Today: 12.2

$\boxed{L6}$

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L6

Angular momentum &

orbital momentum

Today: 12.2

Tuesday: Review

$\boxed{L6}$

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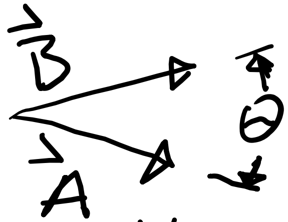
$\boxed{L6}$

Tuesday: Review

Thursday February 4th : Exam #1

Cross product: 

$\vec{A}$ & $\vec{B}$ define a plane. I will call that plane the AB-plane

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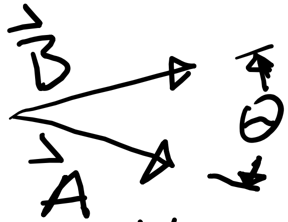

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$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$, where $\hat{n}$ is a unit vector orthogonal to the AB-plane & points in direction according to the right hand rule.

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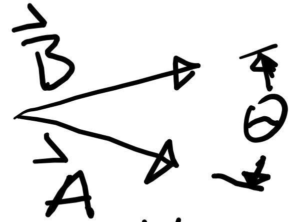

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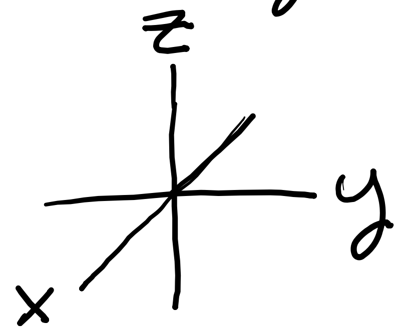
For rectangular coordinates



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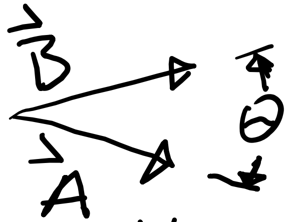

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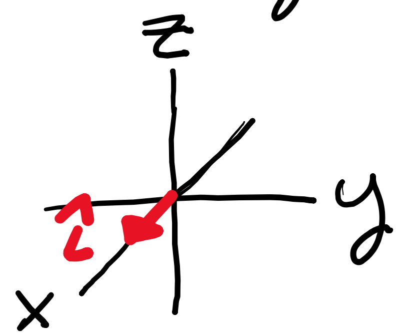

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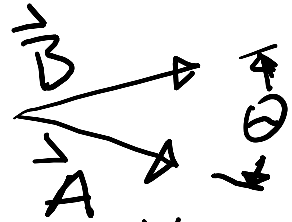

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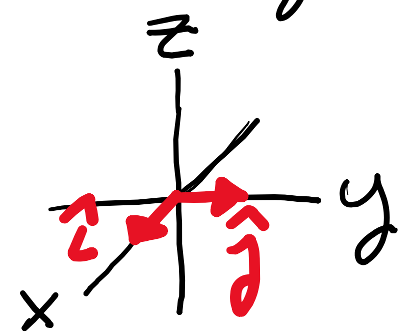

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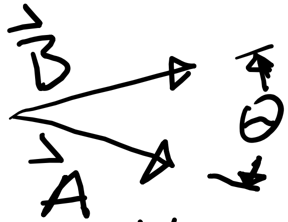

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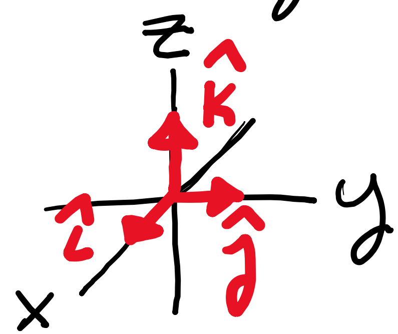

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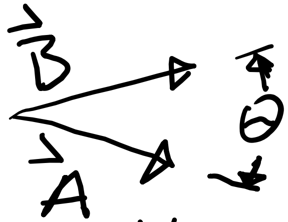

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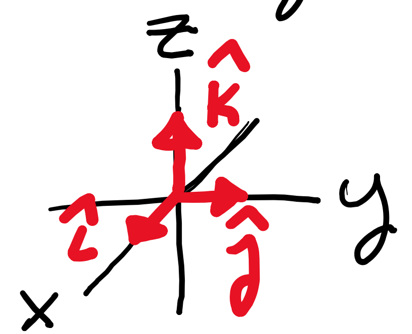
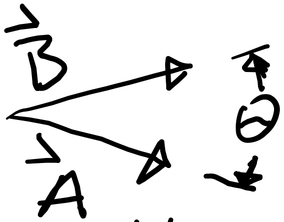


Cross product:  $\vec{A}$ & $\vec{B}$ define a plane. I will call that plane the AB-plane

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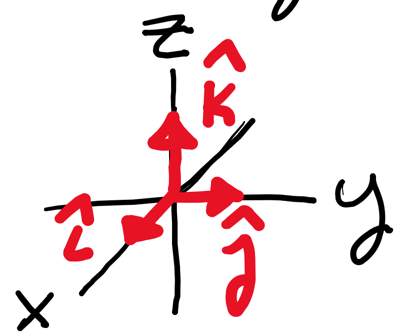

$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$

& $\hat{j}\times\hat{i} = -\hat{k}$, $\hat{k}\times\hat{j} = -\hat{i}$, $\hat{i}\times\hat{k} = -\hat{j}$

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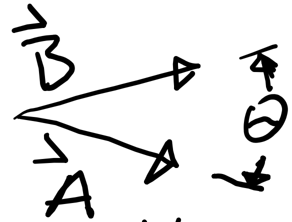

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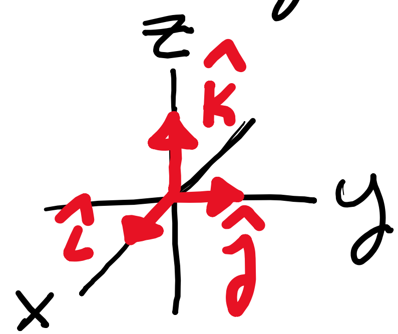

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Also $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$

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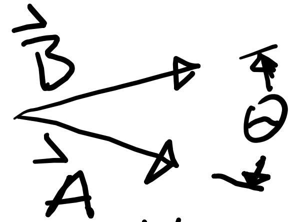

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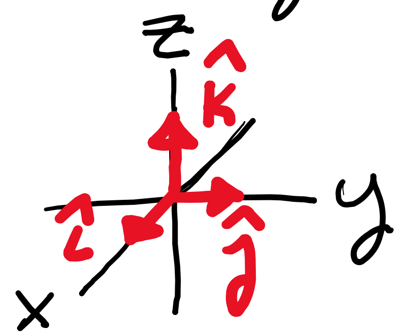
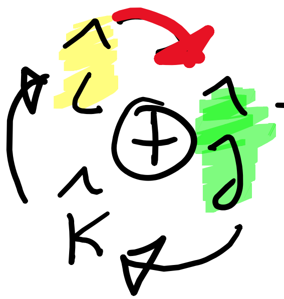


$$\left.\begin{array}{l} \hat{i}\times\hat{j} = \hat{k},\quad \hat{j}\times\hat{k} = \hat{i},\quad \hat{k}\times\hat{i} = \hat{j} \\ \text{\& } \hat{j}\times\hat{i} = -\hat{k},\quad \hat{k}\times\hat{j} = -\hat{i},\quad \hat{i}\times\hat{k} = -\hat{j} \end{array}\right\}$$

Also $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

Notice the structure



$\Rightarrow \quad \hat{i} \times \hat{j} =$

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$\Rightarrow \quad \hat{i} \times \hat{j} = \hat{k}$

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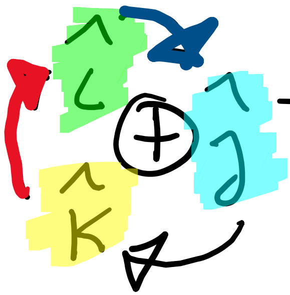

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$$\hat{j} \times \hat{i} = -\hat{k}, \quad \hat{k} \times \hat{j} = -\hat{i}, \quad \hat{i} \times \hat{k} = -\hat{j}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_y B_z - A_z B_y) + \hat{j}(A_z B_y - A_y B_z) + \hat{k}(A_x B_y - A_y B_x)$$

Notice the structure

$\Rightarrow$ $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$, $\hat{k}\times\hat{i}=\hat{j}$

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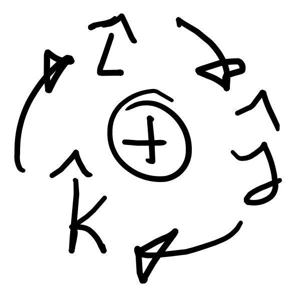
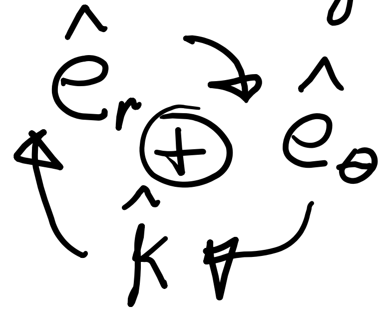
Do you see the



structure?

Same thing holds for cylindrical coordinates, with



$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{k} \\ A_r & A_\theta & A_z \\ B_r & B_\theta & B_z \end{vmatrix}$$

$$= \hat{e}_r(A_\theta B_z - A_z B_\theta) + \hat{e}_\theta(A_z B_r - A_r B_z) + \hat{k}(A_r B_\theta - A_\theta B_r)$$

From last lecture: $\vec{F} = \frac{d}{dt}\vec{L}$.

From last lecture: $\vec{F} = \frac{\partial}{\partial t}\vec{L}$.

Using vector calculus we are going to see that, by taking cross product on both sides with the position vector $\vec{r}$, From last lecture : $\vec{F} = \frac{d}{dt}\vec{L}$.

Using vector calculus we are going to see that, by taking cross product on both sides with the position vector $\vec{r}$, we will obtain

Torque = time rate of change of angular momentum

Create a coordinate system with the origin denoted as O

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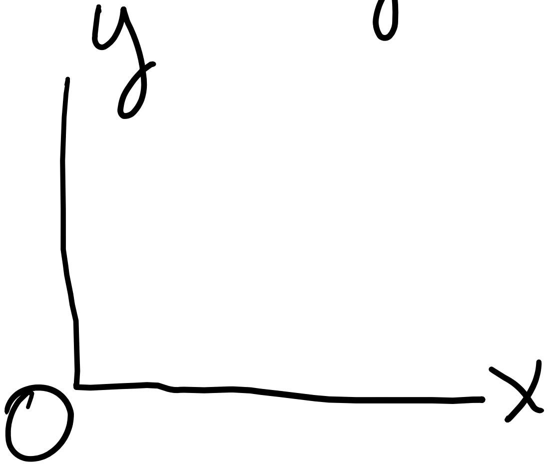
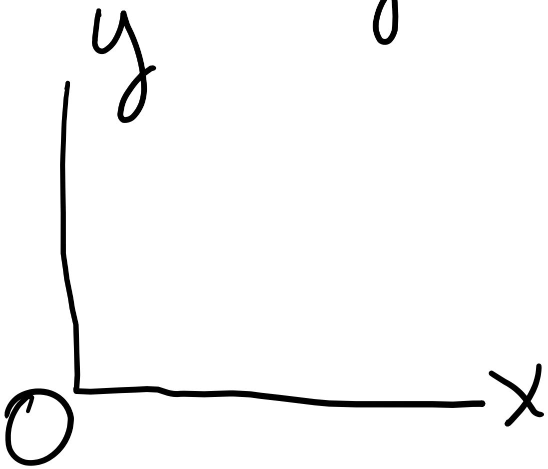


Create a coordinate system with the origin denoted as O. Place an object in our coordinate system thas has linear momentum $\vec{L}$



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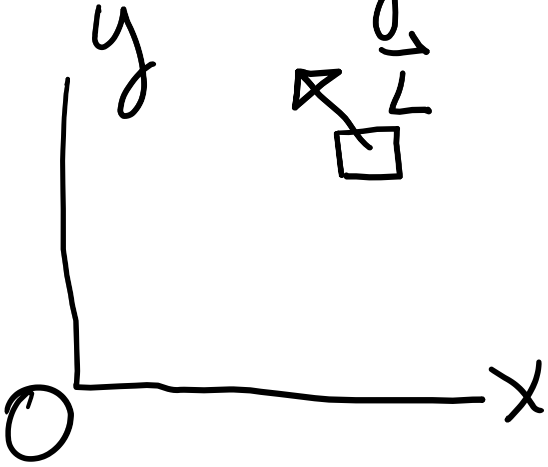
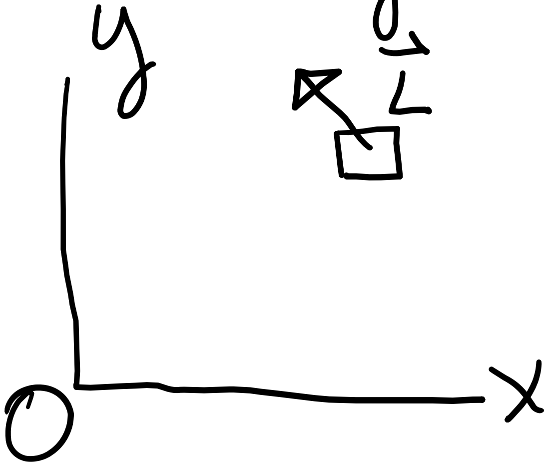


Create a coordinate system with the origin denoted as O. Place an object in our coordinate system thas has linear momentum $\vec{L}$ & force $\vec{F}$ acting on it



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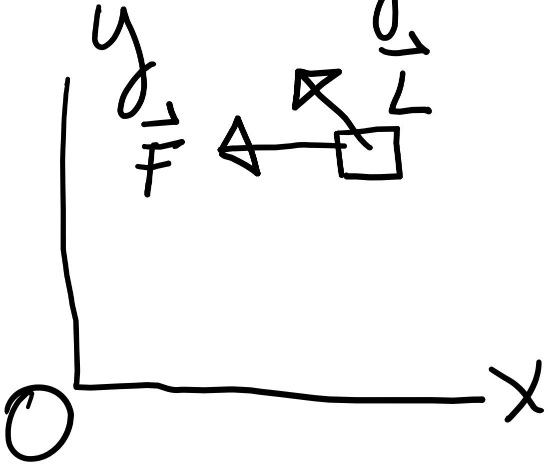

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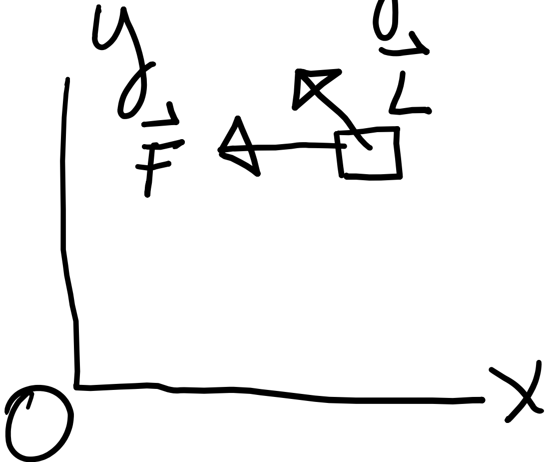

Now draw a position vector $\vec{r}$ from origin to object

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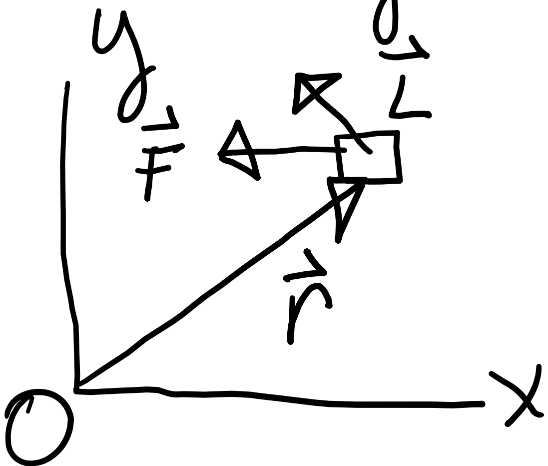

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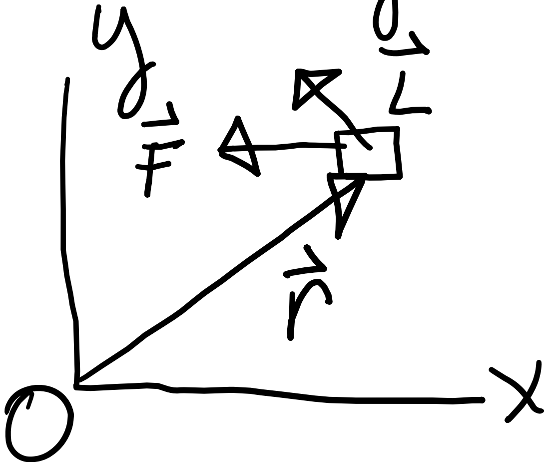

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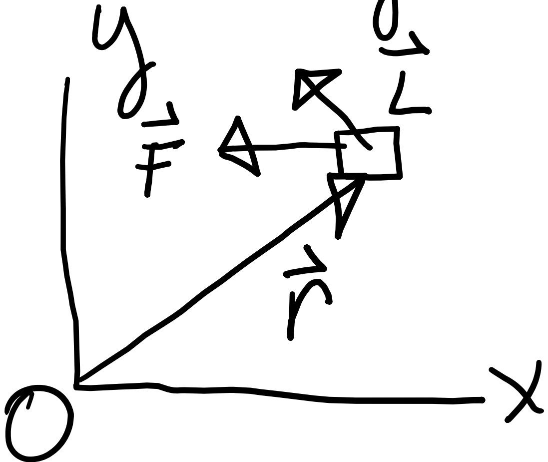

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Note: $\frac{d}{dt}(\vec{r} \times \vec{L}) = \dot{\vec{r}} \times \vec{L} + \vec{r} \times \dot{\vec{L}}$, But $\dot{\vec{r}} \times \vec{L} = \dot{\vec{r}} \times m\dot{\vec{r}} = 0$

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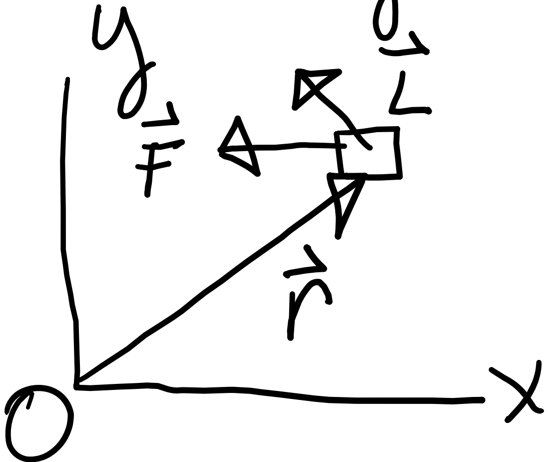

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So $\vec{r} \times \vec{F} = \frac{d}{dt}[\vec{r} \times \vec{L}]$

We have $\vec{r} \times \vec{F} = \frac{d}{dt}(\vec{r} \times \vec{L})$

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Notation & terminology:

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Notation & terminology: Let $\vec{H}_o \equiv \vec{r} \times \vec{L}$

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For single moment: $\vec{M}_o = \frac{d}{dt}\vec{H}_o$

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Central Jorce

Central Force

If $\vec{F}$ is in same or opposite direction as $\vec{r}$

Central Force

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Central force

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$\vec{H} =$ const. and is conserved

A funny example of conserved angular momentum

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particle in straight line at constant speed v.

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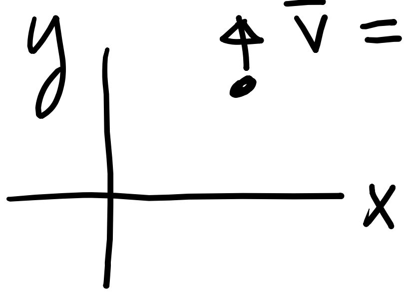
$\vec{v} = v\hat{y}$



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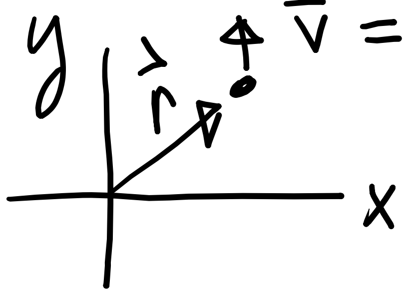

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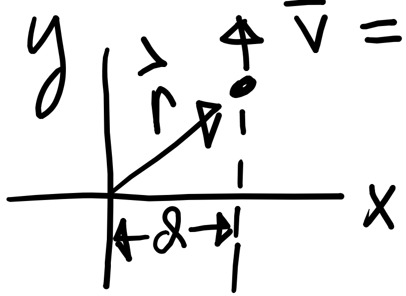

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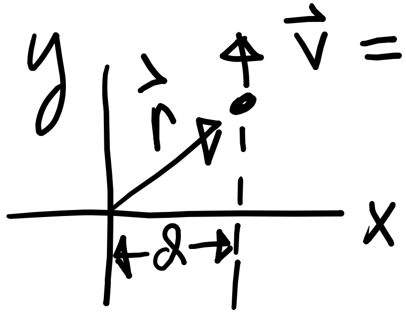

$\vec{v} = v\hat{j}$

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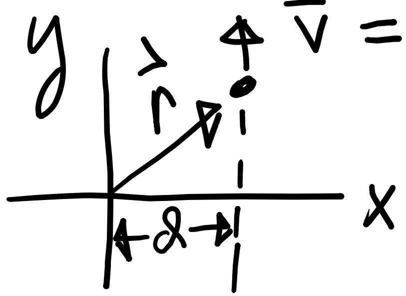

$$\vec{H}_0 = \vec{r} \times \vec{L} = m\vec{r} \times \vec{v}$$

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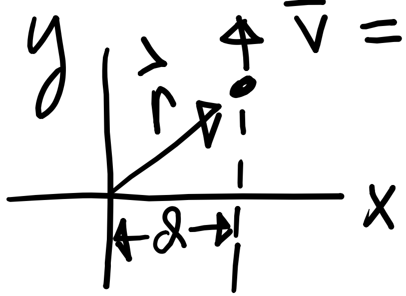

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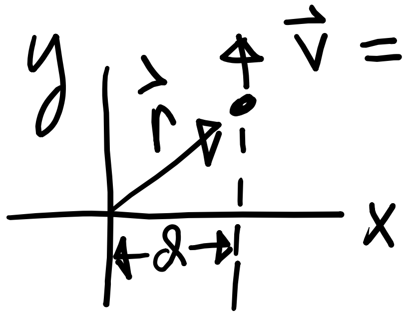

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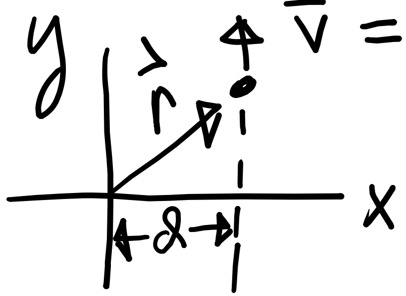

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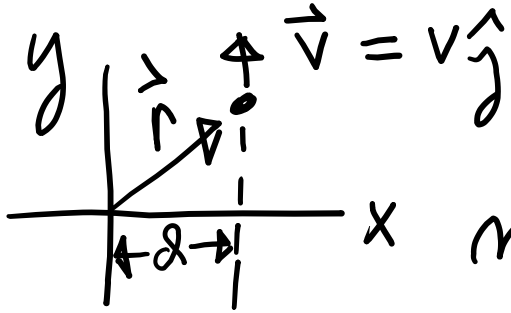

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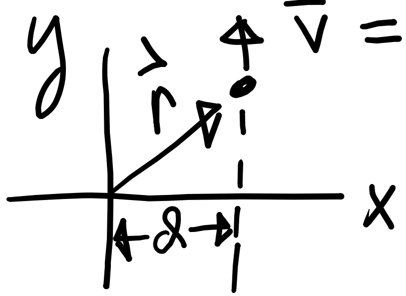

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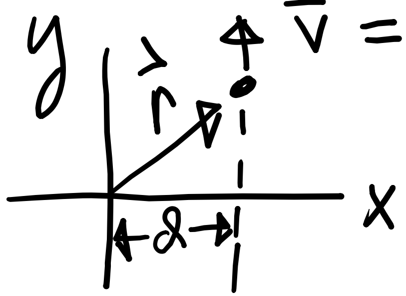

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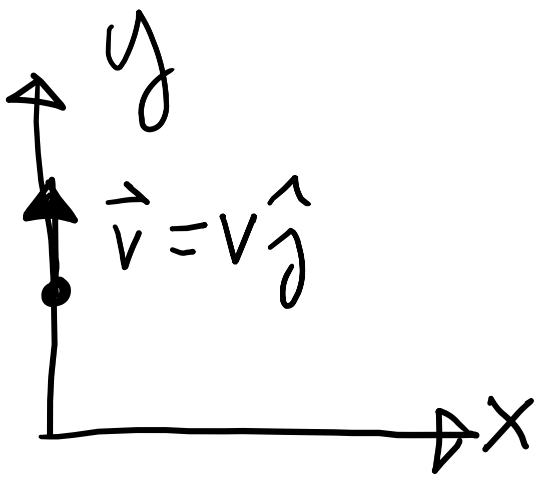
So $\vec{H}_0 = dmv\hat{k}$ We have angular momentum $= \text{const} \neq 0$, <u>but</u> NO rotation is happening.

<u>Note</u>: We can get rid of the angular momentum by choosing a different coordinate system:



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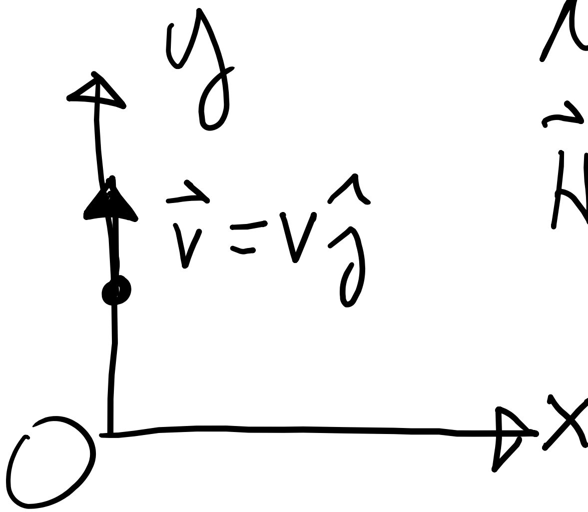

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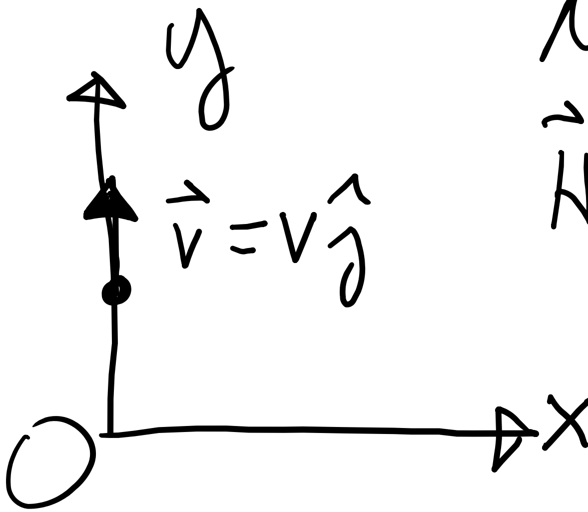

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Choice of coordinate system can be helpful in solving problems

Newtonian gravity

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$$F = G\frac{m_1 m_2}{r^2}$$

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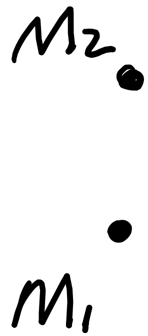

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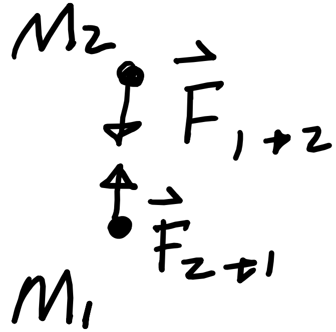

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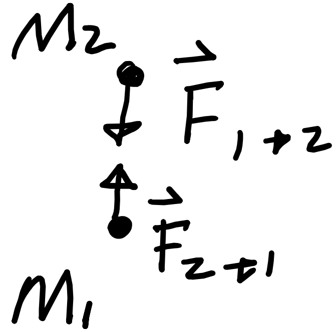

$$\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}$$

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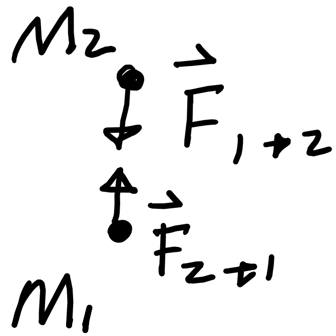

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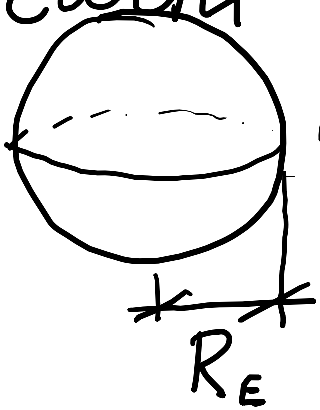
Earth



$\text{Mass} = M_E$

Newtonian gravity

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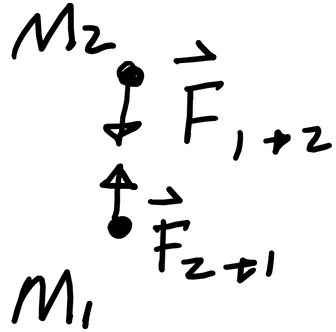

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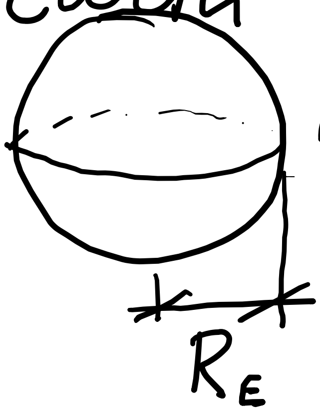

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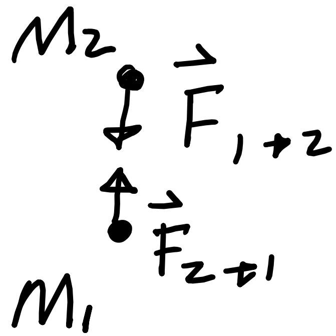

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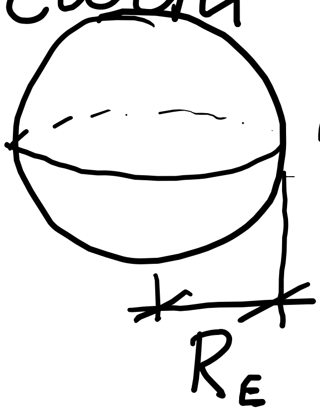

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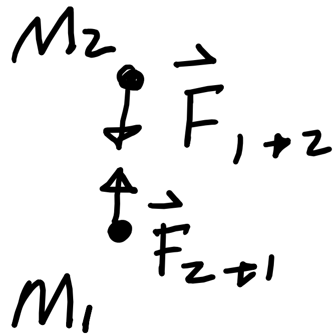

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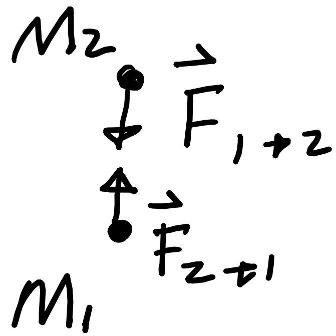

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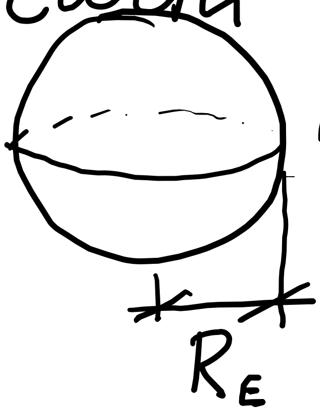

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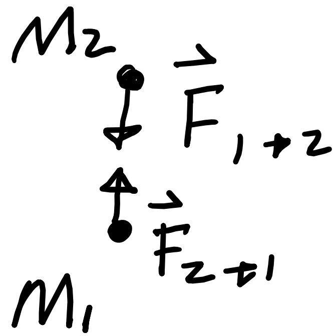

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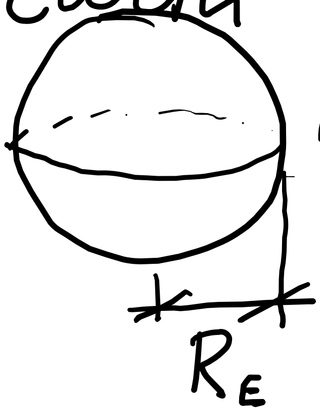

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So $$F(r) \cong \left[\frac{GmM_E}{R_E^2}\right]\left[1-\left(\frac{2}{R_E}\right)(r-R_E)\right]$$ Near Earth

but $r - R_E = h$

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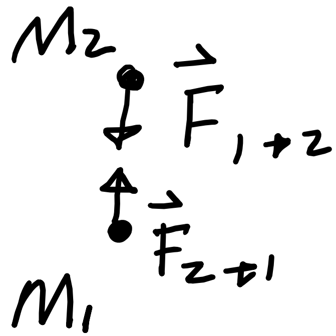

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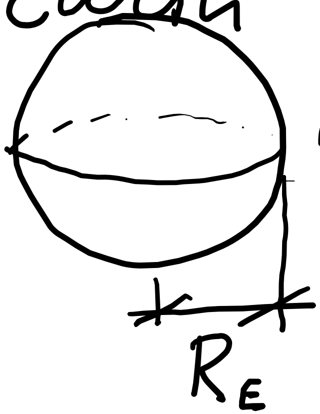

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Near Earth

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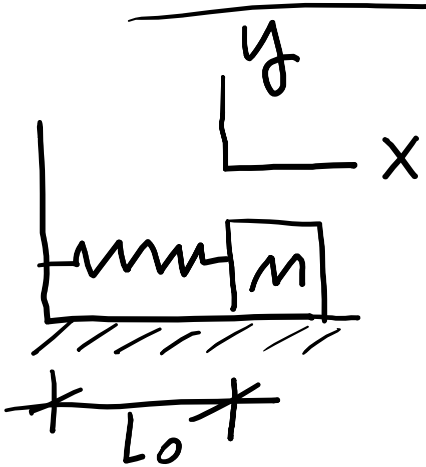
$\Rightarrow h < 3.2 Km$. So, if $h < 3 Km$, then $F = mg$, where $g \equiv G\frac{M_E}{R_E^2}$, is good to within $\frac{1}{10}\%$. But earth is not a sphere & earth is spinning. So g is not as uniform as calculation suggests

Force due to spring (Need for problem 12.90)



Let natural unstretched [nor compressed] length be L_0.

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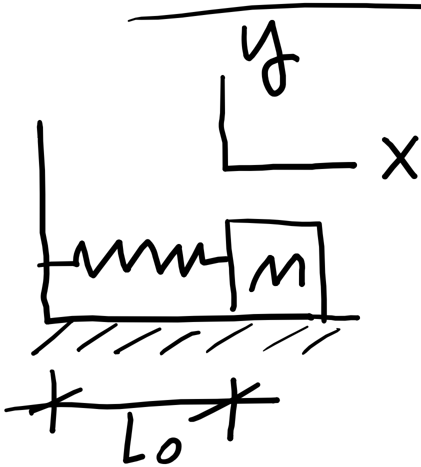

Let natural unstretched [nor compressed] length be L_0. In this case, no spring force F_{sp} on box

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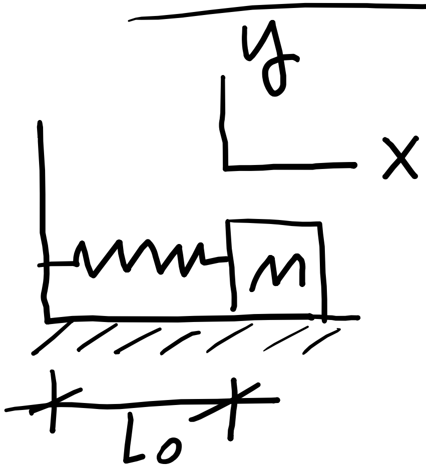

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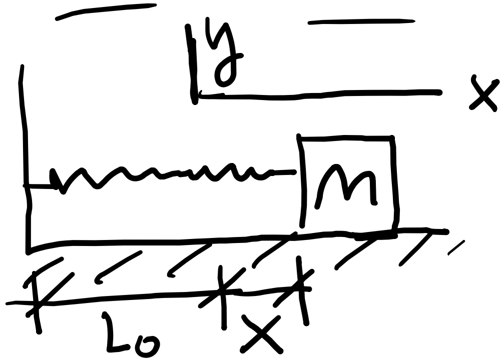

In this case, the spring is stretched and has length $L = L_0 + x$

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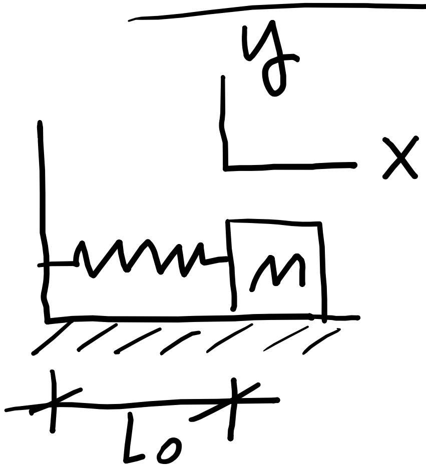

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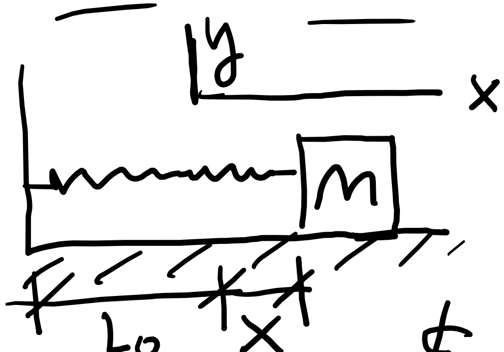
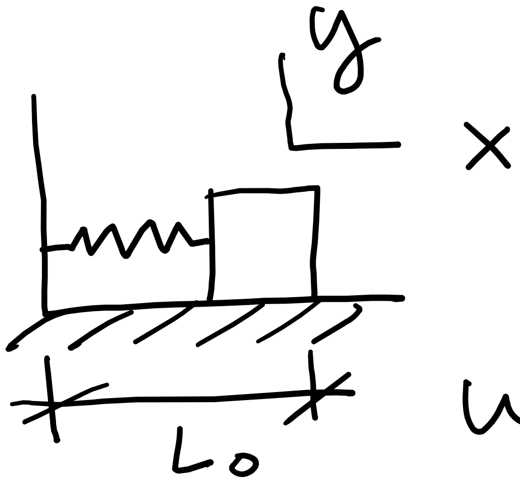
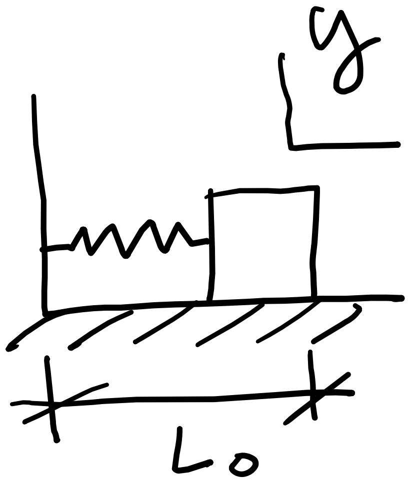


In this case, the spring is stretched and has length $L = L_0 + x$. So $x = L - L_0$ & the force is $F_{sp} = -Kx$, where K is the spring constant.



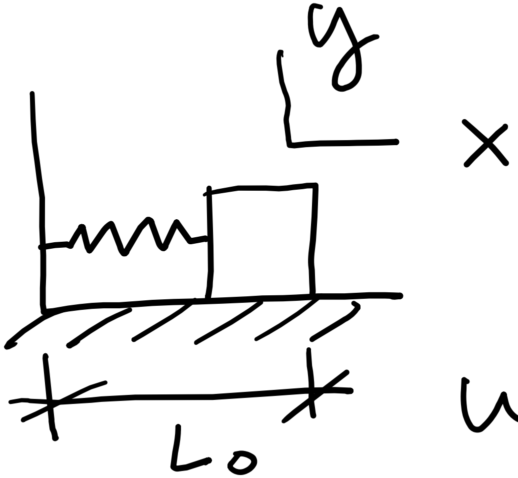
In this case the spring is compressed and has length $L = L_0 + x$, where in this situati

$x < 0$



In this case the spring is compressed and has length $L = L_0 + X$, where in this situati

$X < 0$ & $X = L - L_0$ as before



In this case the spring is compressed and has length $L = L_o + x$, where in this situati

$x < 0$ & $x = L - L_o$ as before

The force is $F_{sp} = -kx$ as before

<u>Example</u>: Massless rod is free to rotate and has a collar with no friction.

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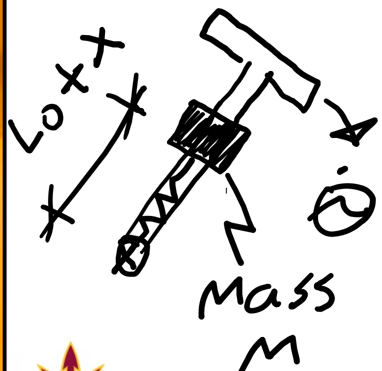
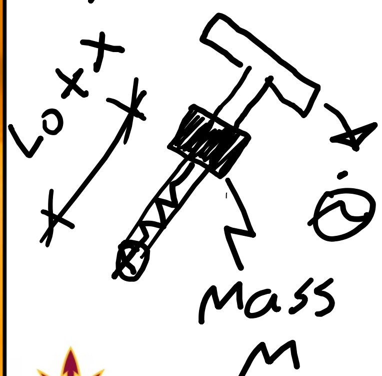
Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_0. Collar starts at position A that is distance $L_0 + X$ from rotation point. Find $\ddot{r}_A$ in terms of L, r_A, M_c:



Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_0. Collar starts at position A that is distance $L_0 + X$ from rotation point. Find $\ddot{r}_A$ in terms of L, r_A, M_c: $\sum F_r = m a_r$



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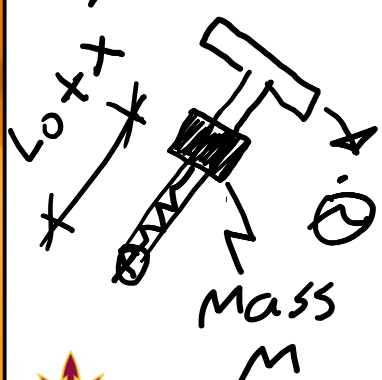

$$\Rightarrow -kx = m(\ddot{r}_A - r_A\dot{\theta}^2)$$

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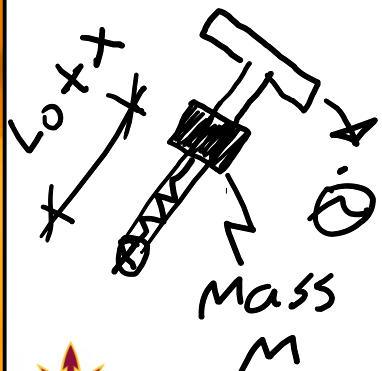
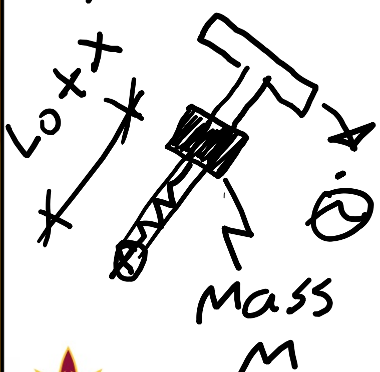


$$\Rightarrow -kx = m(\ddot{r}_A - r_A\dot{\theta}^2) \Rightarrow \ddot{r}_A = -\frac{kx}{m} + r_A\dot{\theta}^2$$

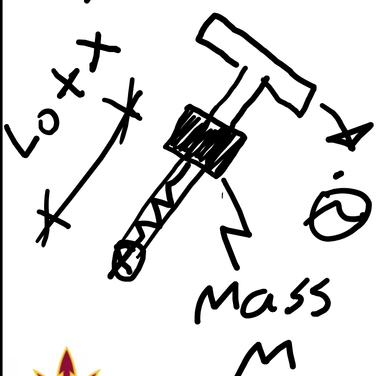
Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_0. Collar starts at position A that is distance $L_0 + x$ from rotation point. Find $\ddot{r}_A$ in terms of L, r_A, M_c: $\sum F_r = m a_r$



$$\Rightarrow -kx = m(\ddot{r}_A - r_A\dot{\theta}^2) \Rightarrow \ddot{r}_A = -\frac{kx}{m} + r_A\dot{\theta}^2$$

But $x = r_A - L_0$

Example: Massless rod is free to rotate and has a collar with no friction. Collar is attached to spring that has opposite side attached to point of rotation. Spring has natural length L_o. Collar starts at position A that is distance $L_o + x$ from rotation point. Find $\ddot{r}_A$ in terms of L, r_A, M_c: $\sum F_r = ma_r$



$$\Rightarrow -kx = m(\ddot{r}_A - r_A\dot{\theta}^2) \Rightarrow \ddot{r}_A = -\frac{kx}{m} + r_A\dot{\theta}^2$$

But $x = r_A - L_o$ So

$$\ddot{r}_A = \frac{-k(r_A - L_o)}{m} + r_A\dot{\theta}^2$$