,ft$ & taking $g = 32\,ft/s^2$

We have $\quad x = v_{x_0} t \Rightarrow t_h = d/v_{x_0}$ &

$$y = \left(-\frac{g}{2}\right)t^2 + v_{0y}t + y_0 \Rightarrow h = \left(-\frac{g}{2}\right)t_h^2 + v_{0y}t_h + y_0$$

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x_0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0$$

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x_0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0 \;, \text{but}$$

$$V_{0x} = V_0 \cos\theta_0 \Rightarrow V_{0x} = V_0\left(\frac{4}{5}\right) = \left(32 * \frac{4}{5}\right)\text{ft}/\text{s}$$

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x_0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0 \text{ , but}$$

$$V_{0x} = V_0 \cos\theta_0 \Rightarrow V_{0x} = V_0\left(\frac{4}{5}\right) = \left(32 * \frac{4}{5}\right)\text{ft/s}$$

<u>Note</u>: $\frac{V_{0y}}{V_{0x}} = \frac{V_0 \sin\theta_0}{V_0 \cos\theta_0} = \tan\theta_0 = \frac{3}{4}$

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x0}^2}\right)d^2 + \left(\frac{V_{0y}}{V_{0x}}\right)d + y_0 \text{ , but}$$

$$V_{0x} = V_0 \cos\theta_0 \Rightarrow V_{0x} = V_0\left(\frac{4}{5}\right) = \left(32 * \frac{4}{5}\right) ft/s$$

Note: $\dfrac{V_{0y}}{V_{0x}} = \dfrac{V_0 \sin\theta_0}{V_0 \cos\theta_0} = \tan\theta_0 = \dfrac{3}{4}$

So $h = \left\{\left[\dfrac{-32 * 5^2}{2 * (32 * 4)^2}\right] 16^2 + \dfrac{3}{4} * 16 + 3\right\} ft$

$$\Rightarrow \quad h = \left(-\frac{g}{2V_{x_0}^2}\right)d^2 + \left(\frac{V_{oy}}{V_{ox}}\right)d + y_0 \;,\; \text{but}$$

$$V_{ox} = V_0\cos\theta_0 \Rightarrow V_{ox} = V_0\left(\frac{4}{5}\right) = \left(32 * \frac{4}{5}\right)\text{ft/s}$$

Note: $\dfrac{V_{oy}}{V_{ox}} = \dfrac{V_0\sin\theta_0}{V_0\cos\theta_0} = \tan\theta_0 = \dfrac{3}{4}$

$$\text{So} \quad h = \left\{\left[\frac{-32 * 5^2}{2*(32*4)^2}\right]16^2 + \frac{3}{4} * 16 + 3\right\}\text{ft}$$

$$\Rightarrow \quad h = \left\{\left[\frac{-25}{2*32*16}\right]16^2 + 12 + 3\right\}\text{ft}$$

$$\Rightarrow \quad h = \left\{-\frac{25}{4} + 15\right\}\text{ft} = 8.75\text{ ft}$$