Today: 12.1

$\boxed{L5}$

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Linear momentum & forces

LS

$\boxed{\text{L5}}$

Today: 12.1

Thursday: 12.2

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$\boxed{L5}$

Angular momentum

&

orbital motion

$\boxed{L5}$

<u>Today</u> : 12.1

<u>Thursday</u> : 12.2

<u>Tuesday February 2nd</u> : Review

LS

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HW #3 (Due Tuesday February 2nd)

12.3, 12.17, 12.37, 12.57, 12.61	§12.1
12.74, 12.78, 12.82, 12.90, 12.91	§12.2

L5

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Thursday: 12.2

Tuesday February 2nd: Review

Thursday February 4th: Exam #1

HW #3 (Due Tuesday February 2nd)

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Dropping some of the notation | Taking $\sum_{i=1}^{N} \vec{F}_i$ to $\sum \vec{F}$

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$$\Rightarrow \boxed{m = \frac{W}{g}}$$

Breaking up Force into separate coordinates

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$\Sigma \vec{F} = m\vec{a} \Rightarrow \quad \Sigma F_t = m\dot{v} \quad \& \quad \Sigma F_n = mv^2/\rho$

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Tangential & normal components

$\Sigma\vec{F}=m\vec{a} \Rightarrow \ \Sigma F_t=m\dot{v}$ & $\Sigma F_n=mv^2/\rho$

Radial & transverse components

$\Sigma\vec{F}=m\vec{a} \Rightarrow \Sigma F_r=m(\ddot{r}-r\dot{\theta}^2)$ &

$\Sigma F_\theta=m(r\ddot{\theta}+2\dot{r}\dot{\theta})$

ASU

Friction

Friction

coefficient of static friction μ_s
coefficient of kinetic friction μ_k

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Friction coefficient of static friction μ_s
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$$\boxed{\begin{array}{l} F_F \leq N\mu_s \text{, static} \\ F_F = N\mu_k \text{, kinetic} \end{array}}$$

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For static case, F_F only provides enough force to keep item static

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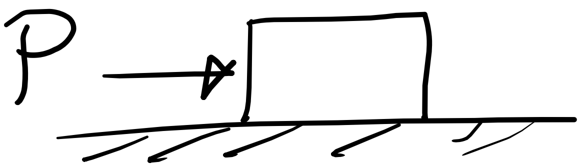
For static case, F_F only provides enough force to keep item static

For both cases, N is the normal force between the two items that are in frictional contact.

Example: Box of mass m on surface being pushed with a force P (shown below). For this problem there is friction between the box & surface



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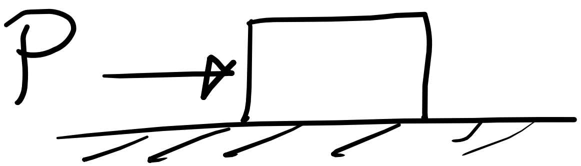
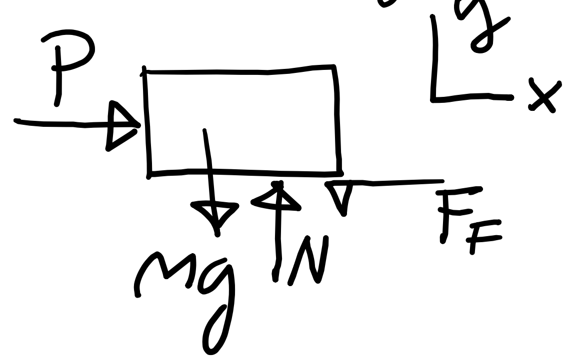


Free body diagram:



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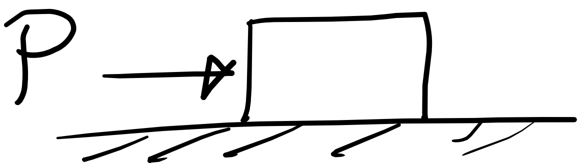

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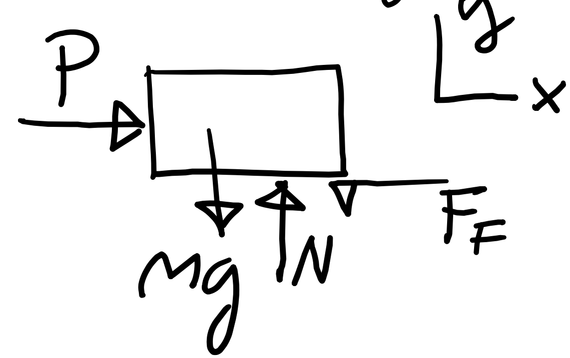

$$\Rightarrow \sum F_x = ma_x \quad \& \quad \sum F_y = ma_y$$

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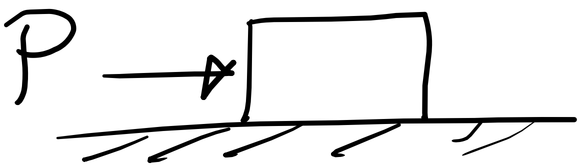

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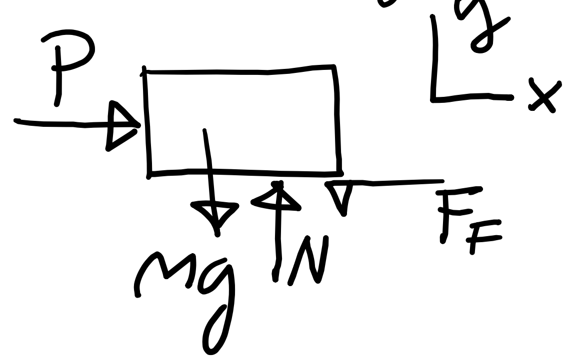

$\Rightarrow$ $\sum F_x = ma_x$ & $\sum F_y = ma_y$

$\Rightarrow$ $P - F_F = ma_x$

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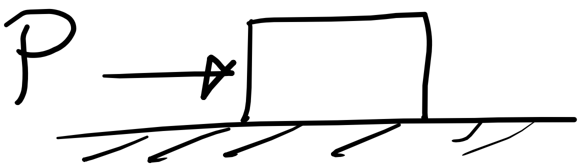

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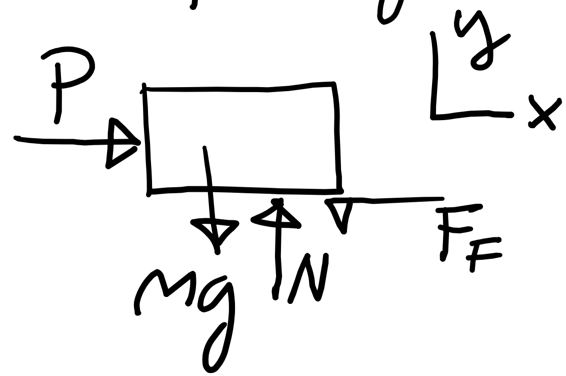

$\Rightarrow \sum F_x = ma_x$ & $\sum F_y = ma_y$

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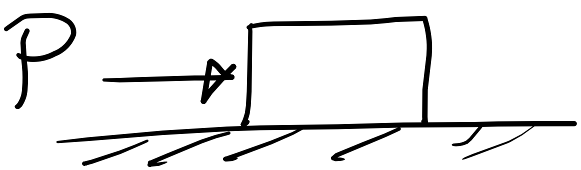

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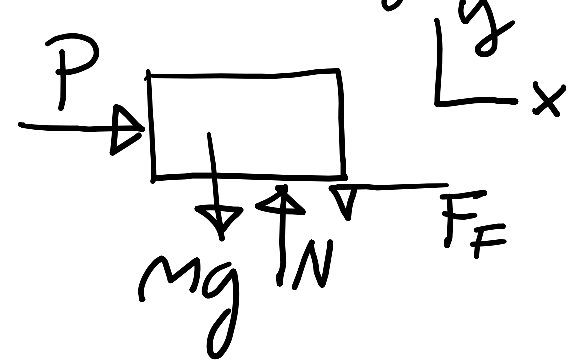

$$\Rightarrow \sum F_x = ma_x \quad \& \quad \sum F_y = ma_y$$

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So far we have $P - F_F = ma_x$ & $N = mg$

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$F_F \leq N\mu_s$ & $N = mg \Rightarrow P \leq mg\mu_K$ <u>IF</u>

$P > mg\mu_K$, we <u>must</u> go to Kinetic case.

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Note: μ_k is always less than μ_s. As an example, if box & surface are both steel, we could have $\mu_s = 0.6$ & $\mu_k = 0.1$

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<u>Kinetic case</u>: $\sum F_x = ma_x \Rightarrow P - F_F = ma_x$

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$\Rightarrow P - mg\mu_k = ma_x \Rightarrow a_x = \frac{P}{m} - g\mu_k$. So, if

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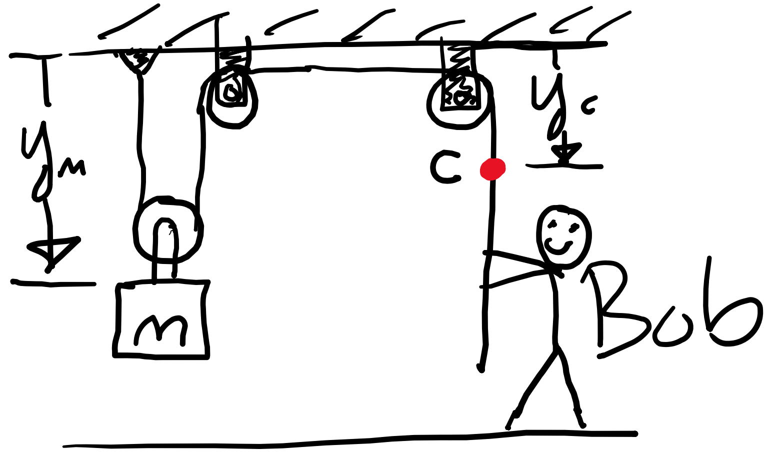
$a_x = (2 - 0.981) m/s^2 = 1.019 \; m/s^2$

Pulley problem:

Bob uses a pulley system to raise a mass m at constant velocity.



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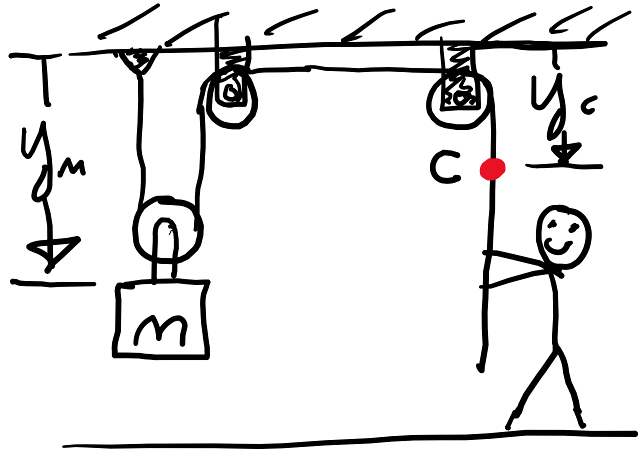

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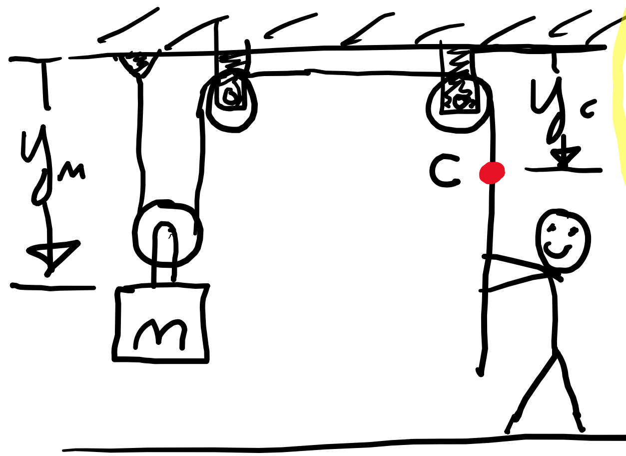

Note: C is just some point on rope Bob is pulling

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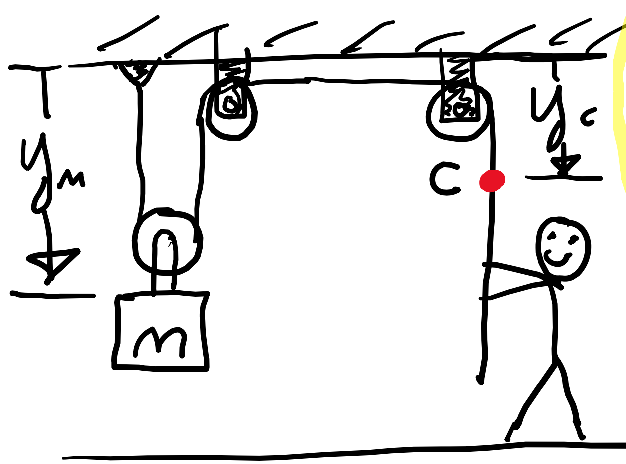

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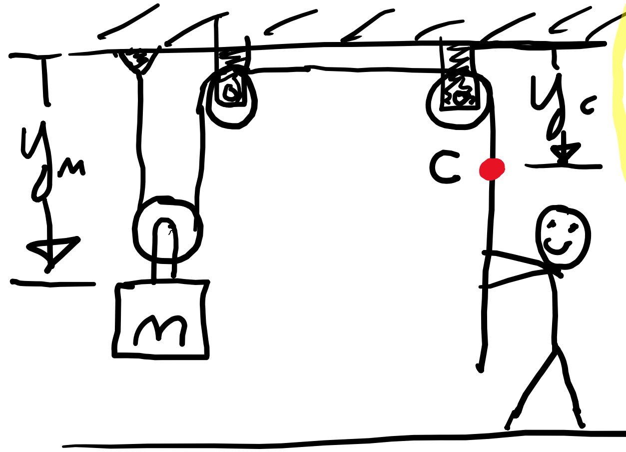

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$\Rightarrow 2v_m + v_c = 0 \Rightarrow v_m = -\frac{1}{2}v_c$

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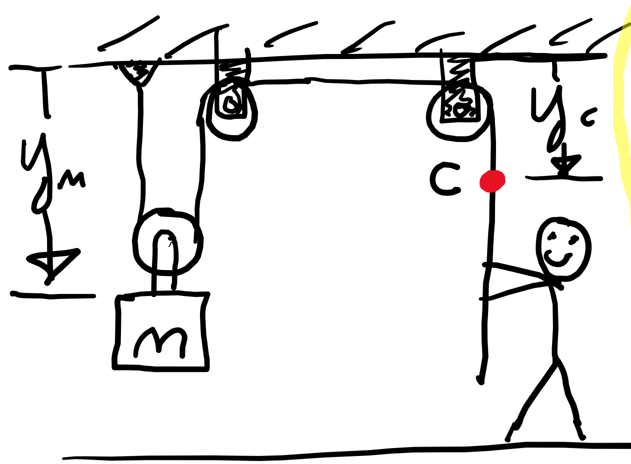

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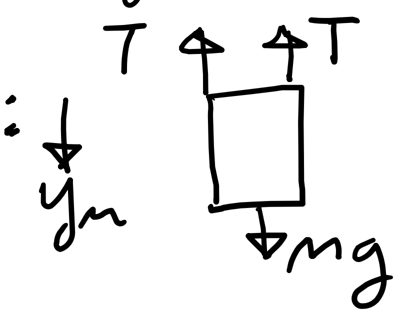
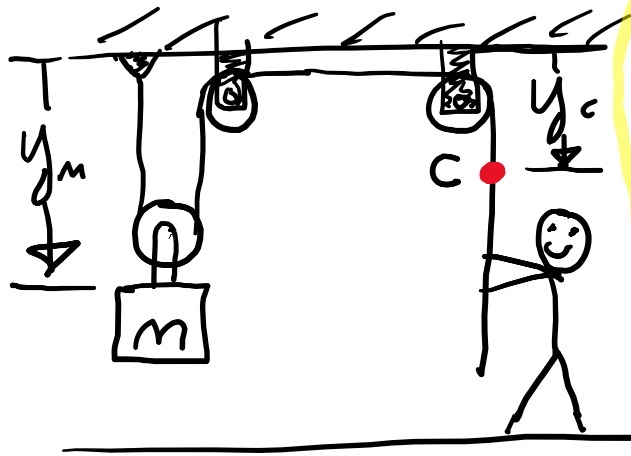
Free body mass m [and pulley]:



Pulley problem:



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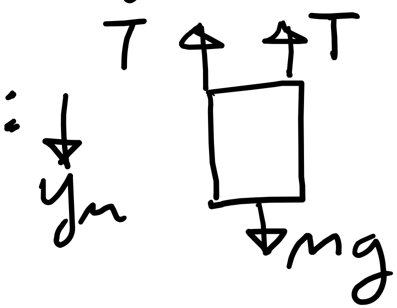
$\Rightarrow 2V_m + V_c = 0 \Rightarrow V_m = -\frac{1}{2}V_c$

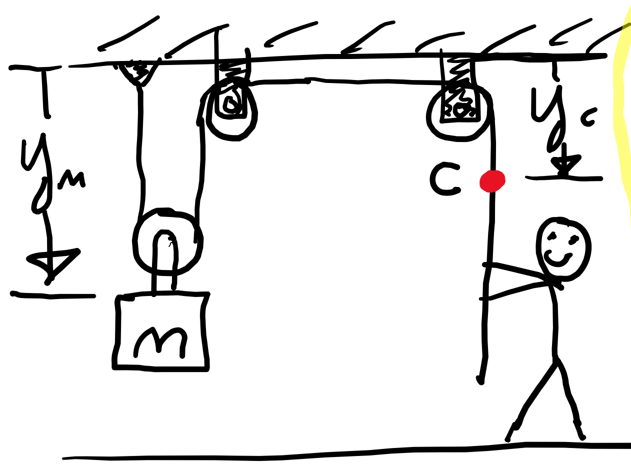
Free body mass m [and pulley]:



$\Rightarrow \Sigma F = ma \Rightarrow -2T + mg = ma_m$

Pulley problem:

Bob uses a pulley system to raise a mass m at constant velocity. Assuming there is no frictional forces, find the tension Bob must provide.



Note: C is just some point on rope Bob is pulling

1^{ST} I'll relate v_c & v_m: $2y_m + y_c = \text{const}$

$\Rightarrow\ 2v_m + v_c = 0 \Rightarrow v_m = -\frac{1}{2}v_c$

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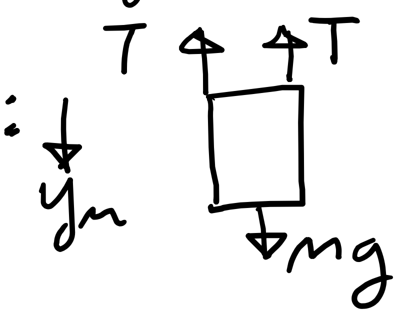

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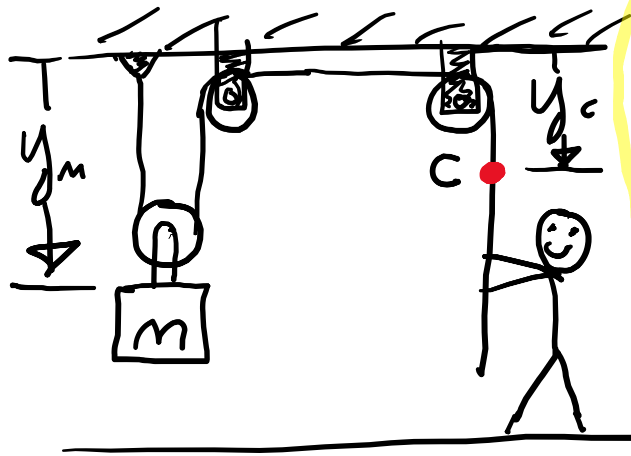

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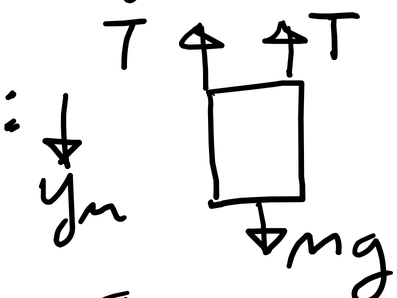

$\Rightarrow \Sigma F = ma \Rightarrow -2T + mg = ma_m$ But $V_m = \text{const}$ so $a_m = 0$ Now $2T = mg$

$\Rightarrow T = \frac{mg}{2}$

So Bob can lift a box weighing mg with $\frac{1}{2}$ that amount of tension

So Bob can lift a box weighing mg with $\frac{1}{2}$ that amount of tension, **but** he will pull on rope at a speed 2 times as large as the speed of the mass he is lifting.

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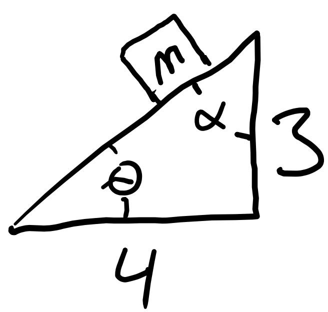
Problem: Box on incline plane.

CASE I: No friction

Find acceleration:



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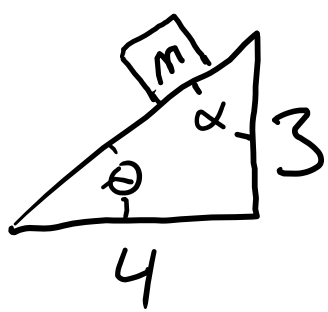

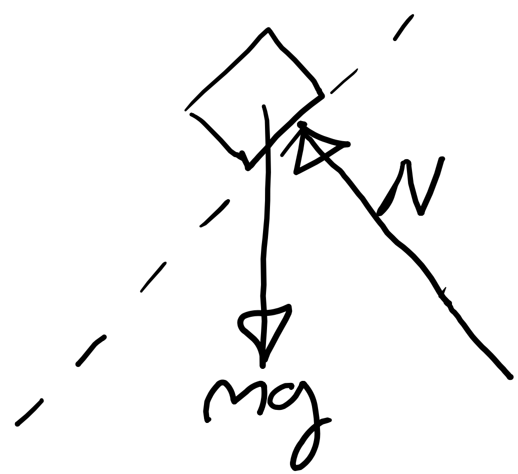

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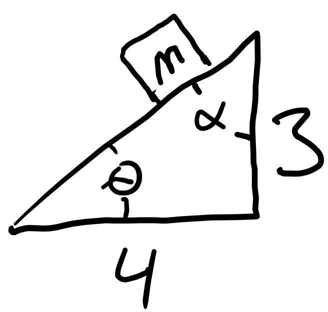

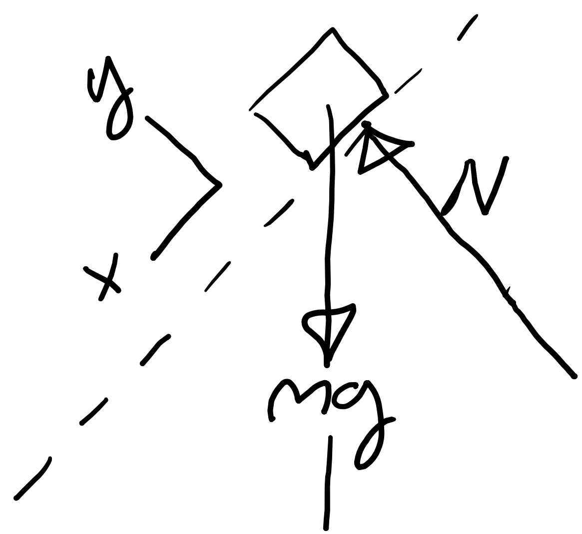

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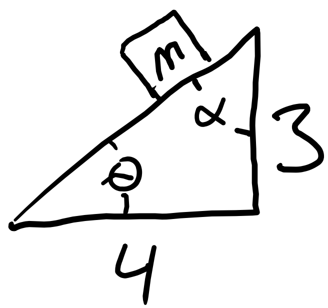

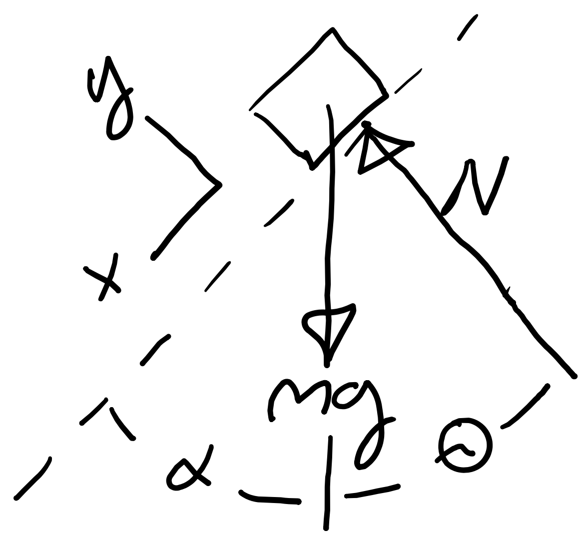

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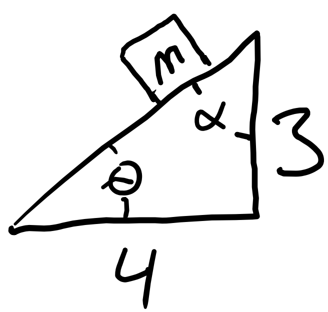

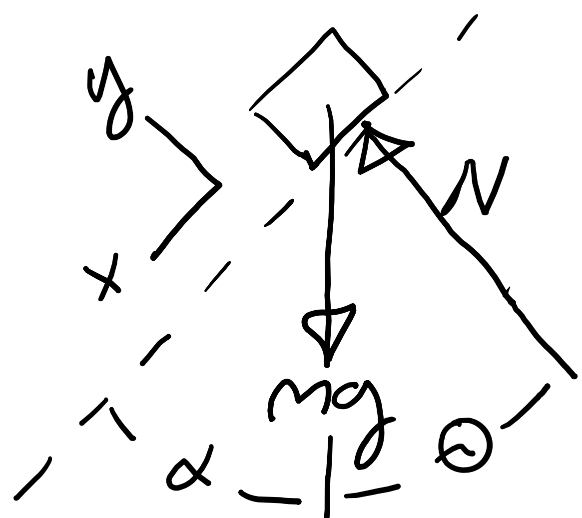

$\Sigma F_x = m a_x$

$\Rightarrow mg\sin\theta = m a_x$

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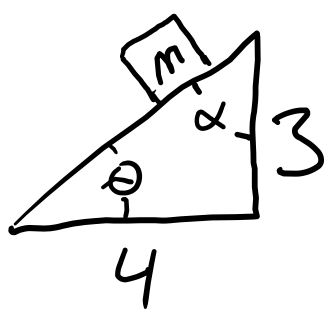

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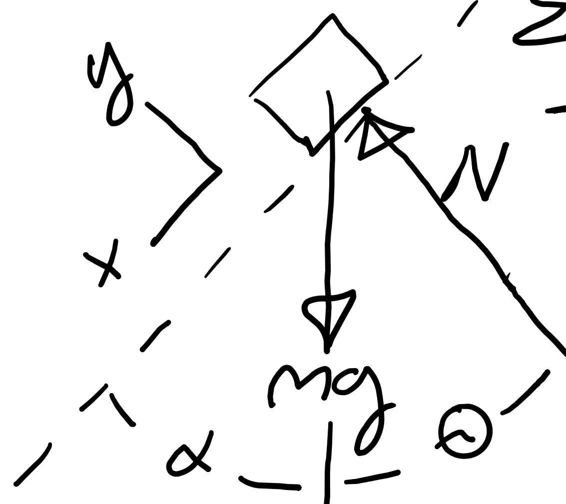

$\sum F_y = ma_y \Rightarrow$

$-mg\cos\theta + N = 0$

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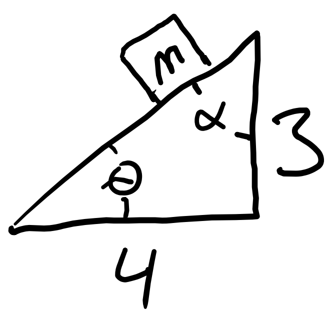

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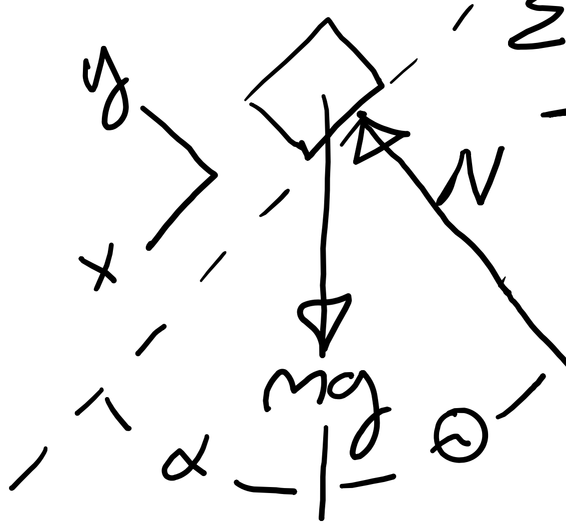

$$\sum F_y = m a_y \Rightarrow$$

$$-mg\cos\theta + N = 0$$

$$\Rightarrow N = mg\cos\theta$$

We have $N = mg\cos\theta \Rightarrow N = \frac{4}{5}mg$

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& $ma_x = mg\sin\theta \Rightarrow$

$$\boxed{\begin{aligned} N &= \tfrac{4}{5}mg \\ a_x &= \tfrac{3}{5}g \end{aligned}}$$

We have $N = mg\cos\theta \Rightarrow$ $\boxed{N = \frac{4}{5}mg \quad a_x = \frac{3}{5}g}$

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CASE II: Kinetic Friction with $\mu_k = 5/8$

We have $N = mg\cos\theta \Rightarrow$ $\boxed{\begin{array}{l} N = \frac{4}{5}mg \\ a_x = \frac{3}{5}g \end{array}}$

$\Sigma ma_x = mg\sin\theta \Rightarrow$

CASE II: Kinetic Friction with $\mu_k = 5/8$

Now $\Sigma F_x = mg\sin\theta - \mu_k N = ma_x$

We have $N = mg\cos\theta \Rightarrow$ $\boxed{N = \frac{4}{5}mg}$

$\& \ ma_x = mg\sin\theta \Rightarrow$ $\boxed{a_x = \frac{3}{5}g}$

CASE II: Kinetic Friction with $\mu_K = 5/8$

Now $\sum F_x = mg\sin\theta - \mu_K N = ma_x$ & as before

$\sum F_y = -mg\cos\theta + N = 0$ so $\boxed{N = mg\cos\theta}$

We have $N = mg\cos\theta \Rightarrow$ $\boxed{N = \frac{4}{5}mg \quad a_x = \frac{3}{5}g}$

$\& \; ma_x = mg\sin\theta \Rightarrow$

CASE II: Kinetic Friction with $\mu_k = 5/8$

Now $\sum F_x = mg\sin\theta - \mu_k N = ma_x$ & as before

$\sum F_y = -mg\cos\theta + N = 0$ so $\boxed{N = mg\cos\theta}$

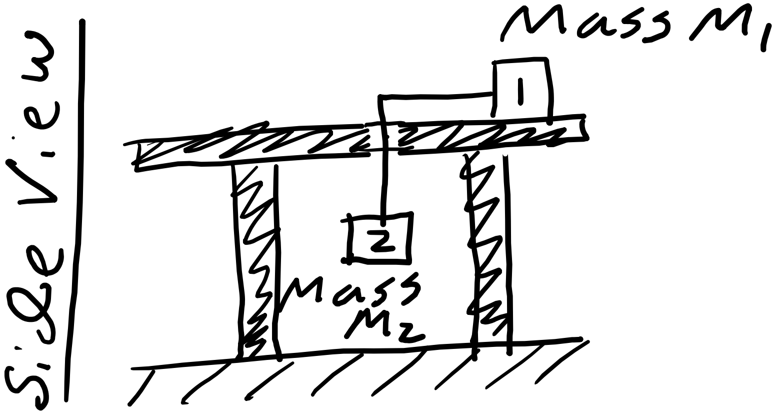
but now $mg\sin\theta - \mu_k mg\cos\theta = ma_x$

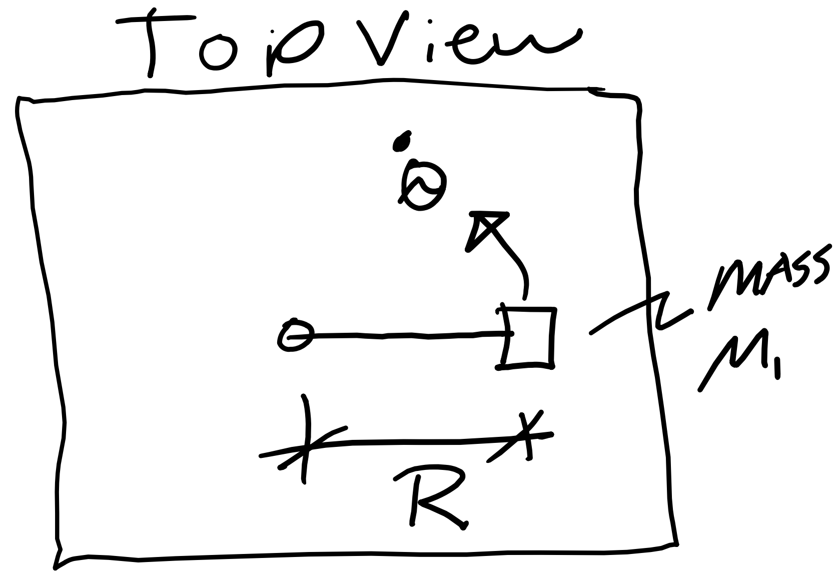
$\Rightarrow g\sin\theta - \mu_k g\cos\theta = a_x \Rightarrow g\left[\frac{3}{5} - \left(\frac{5}{8}\right)\frac{4}{5}\right] = a_x$

$\Rightarrow g\left[\frac{3}{5} - \frac{1}{2}\right] = a_x \Rightarrow g\left[\frac{6}{10} - \frac{5}{10}\right] = a_x$

$\Rightarrow \boxed{a_x = \frac{g}{10}}$

Problem: A string has a mass attached at each end. One side of the string hangs down from a hole in a table. The other end of string rotates on the table surface about the hole.



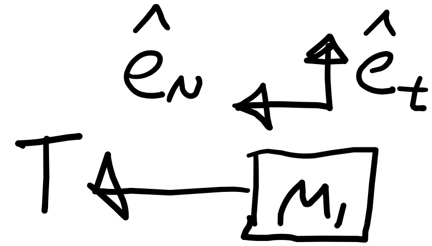


Given that the radius of curvature is a constant $R = 0.981\,m$ & $M_2 = \frac{4}{10} M_1$, find $\dot{\theta}$.

Assume no friction

Free body diagrams:

Mass 1:



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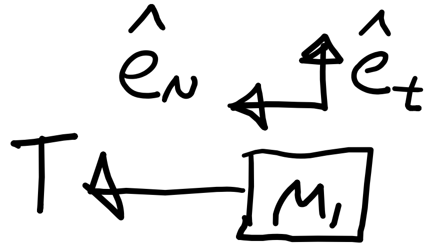
Mass 1: 

$\Sigma F_t = m a_t$

$\& \ \Sigma F_N = m a_N$

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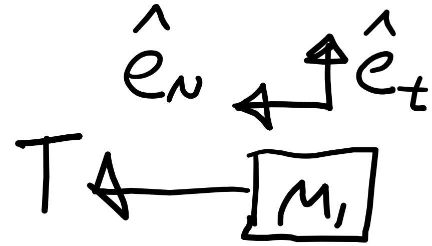

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$$\Rightarrow \quad T = M_1 \frac{V^2}{R} \quad (1)$$

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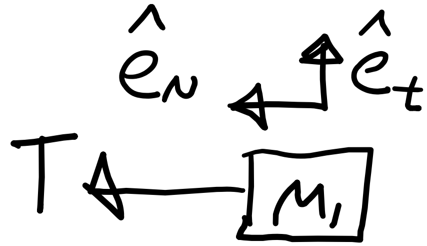

$$\sum F_t = m \cancelto{0}{a_t}$$

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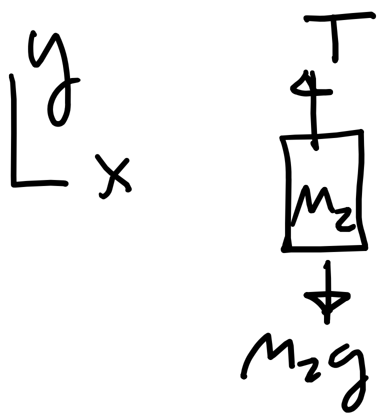
Mass 2:



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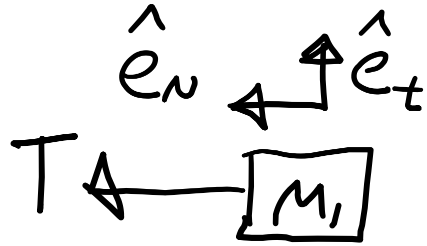
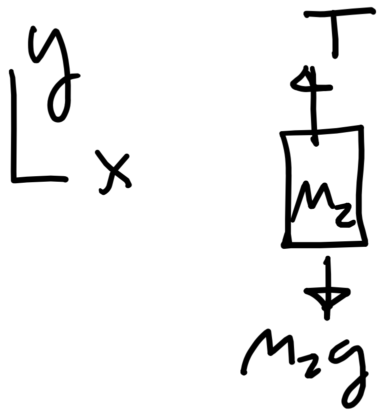


$$\sum F_t = m\cancel{a_t}^{\,0}$$

$$\sum F_N = m a_N \Rightarrow T = m_1 \frac{v^2}{R} \quad (1)$$

Mass 2:



$$\sum F_x = m\cancel{a_x}^{\,0}$$

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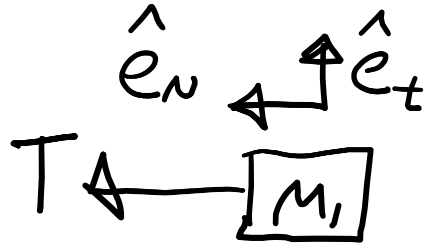

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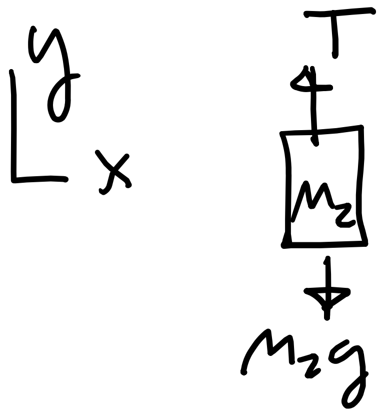

$$\sum F_x = m \cancel{a_x}^{\,0}$$

$$\sum F_y = m \cancel{a_y}^{\,0} \Rightarrow T - m_2 g = 0$$

$$\Rightarrow T = m_2 g \quad (2)$$

Free body diagrams:

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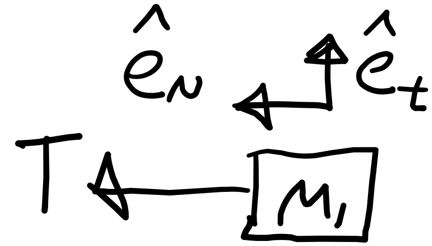

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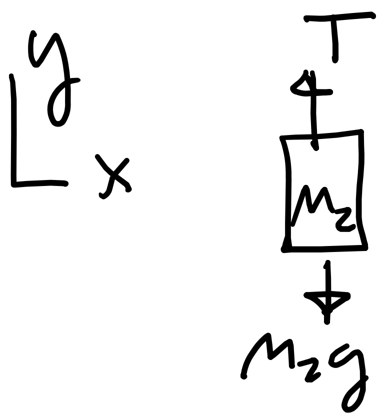
Mass 2:



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Eqn 1 & 2 $\Rightarrow$ $m_1 \frac{v^2}{R} = m_2 g$

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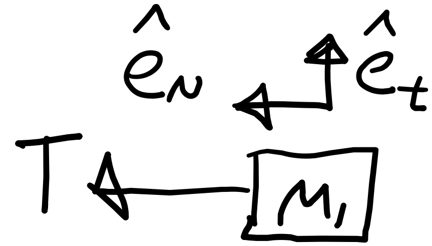

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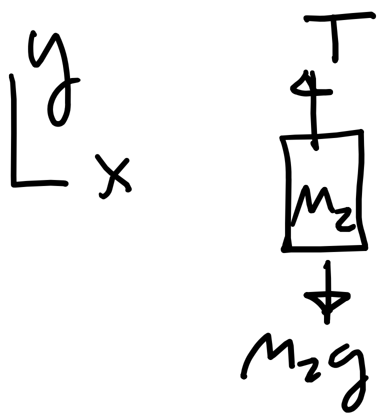
Mass 2:



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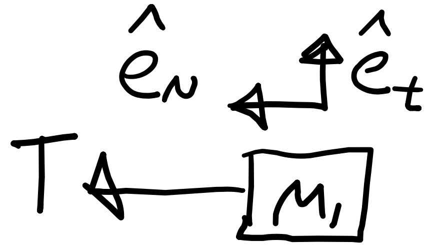

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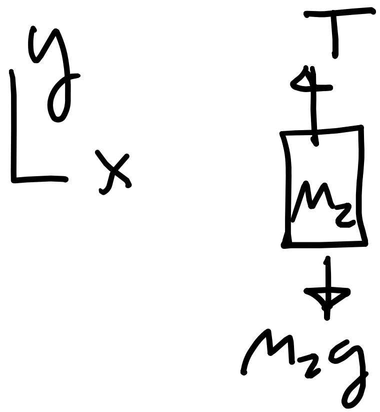

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<u>But</u> $v = R\dot{\theta}$ so $R\dot{\theta}^2 = \frac{m_2}{m_1} g$

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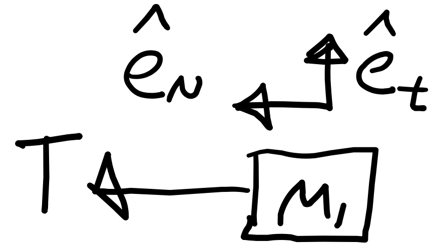

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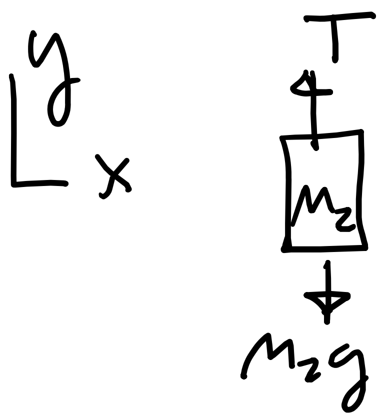
Mass 2:



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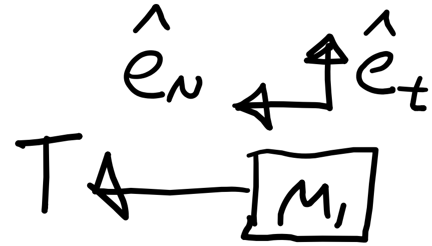

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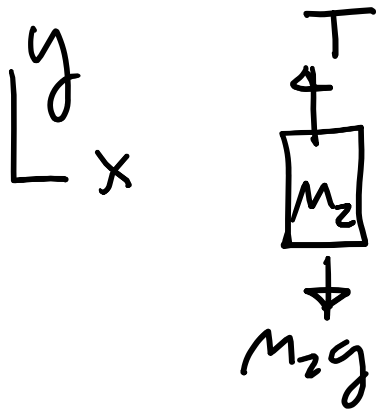
Mass 2:



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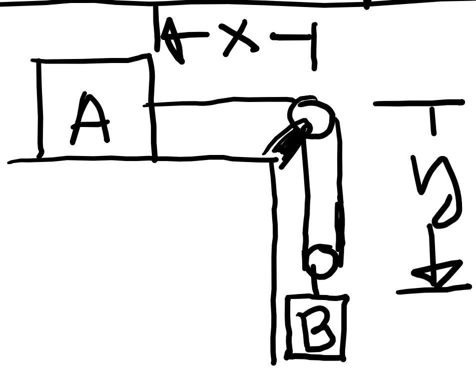
But $v = R\dot{\theta}$ so $R\dot{\theta}^2 = \frac{m_2}{m_1} g \Rightarrow \dot{\theta} = \left[\left(\frac{m_2}{m_1}\right)\left(\frac{g}{R}\right)\right]^{1/2} \Rightarrow$

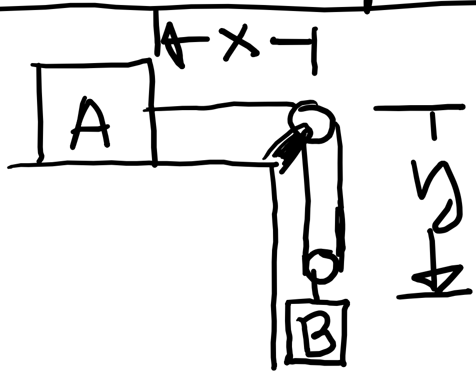
$$\dot{\theta} = \left[\frac{4}{10} * \frac{9.81}{0.981}\right]^{1/2} \frac{\text{rad}}{s} \Rightarrow \boxed{\dot{\theta} = 2\ \text{rad/s}}$$

Another pulley problem:



Another pulley problem: pulleys are massless, $m_A = 100\,kg$, $m_B = 300\,kg$

NO friction. Find all tensions & accelerations



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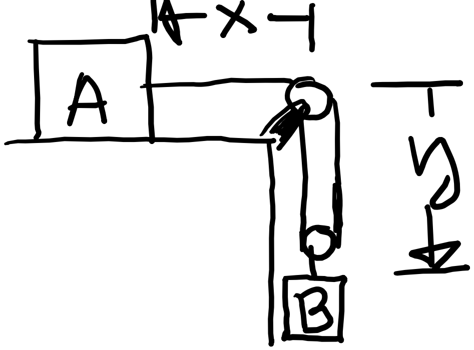
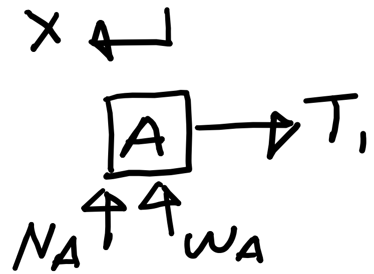
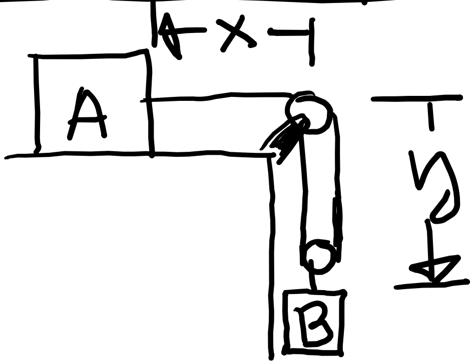


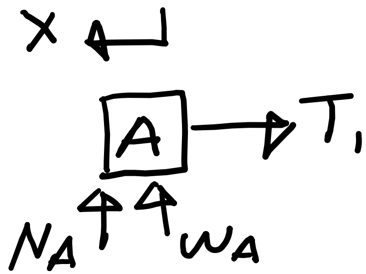
Free Body A:



Another pulley problem: pulleys are massless, $M_A = 100\,\text{kg}$, $M_B = 300\,\text{kg}$. No friction. Find all tensions & accelerations



Free Body A:



$$\Rightarrow -T_1 = M_A a_A$$

$$\Rightarrow a_A = -T_1 / M_A$$

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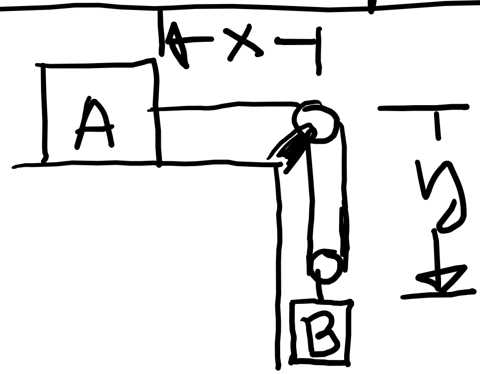

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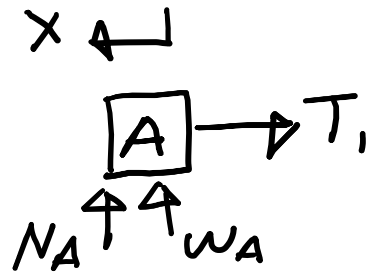

$$\Rightarrow -T_1 = M_A a_A$$

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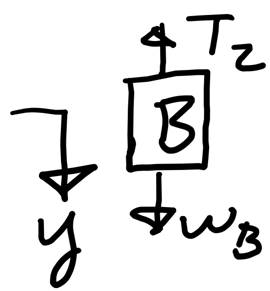
Free Body B:

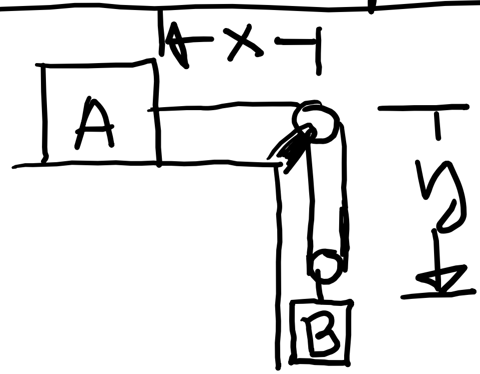


$$\Rightarrow -T_2 + W_B = M_B a_B$$

$$\Rightarrow a_B = \frac{W_B - T_2}{M_B}$$

ASU

Another pulley problem: pulleys are massless, $M_A = 100\,kg$, $M_B = 300\,kg$, no friction. Find all tensions & accelerations



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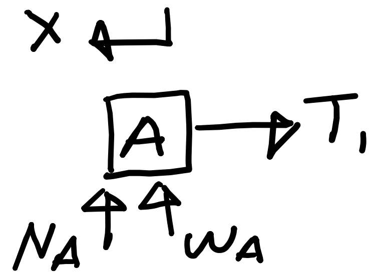

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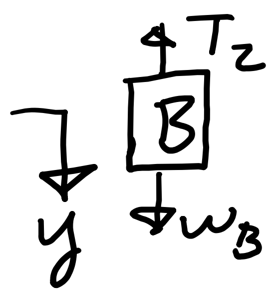

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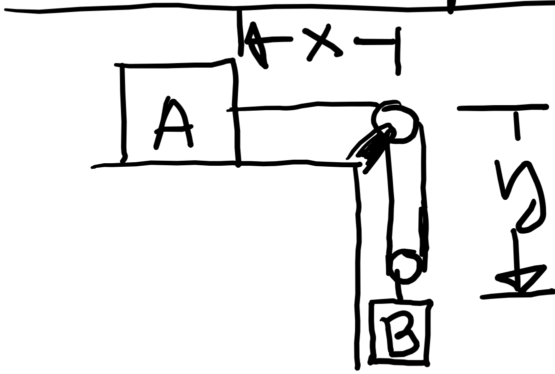

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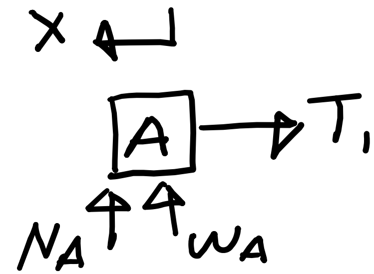

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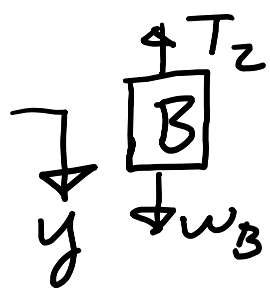

$\Rightarrow -T_2 + W_B = M_B a_B$

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Also $2y + x = \text{const.} \Rightarrow 2a_y + a_x = 0$, But

$a_y = a_B$ & $a_x = a_A \Rightarrow 2a_B = -a_x$

From previous $a_A = \frac{-T_1}{m_A}$, $a_B = \frac{W_B - T_2}{m_B}$ & $2a_B = -a_A$

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So $T_1 = 2\left(\frac{m_A}{m_B}\right)(m_B g - T_2)$

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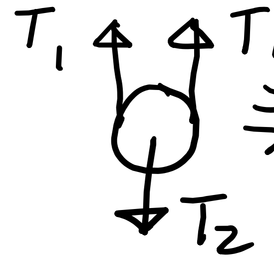

$\Rightarrow T_2 = 2T_1$

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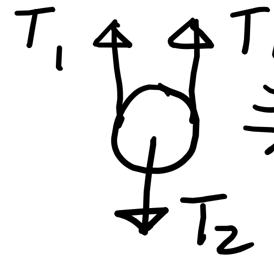

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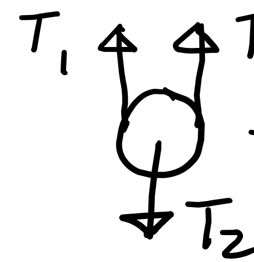


$\Rightarrow T_2 = 2T_1$

So $\quad T_1 = 2\left(\dfrac{M_A}{M_B}\right)(M_B g - 2T_1) \Rightarrow$

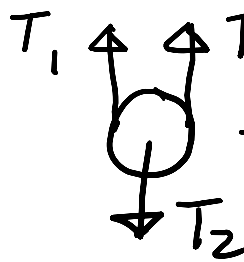
$$4T_1 \frac{M_A}{M_B} + T_1 = 2M_A g \Rightarrow T_1\left[4\frac{M_A}{M_B} + 1\right] = 2M_A g$$

$$\Rightarrow T_1\left[\frac{4M_A + M_B}{M_B}\right] = 2M_A g \Rightarrow T_1 = \frac{2M_A M_B g}{4M_A + M_B}$$

$$\Rightarrow \boxed{T_1 = \left[\frac{2*100*300}{4*100+300}\right] kg*g = 841N}$$

From previous $a_A = \frac{-T_1}{M_A}$, $a_B = \frac{W_B - T_2}{M_B}$ & $2a_B = -a_A$

Now $2\left(\frac{W_B - T_2}{M_B}\right) = \frac{T_1}{M_A}$ But $W_B = M_B g$



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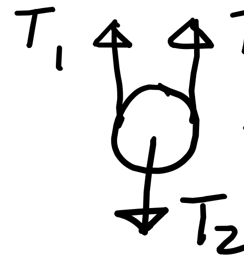

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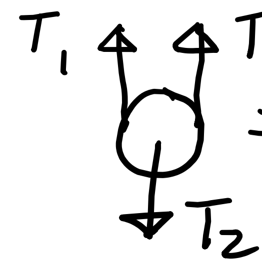
$\Rightarrow \boxed{T_2 = 1682N}$ \& $\boxed{a_A = \frac{-T_1}{M_A} = \frac{-841}{100}\frac{m}{s^2} = -8.41\, m/s^2}$

From previous $a_A = \frac{-T_1}{M_A}$, $a_B = \frac{W_B - T_2}{M_B}$ & $2a_B = -a_A$

Now $2\left(\frac{W_B - T_2}{M_B}\right) = \frac{T_1}{M_A}$ But $W_B = M_B g$

So $T_1 = 2\left(\frac{M_A}{M_B}\right)(M_B g - T_2)$



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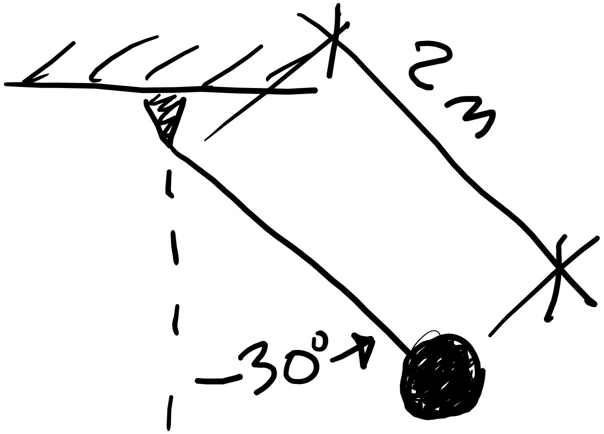
& $\boxed{a_B = -\frac{a_A}{2} = 4.20\, m/s^2}$

Pendulum problem



Given that $T = \frac{2}{5}mg$ at position shown, find v & a

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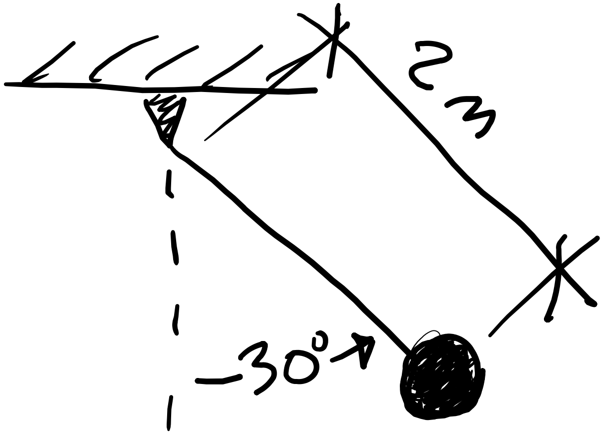
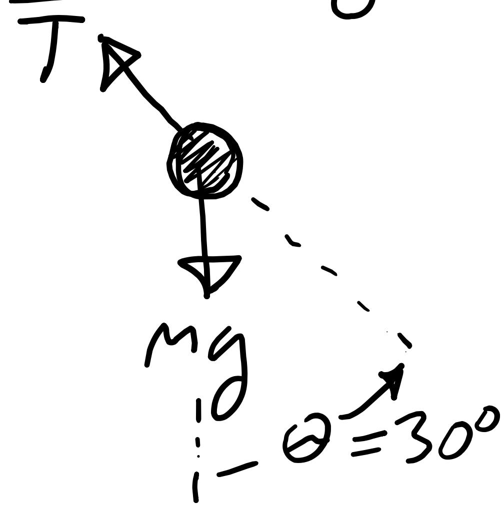


Free body



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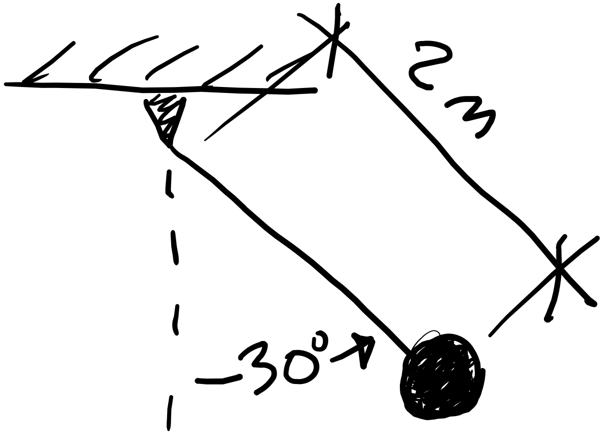

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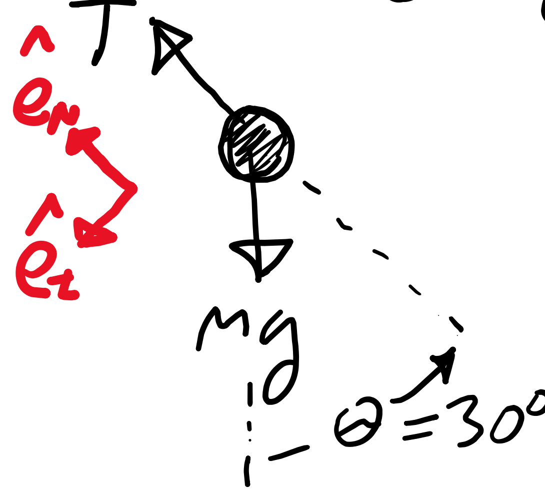

$$\vec{a} = \frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

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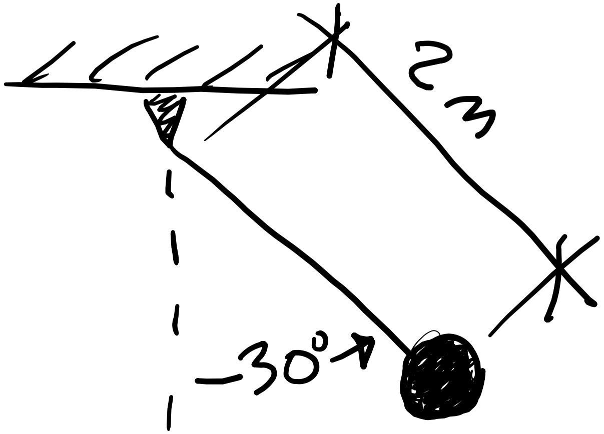

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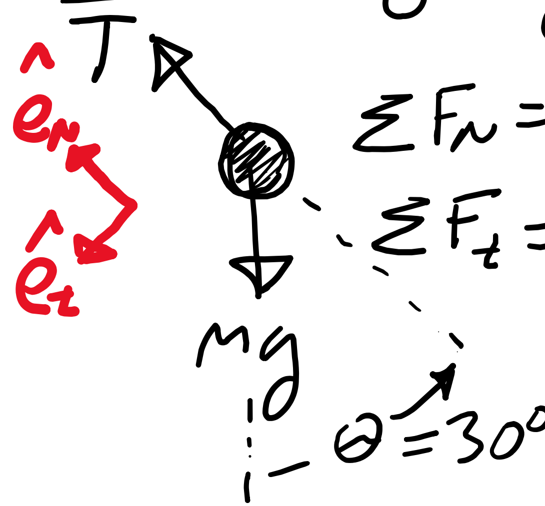
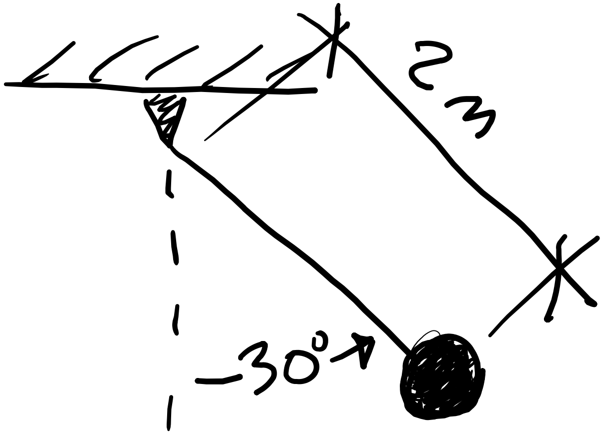


$$\vec{a} = \frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$\Sigma F_n = T - mg\cos\theta = m\frac{v^2}{\rho}$$

$$\Sigma F_t = mg\sin\theta = mv$$

Pendulum problem



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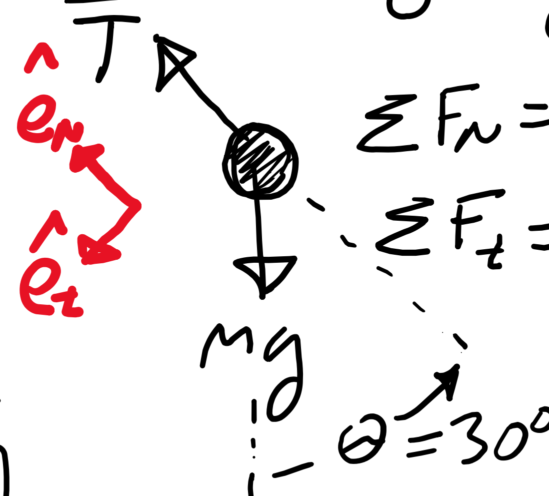


$$\vec{a} = \frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$\sum F_n = T - mg\cos\theta = m\frac{v^2}{\rho}$$

$$\sum F_t = mg\sin\theta = mv$$

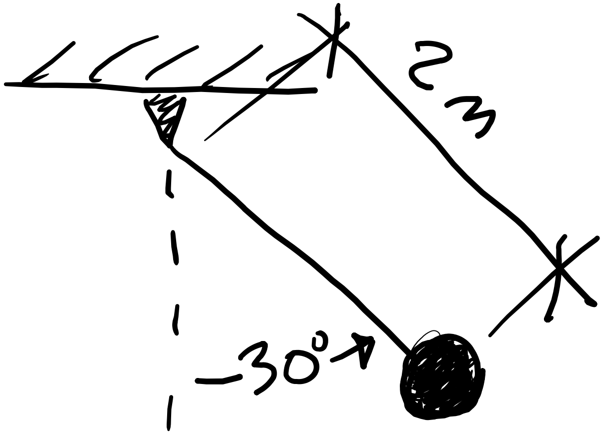
So $m\frac{v^2}{\rho} = \frac{5}{2}mg - mg\cos\theta$

$\Rightarrow v = [(g\rho)(\frac{5}{2} - \cos\theta)]$

$\Rightarrow \boxed{v = 5.66\ m/s}$

Pendulum problem

Given that $T=\frac{2}{5}mg$ at position shown, find v & a



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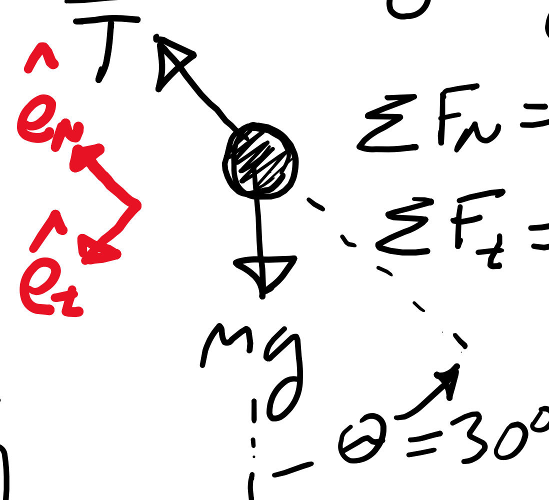

$$\vec a = \frac{dv}{dt}\hat e_t + \frac{v^2}{\rho}\hat e_n$$

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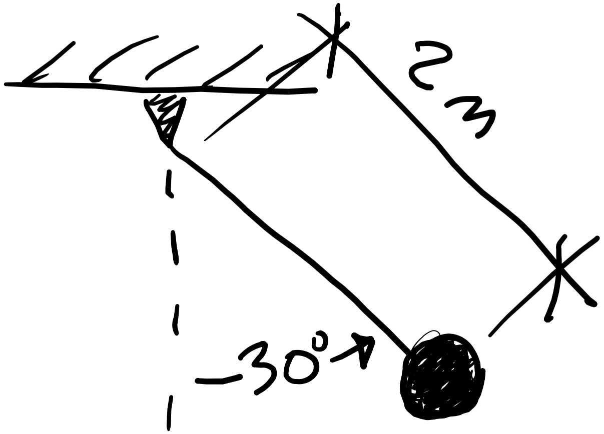
Pendulum problem



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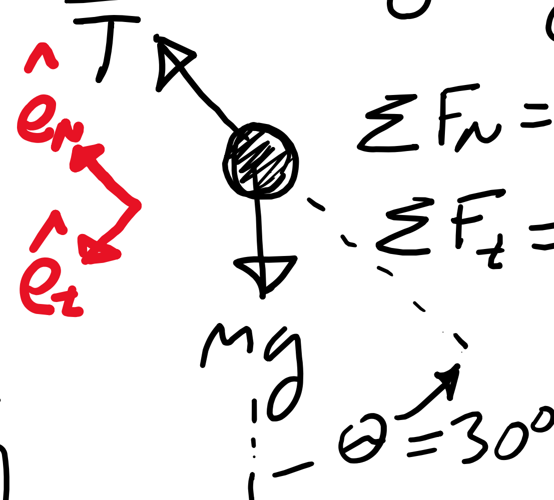

$$\vec{a} = \frac{dv}{dt}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

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So $m\frac{v^2}{\rho} = \frac{5}{2}mg - mg\cos\theta$

$\Rightarrow v = \left[(g\rho)\left(\frac{5}{2} - \cos\theta\right)\right]$

$\Rightarrow \boxed{v = 5.66\ m/s}$ & $m\vec{a} = (T - mg\cos\theta)\hat{e}_n + mg\sin\theta\,\hat{e}_t$

$\Rightarrow \vec{a} = g\left(\frac{5}{2} - \cos 30°\right)\hat{e}_n + g\sin 30°\,\hat{e}_t \Rightarrow$

$$\boxed{\vec{a} = 16.03\frac{m}{s^2}\hat{e}_n + 4.90\frac{m}{s^2}\hat{e}_t}$$