Today: Section 11.5

$\boxed{L4}$

Today: Section 11.5 $\boxed{L4}$

Curvilinear Motion of particles

Today: Section 11.5

L4

Curvilinear Motion of particles

we will learn what these mean

$\vec{v} = \rho\dot{\theta}\hat{e}_t$ & $\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ & $\vec{a} = [\ddot{r} - r\dot{\theta}^2] + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$

ASU

Today: Section 11.5

$\boxed{L4}$

Tuesday: Section 12.1

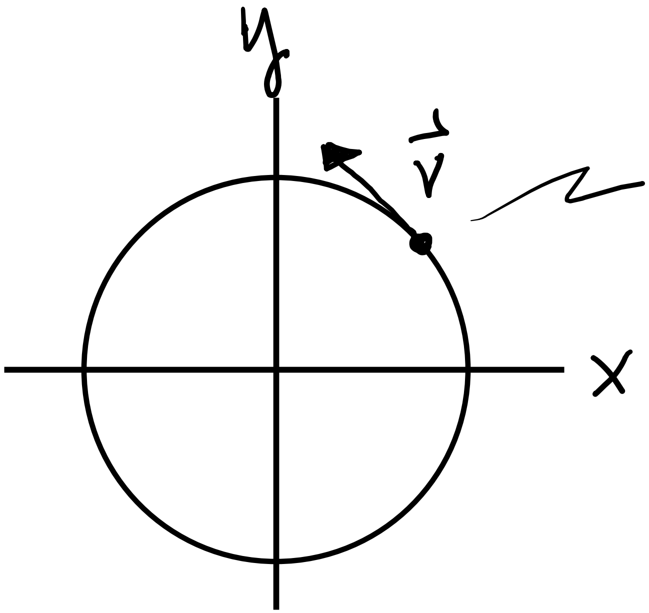
Today: Section 11.5

$\boxed{L4}$

Tuesday: Section 12.1

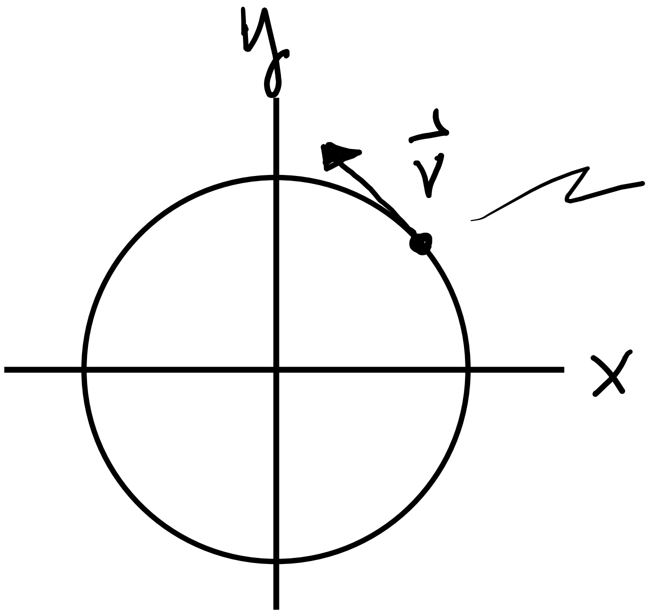
Linear momentum & forces

Uniform circular motion



particle moving at constant speed

Uniform circular motion

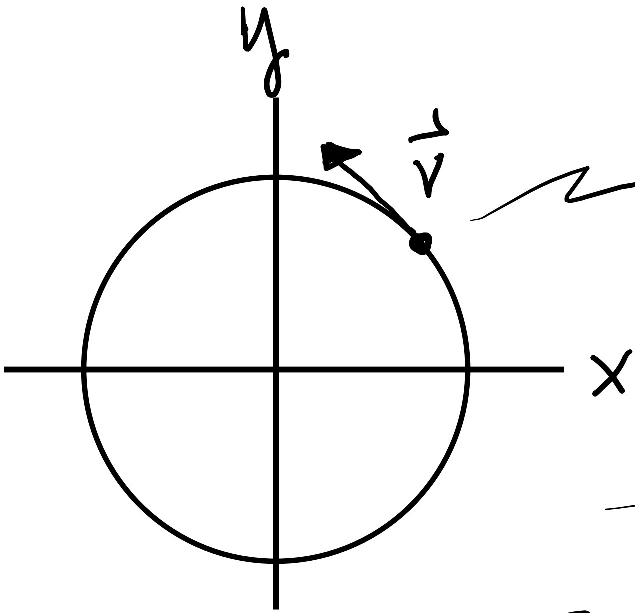


particle moving at constant speed

$|\vec{v}| = \text{const.}$

so $\frac{d|\vec{v}|}{dt} = 0$

Uniform circular motion



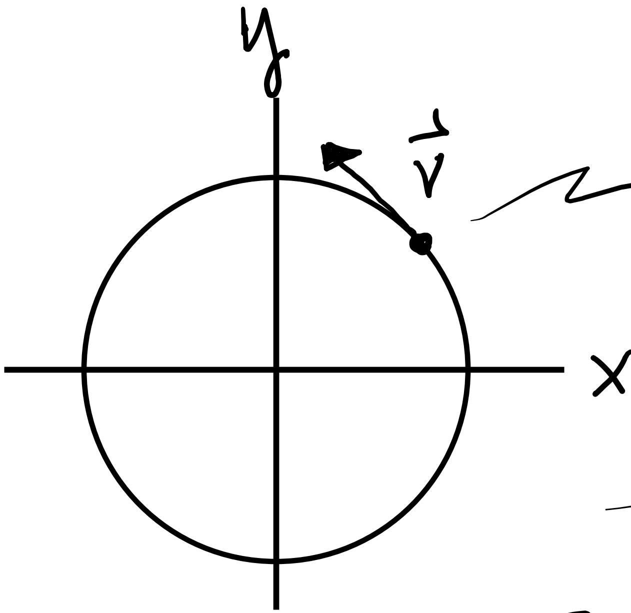
particle moving at constant speed

$|\vec{v}| = \text{const.}$

so $\dfrac{d|\vec{v}|}{dt} = 0$

But the direction **does** change.

Uniform circular motion



particle moving at constant speed

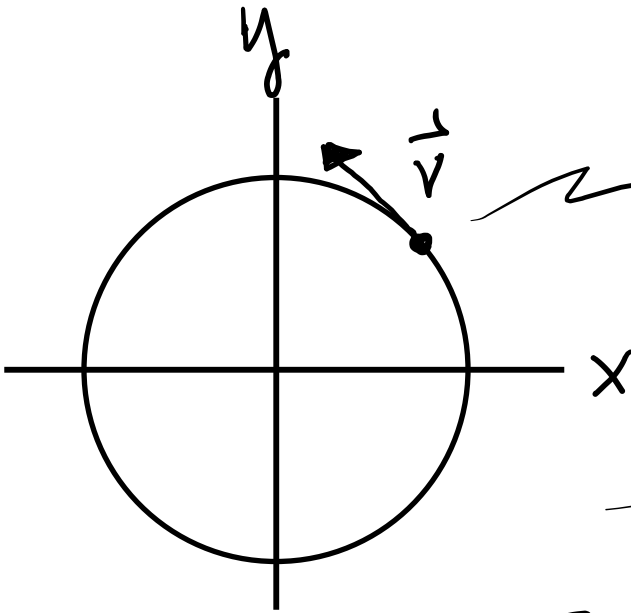
$|\vec{v}| = \text{const.}$

so $\dfrac{d|\vec{v}|}{dt} = 0$

But the direction **does** change.

So $\dfrac{d\vec{v}}{dt} \neq 0$

Uniform circular motion



particle moving at constant speed

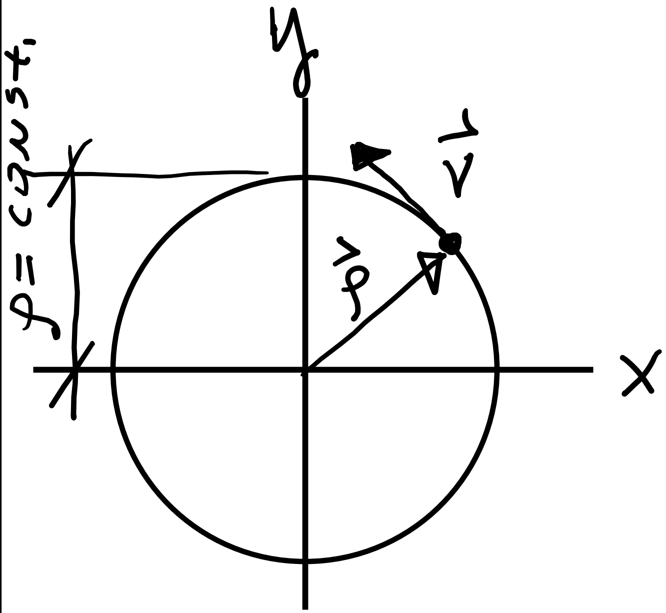
$|\vec{v}| = \text{const.}$

so $\frac{d|\vec{v}|}{dt} = 0$

But the direction does change.

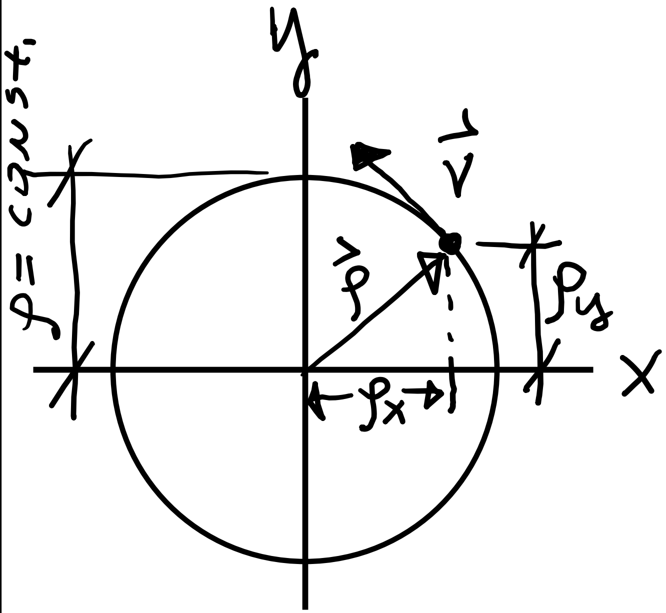
So $\frac{d\vec{v}}{dt} \neq 0$. We want to find an expression $\vec{a} = \frac{d\vec{v}}{dt}$ for uniform circular motion

Uniform circular motion



We can construct a position vector $\vec{\rho}$

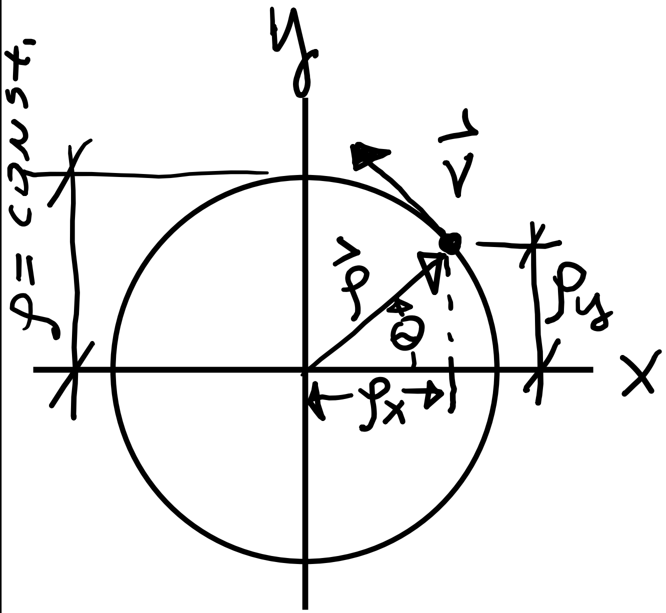
Uniform circular motion



We can construct a position vector $\vec{\rho}$, where

$$\vec{\rho} = \rho_x \hat{i} + \rho_y \hat{j}$$

Uniform circular motion

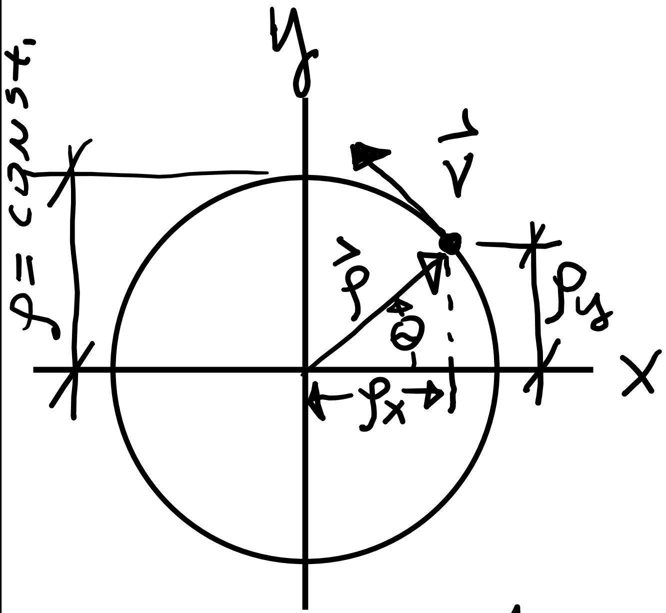


We can construct a position vector $\vec{\rho}$, where

$$\vec{\rho} = \rho_x \hat{\imath} + \rho_y \hat{\jmath} \quad \text{but}$$

$$\rho_x = \rho \cos\theta \ \& \ \rho_y = \rho \sin\theta$$

Uniform circular motion



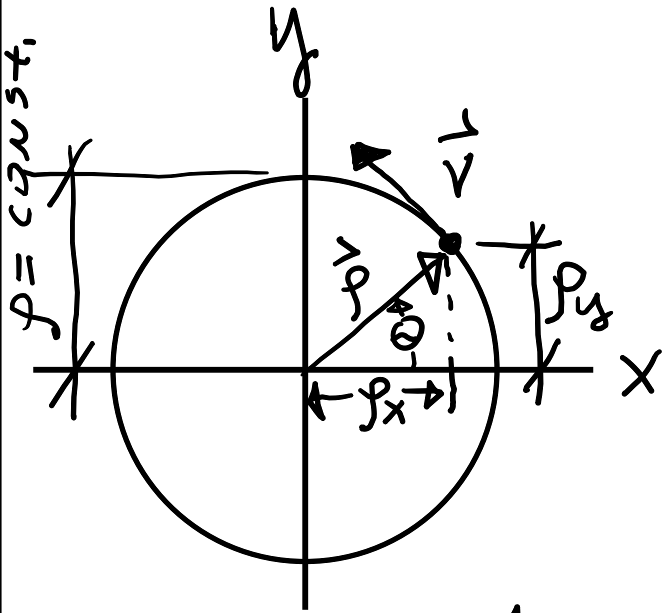
We can construct a position vector $\vec{\rho}$, where

$$\vec{\rho} = \rho_x \hat{i} + \rho_y \hat{j} \quad \text{but}$$

$$\rho_x = \rho\cos\theta \;\&\; \rho_y = \rho\sin\theta$$

So $\vec{\rho} = \rho[\hat{i}\cos\theta + \hat{j}\sin\theta]$

Uniform circular motion



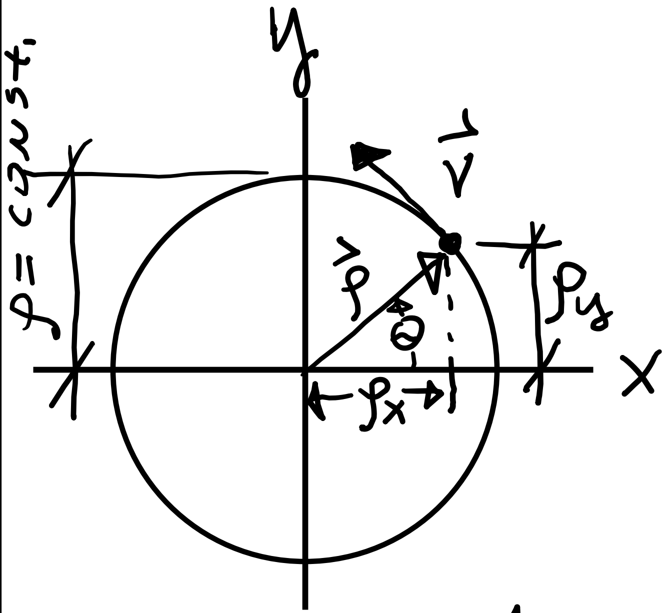
We can construct a position vector $\vec{\rho}$, where

$$\vec{\rho} = \rho_x \hat{i} + \rho_y \hat{j} \quad \text{but}$$

$$\rho_x = \rho\cos\theta \;\&\; \rho_y = \rho\sin\theta$$

So $\vec{\rho} = \rho[\hat{i}\cos\theta + \hat{j}\sin\theta] \Rightarrow \vec{v} = \dot{\vec{\rho}}$

Uniform circular motion



We can construct a position vector $\vec{\rho}$, where

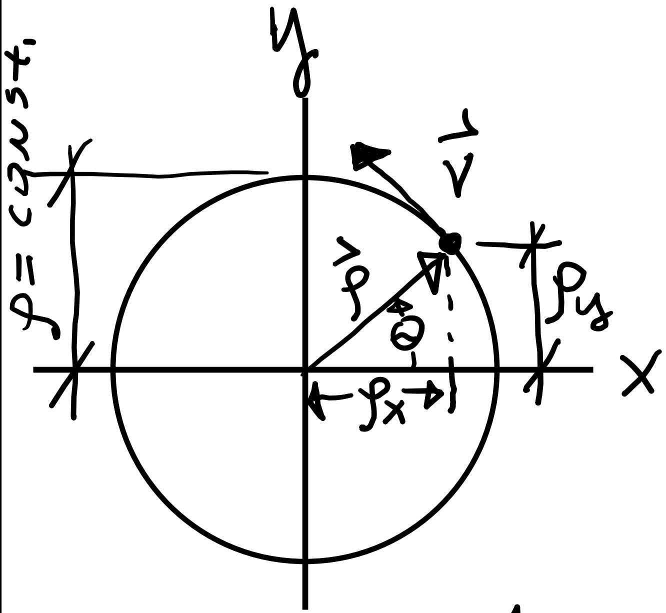
$$\vec{\rho} = \rho_x \hat{i} + \rho_y \hat{j} \quad \text{but}$$

$$\rho_x = \rho \cos\theta \ \ \& \ \ \rho_y = \rho \sin\theta$$

So $\vec{\rho} = \rho[\hat{i}\cos\theta + \hat{j}\sin\theta] \Rightarrow \vec{v} = \dot{\vec{\rho}}$

Now $\vec{v} = \rho[\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}]$

Uniform circular motion



We can construct a position vector $\vec{\rho}$, where

$$\vec{\rho} = \rho_x \hat{i} + \rho_y \hat{j} \quad \text{but}$$

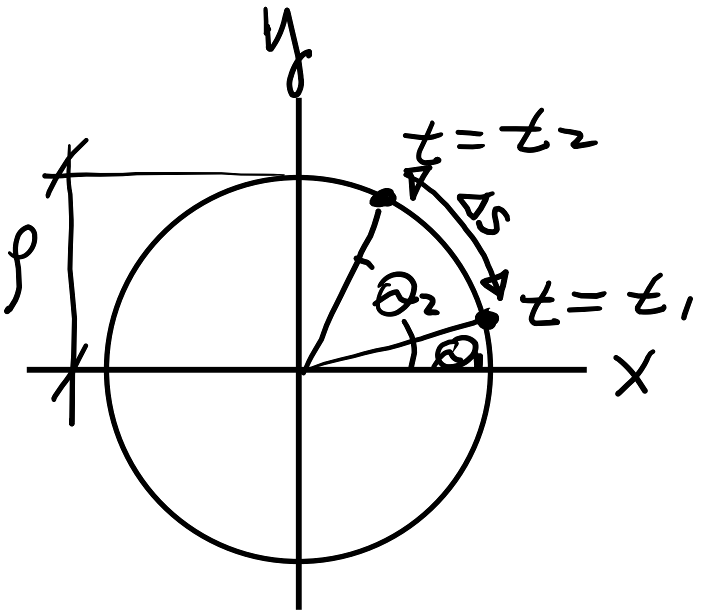
$$\rho_x = \rho\cos\theta \ \& \ \rho_y = \rho\sin\theta$$

So $\vec{\rho} = \rho[\hat{i}\cos\theta + \hat{j}\sin\theta] \Rightarrow \vec{v} = \dot{\vec{\rho}}$

Now $\vec{v} = \rho[\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}]$

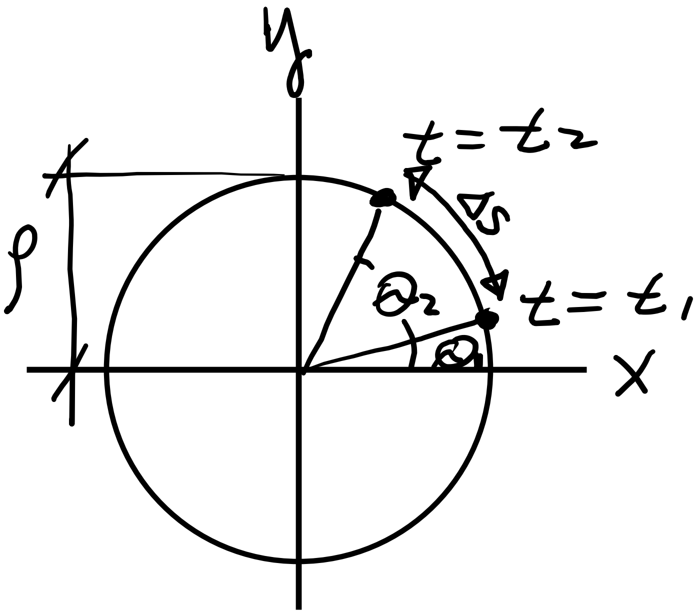
$$\Rightarrow \vec{v} = \rho\dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Uniform circular motion



Since $\Delta s = \rho \Delta \theta$

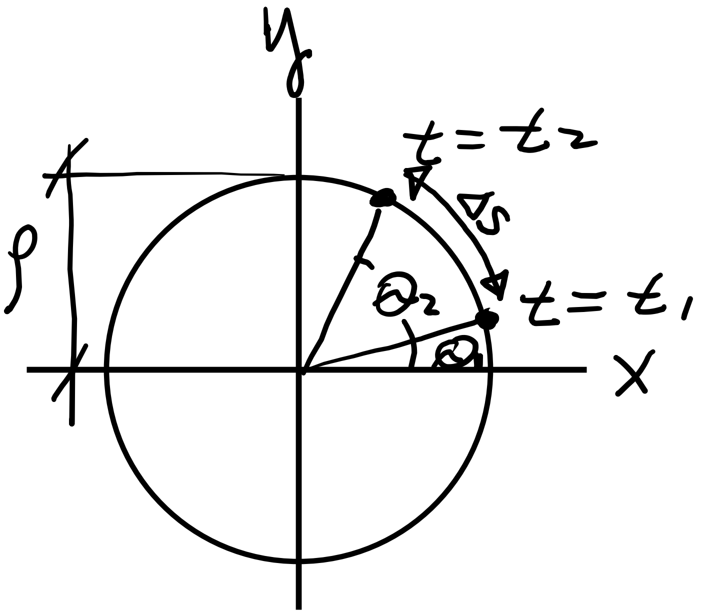
Uniform circular motion



Since $\Delta s = \rho \Delta\theta$ and

$$|\vec{v}|_{AVE} = \frac{\Delta s}{\Delta t} = \rho \frac{\Delta\theta}{\Delta t}$$

Uniform circular motion

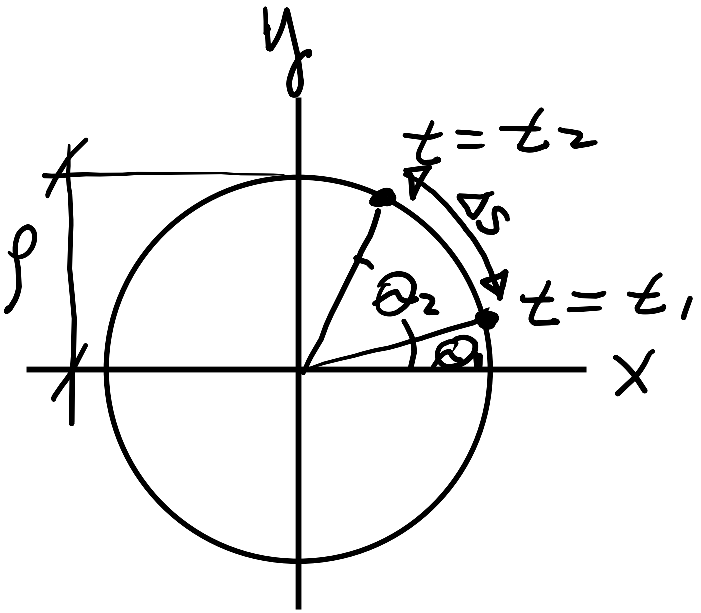


Since $\Delta s = \rho \Delta\theta$ and

$$|\vec{v}|_{AVE} = \frac{\Delta s}{\Delta t} = \rho \frac{\Delta\theta}{\Delta t}$$ then

$$|\vec{v}| = \rho \frac{d\theta}{dt} = \rho\dot{\theta}$$

Uniform circular motion



Since $\Delta s = \rho \Delta\theta$ and

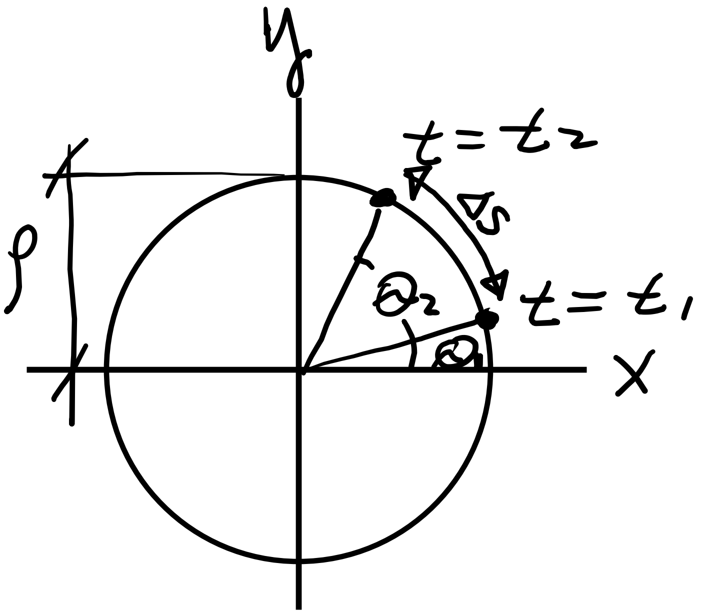
$$|\vec{v}|_{AVE} = \frac{\Delta s}{\Delta t} = \rho \frac{\Delta\theta}{\Delta t} \text{ then}$$

$$|\vec{v}| = \rho \frac{d\theta}{dt} = \rho\dot{\theta}$$

Now $\vec{v} = \rho\dot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]$

{from previous slide}

Uniform circular motion



Since $\Delta s = \rho \Delta\theta$ and

$$|\vec{v}|_{ave} = \frac{\Delta s}{\Delta t} = \rho \frac{\Delta\theta}{\Delta t} \text{ then}$$

$$|\vec{v}| = \rho \frac{d\theta}{dt} = \rho\dot{\theta}$$

Now $\vec{v} = \rho\dot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]$ {from previous slide} becomes

$$\vec{v} = |\vec{v}|\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right], \text{ where}$$

$$|\vec{v}| = \rho\dot{\theta}$$

For position we have $\vec{p} = |\vec{p}|\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$

For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

What does the dot product tell us?

For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

What does the dot product tell us?

$$\vec{p}\cdot\vec{v} = |\vec{p}||\vec{v}|[-\cos\theta\sin\theta + \sin\theta\cos\theta] = 0$$

For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

What does the dot product tell us?

$$\vec{p} \cdot \vec{v} = |\vec{p}||\vec{v}|[-\cos\theta\sin\theta + \sin\theta\cos\theta] = 0$$

$\Rightarrow$ $\vec{p}$ & $\vec{v}$ must be orthogonal to each other.

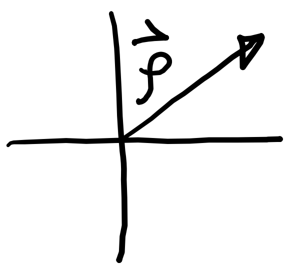
For position we have $\vec{p} = |\vec{p}|\,[\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|\,[-\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta]$

What does the dot product tell us?

$$\vec{p}\cdot\vec{v} = |\vec{p}|\,|\vec{v}|\,[-\cos\theta\sin\theta + \sin\theta\cos\theta] = 0$$

$\Rightarrow$ $\vec{p}$ & $\vec{v}$ must be orthogonal to each other.



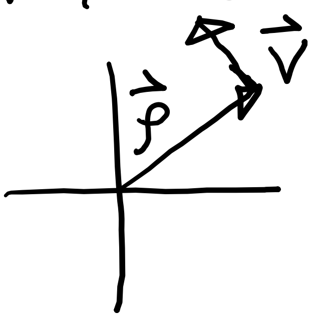
For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

What does the dot product tell us?

$\vec{p}\cdot\vec{v} = |\vec{p}||\vec{v}|[-\cos\theta\sin\theta + \sin\theta\cos\theta] = 0$

$\Rightarrow$ $\vec{p}$ & $\vec{v}$ must be orthogonal to each other.



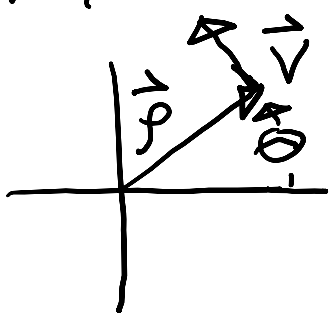
For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

What does the dot product tell us?

$$\vec{p}\cdot\vec{v} = |\vec{p}||\vec{v}|[-\cos\theta\sin\theta + \sin\theta\cos\theta] = 0$$

$\Rightarrow$ $\vec{p}$ & $\vec{v}$ must be orthogonal to each other.



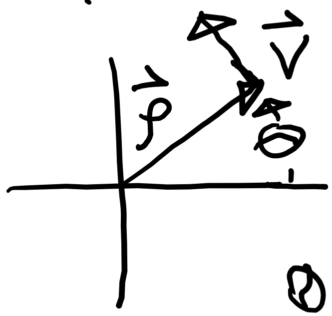
For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

What does the dot product tell us?

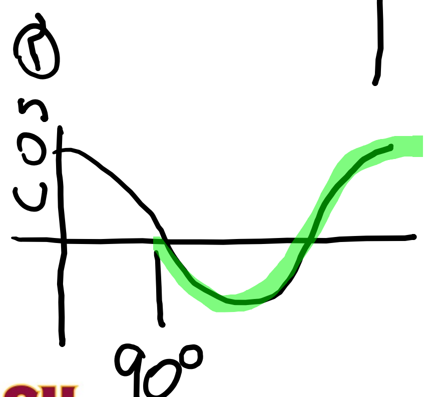
$$\vec{p}\cdot\vec{v} = |\vec{p}||\vec{v}|[-\cos\theta\sin\theta + \sin\theta\cos\theta] = 0$$

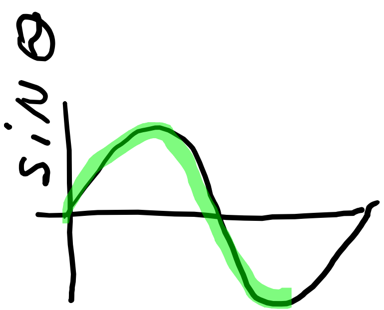
$\Rightarrow$ $\vec{p}$ & $\vec{v}$ must be orthogonal to each other.



Notice that

$\cos(\theta + 90^\circ) = -\sin\theta$





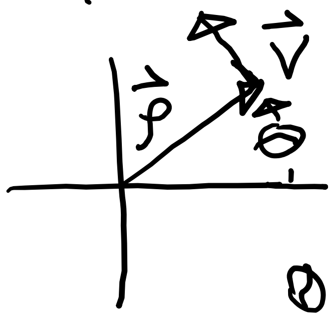
For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

What does the dot product tell us?

$$\vec{p}\cdot\vec{v} = |\vec{p}||\vec{v}|[-\cos\theta\sin\theta + \sin\theta\cos\theta] = 0$$

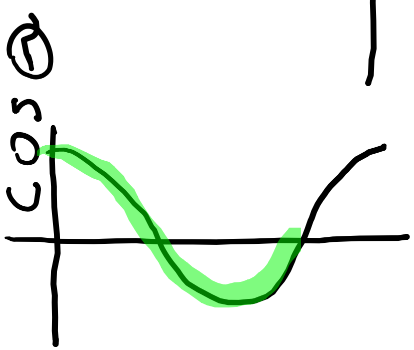
$\Rightarrow$ $\vec{p}$ & $\vec{v}$ must be orthogonal to each other.

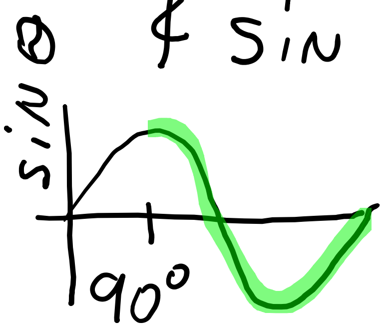


Notice that

$\cos(\theta + 90°) = -\sin\theta$

& $\sin(\theta + 90°) = \cos\theta$



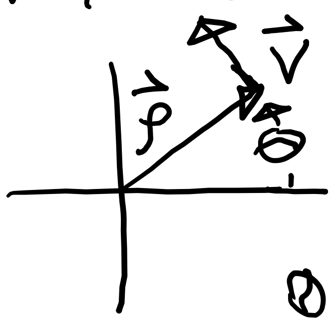


For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$
& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

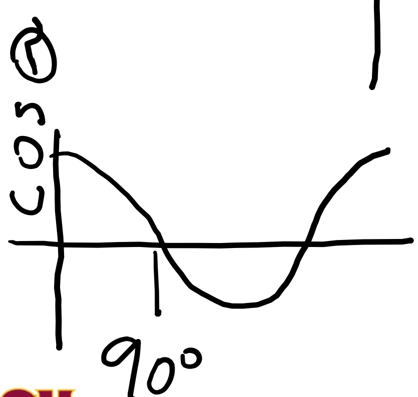
What does the dot product tell us?

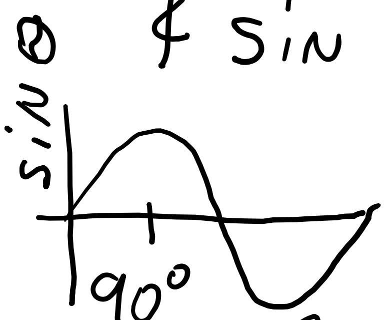
$$\vec{p}\cdot\vec{v} = |\vec{p}||\vec{v}|[-\cos\theta\sin\theta + \sin\theta\cos\theta] = 0$$

$\Rightarrow$ $\vec{p}$ & $\vec{v}$ must be orthogonal to each other.



Notice that
$\cos(\theta + 90^\circ) = -\sin\theta$
& $\sin(\theta + 90^\circ) = \cos\theta$





It looks like our expression for $\vec{v}$ makes sense ☺

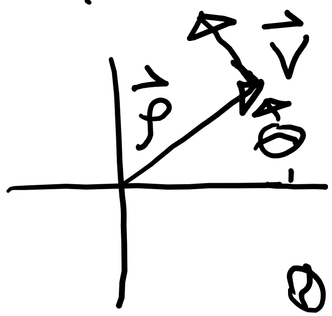
For position we have $\vec{p} = |\vec{p}|[\hat{i}\cos\theta + \hat{j}\sin\theta]$

& for velocity we have $\vec{v} = |\vec{v}|[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

What does the dot product tell us?

$$\vec{p}\cdot\vec{v} = |\vec{p}||\vec{v}|[-\cos\theta\sin\theta + \sin\theta\cos\theta] = 0$$

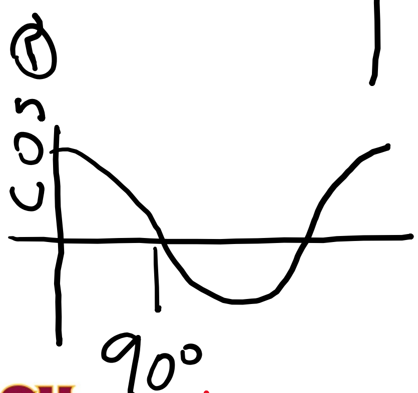
$\Rightarrow$ $\vec{p}$ & $\vec{v}$ must be orthogonal to each other.

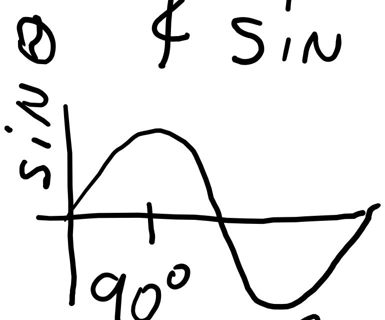


Notice that

$\cos(\theta + 90^\circ) = -\sin\theta$

& $\sin(\theta + 90^\circ) = \cos\theta$





It looks like our expression for $\vec{v}$ makes sense ☺

Now lets find $\vec{a} = d\vec{v}/dt$ $\longrightarrow$

$$\vec{a} = \frac{d}{dt}\left\{\rho\dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]\right\} = \rho\ddot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

$$+\rho\dot{\theta}[-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}] = \rho\ddot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

$$-\rho\dot{\theta}^2[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

But we have

uniform circular motion : $|\vec{v}| = \rho\dot{\theta} = \text{const.}$

$$\vec{a} = \frac{d}{dt}\left\{\rho\dot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]\right\} = \rho\ddot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]$$

$$+ \rho\dot{\theta}\left[-\hat{i}\,(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\,\dot{\theta}\right] = \rho\ddot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]$$

$$-\rho\dot{\theta}^2\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$$

But we have uniform circular motion: $|\vec{v}| = \rho\dot{\theta} = \text{const.}$

$$\Rightarrow \ddot{\theta} = \text{zero}$$

$$\vec{a} = \frac{d}{dt}\left\{\rho\dot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]\right\} = \rho\ddot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]$$

$$+\rho\dot{\theta}\left[-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}\right] = \rho\ddot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]$$

$$-\rho\dot{\theta}^2\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$$

But we have uniform circular motion : $|\vec{v}| = \rho\dot{\theta} = \text{const.}$

$\Rightarrow \ddot{\theta} = \text{zero}$

$$a = \rho\dot{\theta}\left[-\hat{i}\cos\theta - \hat{j}\sin\theta\right]$$

$$\vec{a} = \frac{d}{dt}\left\{\rho\dot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]\right\} = \rho\ddot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right]$$

$$+\rho\dot{\theta}\left[-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}\right] = \rho\ddot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right] - \rho\dot{\theta}^2\left[\hat{i}\cos\theta + \hat{j}\sin\theta\right]$$

But we have uniform circular motion: $|\vec{v}| = \rho\dot{\theta} = \text{const.}$

$\Rightarrow \ddot{\theta} = \text{zero}$ so

$$a = \rho\dot{\theta}\left[-\hat{i}\cos\theta - \hat{j}\sin\theta\right]$$

Compare to $\rho = \rho\left[+\hat{i}\cos\theta + \hat{j}\sin\theta\right]$

$$\vec{a} = \frac{d}{dt}\left\{\rho\dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]\right\} = \rho\ddot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

$$+\rho\dot{\theta}[-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}] = \rho\ddot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta] - \rho\dot{\theta}^2[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

But we have uniform circular motion: $|\vec{v}| = \rho\dot{\theta} = \text{const.}$

$\Rightarrow \ddot{\theta} = \text{zero}$

$$a = \rho\dot{\theta}[-\hat{i}\cos\theta - \hat{j}\sin\theta]$$

Compare to $\rho = \rho[+\hat{i}\cos\theta + \hat{j}\sin\theta]$ So

$\vec{\rho}$ points from center to position

$$\vec{a} = \frac{d}{dt}\left\{\rho\dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]\right\} = \rho\ddot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

$$+\rho\dot{\theta}[-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}] = \rho\ddot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta] - \rho\dot{\theta}^2[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

But we have uniform circular motion: $|\vec{v}| = \rho\dot{\theta} = \text{const.}$

$\Rightarrow \ddot{\theta} = \text{zero}$

$$a = \rho\dot{\theta}[-\hat{i}\cos\theta - \hat{j}\sin\theta]$$

Compare to $\rho = \rho[+\hat{i}\cos\theta + \hat{j}\sin\theta]$ So

$\vec{\rho}$ points from center to position &

$\vec{a}$ points from position to center

Let $\hat{e}_n \equiv \left[-\hat{i}\cos\theta - \hat{j}\sin\theta\right]$

Let $\hat{e}_N \equiv [-\hat{i}\cos\theta - \hat{j}\sin\theta]$

Note: $|\hat{e}_N| = [\hat{e}_N \cdot \hat{e}_N]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

Let $\hat{e}_N \equiv [-\hat{i}\cos\theta - \hat{j}\sin\theta]$

Note: $|\hat{e}_N| = [\hat{e}_N \cdot \hat{e}_N]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

So $\hat{e}_N$ is a unit vector

Let $\hat{e}_N \equiv [-\hat{i}\cos\theta - \hat{j}\sin\theta]$

Note: $|\hat{e}_N| = [\hat{e}_N \cdot \hat{e}_N]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

So $\hat{e}_N$ is a unit vector

Similarly $\hat{e}_t = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ is a unit vector

Let $\hat{e}_n \equiv [-\hat{i}\cos\theta - \hat{j}\sin\theta]$

Note: $|\hat{e}_n| = [\hat{e}_n \cdot \hat{e}_n]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

So $\hat{e}_n$ is a unit vector

Similarly $\hat{e}_t = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ is a unit vector and $\hat{e}_n \cdot \hat{e}_t = 0$

Let $\hat{e}_n \equiv [-\hat{i}\cos\theta - \hat{j}\sin\theta]$

Note: $|\hat{e}_n| = [\hat{e}_n \cdot \hat{e}_n]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

So $\hat{e}_n$ is a unit vector

Similarly $\hat{e}_t = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ is a unit vector and $\hat{e}_n \cdot \hat{e}_t = 0$

Moreover $\vec{V} = V\hat{e}_t$

Let $\hat{e}_n \equiv [-\hat{i}\cos\theta - \hat{j}\sin\theta]$

Note: $|\hat{e}_n| = [\hat{e}_n \cdot \hat{e}_n]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

So $\hat{e}_n$ is a unit vector

Similarly $\hat{e}_t = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ is a unit vector and $\hat{e}_n \cdot \hat{e}_t = 0$

Moreover $\vec{V} = V\hat{e}_t$ & $\vec{a} = a\hat{e}_n$

Note: Since $a = \rho\dot{\theta}^2$ & $V = \rho\dot{\theta}$

Let $\hat{e}_N \equiv [-\hat{\imath}\cos\theta - \hat{\jmath}\sin\theta]$

Note: $|\hat{e}_N| = [\hat{e}_N \cdot \hat{e}_N]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

So $\hat{e}_N$ is a unit vector

Similarly $\hat{e}_t = [-\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta]$ is a unit vector and $\hat{e}_N \cdot \hat{e}_t = \cancel{\theta}$

Moreover $\vec{V} = V\hat{e}_t$ & $\vec{a} = a\hat{e}_N$

Note: Since $a = \rho\dot{\theta}^2$ & $V = \rho\dot{\theta}$ then we can write $a = \frac{V^2}{\rho}$

So, for uniform circular motion $V = \text{Const.}$ & $\rho = \text{const.}$ But

Let $\hat{e}_N \equiv [-\hat{\imath}\cos\theta - \hat{\jmath}\sin\theta]$

Note: $|\hat{e}_N| = [\hat{e}_N \cdot \hat{e}_N]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

So $\hat{e}_N$ is a unit vector

Similarly $\hat{e}_t = [-\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta]$ is a unit vector and $\hat{e}_N \cdot \hat{e}_t = \cancel{\theta}$

Moreover $\vec{v} = v\hat{e}_t$ & $\vec{a} = a\hat{e}_N$

Note: Since $a = \rho\dot{\theta}^2$ & $v = \rho\dot{\theta}$ then we can write $a = \frac{v^2}{\rho}$

So, for uniform circular motion $v = \text{Const.}$ & $\rho = \text{const.}$ But $\vec{v} \neq \text{const.}$ & $\vec{\rho} \neq \text{const.}$

Let $\hat{e}_N \equiv [-\hat{i}\cos\theta - \hat{j}\sin\theta]$

Note: $|\hat{e}_N| = [\hat{e}_N \cdot \hat{e}_N]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

So $\hat{e}_N$ is a unit vector

Similarly $\hat{e}_t = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ is a unit vector and $\hat{e}_N \cdot \hat{e}_t = 0$

Moreover $\vec{v} = v\hat{e}_t$ & $\vec{a} = a\hat{e}_N$

Note: Since $a = \rho\dot{\theta}^2$ & $v = \rho\dot{\theta}$ then we can write $a = \frac{v^2}{\rho}$

So, for uniform circular motion $v = \text{Const.}$ & $\rho = \text{const.}$ But $\vec{v} \neq \text{const.}$ & $\vec{\rho} \neq \text{const.}$

$\vec{a} = \frac{v^2}{\rho}\hat{e}_N$, where $\hat{e}_N$ points towards center

Let $\hat{e}_N \equiv [-\hat{i}\cos\theta - \hat{j}\sin\theta]$

Note: $|\hat{e}_N| = [\hat{e}_N \cdot \hat{e}_N]^{1/2} = [\cos^2\theta + \sin^2\theta]^{1/2} = \sqrt{1} = 1$

So $\hat{e}_N$ is a unit vector

Similarly $\hat{e}_t = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ is a unit vector and $\hat{e}_N \cdot \hat{e}_t = 0$

Moreover $\vec{v} = v\hat{e}_t$ & $\vec{a} = a\hat{e}_N$

Note: Since $a = \rho\dot{\theta}^2$ & $v = \rho\dot{\theta}$ then we can write $a = \frac{v^2}{\rho}$

So, for uniform circular motion $v = \text{const.}$ & $\rho = \text{const.}$ But $\vec{v} \neq \text{const.}$ & $\vec{\rho} \neq \text{const.}$

$\vec{a} = \frac{v^2}{\rho}\hat{e}_N$, where $\hat{e}_N$ points towards center and is orthogonal to $\vec{v} = \rho\dot{\theta}\hat{e}_t$

Non-Uniform circular motion

Non-Uniform circular motion

We now allow $\dot{\theta} \neq const. \Rightarrow \frac{d|\vec{v}|}{dt} \neq 0$

Non-Uniform circular motion

We now allow $\dot{\theta} \neq$ const. $\Rightarrow \frac{d|\vec{v}|}{dt} \neq 0$

This means that

$$\vec{a} = \rho\ddot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right] + \rho\dot{\theta}\left[-\hat{i}\cos\theta - \hat{j}\sin\theta\right]$$

Non-Uniform circular motion

We now allow $\dot{\theta} \neq const. \Rightarrow \frac{d|\vec{v}|}{dt} \neq 0$

This means that

$$\vec{a} = \rho\ddot{\theta}\left[-\hat{i}\sin\theta + \hat{j}\cos\theta\right] + \rho\dot{\theta}^2\left[-\hat{i}\cos\theta - \hat{j}\sin\theta\right]$$

$$\Rightarrow \quad a = \rho\ddot{\theta}\,\hat{e}_t + \rho\dot{\theta}^2\,\hat{e}_n$$

Non-Uniform circular motion

We now allow $\dot{\theta} \neq$ const. $\Rightarrow \frac{d|\vec{v}|}{dt} \neq 0$

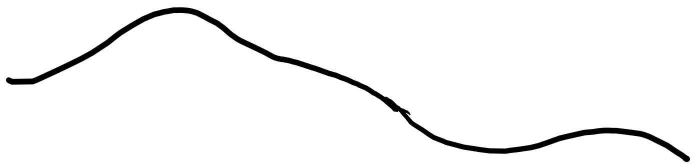
This means that

$$\vec{a} = \rho\ddot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta] + \rho\dot{\theta}^2[-\hat{i}\cos\theta - \hat{j}\sin\theta]$$

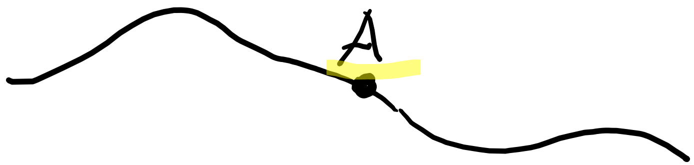
$$\Rightarrow \quad a = \rho\ddot{\theta}\,\hat{e}_t + \rho\dot{\theta}^2\,\hat{e}_n$$

Or $\boxed{\vec{a} = \dot{v}\,\hat{e}_t + \frac{v^2}{\rho}\,\hat{e}_n}$

Now generalize to some arbitrary smooth path

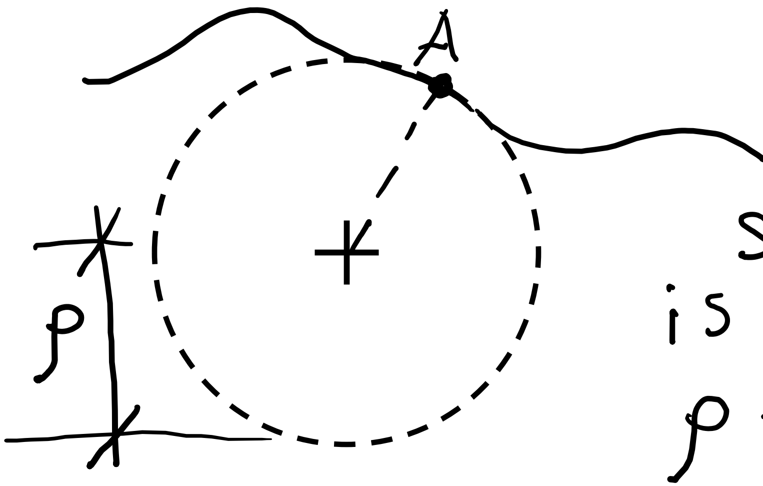


Now generalize to some arbitrary smooth path



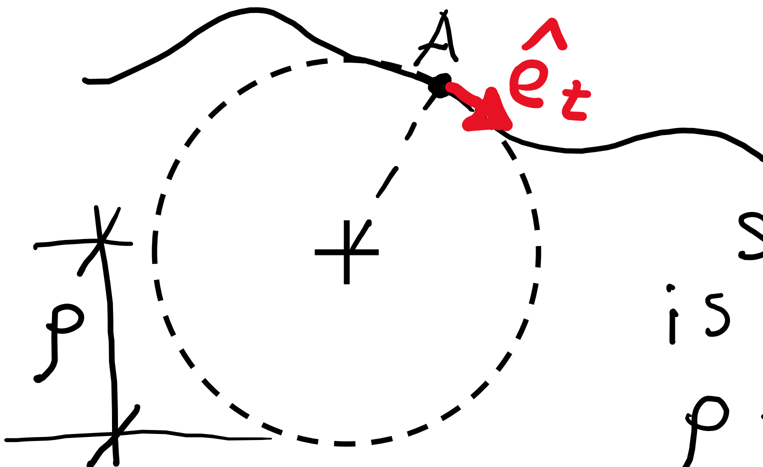
For some point A on some smooth curve

Now generalize to some arbitrary smooth path



For some point A on some smooth curve, there is a radius of curvature $\rho = \text{constant}$ $\left\{ \begin{array}{c} \text{could be} \\ \rho \to \infty \end{array} \right\}$

Now generalize to some arbitrary smooth path

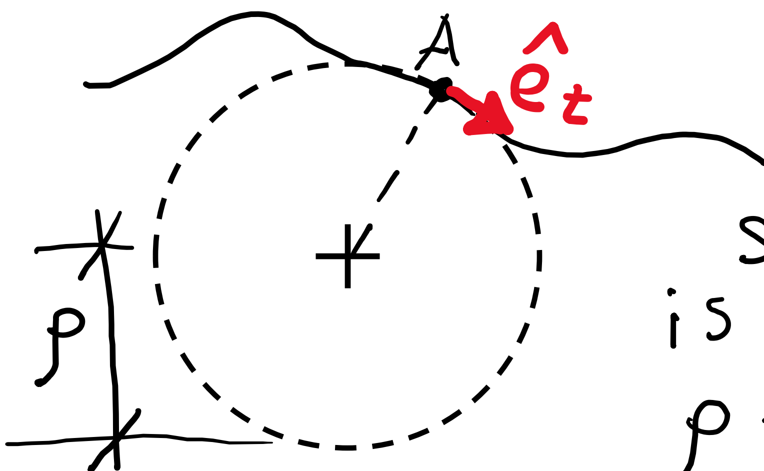


For some point A on some smooth curve, there is a radius of curvature

$\rho =$ constant {could be $\rho \to \infty$}

For a particle located at A & moving along the path

Now generalize to some arbitrary smooth path

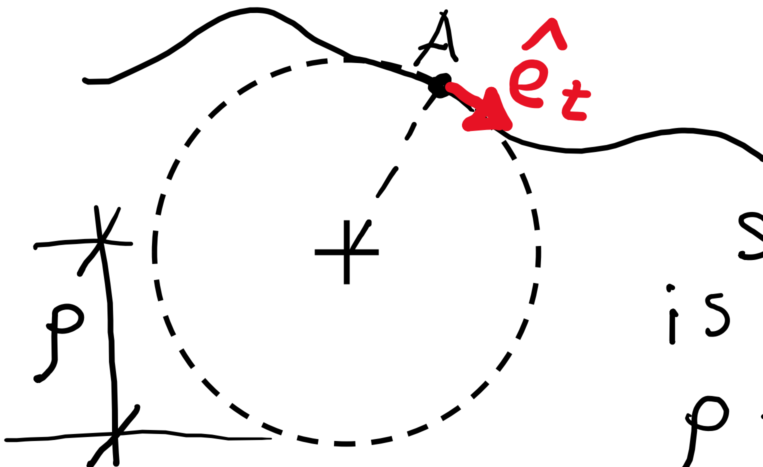


For some point A on some smooth curve, there is a radius of curvature $\rho = \text{constant}$ $\left\{ \begin{array}{l} \text{could be} \\ \rho \to \infty \end{array} \right\}$

For a particle located at A & moving along the path

$$\vec{v} = v\hat{e}_t$$

Now generalize to some arbitrary smooth path

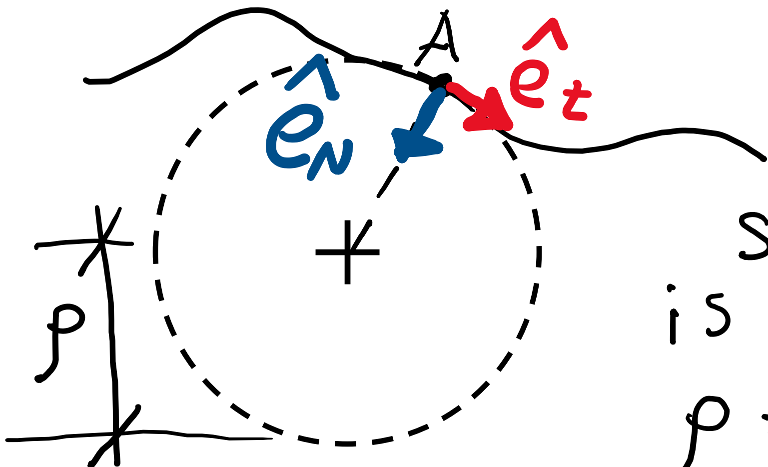


For some point A on some smooth curve, there is a radius of curvature ρ = constant {could be $\rho \to \infty$}

For a particle located at A & moving along the path

$$\vec{v} = v\hat{e}_t \quad \& \quad \vec{a} = \dot{v}\hat{e}_t +$$

Now generalize to some arbitrary smooth path

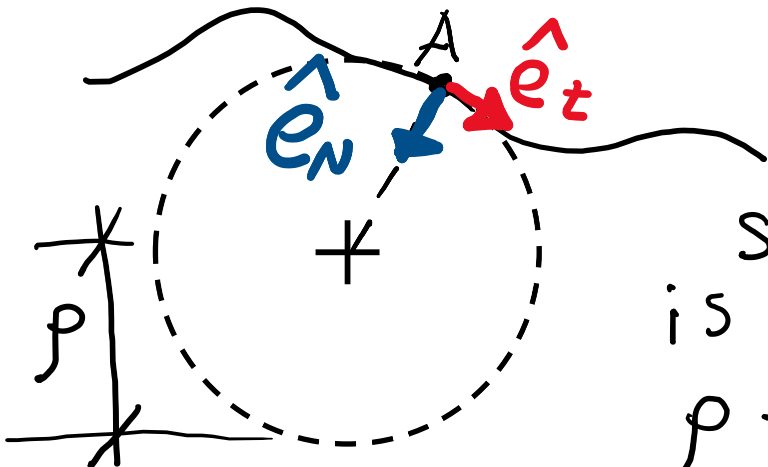


For some point A on some smooth curve, there is a radius of curvature $\rho =$ constant $\left\{ \text{could be } \rho \to \infty \right\}$

For a particle located at A & moving along the path

$$\vec{v} = v\hat{e}_t \quad \& \quad \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_N$$

Now generalize to some arbitrary smooth path



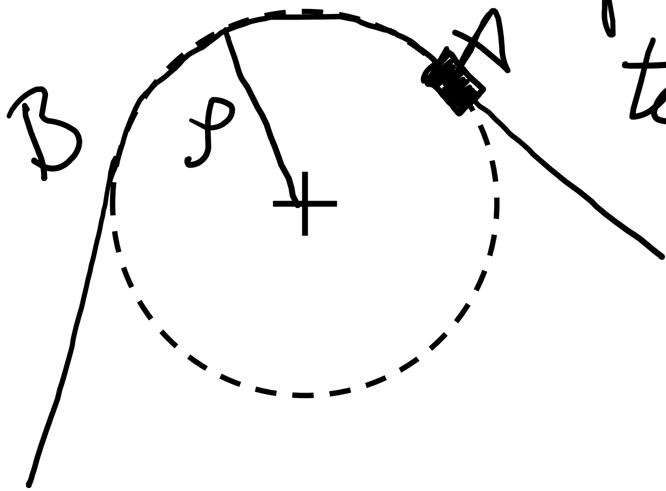
For some point A on some smooth curve, there is a radius of curvature ρ = constant {could be $\rho \to \infty$}

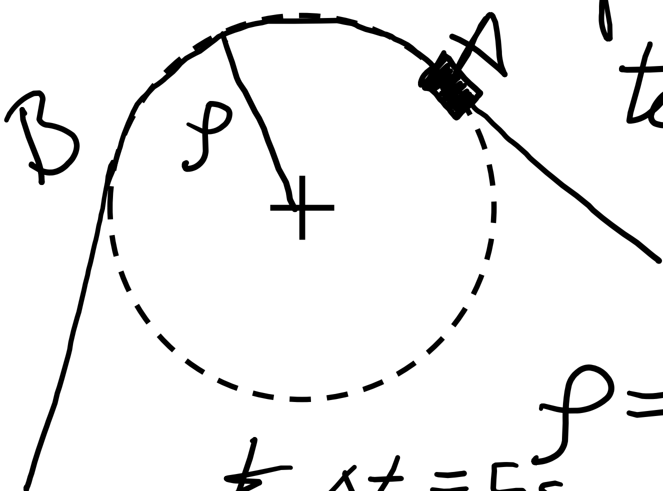
For a particle located at A & moving along the path

$$\vec{v} = v\hat{e}_t \quad \& \quad \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_N$$

It is like having an instantaneous coordinate system for each point along the path

Example: Car hits brakes at point A & causes car to slowdown at a constant rate to point B.



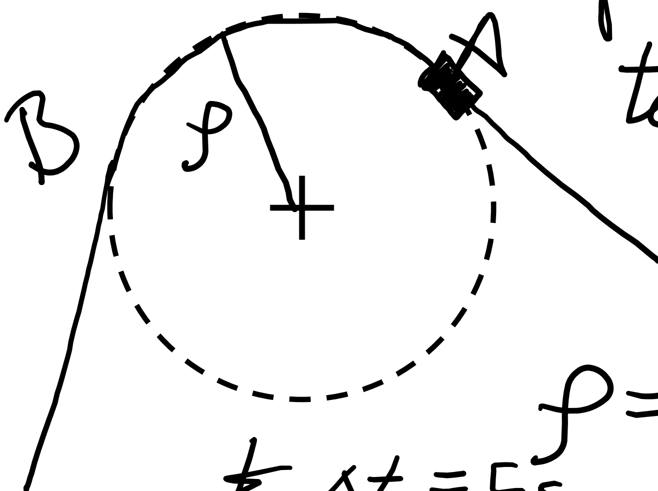


Example: Car hits brakes at point A & causes car to slowdown at a constant rate to point B. Given $\rho = 100m$, $V_A = 20\,m/s$, $V_B = 5\,\frac{m}{s}$ & $\Delta t = 5s$

Find $|\vec{a}|$ at point A:

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

Example: Car hits brakes at point A & causes car to slowdown at a constant rate to point B. Given



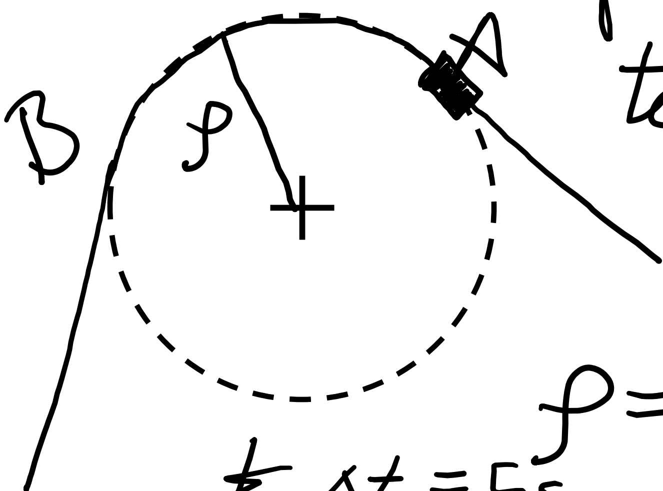
$\rho = 100m$, $V_A = 20\,m/s$, $V_B = 5\,\frac{m}{s}$

& $\Delta t = 5s$

Find $|\vec{a}|$ at point A:

$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$ & $\dot{v} = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{15\,m/s}{5s} = 3\,m/s^2$

<u>Example</u>: Car hits brakes at point A & causes car to slowdown at a constant rate to point B. Given



$\rho = 100m$, $V_A = 20 \, m/s$, $V_B = 5 \, \frac{m}{s}$

& $\Delta t = 5s$

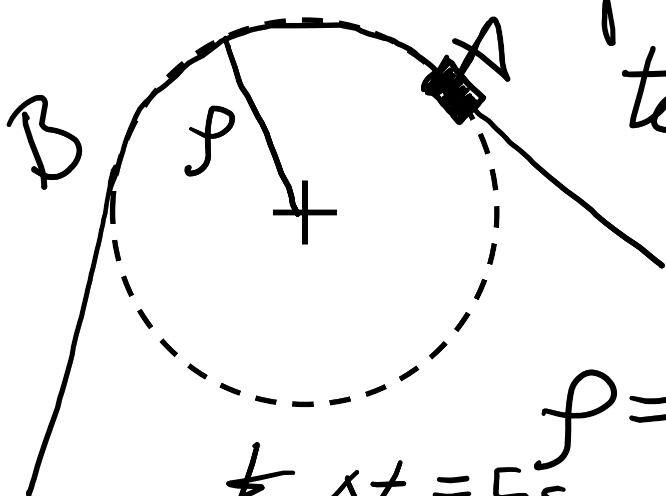
<u>Find $|\vec{a}|$ at point A:</u>

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

& $\dot{v} = \frac{dV}{dt} = \frac{\Delta V}{\Delta t} = \frac{15 \, m/s}{5s} = 3 \, m/s^2$

& $\frac{v^2}{\rho} = \frac{400 \, m^2/s^2}{100m} = 4 \, m/s^2$

Example: Car hits brakes at point A & causes car to slowdown at a constant rate to point B. Given $\rho = 100m$, $v_A = 20 m/s$, $v_B = 5 \frac{m}{s}$ & $\Delta t = 5s$

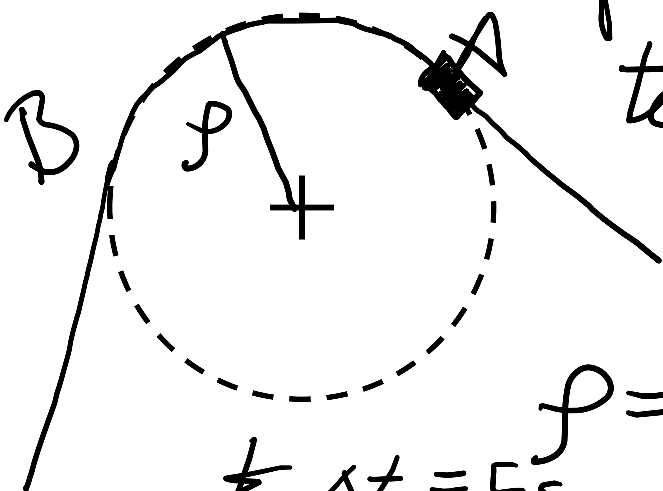


Find $|\vec{a}|$ at point A:

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n \quad \& \quad \dot{v} = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{15 m/s}{5s} = 3 m/s^2$$

$$\& \quad \frac{v^2}{\rho} = \frac{400 m^2/s^2}{100m} = 4 m/s^2 \quad \text{so} \quad \vec{a} = 3\frac{m}{s^2}\hat{e}_t + 4\frac{m}{s^2}\hat{e}_n$$

Example: Car hits brakes at point A & causes car to slowdown at a constant rate to point B. Given



$\rho = 100m$, $v_A = 20 m/s$, $v_B = 5 \frac{m}{s}$ & $\Delta t = 5s$

Find $|\vec{a}|$ at point A:

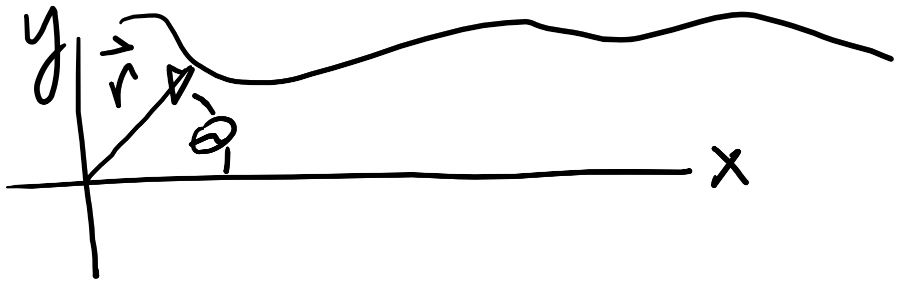
$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

& $\dot{v} = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{15 m/s}{5s} = 3 m/s^2$

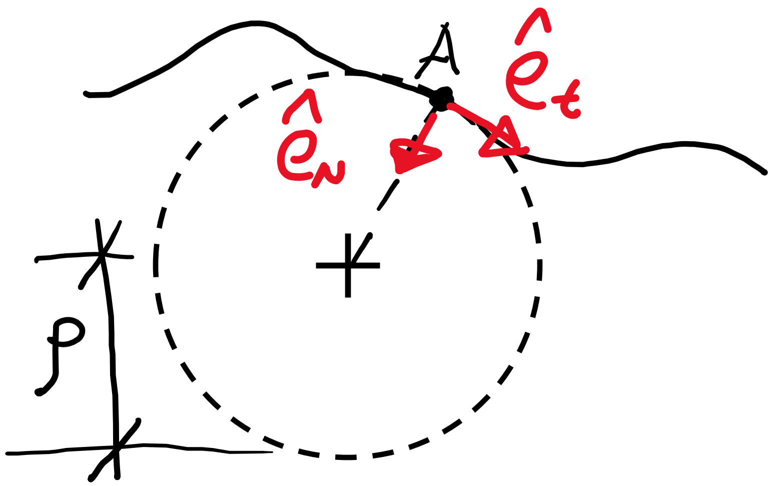
& $\frac{v^2}{\rho} = \frac{400 m^2/s^2}{100m} = 4 m/s^2$ so $\vec{a} = 3\frac{m}{s^2}\hat{e}_t + 4\frac{m}{s^2}\hat{e}_n$

$$\Rightarrow \quad |\vec{a}| = \sqrt{3^2+4^2}\left(\frac{m}{s^2}\right) = 5\frac{m}{s^2}$$

Fixed coordinates in polar form

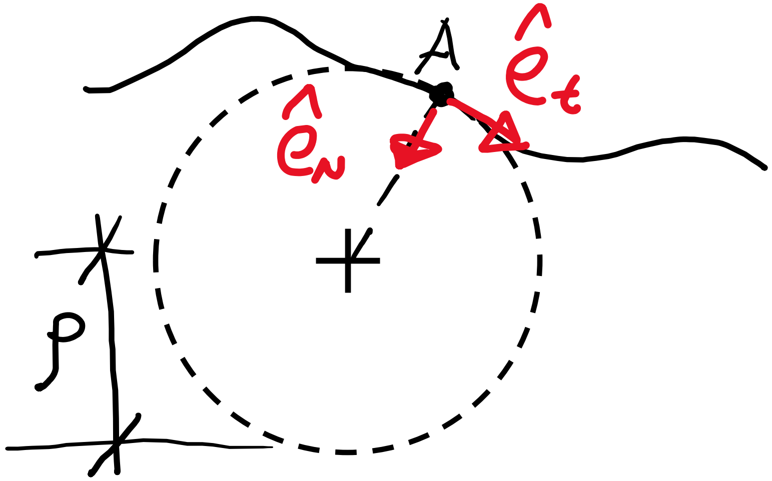


$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$



Previously we found that

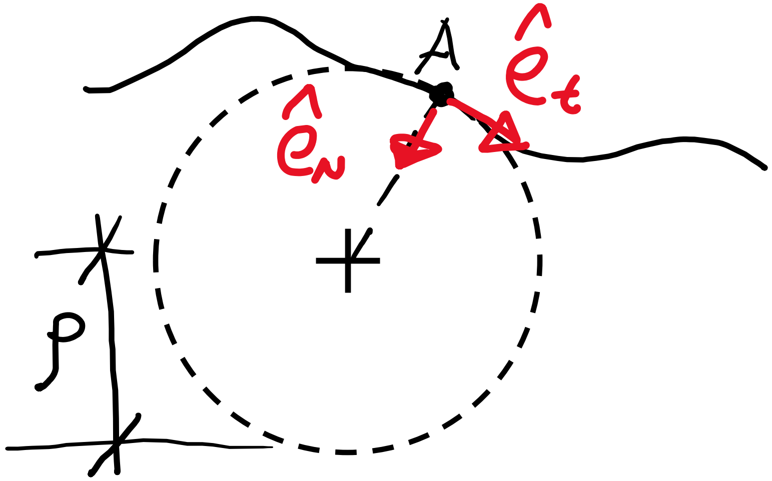
$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t$$



Previously we found that

$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

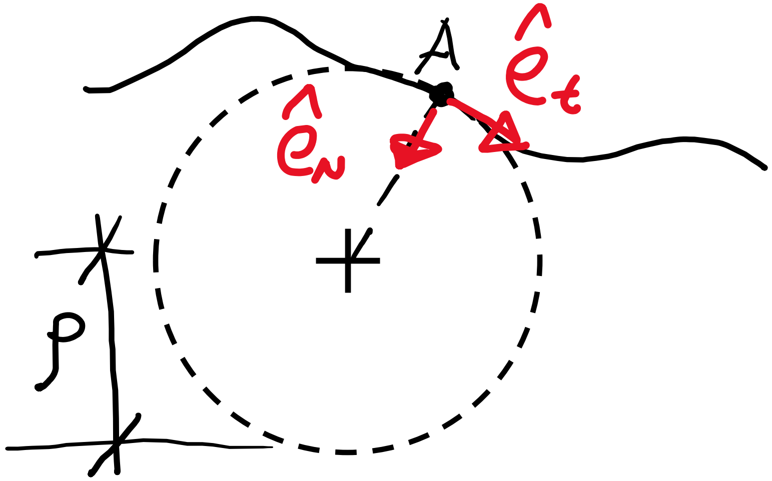
$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\,\hat{e}_t$$



Previously we found that

$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\,\hat{e}_t,$$ where

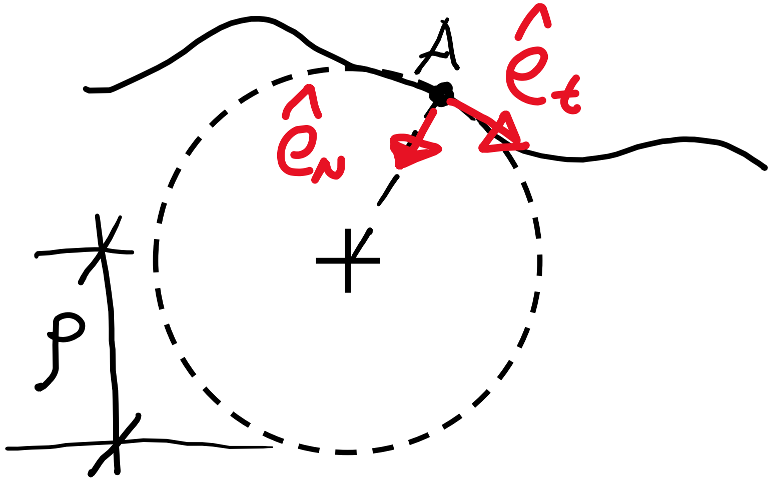


Previously we found that

$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

$$\hat{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\,\hat{e}_t, \text{ where}$$

$\hat{e}_t \equiv$ unit vector in tangential direction



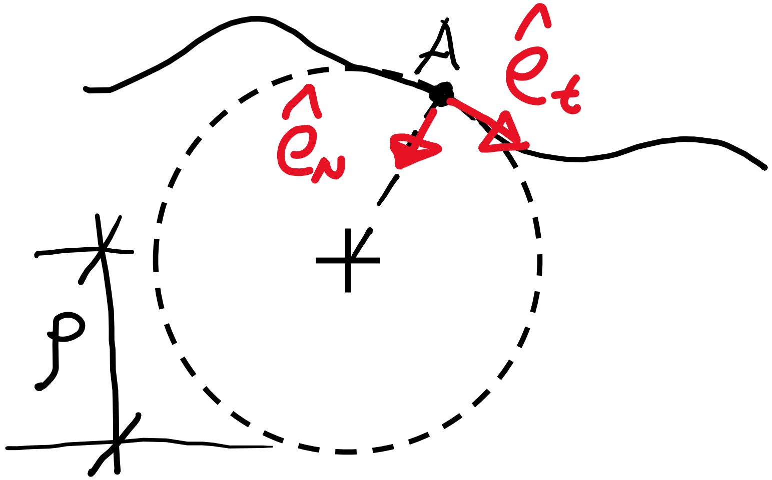
Previously we found that

$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\,\hat{e}_t, \text{ where}$$

$\hat{e}_t \equiv$ unit vector in tangential direction

$\hat{e}_n \equiv$ unit vector in normal direction



Previously we found that

$$\vec{v} = \rho\dot{\theta}\,\hat{e}_t \quad \&$$

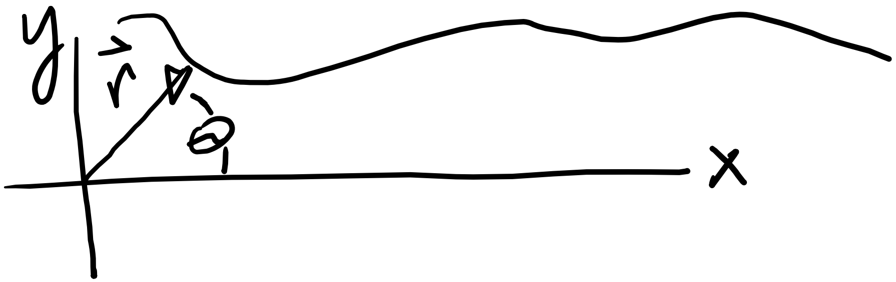
$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\,\hat{e}_t, \text{ where}$$

$\hat{e}_t \equiv$ unit vector in tangential direction

$\hat{e}_n \equiv$ unit vector in normal direction

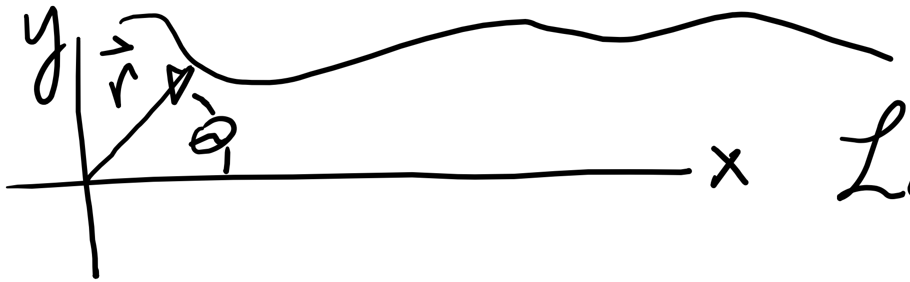
Normal & tangential components

Fixed coordinates in polar form



$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

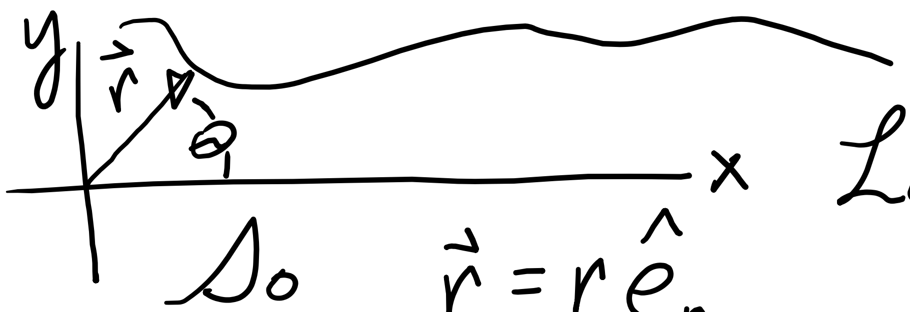
Fixed coordinates in polar form



$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

Fixed coordinates in polar form

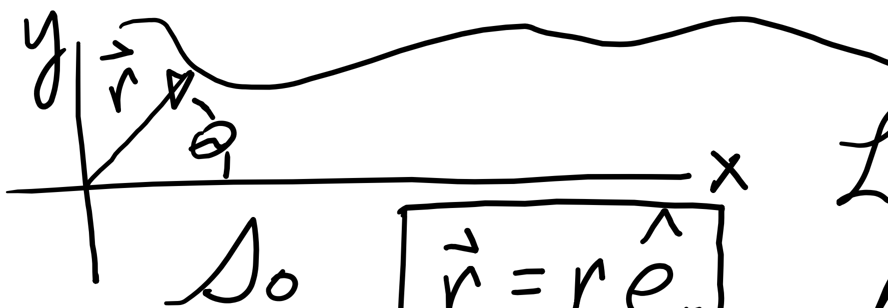


$$\vec{r} = r\left[\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta\right]$$

Let $\hat{e}_r \equiv \left[\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta\right]$

$$\vec{r} = r\,\hat{e}_r$$

Fixed coordinates in polar form



$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

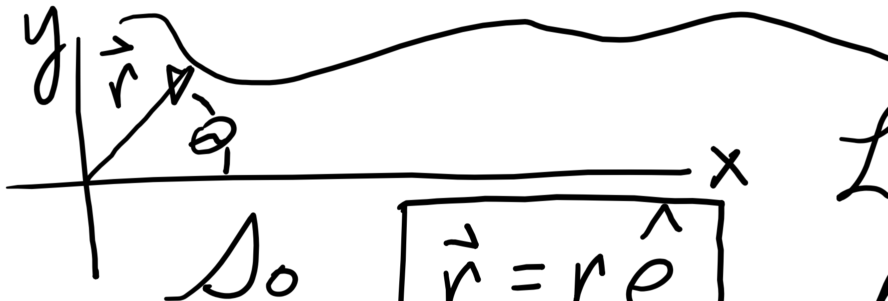
Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

$$\boxed{\vec{r} = r\hat{e}_r}$$

Note:

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}]$$

Fixed coordinates in polar form



$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

$$\boxed{\vec{r} = r\hat{e}_r}$$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

Note:

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Fixed coordinates in polar form

$\vec r = r[\hat i \cos\theta + \hat j \sin\theta]$

Let $\hat e_r \equiv [\hat i \cos\theta + \hat j \sin\theta]$

S_0 $\boxed{\vec r = r\hat e_r}$

Note:

$$\frac{d}{dt}\hat e = [\hat i(-\sin\theta)\dot\theta + \hat j(\cos\theta)\dot\theta] = \dot\theta[-\hat i \sin\theta + \hat j \cos\theta]$$

Let $\hat e_\theta = [-\hat i \sin\theta + \hat j \cos\theta]$

Fixed coordinates in polar form

$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

So $\boxed{\vec{r} = r\hat{e}_r}$

<u>Note:</u>

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Let $\hat{e}_\theta = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ so that $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$

Fixed coordinates in polar form

$$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

So $\boxed{\vec{r} = r\hat{e}_r}$

Note:

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Let $\hat{e}_\theta = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ so that $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$

Note: $\frac{d}{dt}\hat{e}_\theta = [-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}]$

Fixed coordinates in polar form

$$\vec r = r[\hat i\cos\theta + \hat j\sin\theta]$$

Let $\hat e_r \equiv [\hat i\cos\theta + \hat j\sin\theta]$

So $\boxed{\vec r = r\hat e_r}$

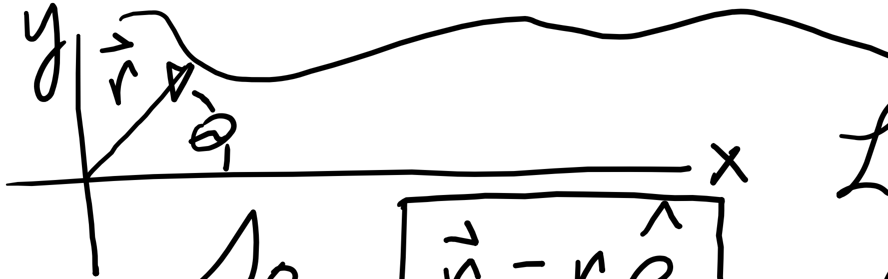
Note:

$$\frac{d}{dt}\hat e = [\hat i(-\sin\theta)\dot\theta + \hat j(\cos\theta)\dot\theta] = \dot\theta[-\hat i\sin\theta + \hat j\cos\theta]$$

Let $\hat e_\theta = [-\hat i\sin\theta + \hat j\cos\theta]$ so that $\frac{d}{dt}\hat e_r = \dot\theta\hat e_\theta$

Note: $\frac{d}{dt}\hat e_\theta = [-\hat i(\cos\theta)\dot\theta - \hat j(\sin\theta)\dot\theta] = -\dot\theta\hat e_r$

Fixed coordinates in polar form



$\vec{r} = r[\hat{i}\cos\theta + \hat{j}\sin\theta]$

Let $\hat{e}_r \equiv [\hat{i}\cos\theta + \hat{j}\sin\theta]$

So $\boxed{\vec{r} = r\hat{e}_r}$

Note:

$$\frac{d}{dt}\hat{e} = [\hat{i}(-\sin\theta)\dot{\theta} + \hat{j}(\cos\theta)\dot{\theta}] = \dot{\theta}[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Let $\hat{e}_\theta = [-\hat{i}\sin\theta + \hat{j}\cos\theta]$ so that $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$

Note: $\frac{d}{dt}\hat{e}_\theta = [-\hat{i}(\cos\theta)\dot{\theta} - \hat{j}(\sin\theta)\dot{\theta}] = -\dot{\theta}\hat{e}_r$

So $\boxed{\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r}$

Fixed coordinates in polar form

$\vec r = r[\hat i \cos\theta + \hat j \sin\theta]$

Let $\hat e_r \equiv [\hat i \cos\theta + \hat j \sin\theta]$

So $\boxed{\vec r = r\hat e_r}$

Note:

$$\frac{d}{dt}\hat e = [\hat i(-\sin\theta)\dot\theta + \hat j(\cos\theta)\dot\theta] = \dot\theta[-\hat i \sin\theta + \hat j \cos\theta]$$

Let $\hat e_\theta = [-\hat i \sin\theta + \hat j \cos\theta]$ so that $\frac{d}{dt}\hat e_r = \dot\theta \hat e_\theta$

Note: $\frac{d}{dt}\hat e_\theta = [-\hat i(\cos\theta)\dot\theta - \hat j(\sin\theta)\dot\theta] = -\dot\theta \hat e_r$

So $\boxed{\frac{d}{dt}\hat e_\theta = -\dot\theta \hat e_r}$ With this bit of Vector calculus out of the way, we can now easily determine $\vec v$ & $\vec a$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\,\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\,\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r}$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r]$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\,\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\,\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}\left[r\hat{e}_r\right] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

& $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta]$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \; \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r +$$

ASU

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r +$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta +$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$
$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta$$

ASU

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta$$

$$- r\dot{\theta}^2\hat{e}_r$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \; \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \underline{\ddot{r}\hat{e}_r} + \underline{\underline{\dot{r}\dot{\theta}\hat{e}_\theta}} + \underline{\underline{\dot{r}\dot{\theta}\hat{e}_\theta}} + \underline{\underline{r\ddot{\theta}\hat{e}_\theta}}$$

$$\underline{-r\dot{\theta}^2\hat{e}_r}$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta$$

$$-r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta$$

$$-r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}e_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}e_\theta + r\ddot{\theta}e_\theta - r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

$$\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

from previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}e_\theta +$$

$$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}e_\theta + r\ddot{\theta}e_\theta$$

$$-r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

$$\boxed{\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta}$$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\text{\&}\ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

$$\boxed{\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta}$$

Note: If $\dot{r} = 0$, then

$$\vec{v} = r\dot{\theta}\hat{e}_\theta$$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

So $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

$$\boxed{\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta}$$

Note: If $\dot{r} = 0$, then

$$\vec{v} = r\dot{\theta}\hat{e}_\theta \Rightarrow v = r\dot{\theta}$$

ASU

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

& $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta +$
$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta$
$-r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$

so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

$$\boxed{\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta}$$

Note: If $\dot{r} = 0$, then

$\vec{v} = r\dot{\theta}\hat{e}_\theta \Rightarrow v = r\dot{\theta}$ & $\vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}e_\theta$

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

& $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}e_\theta +$
$r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta$
$-r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$

so $\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$ &

$$\boxed{\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta}$$

Note: If $\dot{r} = 0$, then

$\vec{v} = r\dot{\theta}\hat{e}_\theta \Rightarrow v = r\dot{\theta}$ & $\vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}e_\theta \Rightarrow$
$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$

ASU

From previous $\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta$ & $\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$

$$\vec{v} = \frac{d}{dt}\vec{r} = \frac{d}{dt}[r\hat{e}_r] = \dot{r}\hat{e}_r + r\frac{d}{dt}\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\& \ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r + r\dot{\theta}e_\theta] = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

so

$$\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta} \quad \&$$

$$\boxed{\vec{a} = [\ddot{r} - r\dot{\theta}^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta}$$

Note: If $\dot{r} = 0$, then

$$\vec{v} = r\dot{\theta}\hat{e}_\theta \Rightarrow v = r\dot{\theta} \ \& \ \vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}e_\theta \Rightarrow$$

$$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$$

So

<u>polar with $\dot{r} = 0$</u>

$\vec{V} = r\dot{\theta}\hat{e}_{\theta}$

<u>Normal & tangential</u>

$\vec{V} = \rho\dot{\theta}\hat{e}_{t}$

So

polar with $\dot{r} = 0$

$$\vec{v} = r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$$

Normal & tangential

$$\vec{v} = \rho\dot{\theta}\hat{e}_t$$

$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$$

So

polar with $\dot{r} = 0$

$\vec{v} = r\dot{\theta}\hat{e}_\theta$

$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$

Normal & tangential

$\vec{v} = \rho\dot{\theta}\hat{e}_t$

$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$

Very Similar when $\dot{r} = 0$

So

<u>polar with $\dot{r} = 0$</u>

$\vec{v} = r\dot{\theta}\hat{e}_\theta$

$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$

<u>Normal & tangential</u>

$\vec{v} = \rho\dot{\theta}\hat{e}_t$

$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$

Making sense of $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

So

polar with $\dot{r} = 0$

$\vec{v} = r\dot{\theta}\hat{e}_\theta$

$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$

Normal & tangential

$\vec{v} = \rho\dot{\theta}\hat{e}_t$

$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$

Making sense of $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

Goal: To understand each term

So

<u>polar with $\dot{r} = \cancel{\theta}$</u>

$\vec{V} = r\dot{\theta}\hat{e}_\theta$

$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$

<u>Normal & tangential</u>

$\vec{V} = \rho\dot{\theta}\hat{e}_t$

$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$

Making sense of $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

Goal: To understand each term

Solution: Look at special cases

So

polar with $\dot{r} = 0$

$$\vec{v} = r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$$

Normal & tangential

$$\vec{v} = \rho\dot{\theta}\hat{e}_t$$

$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$$

Making sense of $\vec{a} = (\ddot{r} - \cancelto{0}{r\dot{\theta}^2})\hat{e}_r + (\cancelto{0}{r\ddot{\theta}} + \cancelto{0}{2\dot{r}\dot{\theta}})\hat{e}_\theta$

Goal: To understand each term

Solution: Look at special cases

CASE I: θ = constant

So

polar with $\dot{r} = 0$

$\vec{v} = r\dot{\theta}\hat{e}_\theta$

$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$

Normal & tangential

$\vec{v} = \rho\dot{\theta}\hat{e}_t$

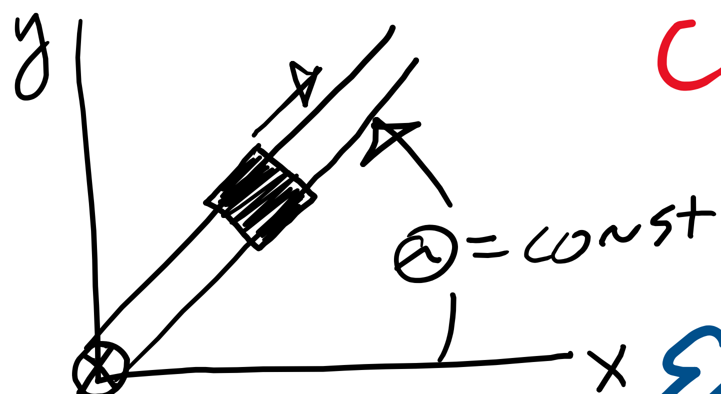
$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + \dot{v}\hat{e}_t$

Making sense of $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

Goal: To understand each term

Solution: Look at special cases

CASE I: $\theta = \text{constant} \Rightarrow \vec{a} = \ddot{r}\hat{e}_r$



Collar on shaft fixed at constant angle θ

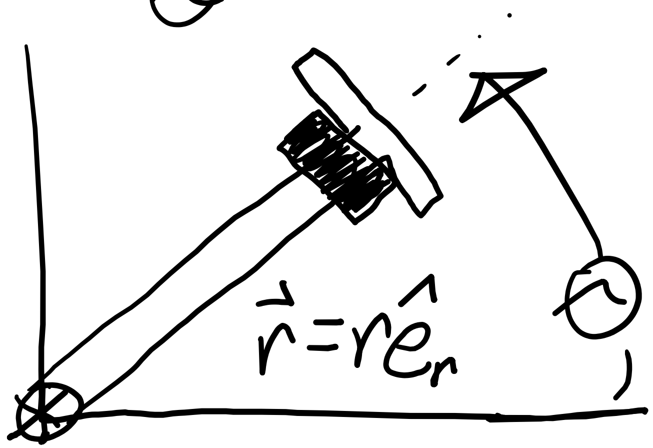
Easiest to understand

CASE II: $r = \text{const.}$ & $\dot{\theta} = \text{const.}$

CASE II: $r = \text{const.}$ & $\dot{\theta} = \text{const.}$

[uniform circular motion]

$\dot{\theta} = \text{constant}$

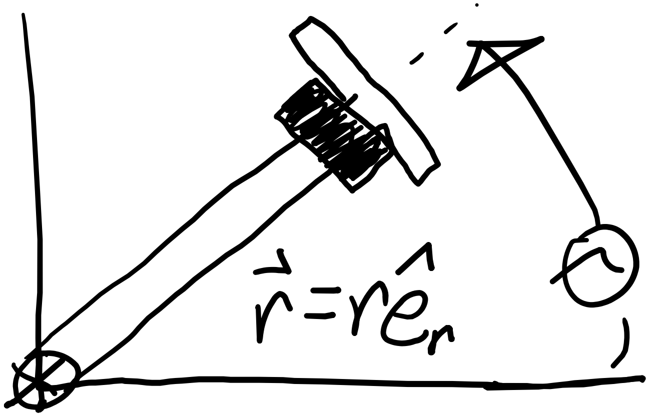


CASE II: $r = \text{const.}$ & $\dot{\theta} = \text{const.}$

[uniform circular motion]

$\dot{\theta} = \text{constant}$

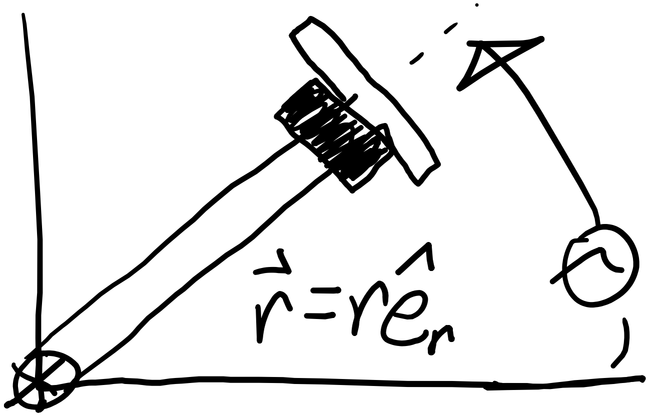
$\Delta s = r\Delta\theta$: Distance traveled



CASE II: $r = \text{const.}$ & $\dot{\theta} = \text{const.}$

[uniform circular motion]

$\dot{\theta} = \text{constant}$



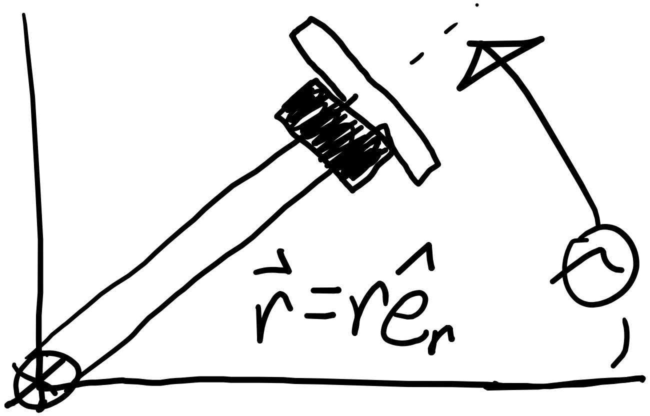
$\Delta s = r\Delta\theta$: Distance traveled

$\vec{v} = r\dot{\theta}\hat{e}_\theta$

CASE II: $r = \text{const.}$ & $\dot{\theta} = \text{const.}$

[uniform circular motion]

$\dot{\theta} = \text{constant}$



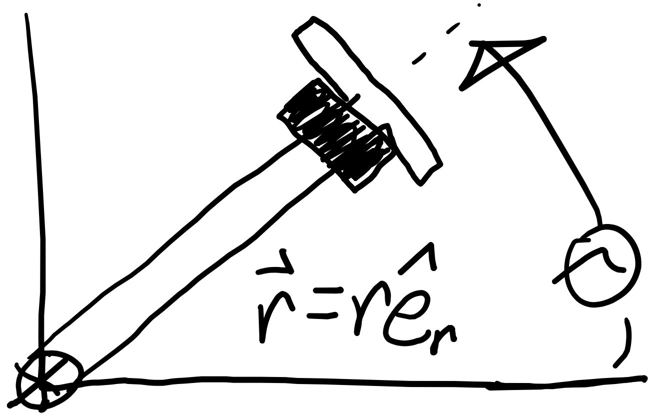
$\Delta s = r\Delta\theta$: Distance traveled

$\vec{v} = r\dot{\theta}\hat{e}_\theta$ & $\vec{a} = -r\dot{\theta}^2\hat{e}_r$

CASE II: $r = \text{const.}$ & $\dot{\theta} = \text{const.}$

[uniform circular motion]

$\dot{\theta} = \text{constant}$



$\Delta s = r\Delta\theta$: Distance traveled

$\vec{v} = r\dot{\theta}\hat{e}_\theta$ & $\vec{a} = -r\dot{\theta}^2\hat{e}_r$

acceleration to stay in circular motion

CASE III: $r = \text{const.}$ & $\ddot{\theta} = \text{const.}$

CASE III: $r = \text{const.}$ & $\ddot{\theta} = \text{const.}$

$$\Rightarrow \vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta$$

CASE III: $r = \text{const.}$ & $\ddot{\theta} = \text{const.}$

$$\Rightarrow \vec{a} = \underbrace{-r\dot{\theta}^2\hat{e}_r}_{\text{keeps motion circular}} + r\ddot{\theta}\hat{e}_\theta$$

CASE III: $r = \text{const.}$ & $\ddot{\theta} = \text{const.}$

$$\Rightarrow \vec{a} = \underbrace{-r\dot{\theta}^2\hat{e}_r}_{\text{keeps motion circular}} + r\ddot{\theta}\hat{e}_\theta$$

, $r\ddot{\theta}$ is the increase (or decrease) in $|\vec{V}|$

CASE III : $r = \text{const.}$ & $\ddot{\theta} = \text{const.}$

$\Rightarrow \vec{a} = \underbrace{-r\dot{\theta}^2\hat{e}_r}_{\text{keeps motion circular}} + r\ddot{\theta}\hat{e}_\theta$, $r\ddot{\theta}$ is the increase (or decrease) in $|\vec{V}|$ since $|\vec{V}| = r\dot{\theta} \Rightarrow \frac{dV}{dt} = r\ddot{\theta}$

CASE III: $r = \text{const.}$ & $\ddot{\theta} = \text{const.}$

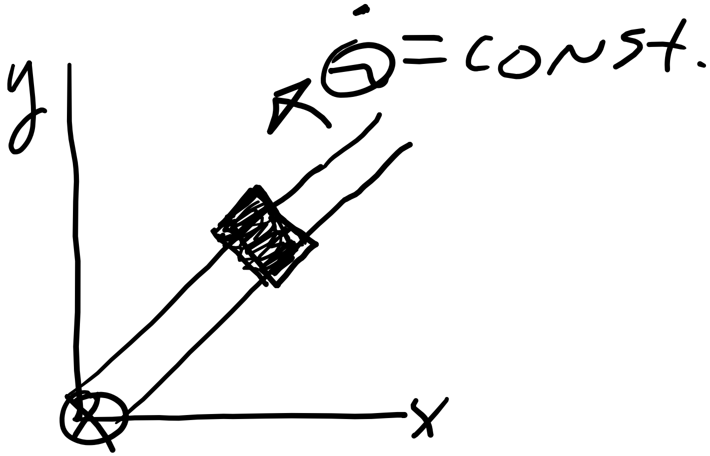
$$\Rightarrow \vec{a} = \underbrace{-r\dot{\theta}^2\hat{e}_r}_{\text{keeps motion circular}} + r\ddot{\theta}\hat{e}_\theta$$

, $r\ddot{\theta}$ is the increase (or decrease) in $|\vec{V}|$ since $|\vec{V}| = r\dot{\theta} \Rightarrow \frac{dv}{dt} = r\ddot{\theta}$

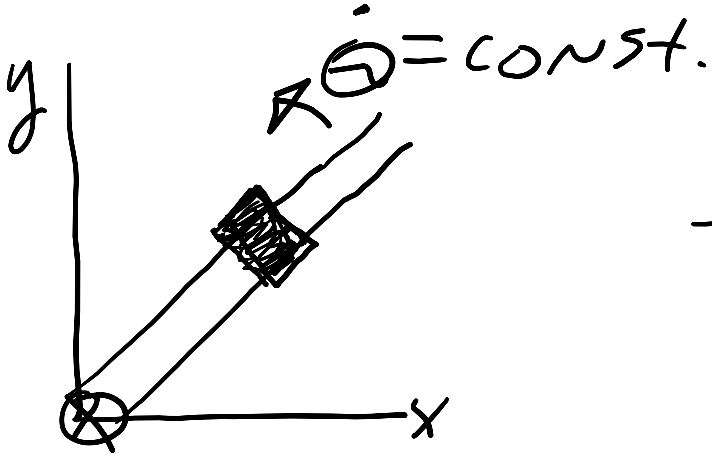
particle moves in fixed circle <u>but</u> goes faster (or slower) as time passes

CASE IV: $\dot{\theta} = \text{const.}$ & NO acceleration in $\hat{e}_r$ direction.

CASE IV: $\dot{\theta}$ = const. & NO acceleration in $\hat{e}_r$ direction.

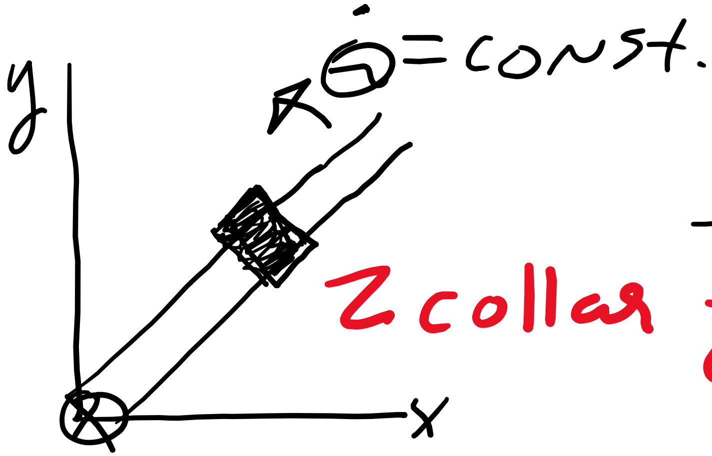


CASE IV: $\dot{\theta} = \text{const.}$ & NO acceleration in $\hat{e}_r$ direction.



Since $\vec{F} = m\vec{a}$, we need force in $\hat{e}_r$ direction to get acceleration in $\hat{e}_r$

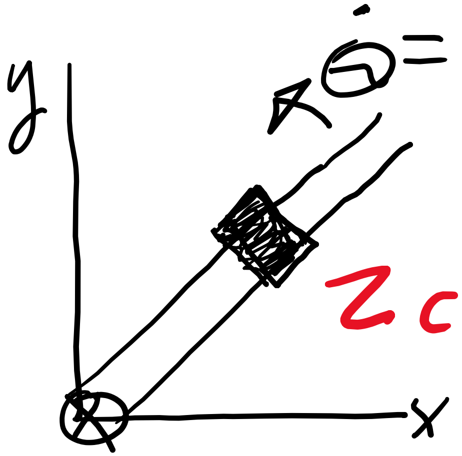
CASE IV: $\dot{\theta} = \text{const.}$ & NO acceleration in $\hat{e}_r$ direction.



Since $\vec{F} = m\vec{a}$, we need force in $\hat{e}_r$ direction to get acceleration in $\hat{e}_r$

collar free to move on shaft

CASE IV: $\dot{\theta} = \text{const.}$ & NO acceleration in $\hat{e}_r$ direction.



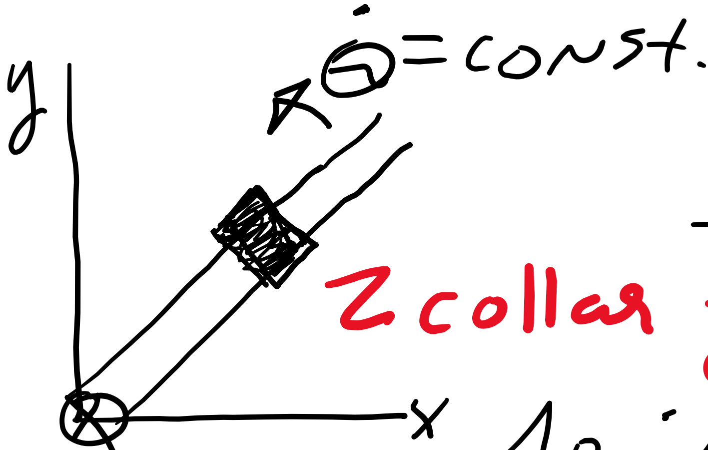
Since $\vec{F} = m\vec{a}$, we need force in $\hat{e}_r$ direction to get acceleration in $\hat{e}_r$

collar free to move on shaft

So, if no force in $\hat{e}_r$ then

$$\vec{a} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{\text{zero}}\hat{e}_r + (\underbrace{r\ddot{\theta}}_{\text{zero since } \dot{\theta} = \text{const}} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

CASE IV: $\dot{\theta}$ = const. & NO acceleration in $\hat{e}_r$ direction.



Since $\vec{F} = m\vec{a}$, we need force in $\hat{e}_r$ direction to get acceleration in $\hat{e}_r$

collar free to move on shaft

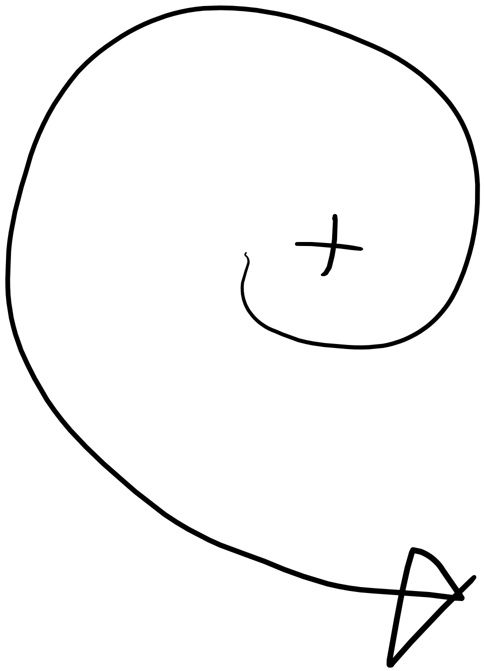
So, if no force in $\hat{e}_r$ then

$$\vec{a} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{\text{zero}}\hat{e}_r + (\cancel{r\ddot{\theta}} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

zero since $\dot{\theta}$ = const

$$\Rightarrow \boxed{\vec{a} = 2\dot{r}\dot{\theta}\hat{e}_\theta}$$

Motion looks like



Just as you would expect for the motion of a collar that is free to move on a rotating shaft.