Today: 13.1, 13.2

L8

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$\boxed{L8}$

Work & kinetic energy

Today: 13.1, 13.2

L8

Work & kinetic energy

Conservation of energy

Today: 13.1, 13.2

Thursday: 13.2, 13.3

$\boxed{L8}$

Today: 13.1, 13.2

Thursday: 13.2, $\underbrace{13.3}_{\text{Impulse \& momentum}}$

L8

Today: 13.1, 13.2

Thursday: 13.2, 13.3

L8

HW #4:

13.2a, 13.11, 13.19a, 13.45	§13.1
13.62a, 13.64, 13.72, 13.100	§13.2
13.120, 13.134	§13.3

ASU

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$$\left[\Sigma \vec{F} = m\vec{a} \oplus \text{Vector calculus}\right] \Rightarrow$$

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$\boxed{\text{1-d case}}$ $\Sigma F = ma$

$\boxed{\text{1-D case}}$ $\Sigma F = ma$ But $a = V\frac{\partial V}{\partial x}$

$\boxed{\text{1-d case}}$ $\Sigma F = ma$ But $a = v\frac{dv}{dx}$

So $\Sigma F = m v \frac{dv}{dx}$

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So $\Sigma\int F\,dx = m\int v\frac{dv}{dx}\,dx \Rightarrow \Sigma\int_{x_1}^{x_2} F\,dx = m\int_{v_1}^{v_2} v\,dv$

$\Rightarrow \Sigma\int_{x_1}^{x_2} F\,dx = \frac{1}{2}mv^2\Big|_{v_1}^{v_2}$

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ASU

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Define: $\quad U_{1\to 2} \equiv$ Work from point 1 to point 2

& $U_{1\to 2} \equiv \sum\int_{x_1}^{x_2} F\,dx$

& $T \equiv$ Kinetic energy

& $T \equiv \tfrac{1}{2}mv^2$

Now $\quad U_{1\to 2} = T_2 - T_1 \;$ or $\; U_{1\to 2} = \Delta T \;$ or $\; T_1 + U_{1\to 2} = T_2$

For 3-d case $U_{1\to2}$ generalizes as

$$U_{1\to2} = \sum \int \vec{F} \cdot d\vec{r}$$

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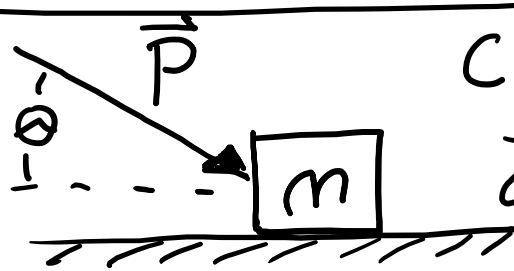
<u>Example</u>:



Coefficient of static friction $= \mu_k$

Box starts at rest & moves distance L. Find T_2

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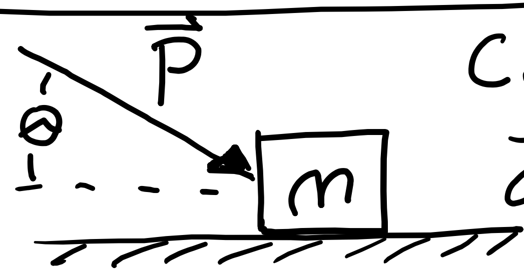

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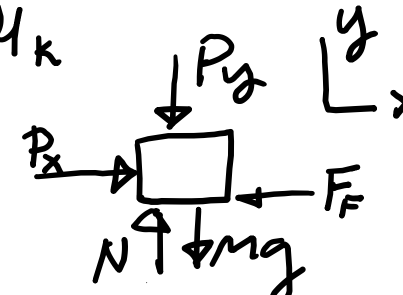

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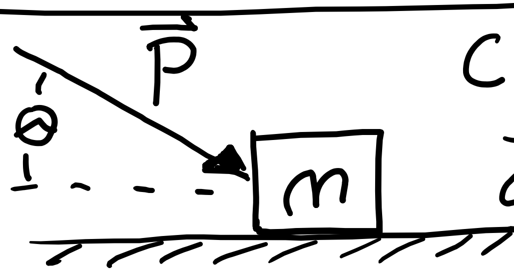

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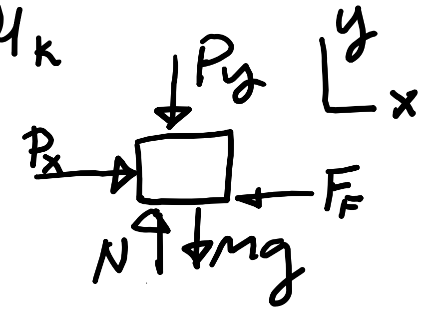

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$$\sum F_x = P_x - F_F = P_x - N\mu_k$$

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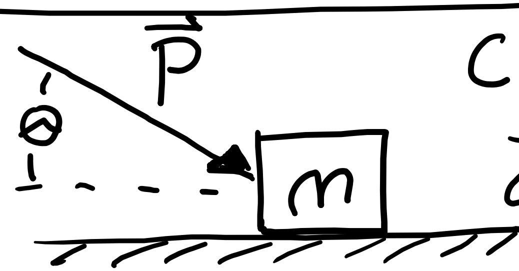

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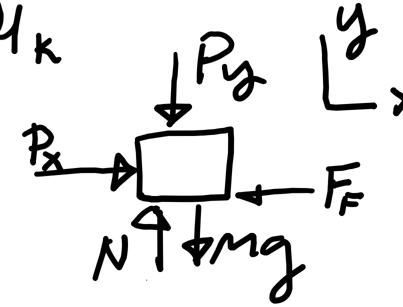

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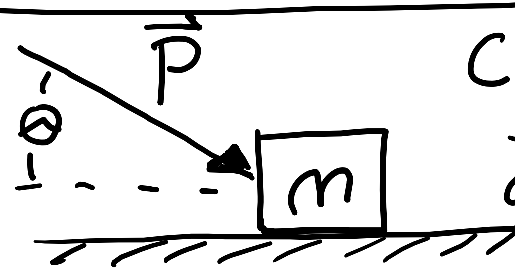

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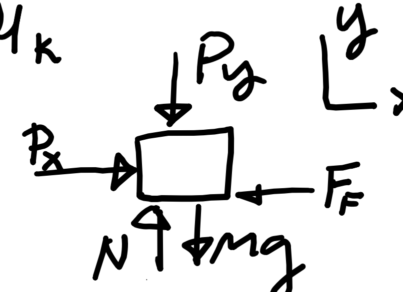

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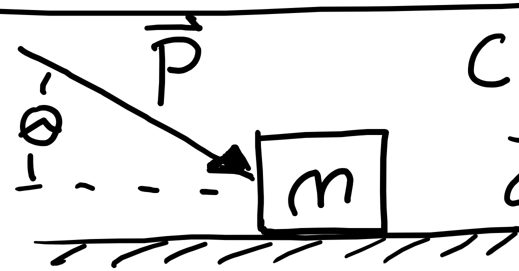

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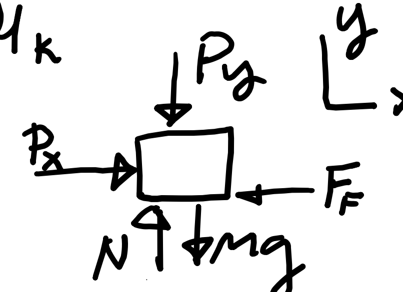

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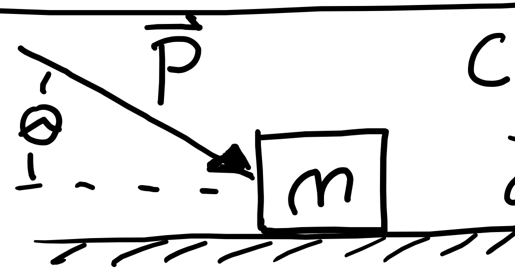

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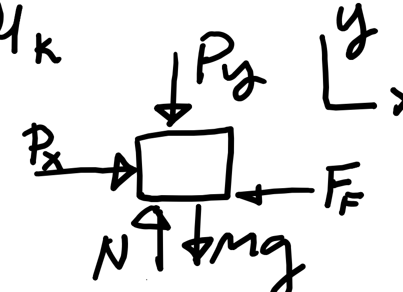

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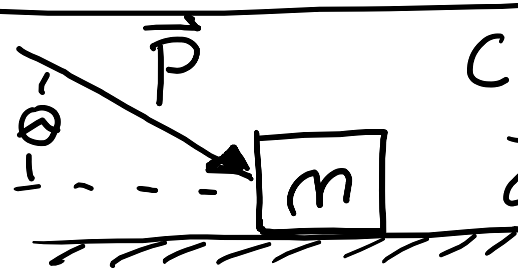

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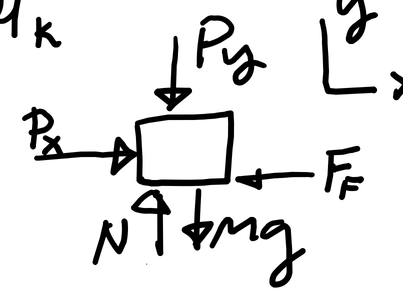

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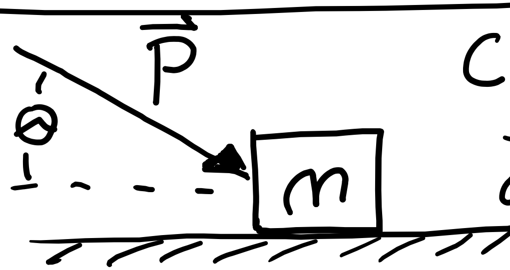

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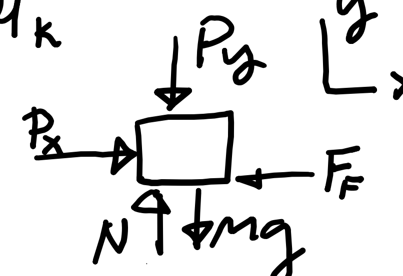

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If no friction $\mu_k = 0$

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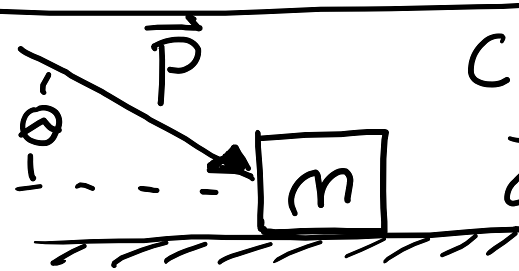

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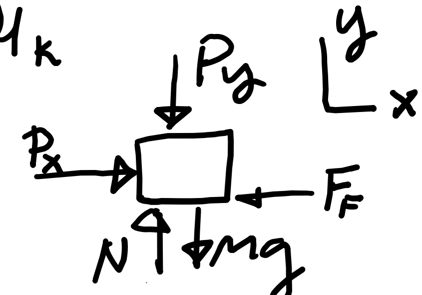

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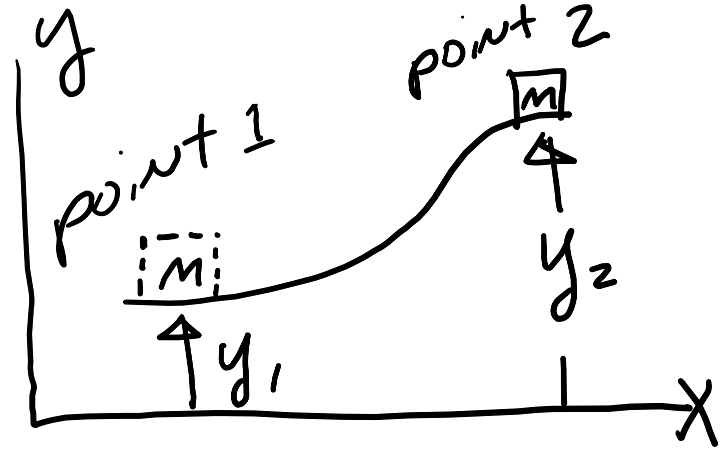
If no friction $\mu_k = 0$ & $T_2 = PL\cos\theta$

Gravity Near earth

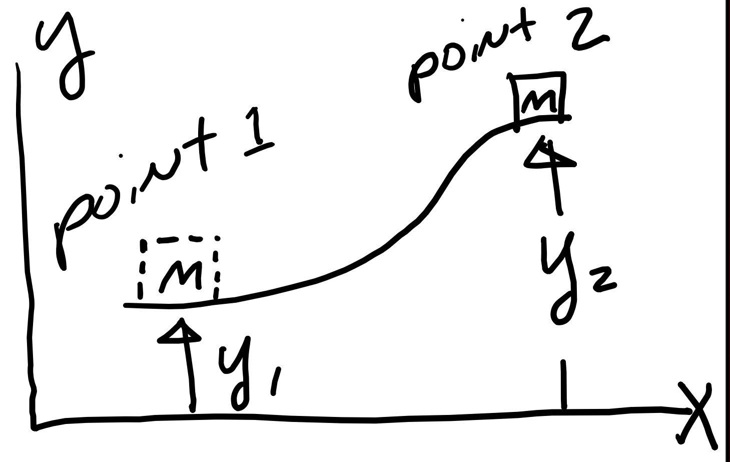
Gravity Near earth



Gravity Near earth

We want the worke **due to gravity**

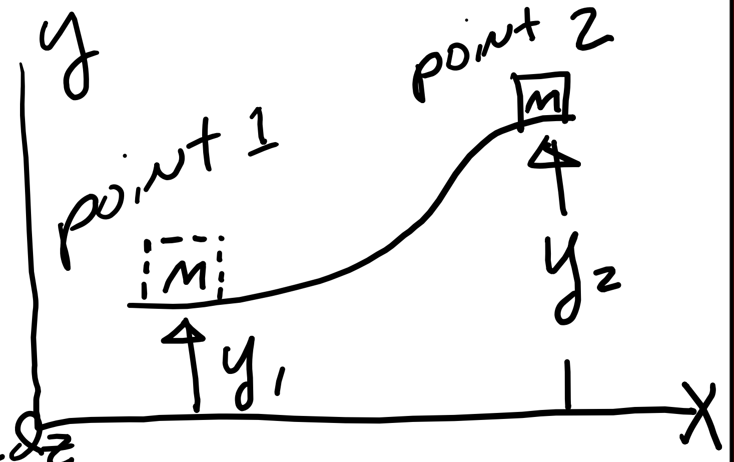
$$U^{(g)}_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$



Gravity Near earth

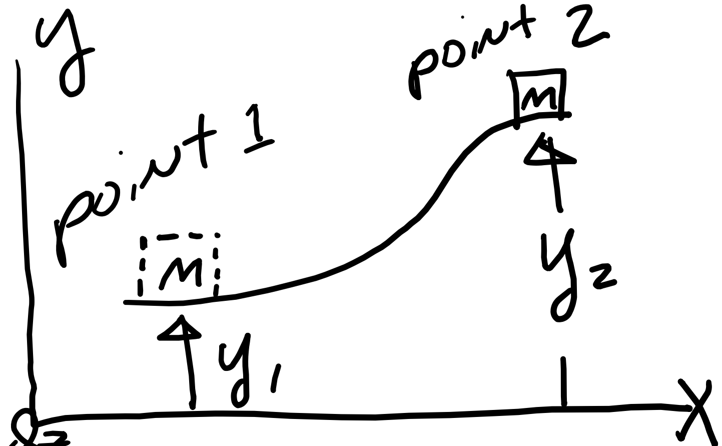
We want the worke **due to gravity**

$$U^{(g)}_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$$



Gravity Near earth

We want the work due **to gravity**



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At any point along curve $F_x = 0, F_y = -mg, F_z = 0$

Gravity Near earth

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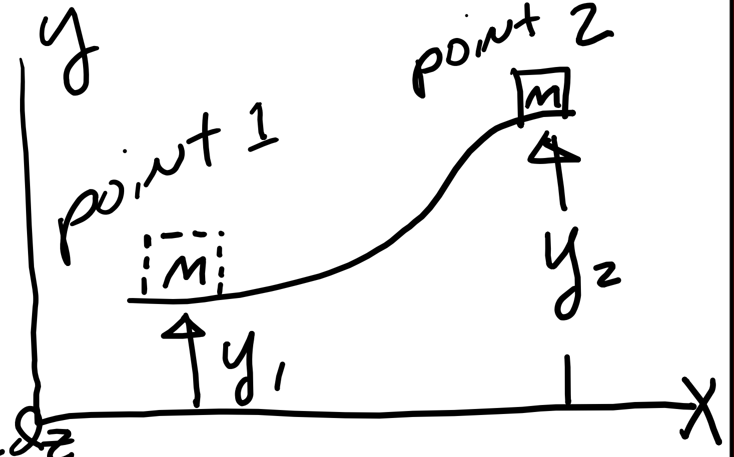
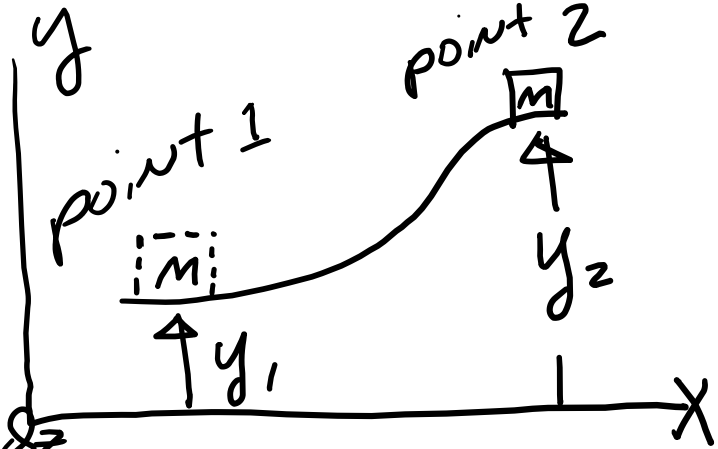


$$U_{1\to2}^{(g)} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$$

At any point along curve $F_x = 0$, $F_y = -mg$, $F_z = 0$

So $U_{1\to2} = \int_{y_1}^{y_2} F_y\,dy$

Gravity Near earth



We want the work **due to gravity**

$$U^{(g)}_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$$

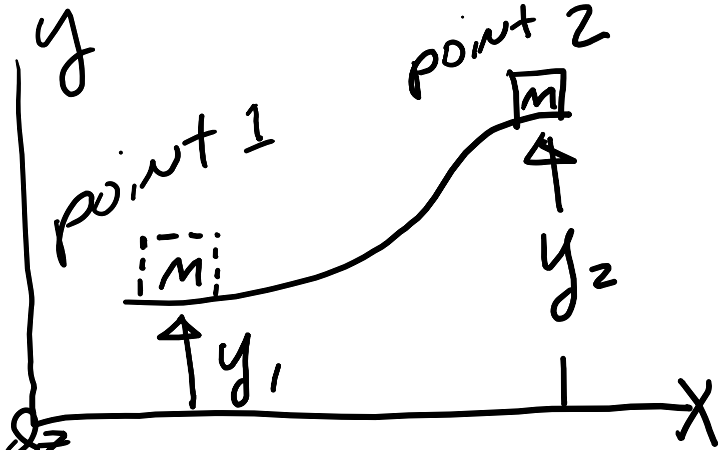
At any point along curve $F_x = 0, F_y = -mg, F_z = 0$

So $U_{1\to 2} = \int_{y_1}^{y_2} F_y\,dy = -mg\int_{y_1}^{y_2} dy = \underbrace{-mg\Delta y}$

decrease in kinetic energy

Gravity Near earth

We want the work due to gravity



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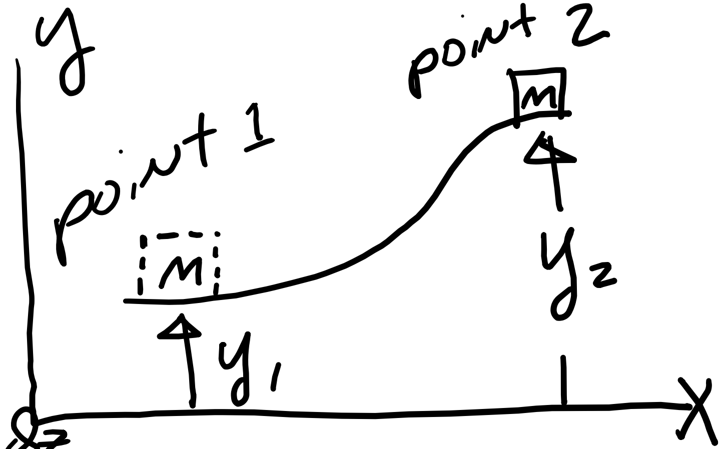
decrease in kinetic energy

Perhaps a bit more natural to go from point 2 to point 1

Gravity Near earth

We want the work **due to gravity**

$$U^{(g)}_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$$



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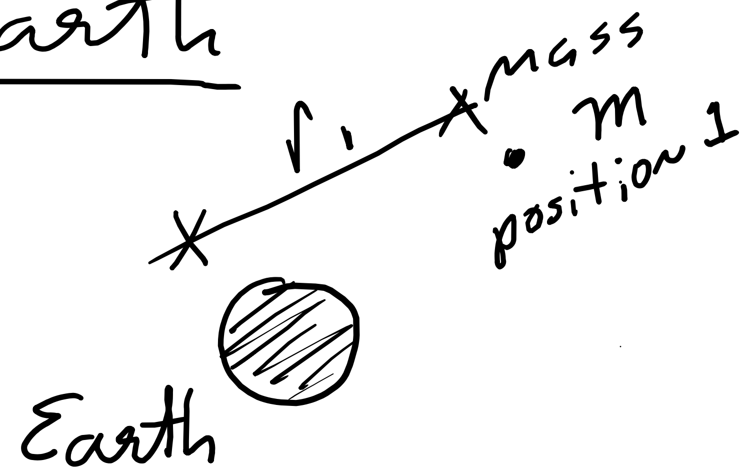
So $$U_{1\to 2} = \int_{y_1}^{y_2} F_y\,dy = -mg\int_{y_1}^{y_2} dy = \underbrace{-mg\Delta y}_{\text{decrease in kinetic energy}}$$

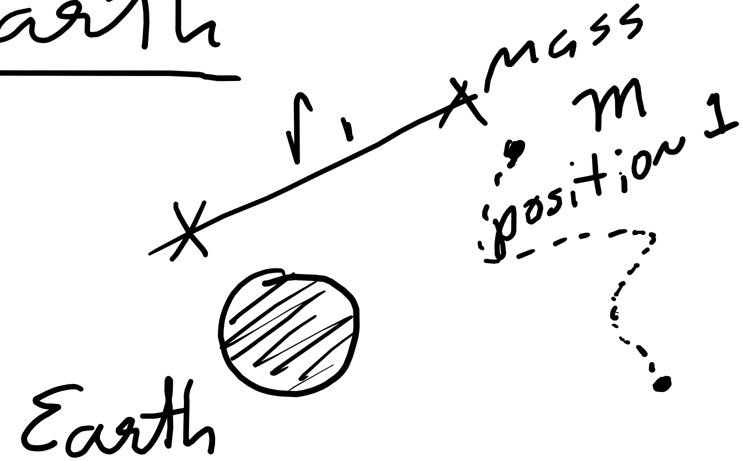
Perhaps a bit more natural to go from point 2 to point 1. In that case

$U_{2\to 1} = +mg\Delta y \Rightarrow$ Increase in kinetic energy

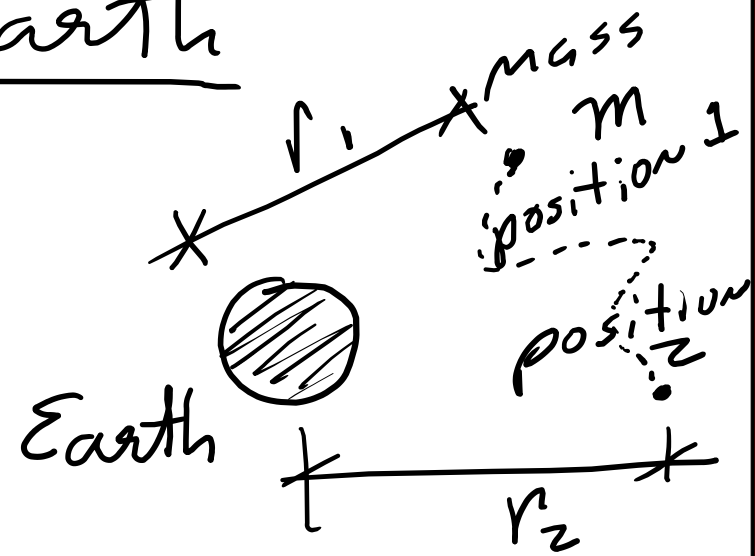
Far from earth



Far from earth

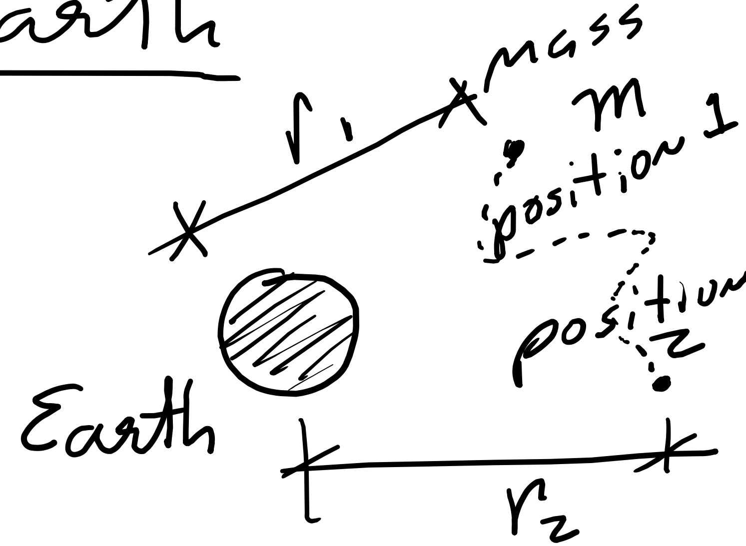


Far from earth



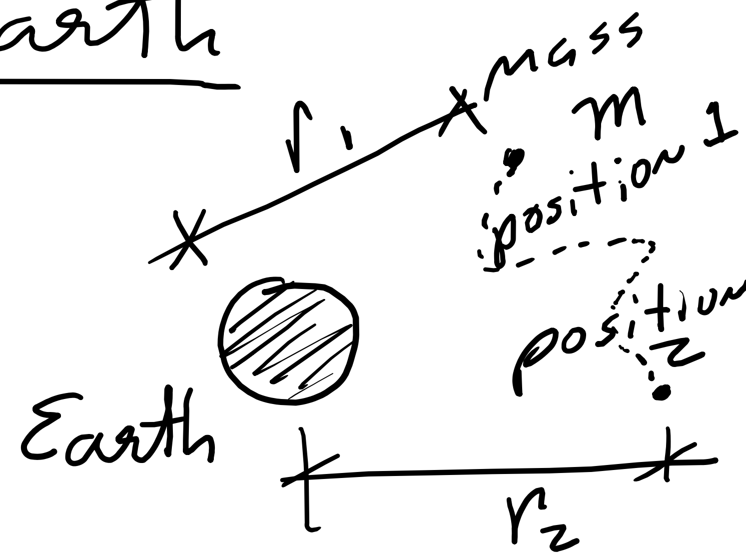
Far from earth

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$



Far from earth

$$U_{1\to2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} \Rightarrow U_{1\to2} = \int_{r_1}^{r_2} F_r\, dr$$

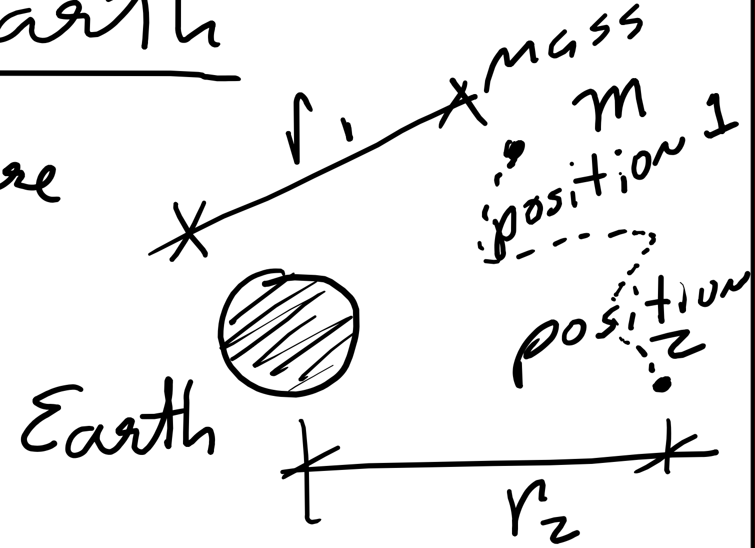


Far from earth

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} \Rightarrow U_{1\to 2} = \int_{r_1}^{r_2} F_r\, dr \text{, where}$$

$$F_r = -G\frac{M_E m}{r^2} \Rightarrow$$

$$U_{1\to 2} = -GM_E m\int_{r_1}^{r_2}\frac{dr}{r^2}$$



Far from earth

$$U_{1\to2} = \int_{\vec r_1}^{\vec r_2} \vec F \cdot d\vec r \Rightarrow U_{1\to2} = \int_{r_1}^{r_2} F_r\, dr \text{, where}$$

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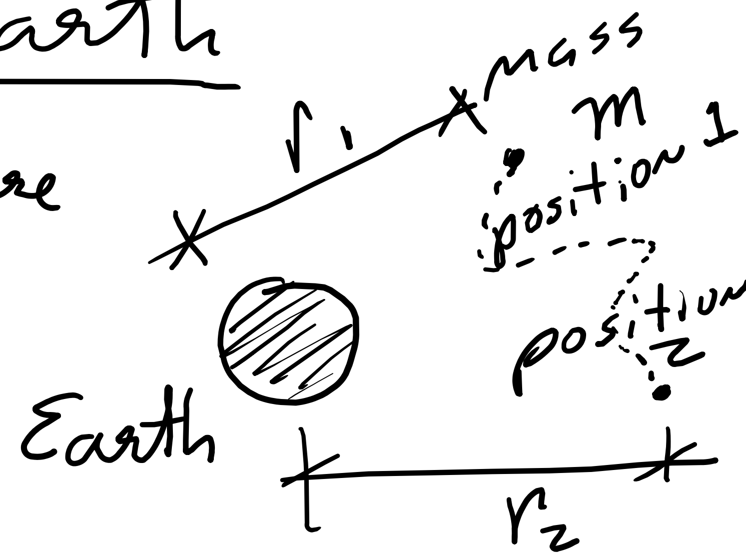
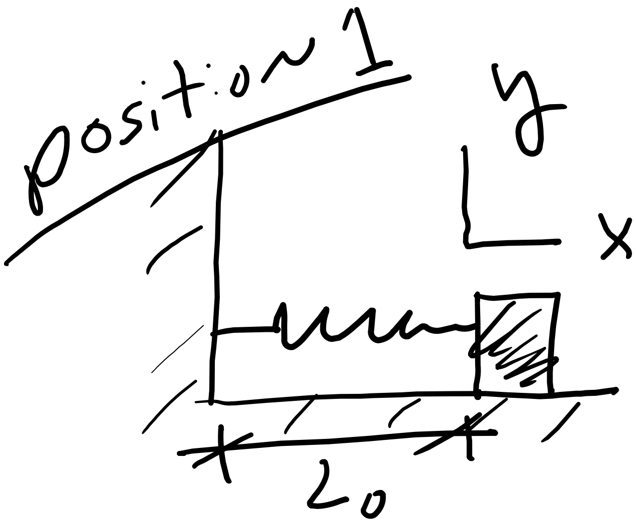
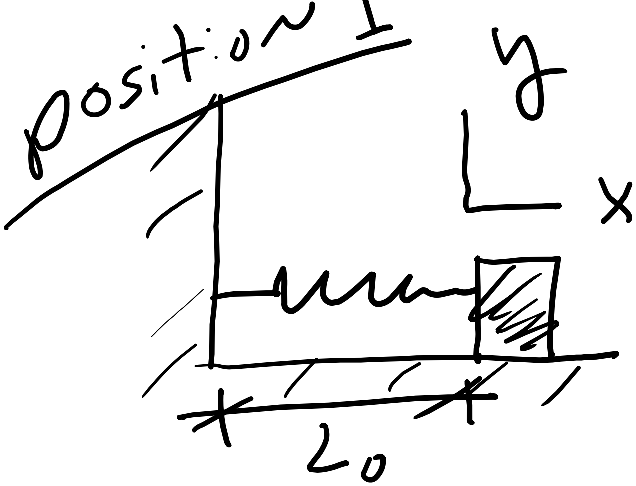
$$\Rightarrow \quad U_{1\to2} = (GM_E m)\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

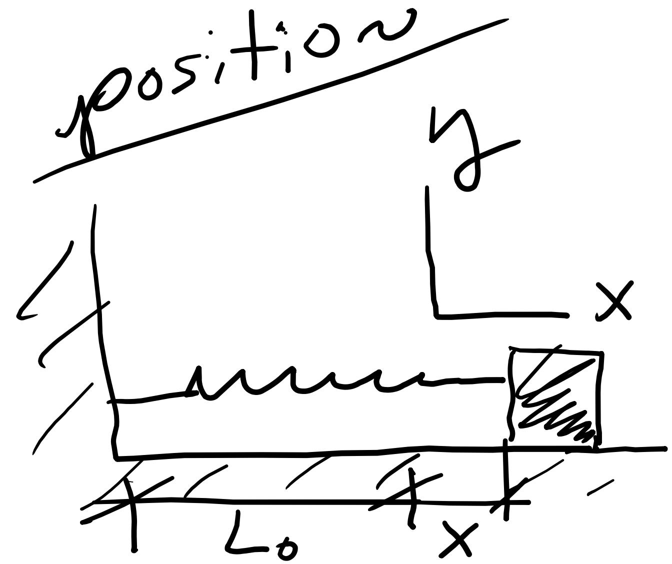


Spring

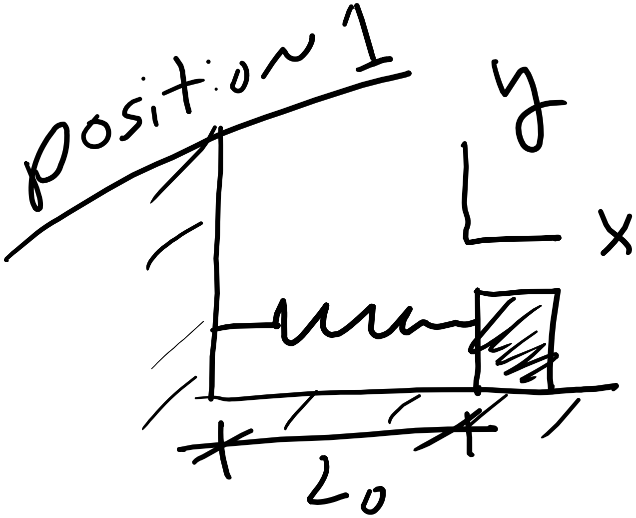


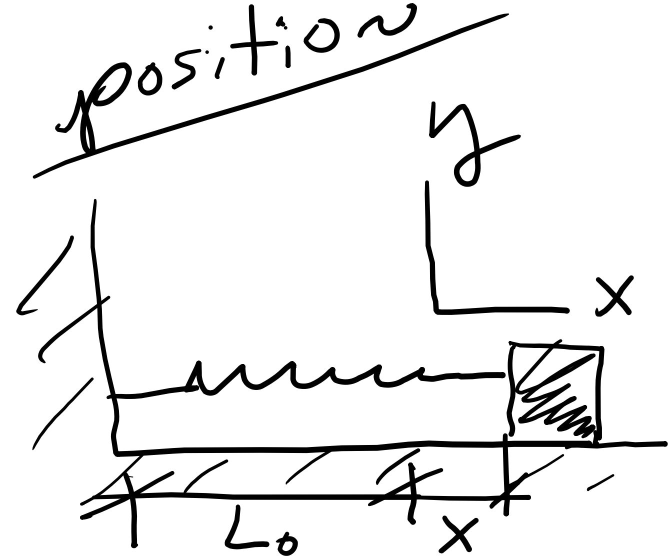
Spring





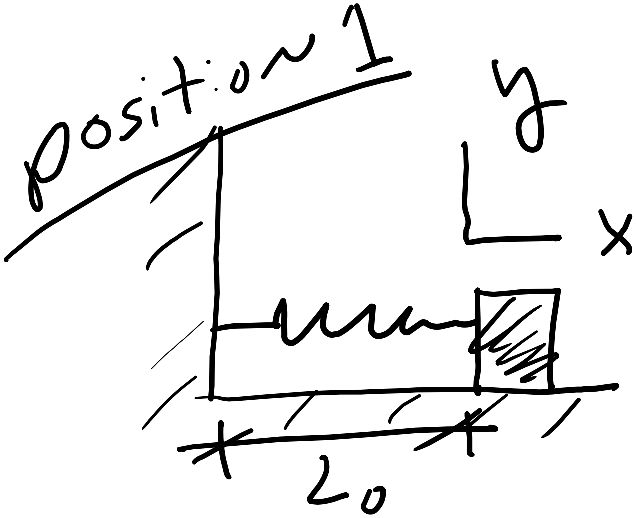
Spring

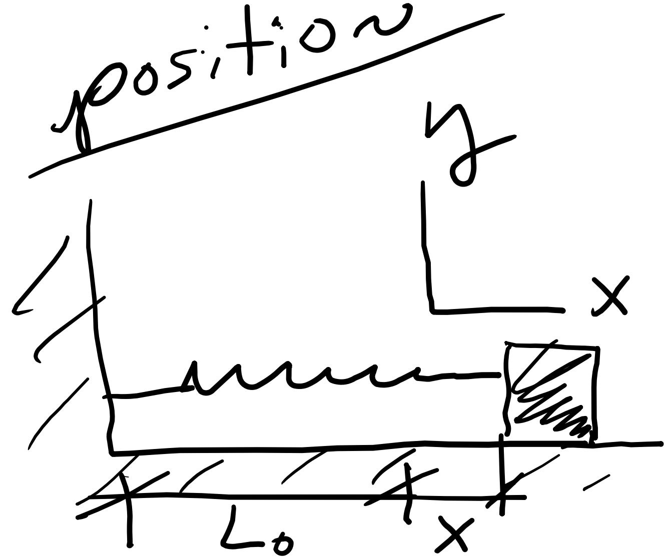




Here $\vec{F} = -kx\hat{i}$

Spring

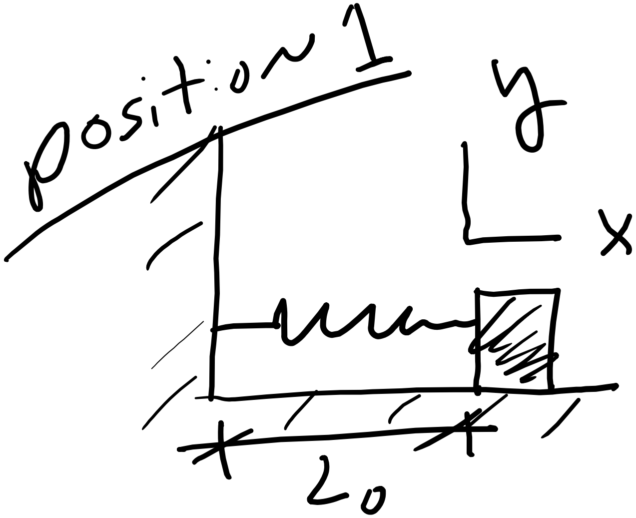


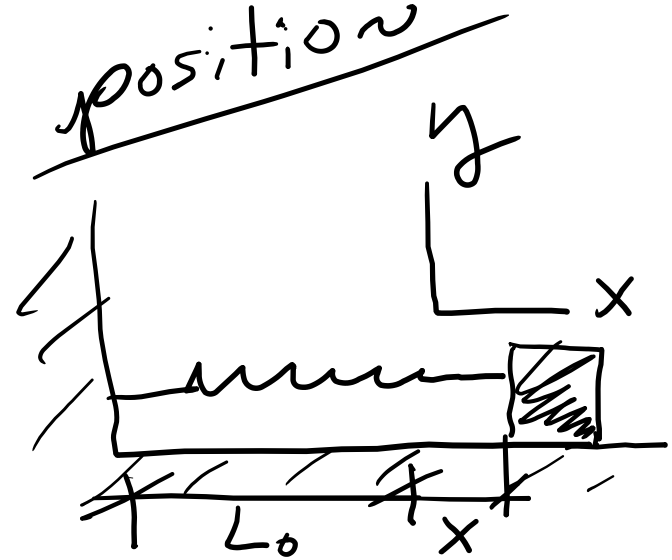


Here $\vec{F} = -kx\hat{i} \Rightarrow$

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

Spring





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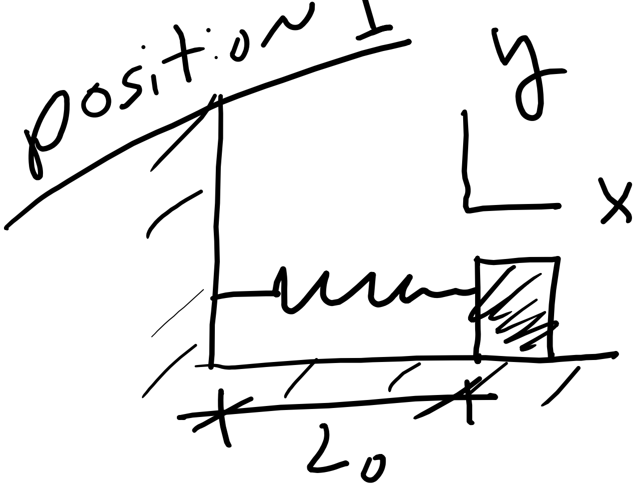
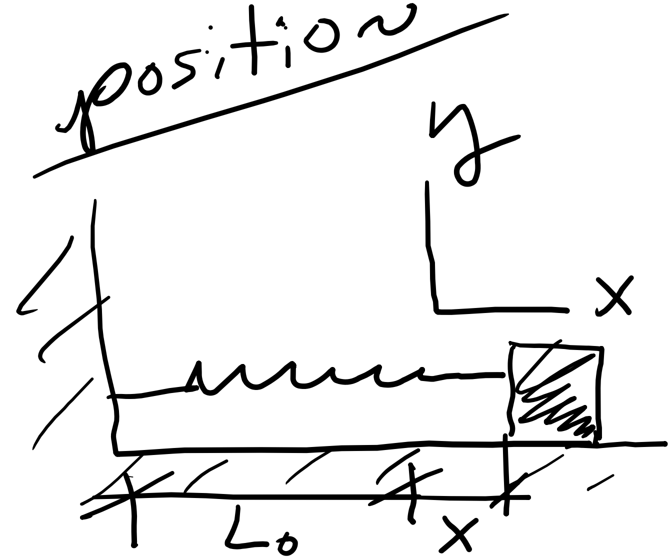
$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x \, dx$$

Spring





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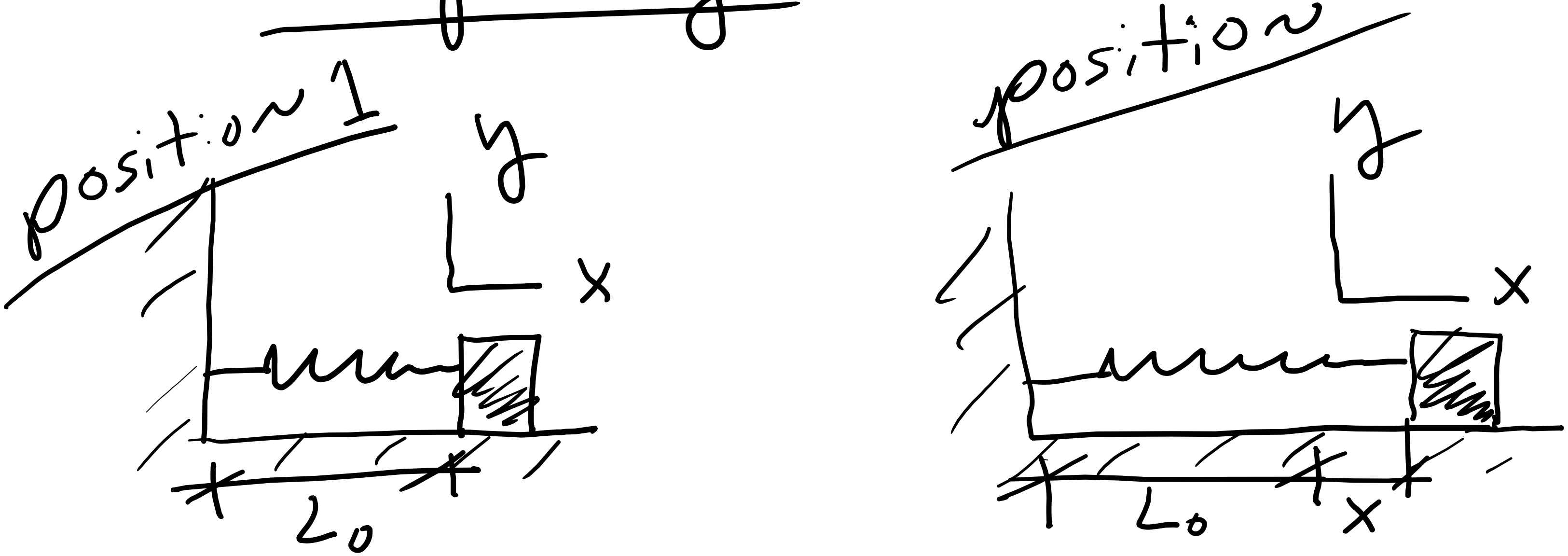
Spring



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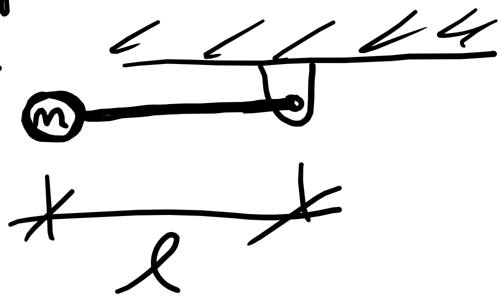
$$U_{1\to 2} = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

Pendulum problem

position 1

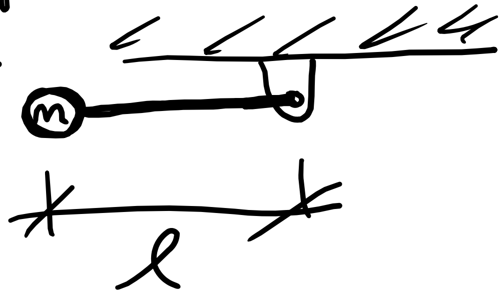
$V_1 = \cancel{0}$



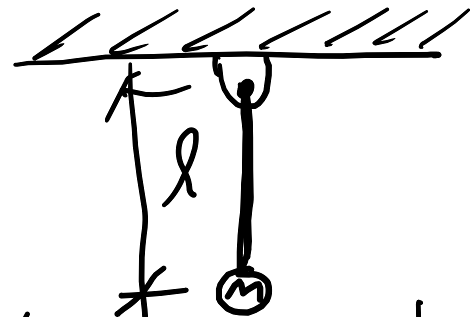
Pendulum problem

position 1

$V_1 = 0$

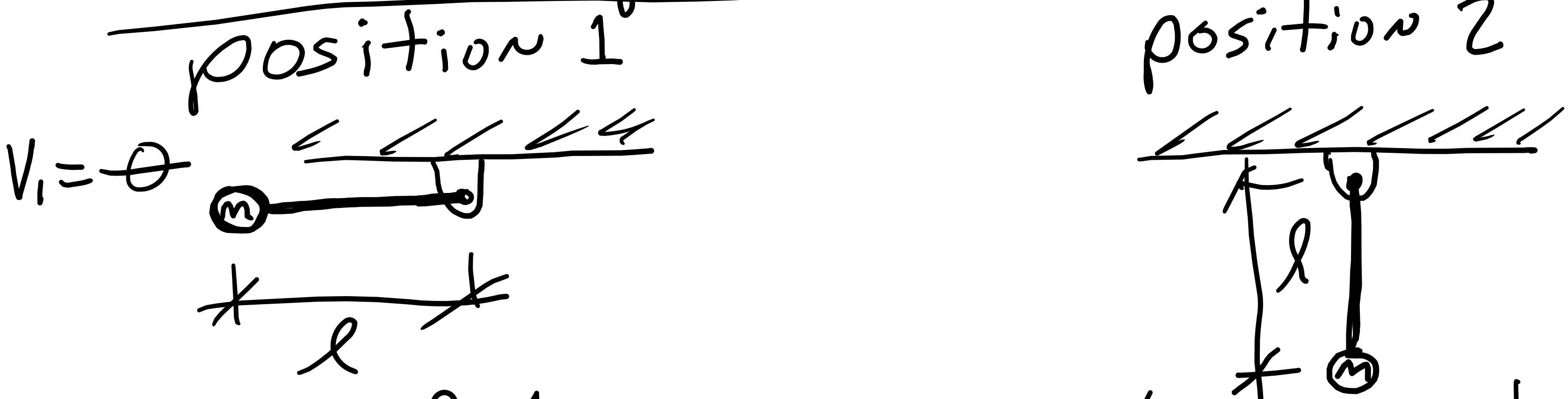


position 2



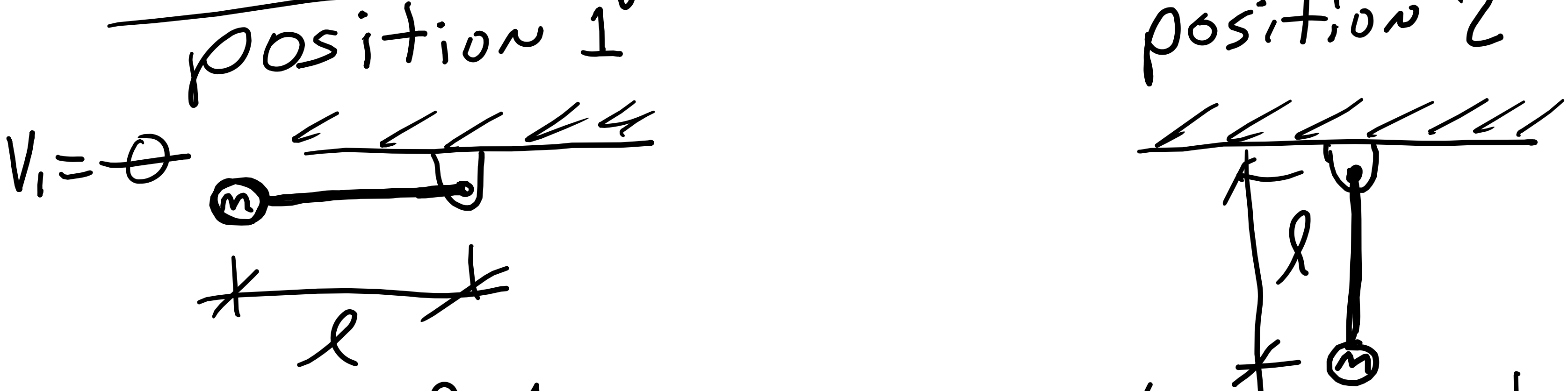
Assume rod has no mass & system has no friction

Pendulum problem



Assume rod has no mass & system has no friction. Find speed of mass at position 2:

Pendulum problem



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Pendulum problem

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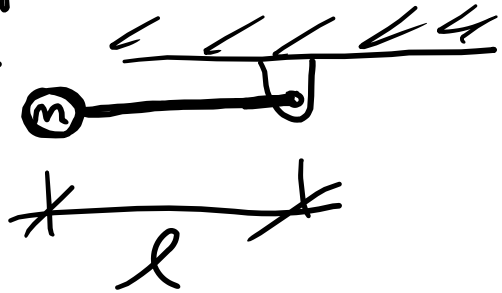
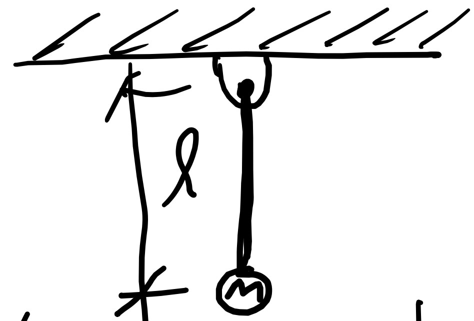


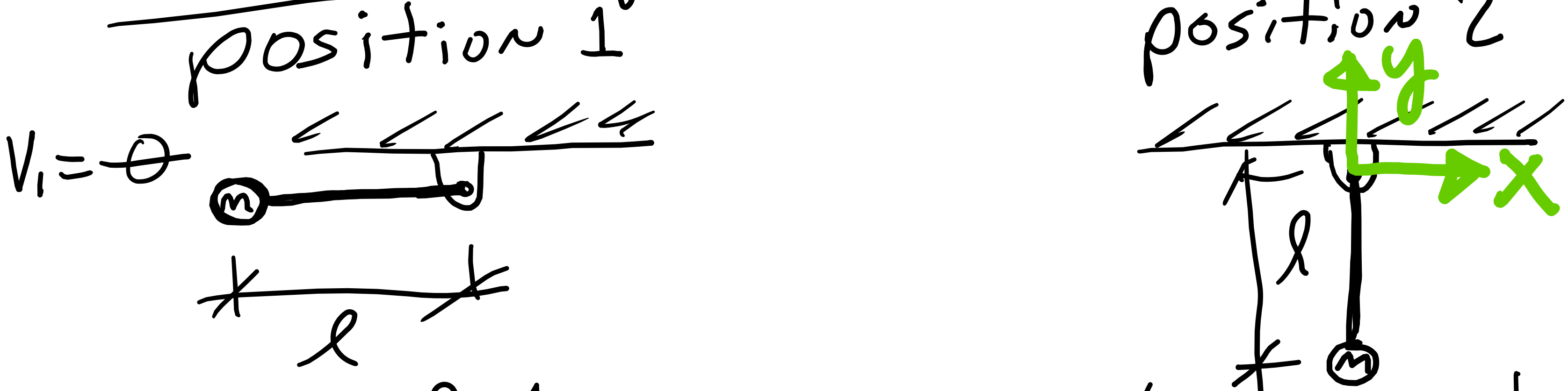
position 2



Assume rod has no mass & system has no friction. Find speed of mass at position 2:

$$U_{1\to2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dx$$

Pendulum problem



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Pendulum problem

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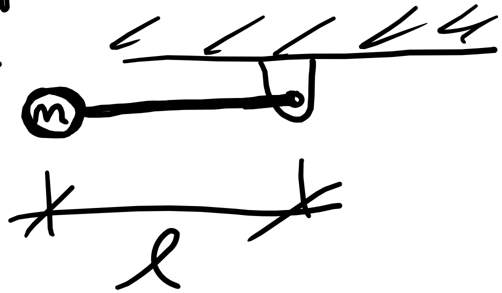
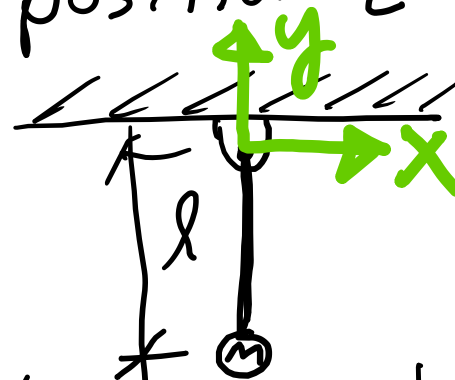


position 2



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so $U_{1\to 2} = -mg\int_0^{-\ell} dy$

Pendulum problem

position 1

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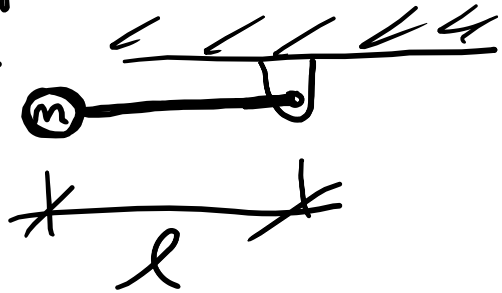
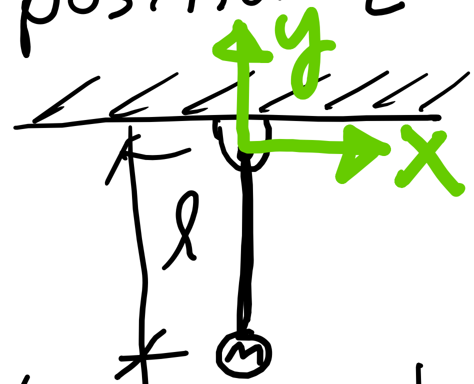


position 2



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Pendulum problem

position 1

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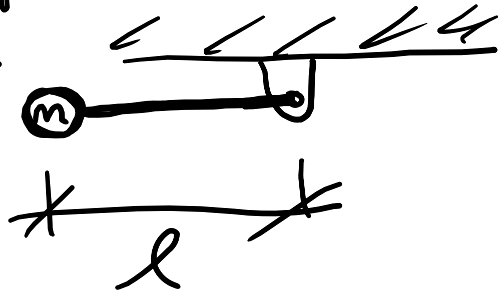
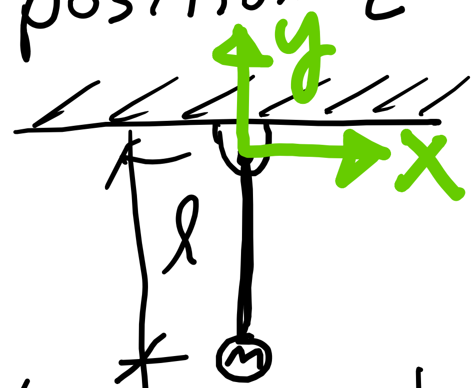


position 2



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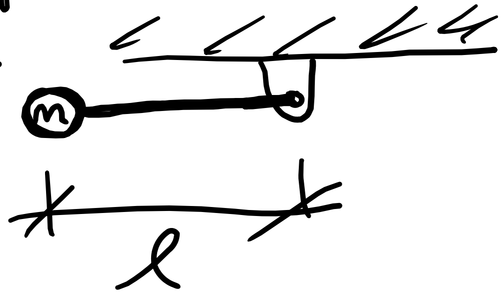
so $U_{1\to 2} = -mg\int_0^{-\ell} dy = (-mg)(-\ell) = mg\ell$ &

since $U_{1\to 2} = \Delta T$

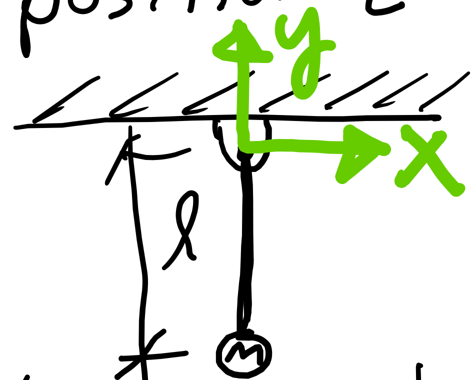
Pendulum problem

position 1

$V_1 = 0$



position 2



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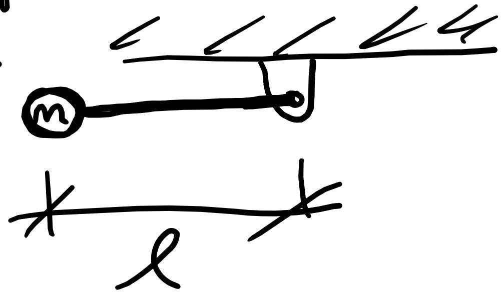
since $U_{1\to2} = \Delta T$ then $mg\ell = T_2$

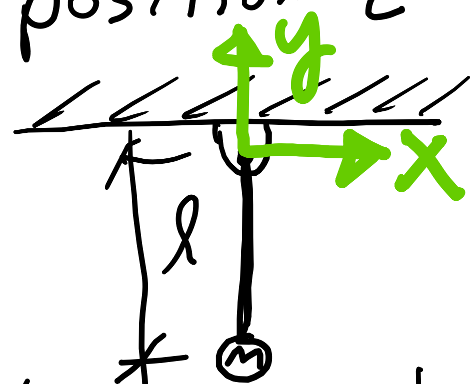
Pendulum problem

position 1

$V_1 = 0$



position 2



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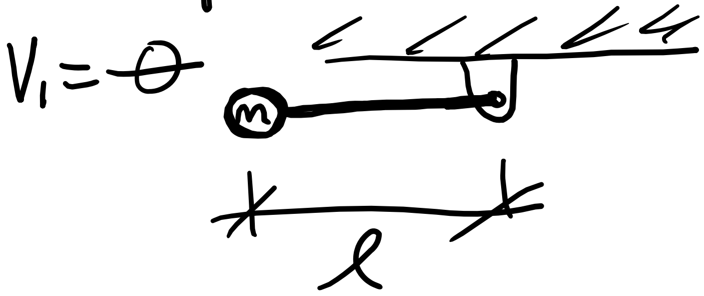
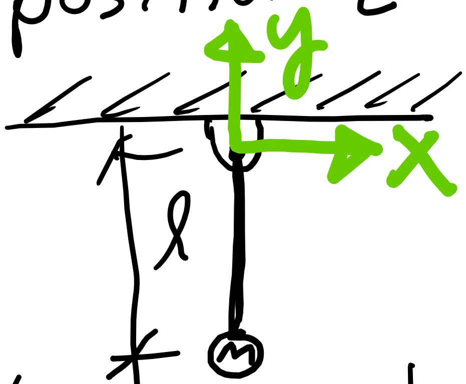
since $U_{1\to2} = \Delta T$ then $mg\ell = T_2$ But $T_2 = \frac{1}{2} m V_2^2$

Pendulum problem

position 1



position 2



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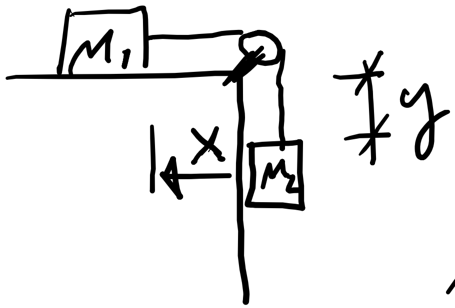
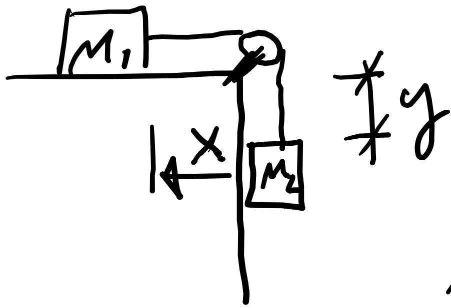
since $U_{1\to 2} = \Delta T$ then $mg\ell = T_2$ But $T_2 = \frac{1}{2}mV_2^2$

so $\frac{1}{2}mV_2^2 = mg\ell \Rightarrow \boxed{V_2 = \sqrt{2g\ell}}$

<u>Pulley problem</u>: system starts at rest. Find v_F after m_2 moves a distance L. Assume no friction



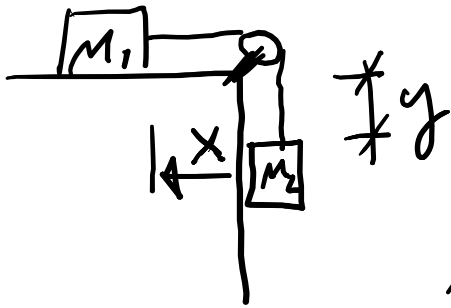
Pulley problem: system starts at rest. Find v_F after m_2 moves a distance L.



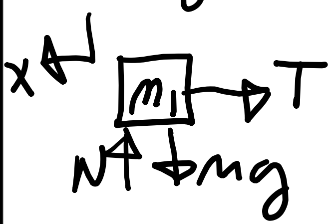
Assume no friction

$x + y = \text{const} \Rightarrow \quad \Delta y = -\Delta x = L$

Pulley problem: system starts at rest. Find v_F after m_2 moves a distance L. Assume no friction



$$x + y = \text{const} \Rightarrow \Delta y = -\Delta x = L$$



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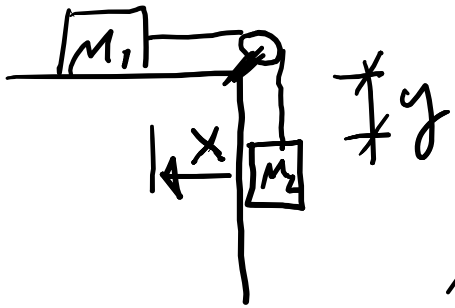
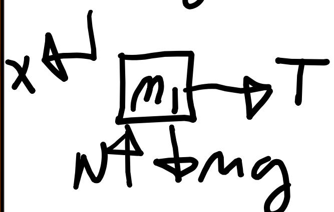


Assume no friction

$x + y = \text{const} \Rightarrow \Delta y = -\Delta x = L$



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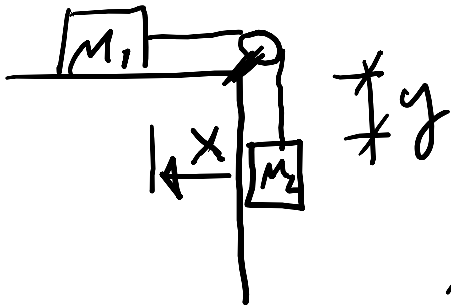

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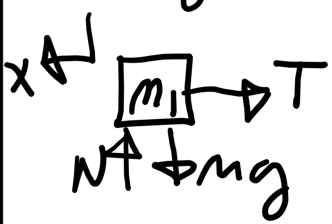

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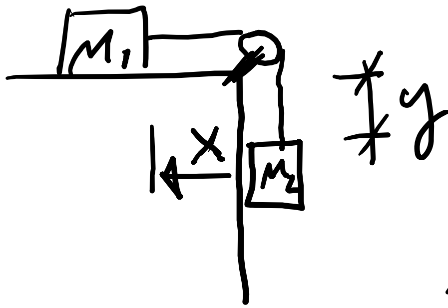

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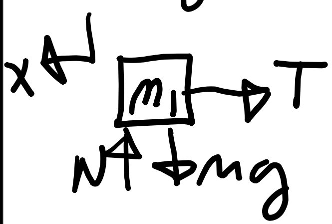

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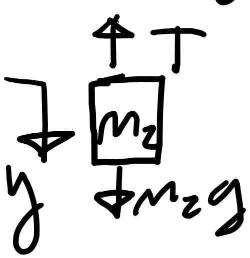

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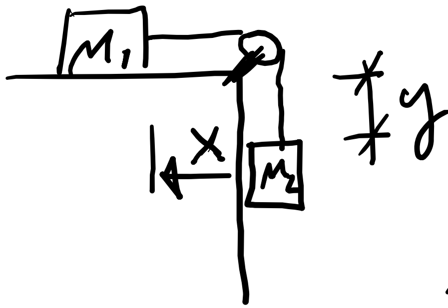

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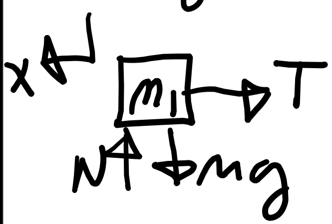

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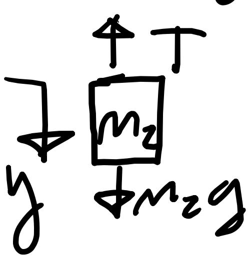

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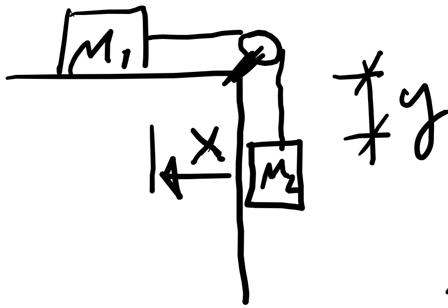

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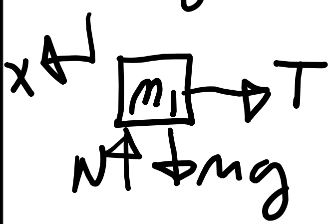

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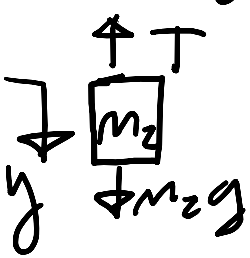

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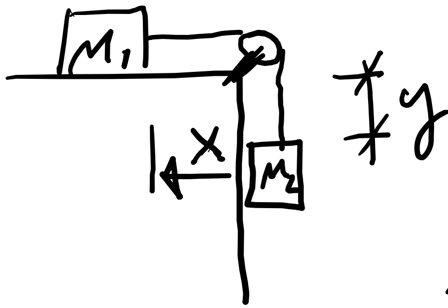

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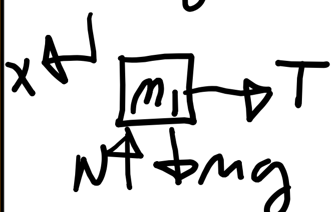

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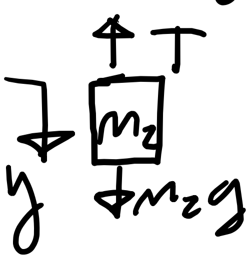

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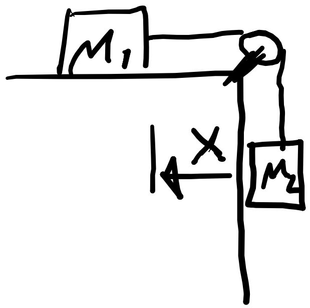

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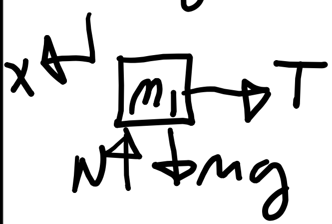

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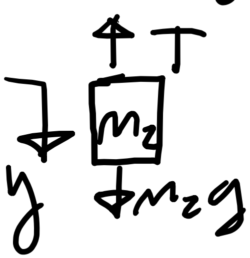

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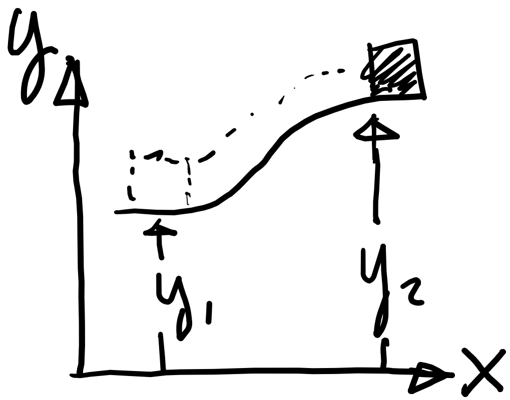
so $v_F = \sqrt{\dfrac{2 m_2 g L}{m_1 + m_2}}$

Earlier we saw that the work due to gravity along some curved path is



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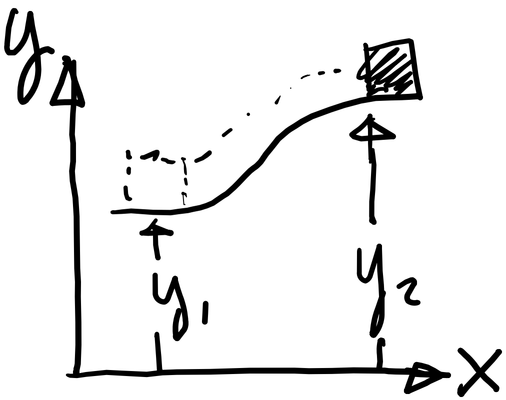

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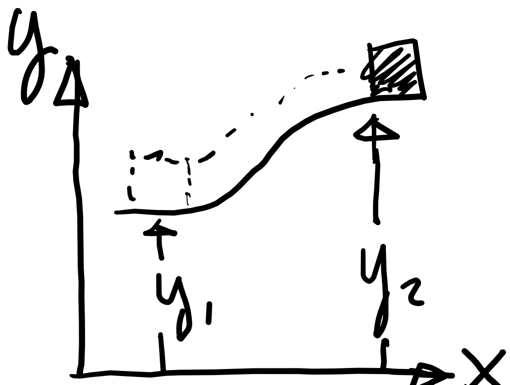

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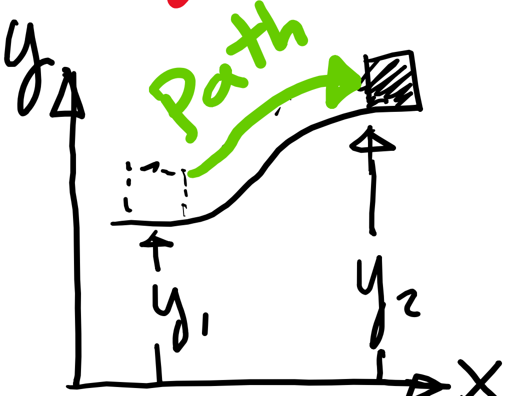

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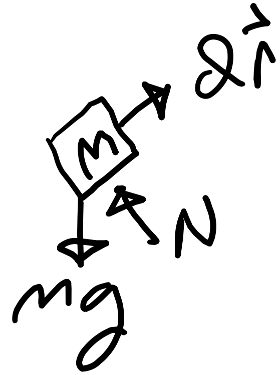
For our problem, we need to integrate the sum of forces over the path shown.

Take an arbitrary position along the path.

Take an arbitrary position along the path. Free body:



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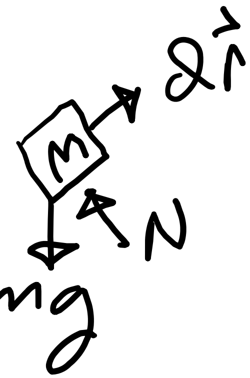

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ASU

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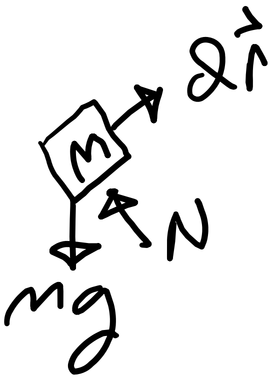

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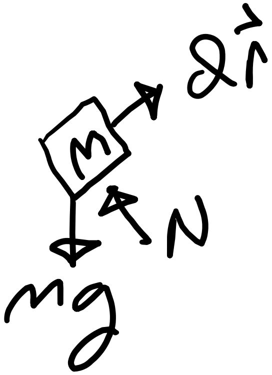

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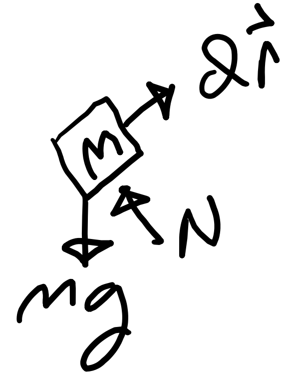

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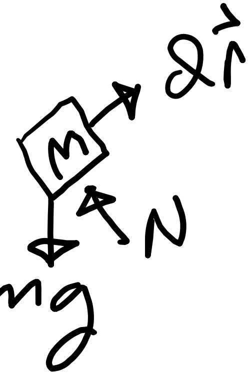

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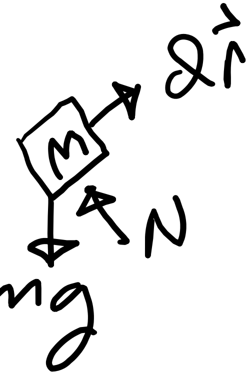

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Since $\sum \vec{F} \cdot d\vec{r} = -mg\,dy$, then

$U_{1\to 2} = -mg\Delta y$.

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$U_{1\to 2} = -mg\Delta y$. This is the work that gravity has done to the object.

The force perpindicular to the motion did not matter & the actual path taken was not important.

We call such a force, where the work done by that force is independent of the path, a

Conservative Force!!

we found that gravity was a conservative force {work performed by that force is independent of the path taken}

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What Kind of forces are conservative?

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Non-conservative forces?

* Mostly friction

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What kind of forces are conservative?

* Gravity
* Ideal springs [only kind we will work with]
* Electric-force [not used in this course]

Non-conservative forces?

* Mostly friction
* Anything that generates heat

3d case

$\sum \vec{F} = m\vec{a} \Rightarrow \sum \vec{F}\cdot d\vec{r} = m\vec{a}\cdot d\vec{r}$, But

$d\vec{r} = \frac{d\vec{r}}{dt}dt = \vec{v}\,dt$ & $\vec{a} = \frac{d\vec{v}}{dt}$

so $\sum \vec{F}\cdot d\vec{r} = m\vec{v}\cdot\left(\frac{d\vec{v}}{dt}\right)dt$ & since

$d\vec{v} = \left(\frac{d\vec{v}}{dt}\right)dt$, then $\int \sum \vec{F}\cdot d\vec{r} = \int m\vec{v}\cdot d\vec{v}$

$= m\left\{\int V_x\,dV_x + \int V_y\,dV_y + \int V_z\,dV_z\right\} \Rightarrow \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = \frac{m}{2}\left(V_x^2 + V_y^2 + V_z^2\right)\Big|_{V_I}^{V_F}$

$= \left(\frac{m}{2}\right)\left\{V_F^2 - V_I^2\right\} \Rightarrow \sum \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = T_2 - T_1$ or

$U_{1\to 2} = \Delta T$ with $U_{1\to 2} \equiv \sum \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$