Today: 13.2, 13.3

$\boxed{L9}$

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$\boxed{L9}$

Conservation of energy

Today: $\underbrace{13.2}_{\text{Conservation of energy}}$, $\underbrace{13.3}_{\text{Impulse \& momentum}}$ $\boxed{L9}$

Today: 13.2, 13.3

Tuesday: 13.4, 14.1

L9

ASU

Today: 13.2, 13.3 $\boxed{L9}$

Tuesday: $\underbrace{13.4}_{\text{Impacts}}$, 14.1

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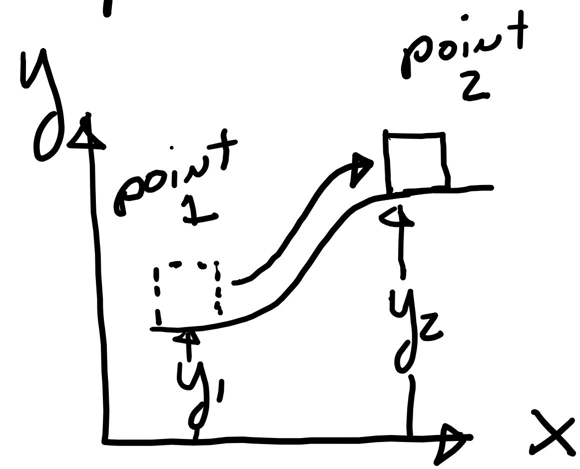
$\underbrace{13.4}_{\text{Impacts}}$, $\underbrace{14.1}_{\text{System of particles, Newton's 2nd law}}$

Going back to our box problem:



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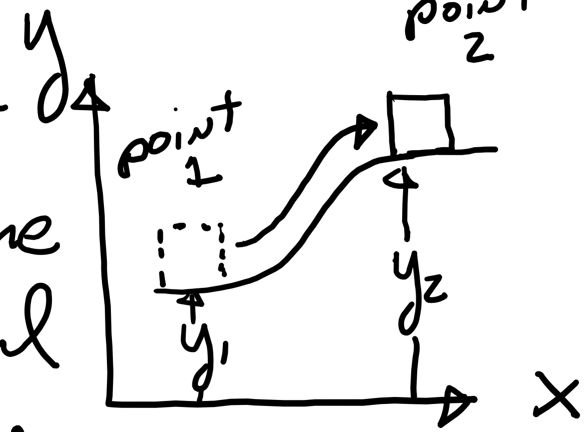
This time Bob pushes the box up the hill such that the initial kinetic energy = final kinetic energy = zero {at rest at points 1 & 2}.



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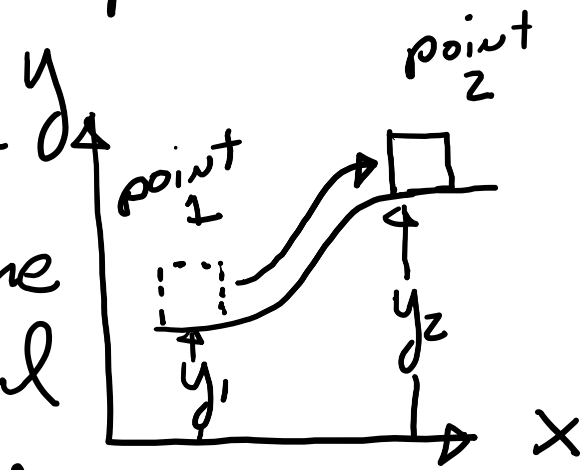

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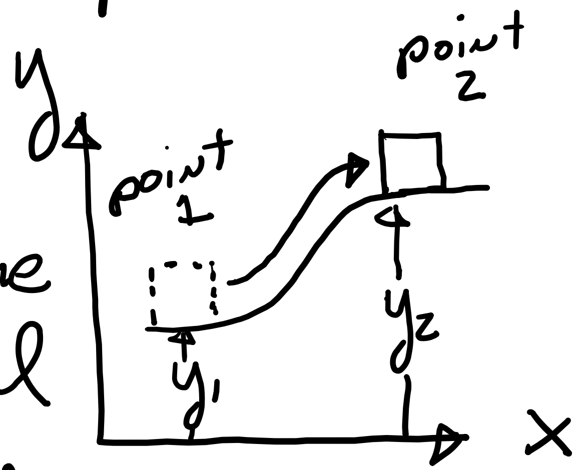

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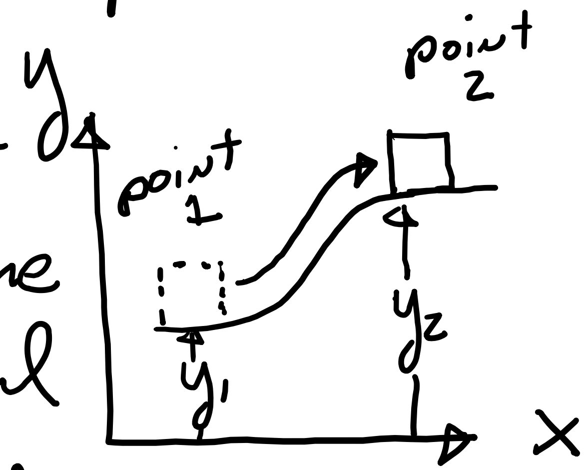

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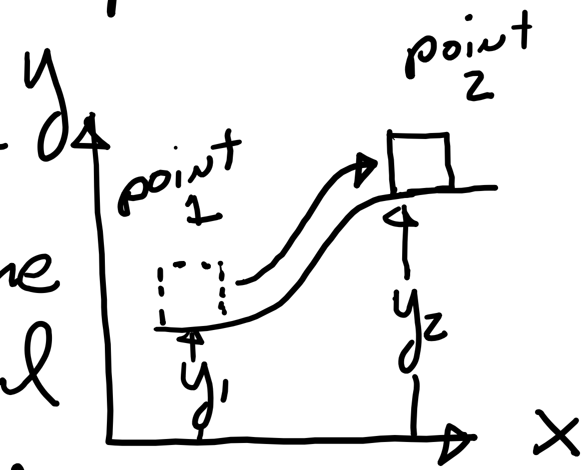

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<u>A new point of view</u>: Let's say that Bob put energy into the box. At point 2, the box has gained "Potential Energy"

Let $\left(\begin{array}{c}\text{potential energy} \\ \text{at point } i\end{array}\right) \equiv V_i$

Let (potential energy at point i) $\equiv V_i$

Now $\underset{\text{Bob}}{U_{1\to 2}} = -\underset{\text{Gravity}}{U_{1\to 2}} = V_2 - V_1$

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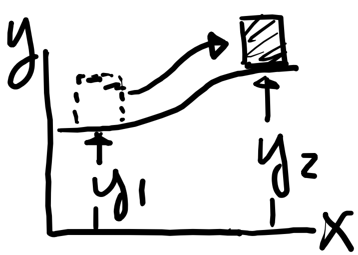
<u>**Please notice**</u>: The <u>difference</u> in potential energy is relatable to work put in.



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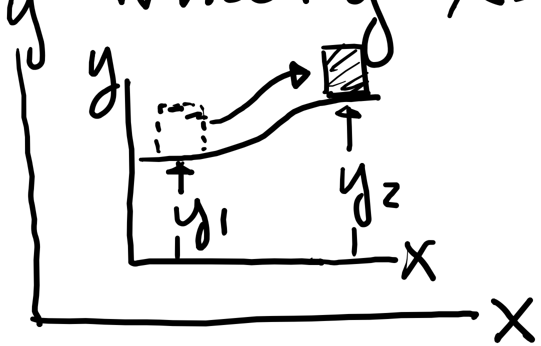
What if Alice had a coordinate system $x'y'$



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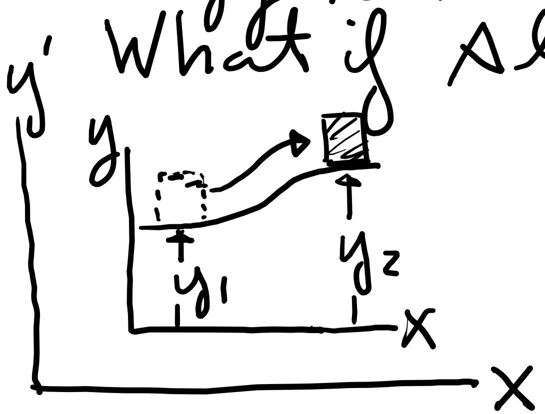

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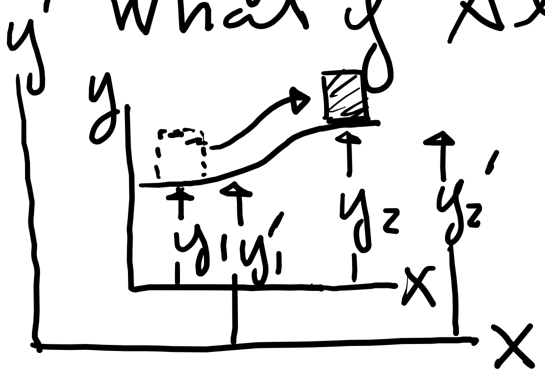

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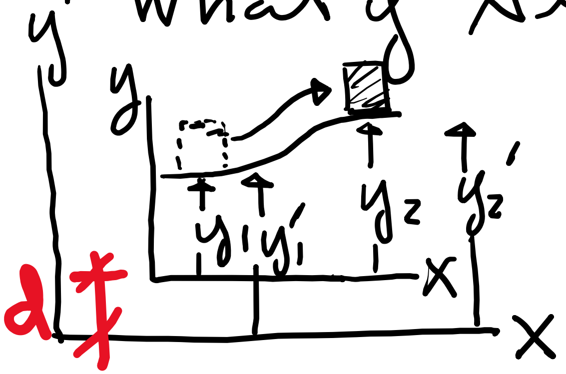

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<u>But</u> $y_1' = y_1 + d$ & $y_2' = y_2 + d$

So $\underset{Bob}{U_{1\to 2}} = mg(y_2' - y_1') = mg(y_2 - y_1)$

So $U_{1\to 2 \atop \text{Bob}} = mg(y_2' - y_1') = mg(y_2 - y_1)$

& we agree on the <u>difference</u> in potential energy

So $U_{\substack{1\to 2 \\ Bob}} = mg(y_2' - y_1') = mg(y_2 - y_1)$

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You can always add a constant to potential energy as long as it is the same constant for <u>all</u> points.

Potential energy:

Gravity : $V_g = mgy$, Near earth

Potential energy:

Gravity : $\begin{cases} V_g = mgy \,, & \text{Near earth} \\ V_g = -\dfrac{GmM_E}{r} \,, & \text{far from earth} \end{cases}$

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For any conservative forces present,

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Conservation of energy

For any conservative forces present, then

$$V_1 + T_1 + U_{1\to 2}^{NC} = V_2 + T_2$$

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---

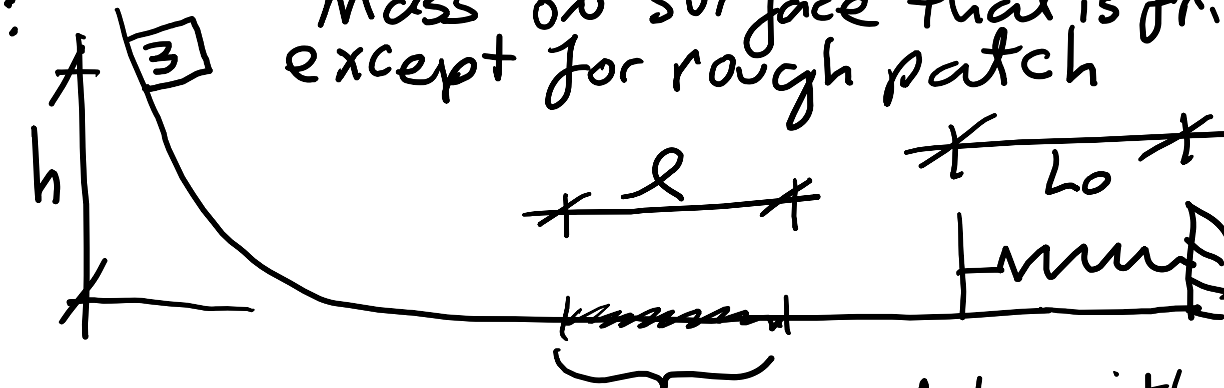
Conservation of energy

If any conservative forces present, then

$V_1 + T_1 + U_{1\to2}^{NC} = V_2 + T_2$, where $U_{1\to2}^{NC}$ is the work due to non-conservative forces

Example:

Mass on surface that is frictionless except for rough patch



Rough patch with $\mu_k \neq 0$

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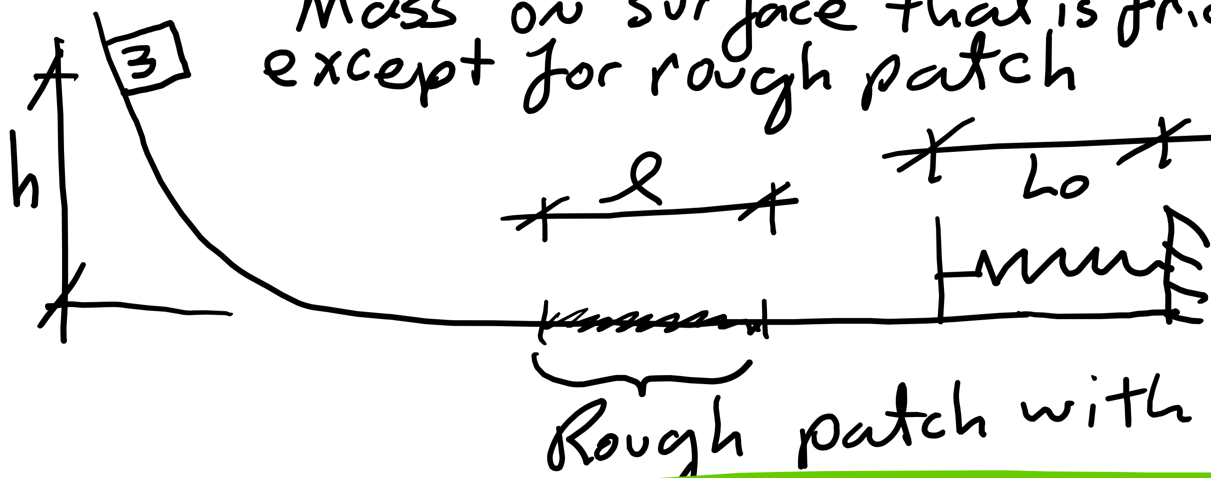

Mass initially at rest, then let go. How far can spring be compressed?

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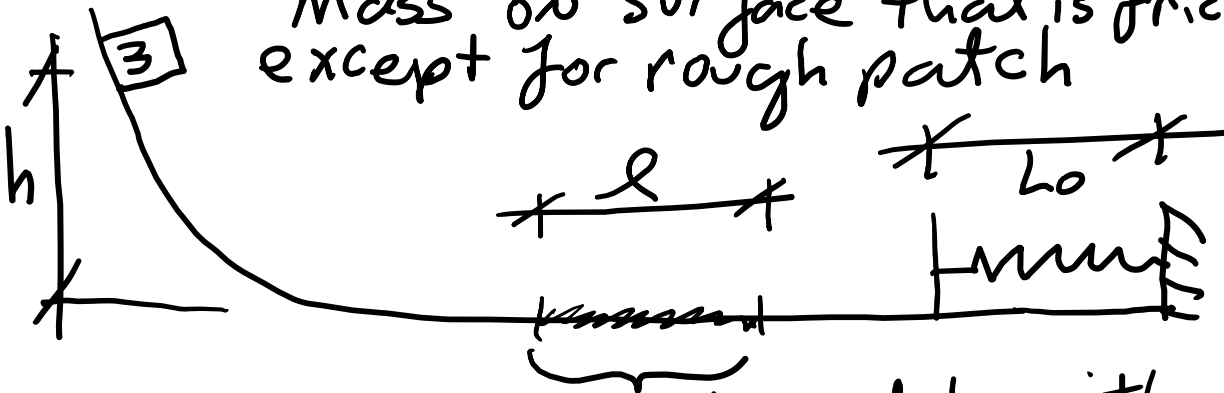

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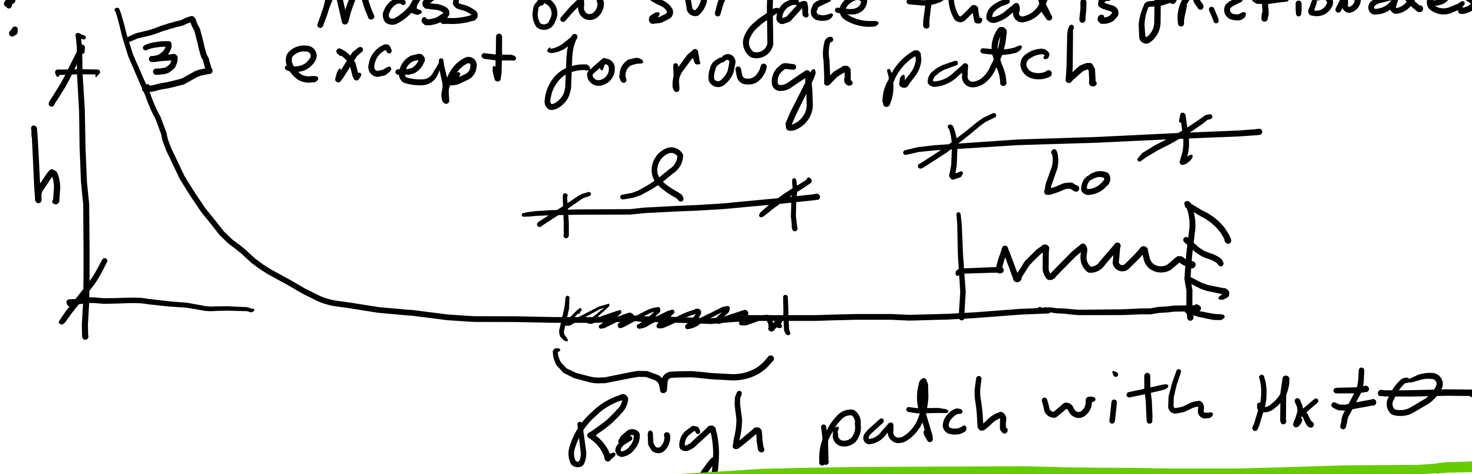

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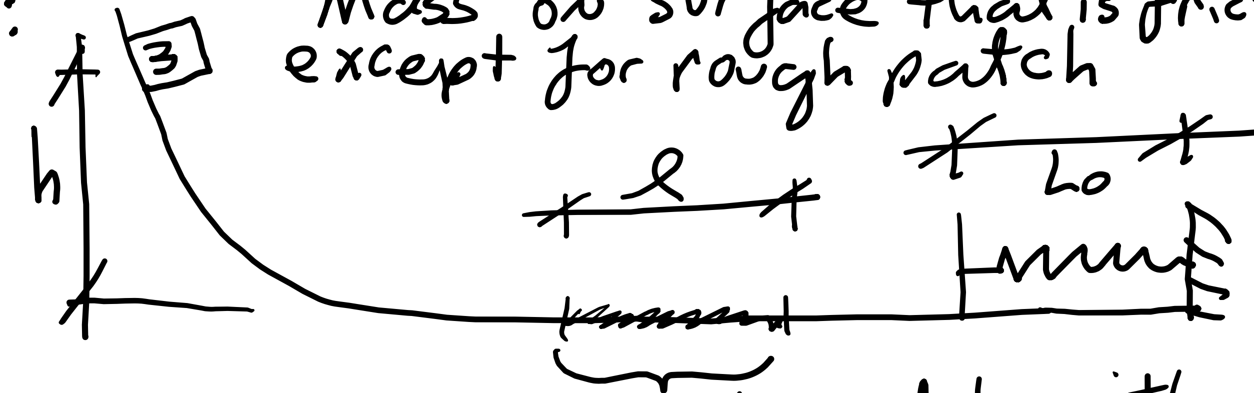

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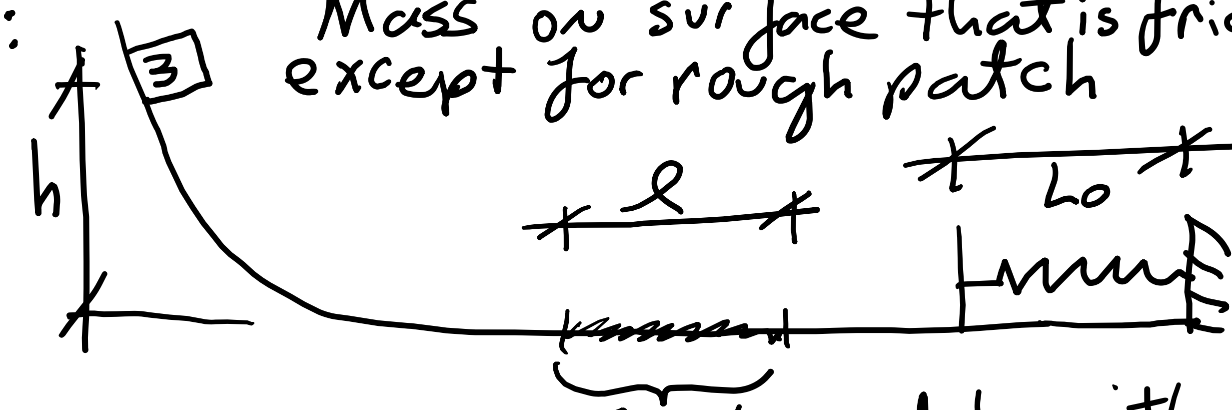

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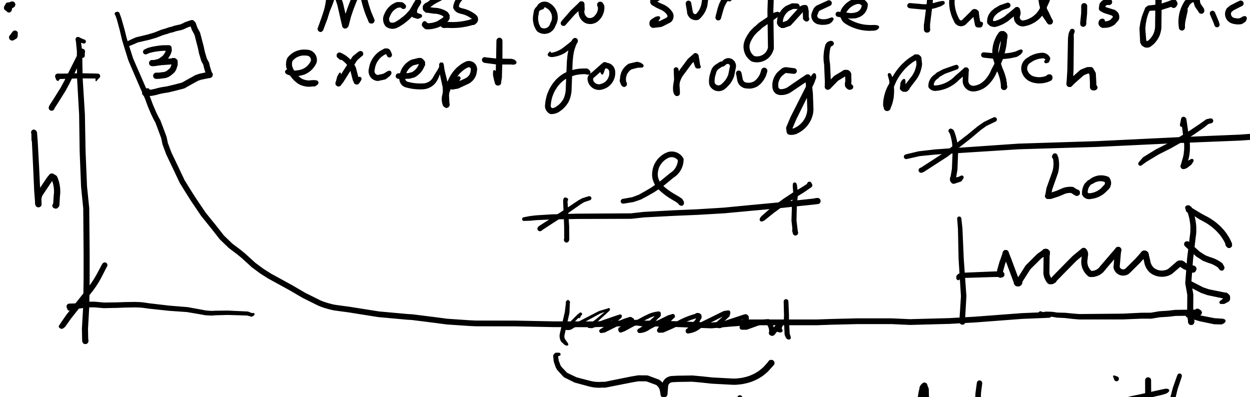

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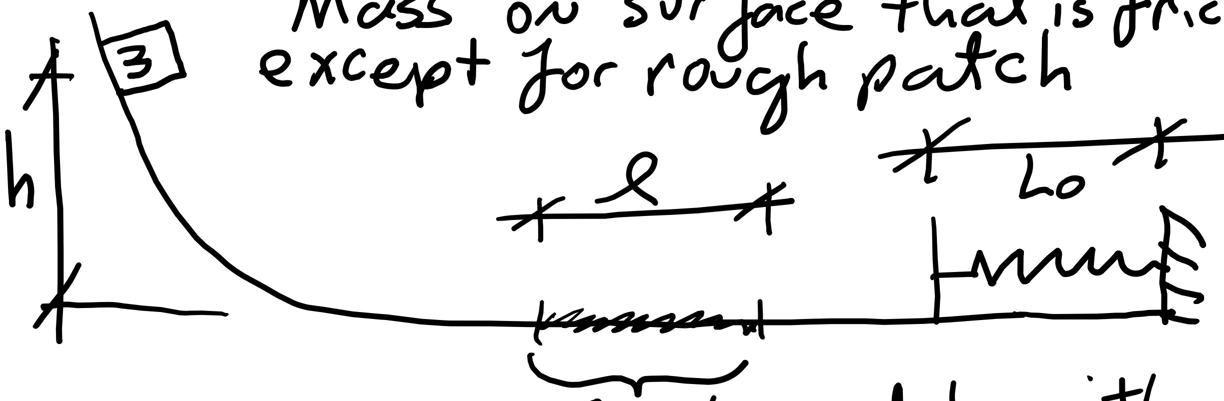

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Notice the sign!!

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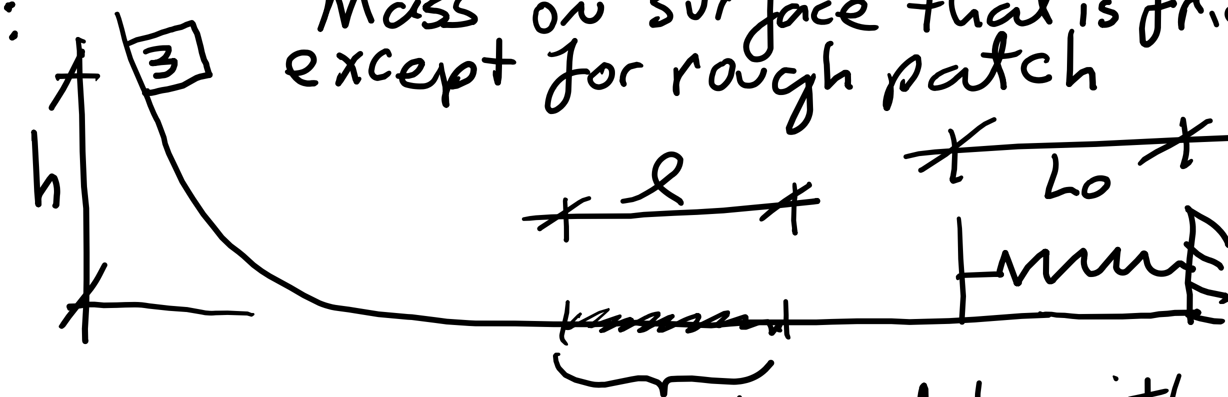

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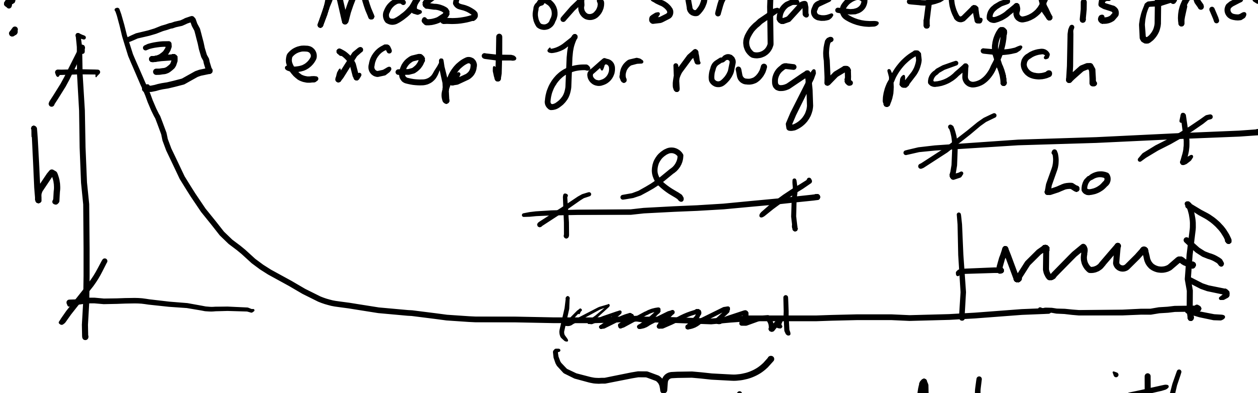

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So $\quad mgh - mg\mu_k \ell = \tfrac{1}{2}kx^2$

$$\Rightarrow \quad x = \left[\left(\frac{2mg}{k}\right)(h - \mu_k \ell)\right]^{1/2}$$

Impulse & momentum

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For multiple impulses:

$$\sum \vec{I}_{mp} = \Delta\vec{L}$$

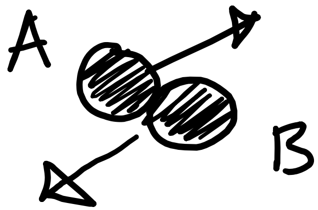
For multiple particles:

$$\sum \vec{I}_{mp} = \sum \Delta\vec{L}$$

Internal impulses

Internal impulses

Example: Two particles collide and there is NO external impulses provided.



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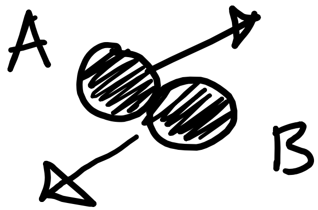

Particle A & B will interact,

Internal impulses

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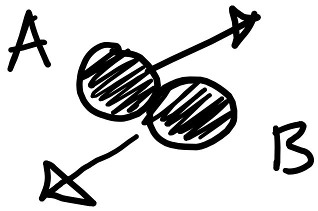

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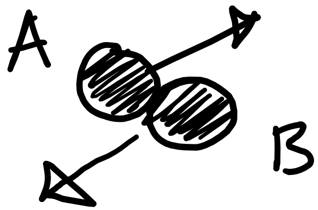

Particle A & B will interact, where the force of A on B $\equiv \vec{F}_{A \to B}$ & force of B on A $\equiv \vec{F}_{B \to A}$.

Internal impulses

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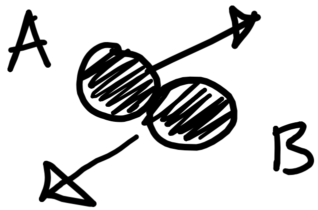

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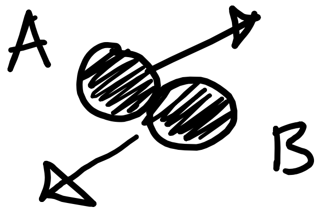
The interaction time can be taken as $\Delta t = t_F - t_I$.

Using our new equation: $\Sigma \vec{Imp} = \Sigma \Delta \vec{L}$

<u>Internal impulses</u> Example: Two particles collide and there is <u>NO</u> external impulses provided.



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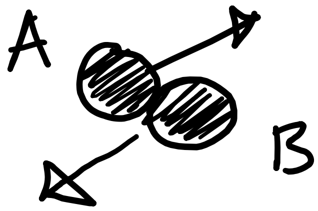
$$\int \vec{F}_{A \to B} dt + \int \vec{F}_{B \to A} dt = \Delta \vec{L}_A + \Delta \vec{L}_B$$

Internal impulses

Example: Two particles collide and there is NO external impulses provided.



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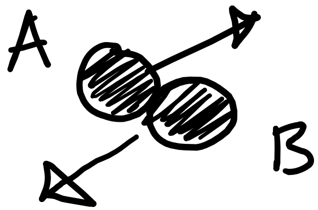
$\int \vec{F}_{A\to B} dt + \int \vec{F}_{B\to A} dt = \Delta \vec{L}_A + \Delta \vec{L}_B$ **But** $\vec{F}_{A\to B} = -\vec{F}_{B\to A}$

Internal impulses Example: Two particles collide and there is NO external impulses provided.



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The interaction time can be taken as $\Delta t = t_F - t_I$.

Using our new equation: $\Sigma \vec{Imp} = \Sigma \Delta \vec{L}$ we obtain

$\int \vec{F}_{A\to B} dt + \int \vec{F}_{B\to A} dt = \Delta \vec{L}_A + \Delta \vec{L}_B$ But $\vec{F}_{A\to B} = -\vec{F}_{B\to A}$ so

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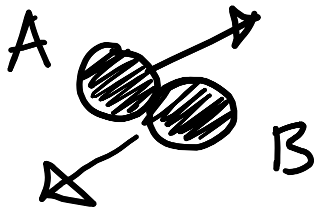
Example: Two particles collide and there is NO external impulses provided.



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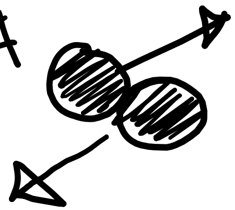

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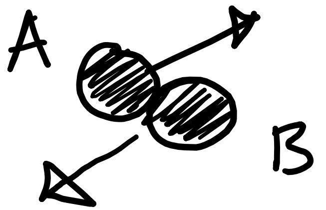

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$\int \vec{F}_{A\to B}\,dt + \int \vec{F}_{B\to A}\,dt = \Delta \vec{L}_A + \Delta \vec{L}_B$ <u>But</u> $\vec{F}_{A\to B} = -\vec{F}_{B\to A}$ so

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The momentum of $\vec{L}_A$ & $\vec{L}_B$ changed ($\vec{L}_{AI} \neq \vec{L}_{AF}$ & $\vec{L}_{BI} \neq \vec{L}_{BF}$)

<u>But</u> the sum: $\vec{L}_A + \vec{L}_B = \text{constant}$. $\longrightarrow$

So, no external impulses $\Rightarrow$ conservation of linear momentum

We can also write $\sum \vec{F}_{AVE} \Delta t = \Delta \vec{L}$ from the mean-value theorem of integrals

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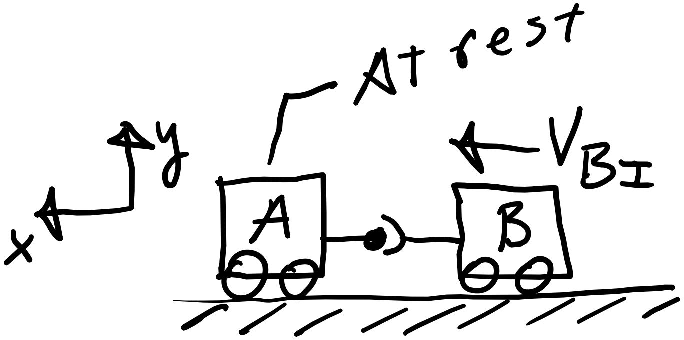
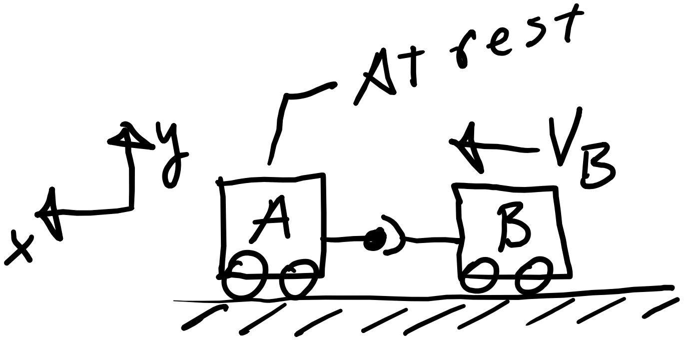
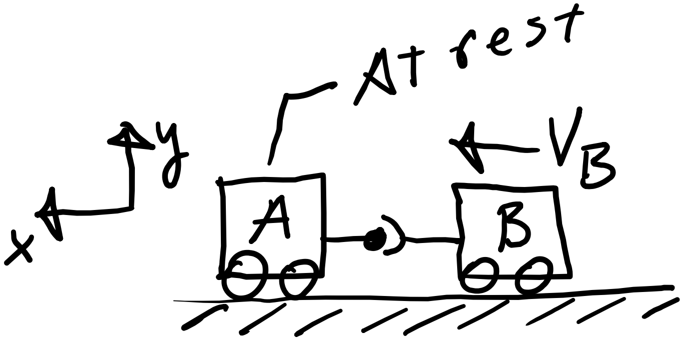

At rest
V_{BI}
A
B
x
y



$$\vec{L}_{A_1} + \vec{L}_{B_1} = \vec{L}_{A_2} + \vec{L}_{B_2}$$



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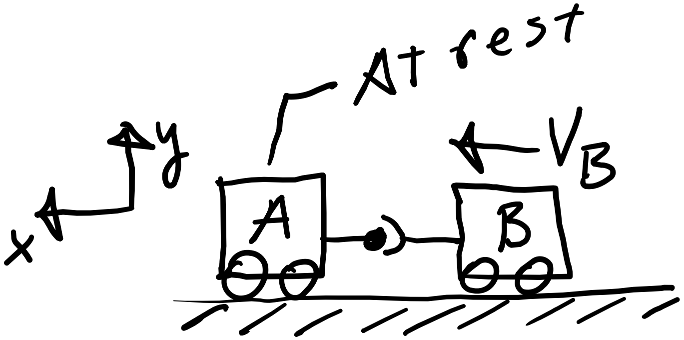

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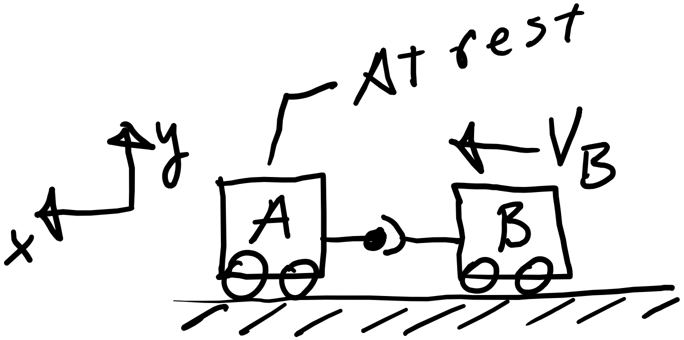

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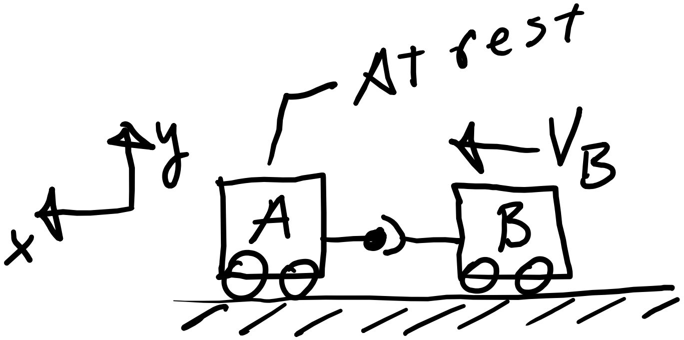

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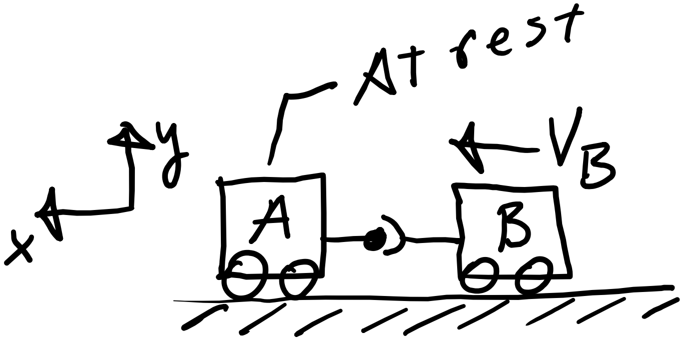

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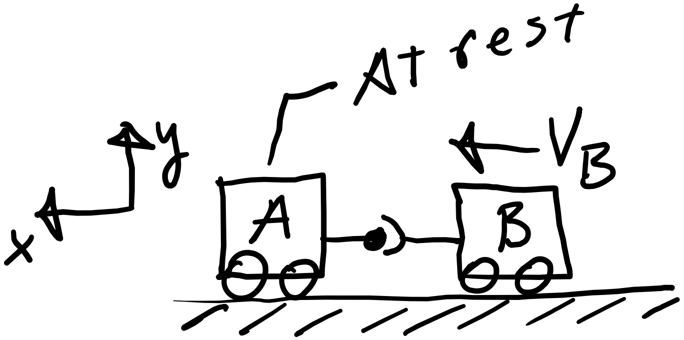

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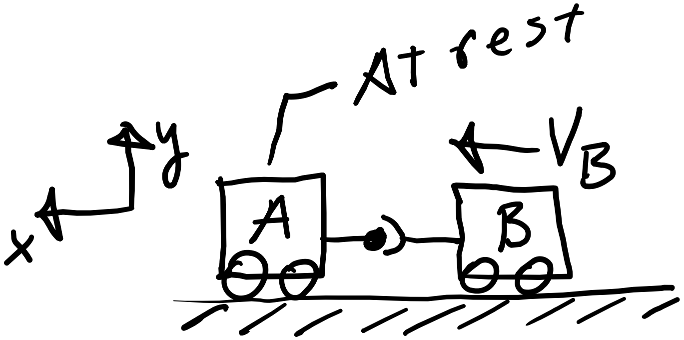
<u>Find time to come to rest:</u>



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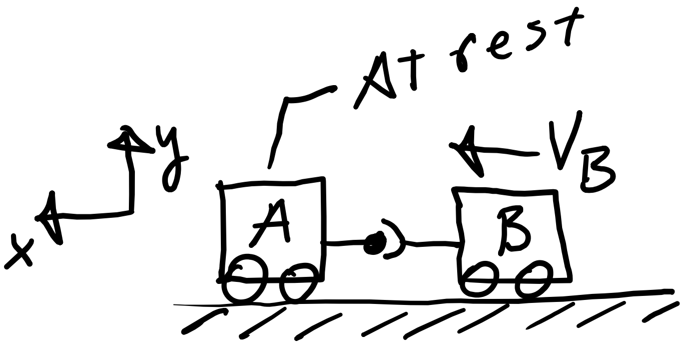

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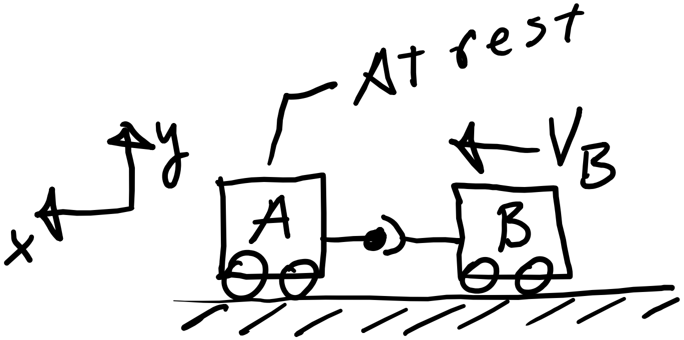

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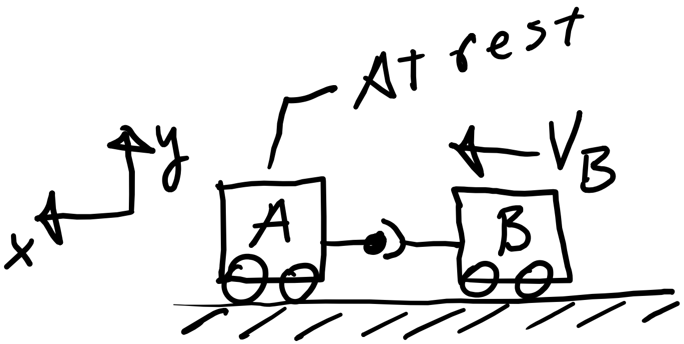

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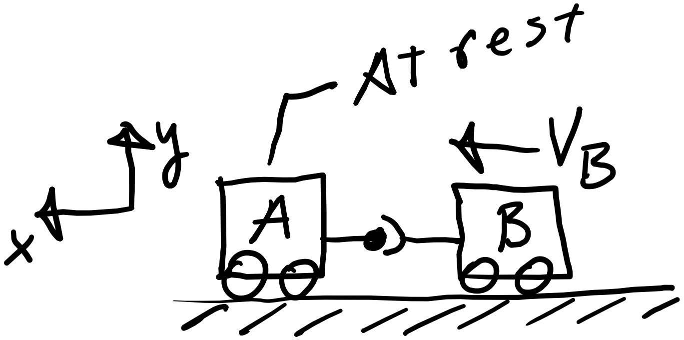

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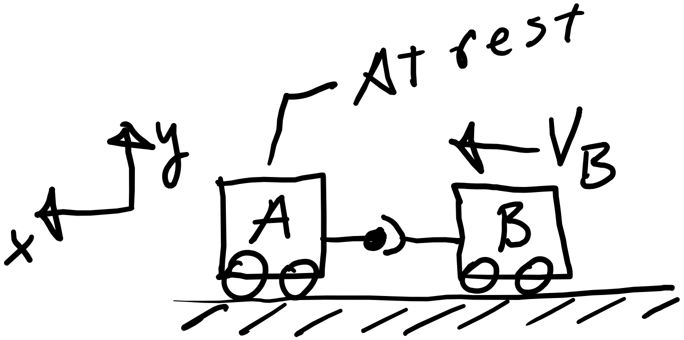

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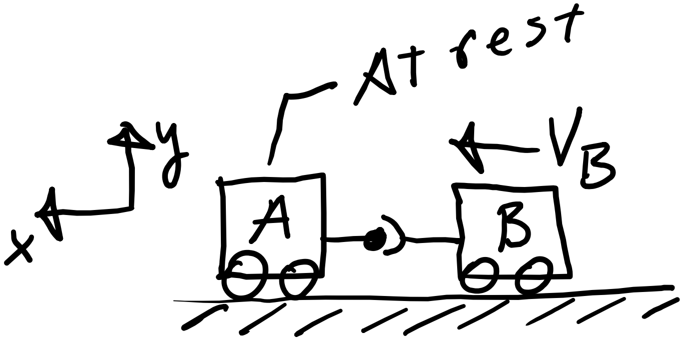

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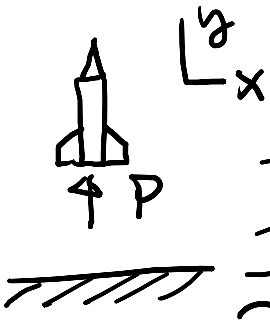
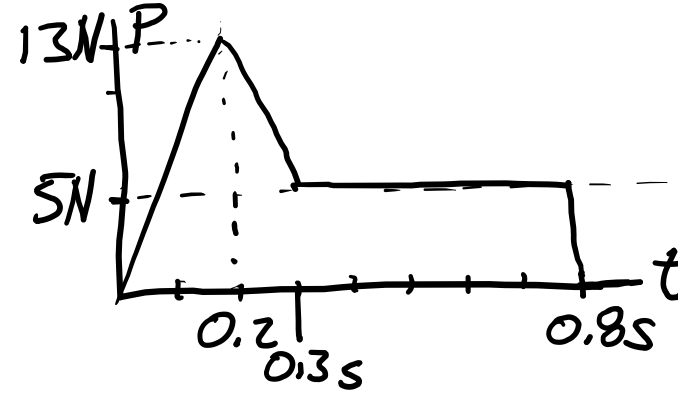
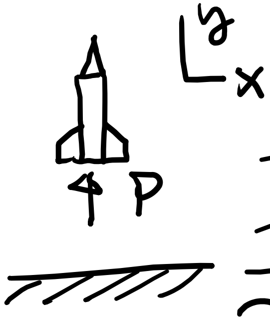


A 60g rocket has a thrust as given in plot.



Find speed at end of thrust:



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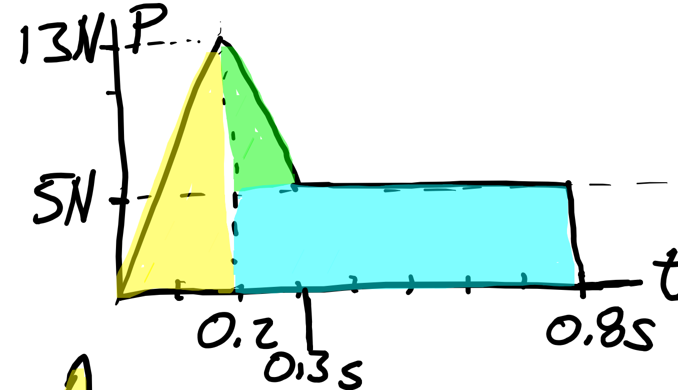
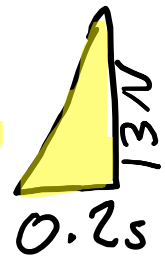
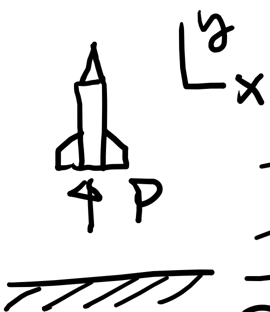


Find speed at end of thrust:

Area of 1st triangle



$= \left(\frac{0.2s}{2}\right) 13N = 1.3 Ns$



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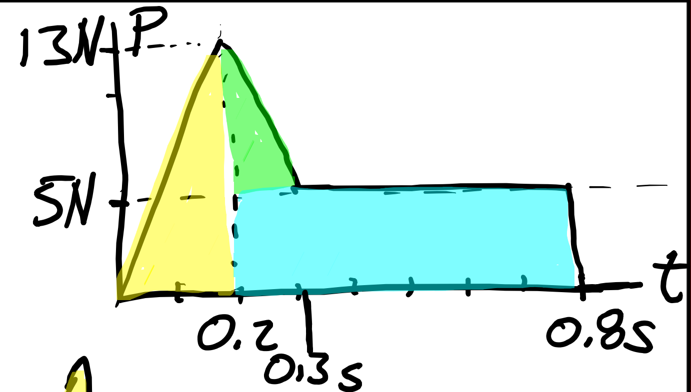
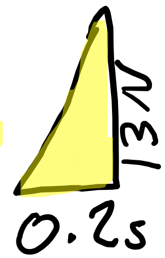
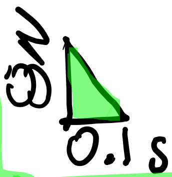


Area of 1st triangle

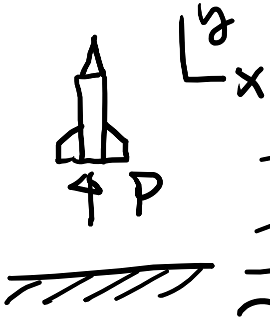


$$= \left(\frac{0.2s}{2}\right)13N = 1.3Ns$$

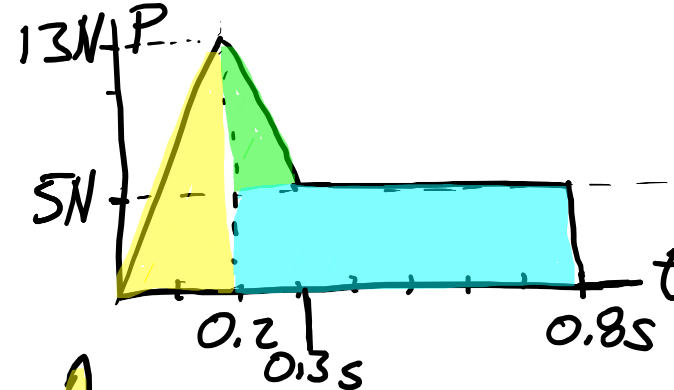
Area of 2nd triangle



$$= \left(\frac{0.1s}{2}\right)8N = 0.4Ns$$

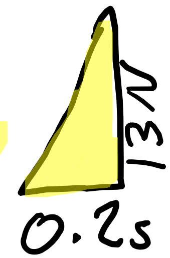


A 60g rocket has a thrust as given in plot.



Find speed at end of thrust:

Area of 1st triangle

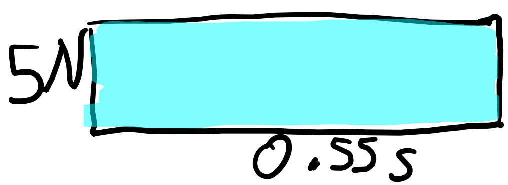


$= \left(\frac{0.2s}{2}\right) 13N = 1.3 Ns$

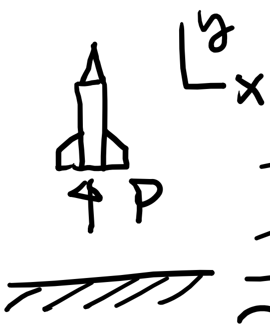
Area of 2nd triangle

$= \left(\frac{0.1s}{2}\right) 8N = 0.4 Ns$

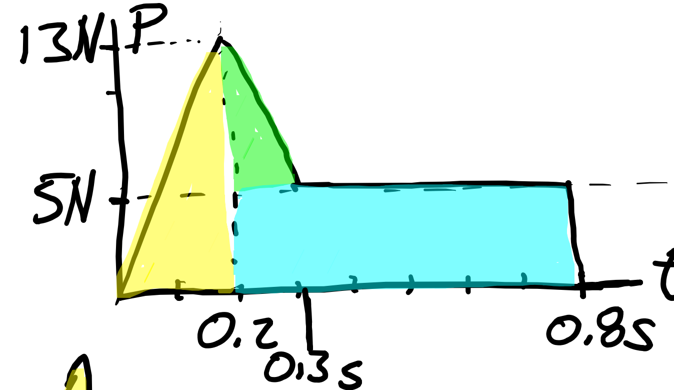
Area of rectangle



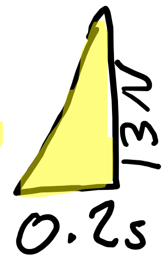
$= (0.6\ s) 5N = 3.0 Ns$



A 60g rocket has a thrust as given in plot. Find speed at end of thrust:



Area of 1^{ST} triangle

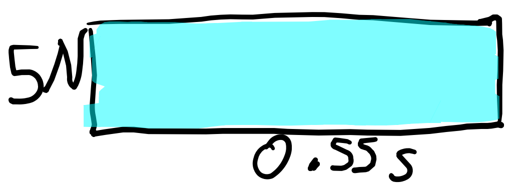


$$= \left(\frac{0.2s}{2}\right) 13N = 1.3 Ns$$

Area of 2^{nd} triangle

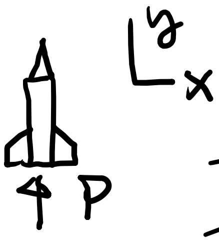
$$= \left(\frac{0.1s}{2}\right) 8N = 0.4 Ns$$

Area of rectangle

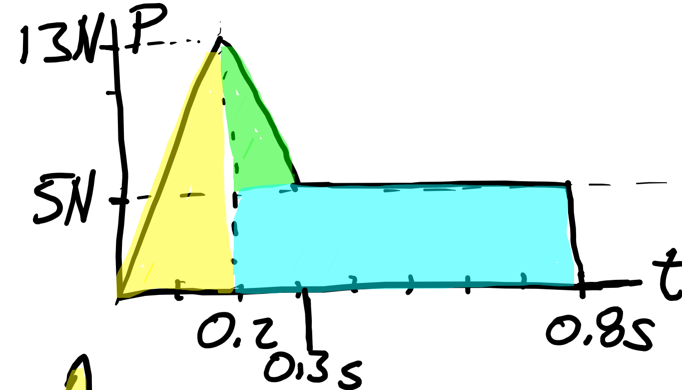


$$= (0.6\ s)\, 5N = 3.0 Ns$$

$$\int_0^{0.8s} \vec{P}\, dt = \hat{j}(\underline{1.3} + \underline{0.4} + \underline{3.0}) Ns = 4.7 \hat{j}\, Ns$$

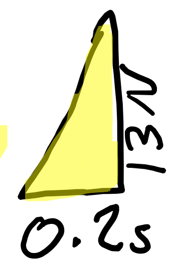


A 60g rocket has a thrust as given in plot.



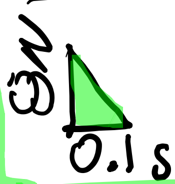
Find speed at end of thrust:

Area of 1st triangle



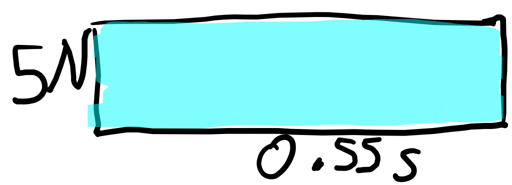
$= \left(\frac{0.2s}{2}\right) 13N = 1.3 Ns$

Area of 2nd triangle



$= \left(\frac{0.1s}{2}\right) 8N = 0.4 Ns$

Area of rectangle



$= (0.6\ s) 5N = 3.0 Ns$

$$\int_0^{0.8s} \vec{P} dt = \hat{j}(1.3 + 0.4 + 3.0) Ns = 4.7 \hat{j} Ns$$

Just need to add in gravity

Gravity

$$\int_0^{0.8s} \vec{F}_g \, dt$$

Gravity

$$\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$$

$$0.06\,kg * 9.81\,m/s^2 * 0.8s(-\hat{j}) = 0.471\,Ns\,(-\hat{j})$$

Gravity

$$\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$$

$$0.06kg * 9.81 m/s^2 * 0.8s(-\hat{j}) = 0.471 Ns (-\hat{j})$$

Now

$$\sum \overrightarrow{I_{mp}} = \int_0^{0.8s} F_g \, dt + \int_0^{0.8s} P \, dt$$

Gravity $\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$

$0.06kg * 9.81 m/s^2 * 0.8s (-\hat{j}) = 0.471 Ns (-\hat{j})$

Now $\sum \vec{I}_{mp} = \int_0^{0.8s} F_g \, dt + \int_0^{0.8s} P \, dt$

$= (-0.471 + 4.7) \hat{j} Ns$

$= 4.229 \hat{j} N \cdot s$

Gravity $\int_0^{0.8s} \vec{F_g}\,dt = (-\hat{j})mg * 0.8s =$

$0.06\,kg * 9.81\,m/s^2 * 0.8s(-\hat{j}) = 0.471\,Ns\,(-\hat{j})$

Now $\sum \vec{I}_{mp} = \int_0^{0.8s} F_g\,dt + \int_0^{0.8s} P\,dt$

$= (-0.471 + 4.7)\hat{j}\,Ns$

$= 4.229\,\hat{j}\,N\cdot s$

So $\sum \vec{I}_{mp} = \Delta \vec{L}$

ASU

<u>Gravity</u> $\int_0^{0.8s} \vec{F}_g\, dt = (-\hat{j}) mg * 0.8s =$

$0.06\,kg * 9.81\,m/s^2 * 0.8s(-\hat{j}) = 0.471\,Ns(-\hat{j})$

Now $\sum \overrightarrow{I_{mp}} = \int_0^{0.8s} F_g\, dt + \int_0^{0.8s} P\, dt$

$= (-0.471 + 4.7)\hat{j}\,Ns$

$= 4.229\,\hat{j}\,N\cdot s$

So $\sum \overrightarrow{I_{mp}} = \Delta \vec{L} \Rightarrow \sum \overrightarrow{I_{mp}} = \vec{L}_F - \vec{L}_I$

Gravity $\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$

$$0.06\,kg * 9.81\,m/s^2 * 0.8s(-\hat{j}) = 0.471\,Ns\,(-\hat{j})$$

Now $\sum \vec{I}_{mp} = \int_0^{0.8s} F_g \, dt + \int_0^{0.8s} P \, dt$

$$= (-0.471 + 4.7)\hat{j}\,Ns$$

$$= 4.229\,\hat{j}\,N \cdot s$$

So $\sum \vec{I}_{mp} = \Delta \vec{L} \Rightarrow \sum \vec{I}_{mp} = \vec{L}_F - \cancelto{0}{\vec{L}_I}$

$\Rightarrow \sum \vec{I}_{mp} = \vec{L}_F = m\vec{V}_F$

Gravity $\int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$

$$0.06\,kg * 9.81\,m/s^2 * 0.8s(-\hat{j}) = 0.471\,Ns\,(-\hat{j})$$

Now $\sum \vec{I}_{mp} = \int_0^{0.8s} F_g \, dt + \int_0^{0.8s} P \, dt$

$$= (-0.471 + 4.7)\hat{j}\,Ns$$

$$= 4.229\,\hat{j}\,N \cdot s$$

So $\sum \vec{I}_{mp} = \Delta \vec{L} \Rightarrow \sum \vec{I}_{mp} = \vec{L}_F - \cancelto{0}{\vec{L}_I}$

$$\Rightarrow \sum \vec{I}_{mp} = \vec{L}_F = m\vec{V}_F \Rightarrow \vec{V}_F = \frac{\sum \vec{I}_{mp}}{m} = \frac{4.23\,Ns\,\hat{j}}{0.06\,kg}$$

$$\underline{\text{Gravity}} \quad \int_0^{0.8s} \vec{F}_g \, dt = (-\hat{j}) mg * 0.8s =$$

$$0.06\,kg * 9.81\,m/s^2 * 0.8s(-\hat{j}) = 0.471\,Ns\,(-\hat{j})$$

$$\text{Now} \quad \sum \vec{I}_{mp} = \int_0^{0.8s} F_g \, dt + \int_0^{0.8s} P \, dt$$

$$= (-0.471 + 4.7)\hat{j}\,Ns$$

$$= 4.229\,\hat{j}\,N \cdot s$$

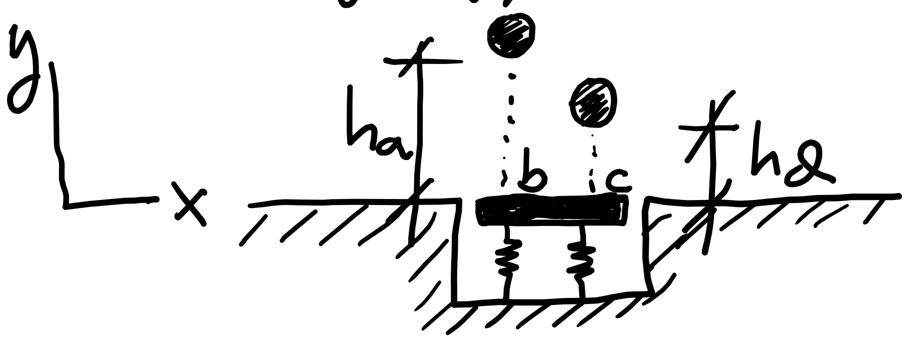
$$\text{So} \quad \sum \vec{I}_{mp} = \Delta \vec{L} \Rightarrow \sum \vec{I}_{mp} = \vec{L}_F - \cancelto{0}{\vec{L}_I}$$

$$\Rightarrow \sum \vec{I}_{mp} = \vec{L}_F = m\vec{V}_F \Rightarrow \vec{V}_F = \frac{\sum \vec{I}_{mp}}{m} = \frac{4.23\,Ns\,\hat{j}}{0.06\,kg}$$

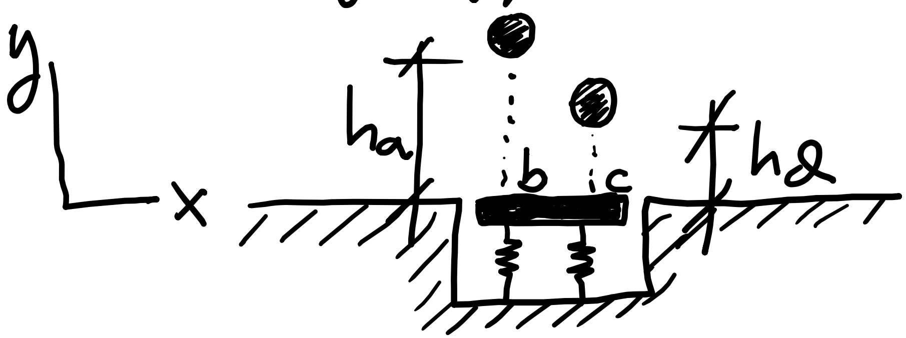
$$\Rightarrow \sum \vec{I}_{mp} = \left(70.5\,\frac{m}{s}\right)\hat{j}$$

ASU

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.

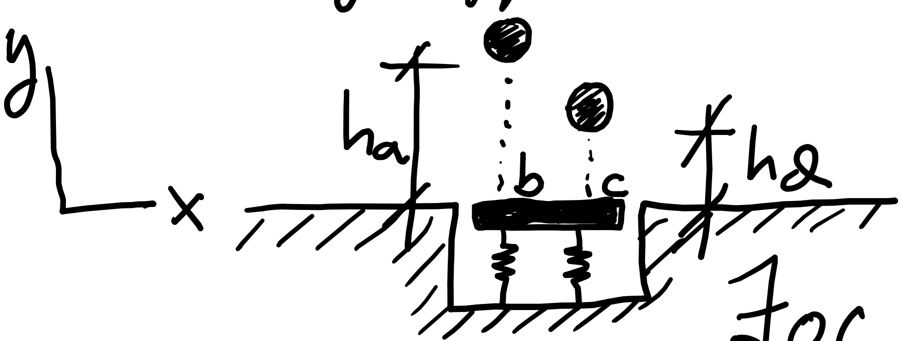


Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



Find $\vec{v}$ of plate right after collision with ball

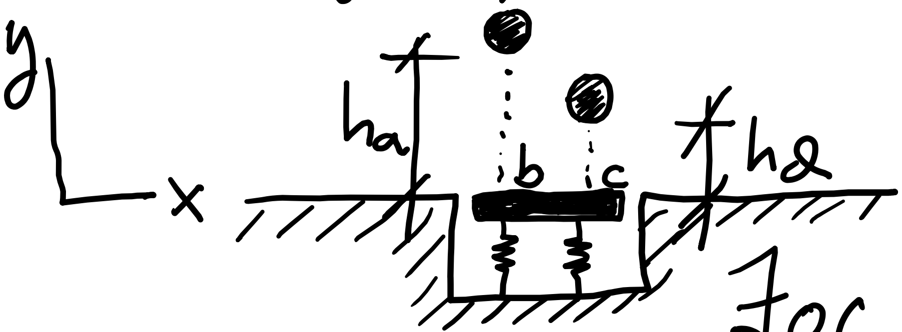
Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



Find $\vec{v}$ of plate right after collision with ball

For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



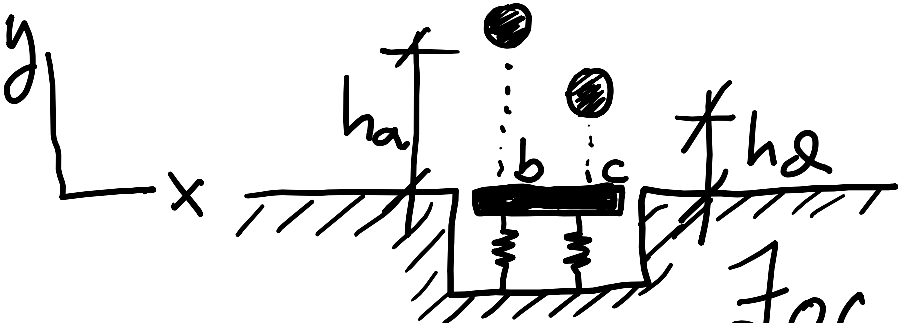
Find $\vec{v}$ of plate right after collision with ball

For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Conservation of momentum $\Rightarrow$

$$\Delta \vec{L}_{Ball} + \Delta \vec{L}_{plate} = 0$$

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



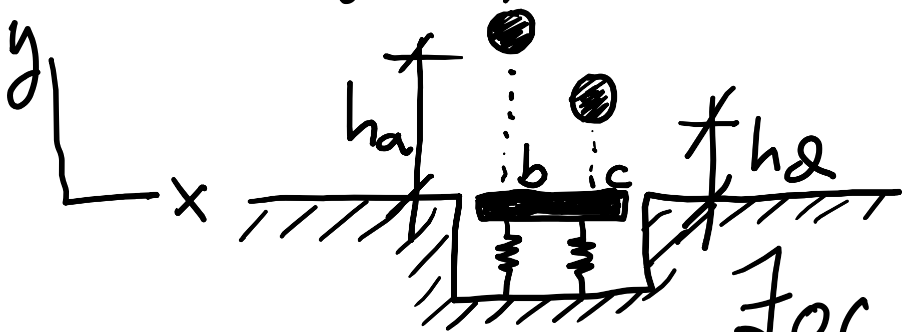
Find $\vec{v}$ of plate right after collision with ball

For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Conservation of momentum $\Rightarrow$

$$\Delta\vec{L}_{Ball} + \Delta\vec{L}_{plate} = 0 \Rightarrow \Delta\vec{L}_p = -\Delta\vec{L}_B$$

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



Find $\vec{v}$ of plate right after collision with ball

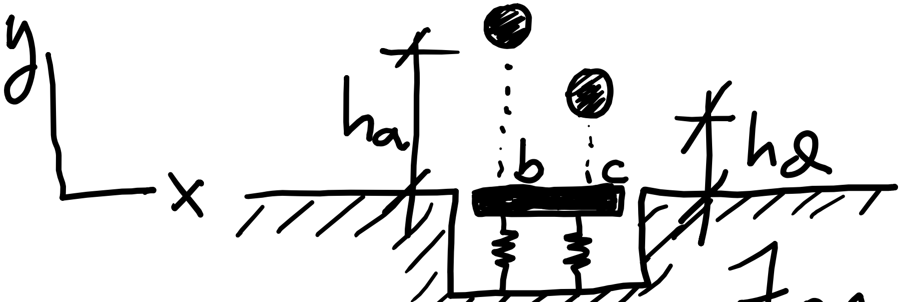
For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Conservation of momentum $\Rightarrow$

$$\Delta \vec{L}_{Ball} + \Delta \vec{L}_{plate} = 0 \Rightarrow \Delta \vec{L}_p = -\Delta \vec{L}_B \Rightarrow$$

$$m_p v_{pF} - m_p v_{pI} = -\Delta \vec{L}_B$$

Ball is dropped on plate that is on springs. The ball starts at h_a & after bouncing off plate, reaches height h_d.



Find $\vec{v}$ of plate right after collision with ball

For this problem we will assume that during the collision Δt is so small that the plate does not move during Δt.

Conservation of momentum $\Rightarrow$

$$\Delta\vec{L}_{Ball} + \Delta\vec{L}_{plate} = 0 \Rightarrow \Delta\vec{L}_p = -\Delta\vec{L}_B \Rightarrow$$

$$m_p v_{pF} - \cancelto{0}{m_p v_{pI}} = -\Delta\vec{L}_B \Rightarrow v_{pF} = \frac{-\Delta\vec{L}_B}{m_p}$$

So far we have $\vec{v}_{pF} = -\Delta\vec{L}_B / m_p$

So far we have $\vec{V}_{PF} = -\Delta\vec{L}_B/m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

So far we have $\vec{V}_{PF} = -\Delta\vec{L}_B / m_p$.
need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + T_a = V_b + T_b$

So far we have $\vec{v}_{pF} = -\Delta\vec{L}_B/m_p$.

Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancelto{0}{T_a} = \cancelto{0}{V_b} + T_b$

So far we have $\vec{V}_{PF} = -\Delta\vec{L}_B / m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $\overset{0}{\cancel{V_a}} + T_a = \overset{0}{\cancel{V_b}} + T_b$

<u>Note</u>: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$

So far we have $\vec{V}_{pF} = -\Delta \vec{L}_B / m_p$.

Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancelto{0}{T_a} = \cancelto{0}{V_b} + T_b$

<u>Note</u>: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

So far we have $\vec{V}_{pF} = -\Delta \vec{L}_B / m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a} = \cancel{V_b} + T_b$

$\cancel{T_a} \to 0$, $\cancel{V_b} \to 0$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow \quad L_B^2 = 2m_B^2 g h_a$

So far we have $\vec{v}_{pF} = -\Delta\vec{L}_B/m_p$.
Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a} = \cancel{V_b} + T_b$ $\quad 0 \quad 0$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$V_c + T_c = V_d + T_d$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B/m_p$.

need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancelto{0}{T_a} = \cancelto{0}{V_b} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$\cancelto{0}{V_c} + T_c = V_d + \cancelto{0}{T_d} \Rightarrow \frac{L_c^2}{2m} = mgh_d$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B/m_p$.

need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancelto{0}{T_a} = \cancelto{0}{V_b} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$\cancelto{0}{V_c} + T_c = V_d + \cancelto{0}{T_d} \Rightarrow \frac{L_c^2}{2m} = mgh_d \Rightarrow L_c^2 = 2m_B^2 g h_d$

So far we have $\vec{v}_{pF} = -\Delta \vec{L}_B / m_p$.
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Momentum of ball at b: $V_a + \cancelto{0}{T_a} = \cancelto{0}{V_b} + T_b$

Note: $T = \frac{1}{2} m v^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2 m_B^2 g h_a$. Momentum of ball at c:

$\cancelto{0}{V_c} + T_c = V_d + \cancelto{0}{T_d} \Rightarrow \frac{L_c^2}{2m} = m g h_d \Rightarrow L_c^2 = 2 m_B^2 g h_d$

So $\vec{L}_b = m\sqrt{2 g h_a}\,(-\hat{j})$ & $L_c = m\sqrt{2 g h_d}\,(+\hat{j})$

So far we have $\vec{v}_{pF} = -\Delta \vec{L}_B / m_p$.

Need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancelto{0}{T_a} = \cancelto{0}{V_b} + T_b$

Note: $T = \frac{1}{2} m v^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2 m_B^2 g h_a$. Momentum of ball at c:

$\cancelto{0}{V_c} + T_c = V_d + \cancelto{0}{T_d} \Rightarrow \frac{L_c^2}{2m} = m g h_d \Rightarrow L_c^2 = 2 m_B^2 g h_d$

So $\vec{L}_b = m\sqrt{2 g h_a}\,(-\hat{j})$ & $L_c = m\sqrt{2 g h_d}\,(+\hat{j})$

Now $\Delta \vec{L}_B = \vec{L}_c - \vec{L}_b$

So far we have $\vec{v}_{pF} = -\Delta\vec{L}_B/m_p$.

need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancel{T_a} = \cancel{V_b} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

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$\cancel{V_c} + T_c = V_d + \cancel{T_d} \Rightarrow \frac{L_c^2}{2m} = mgh_d \Rightarrow L_c^2 = 2m_B^2 g h_d$

So $\vec{L}_b = m\sqrt{2gh_a}(-\hat{j})$ & $L_c = m\sqrt{2gh_d}(+\hat{j})$

Now $\Delta\vec{L}_B = \vec{L}_c - \vec{L}_b = (m\sqrt{2g})(h_d + h_a)\hat{j}$

So far we have $\vec{V}_{pF} = -\Delta\vec{L}_B/m_p$.
need find L_B when first touching the plate [time b] & L_B when leaving plate at time c.

Momentum of ball at b: $V_a + \cancelto{0}{T_a} = \cancelto{0}{V_b} + T_b$

Note: $T = \frac{1}{2}mv^2 = \frac{L^2}{2m}$ so $m_B g h_a = \frac{L_{B_b}^2}{2m}$

$\Rightarrow L_B^2 = 2m_B^2 g h_a$. Momentum of ball at c:

$\cancelto{0}{V_c} + T_c = V_d + \cancelto{0}{T_d} \Rightarrow \frac{L_c^2}{2m} = mgh_d \Rightarrow L_c^2 = 2m_B^2 g h_d$

So $\vec{L}_b = m\sqrt{2gh_a}(-\hat{j})$ & $L_c = m\sqrt{2gh_d}(+\hat{j})$

Now $\Delta\vec{L}_B = \vec{L}_c - \vec{L}_b = (m_B\sqrt{2g})(\sqrt{h_d} + \sqrt{h_a})\hat{j}$ So

$$\boxed{\vec{V}_{pF} = \frac{-\Delta L_B}{m_p} = \left(\frac{m_B}{m_p}\right)\sqrt{2g}\,(\sqrt{h_d}+\sqrt{h_a})(-\hat{j})}$$